

Related topics

Parahelium, orthohelium, exchange energy, spin, angular momentum, spinorbital angular momentum-interaction, singlet series, triplet series, selection rules, forbidden transitions, energy level, excitation energy.

Principle and task

The prism spectrometer is calibrated with the aid of the He spectrum. The wavelengths of the spectral lines of Hg, Cd and Zn are determined.

Equipment

Spectrometer/goniom. w. vernier	35635.02	1
Spectral lamp He, pico 9 base	08120.03	1
Spectral lamp Hg 100, pico 9 base	08120.14	1
Spectral lamp Cd, pico 9 base	08120.01	1
Spectral lamp Zn, pico 9 base	08120.11	1
Power supply for spectral lamps	13662.93	1
Lamp holder, pico 9, f. spectr. lamps	08119.00	1
Tripod base -PASS-	02002.55	1

Problems

1. Calibration of the prism spectrometer using the He spectrum.
2. Determination of the most intense spectral lines of Hg, Cd and Zn.

Set-up and procedure

The experimental set up is as shown in Fig. 1. The spectrometer/goniometer and the prism must be set up and adjusted in accordance with the operating instructions.

The spectral lamps attain their maximum light intensity after a warm-up period of approx. 5 min. The lamp housing should be set up so as to ensure free circulation of air through the ventilator slit. Before changing the spectral lamps they must be allowed to cool since the paper towels or cloths used for this operation might otherwise stick to the glass. The illuminated scale is used for recording the spectra.

Theory and evaluation

When light of wavelength λ passes through a prism, it is deviated. The angle of deviation depends on the geometry of the prism and on the angle of incidence. The refractive index of a prism depends on the wavelength and thus also on the angle of deviation. Fig. 2 shows the calibration curve for the He spectrum (dispersion curve), obtained at the angle of minimum deviation.

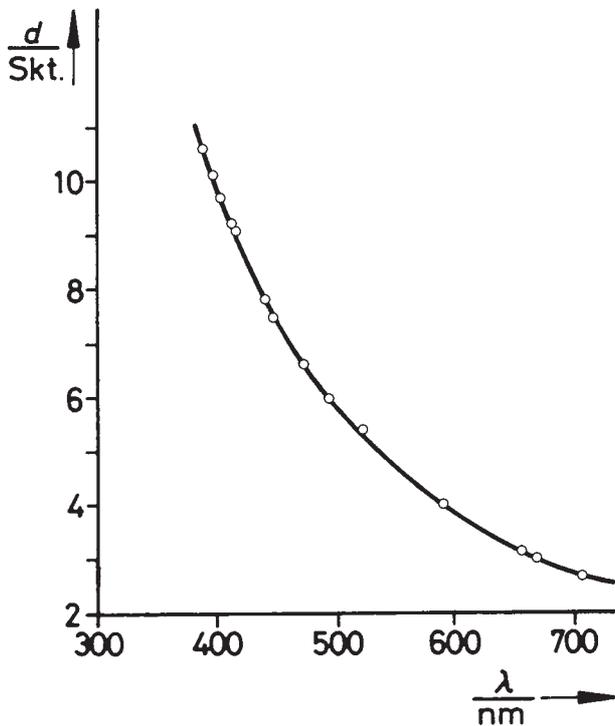
Excitation of atoms results from electron impact. The energy difference produced when electrons revert from the excited state E_0 is emitted as a photon with a frequency f .

$$hf = E_1 - E_0$$

Fig.1: Experimental set up for measuring the spectra of Hg, Cd and Zn.



Fig. 2: Calibration curve of the prism spectrometer.



where:

$$h = \text{Planck's constant} \\ = 6.63 \cdot 10^{-34} \text{ joule-seconds.}$$

The Hamiltonian operator (non-relativistic) for the two electrons 1 and 2 of the He atom is:

$$H = -\frac{\hbar^2}{2m}\Delta_1 - \frac{\hbar^2}{2m}\Delta_2 - \frac{2e^2}{|\vec{r}_1|} - \frac{2e^2}{|\vec{r}_2|} + \frac{e^2}{|\vec{r}_1 - \vec{r}_2|}$$

$$\text{where } \hbar = \frac{h}{2\pi},$$

m and e represent the mass and charge of the electron respectively,

$$\Delta_i = \frac{d^2}{dx_i^2} + \frac{d^2}{dy_i^2} + \frac{d^2}{dz_i^2}$$

is the Laplace operator, and \vec{r}_i is the position of the i -th electron. The Spin-orbit interaction energy

$$E_{\text{so}} \propto \frac{Z^4}{4 \cdot (137)^2}$$

was ignored in the case of the nuclear charge $Z = 2$ of helium, because it is small when Z is small.

If we consider $\frac{e^2}{|\vec{r}_1 - \vec{r}_2|}$ as the electron-electron interaction term, then the eigenvalues of the Hamiltonian operator without interaction are those of the hydrogen atom:

$$E_{n,m}^0 = -\frac{me^4}{8h^2} \left(\frac{1}{n^2} + \frac{1}{m^2} \right) \\ n, m = 1, 2, 3, \dots$$

As the transition probability for simultaneous two-electron excitation is very much less than that for one-electron excitation, the energy spectrum of the undisturbed system is:

$$E_{l,m}^0 = -\frac{me^4}{8h^2} \left(1 + \frac{1}{m^2} \right) \quad m = 1, 2$$

The interaction term removes the angular momentum degeneracy of the pure hydrogen spectrum and the exchange energy degeneracy. There results an energy adjustment:

$$E_{nl\pm}^1 = \langle \phi_{nl\alpha}^\pm | \frac{e^2}{|\vec{r}_1 - \vec{r}_2|} | \phi_{nl\alpha}^\pm \rangle = C_{nl} \pm A_{nl}$$

in which $\phi_{nl\alpha}^\pm$ are the antisymmetrized undisturbed 2-particle wave functions with symmetrical (ϕ^+) or antisymmetrical (ϕ^-) position component, l^* is the angular momentum quantum number, and α is the set of the other quantum numbers required.

In the present case, the orbital angular momentum of the single electron l is equal to the total angular momentum of the two electrons L , since only one-particle excitations are being considered and the second electron remains in the ground state ($l = 0$).

C_{nl} and A_{nl} are the Coulomb and exchange energy respectively. They are positive. Coupling the orbital angular momentum L with the total spin S produces for $S = 0$, i.e. ϕ^+ , a singlet series and for $S = 1$, i.e. ϕ^- , a triplet series. Because of the lack of spin-orbit interaction, splitting within a triplet is slight. As the disturbed wave functions are eigenfunctions for S^2 and as S^2 interchanges with the dipole operator, the selection rule

$$\Delta S = 0$$

(which is characteristic for 2-electron systems with a low nuclear charge number) results and forbids transitions between the triplet and singlet levels.

In addition, independent of the spin-orbit interaction, the selection rule for the total angular momentum

$$\Delta J = 0, \pm 1$$

applies except where

$$J = 0 \rightarrow J' = 0.$$

If the spin-orbit interaction is slight, then

$$\Delta L = 0, \pm 1$$

applies.

Detailed calculations produce the helium spectrum of Fig. 3.

Fig. 3: Spectrum of helium.

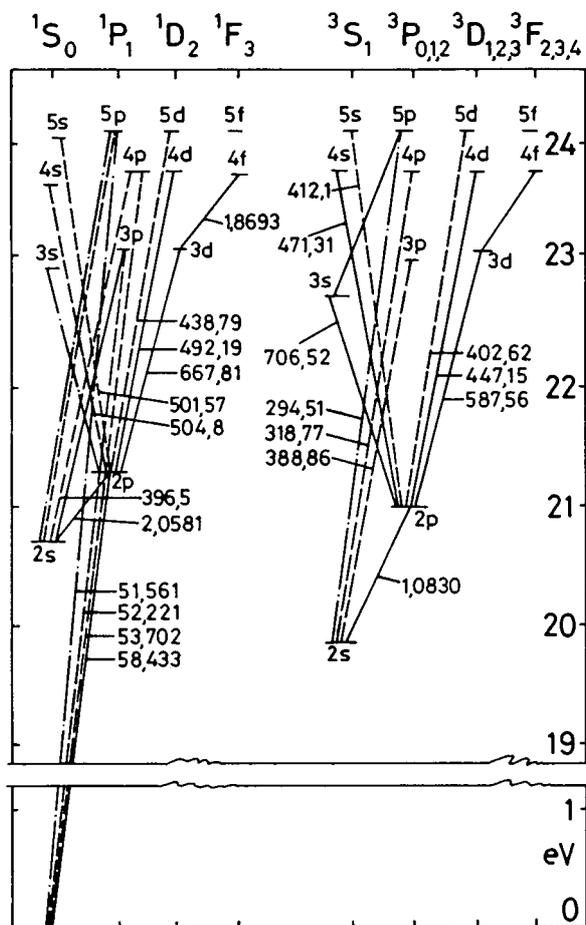
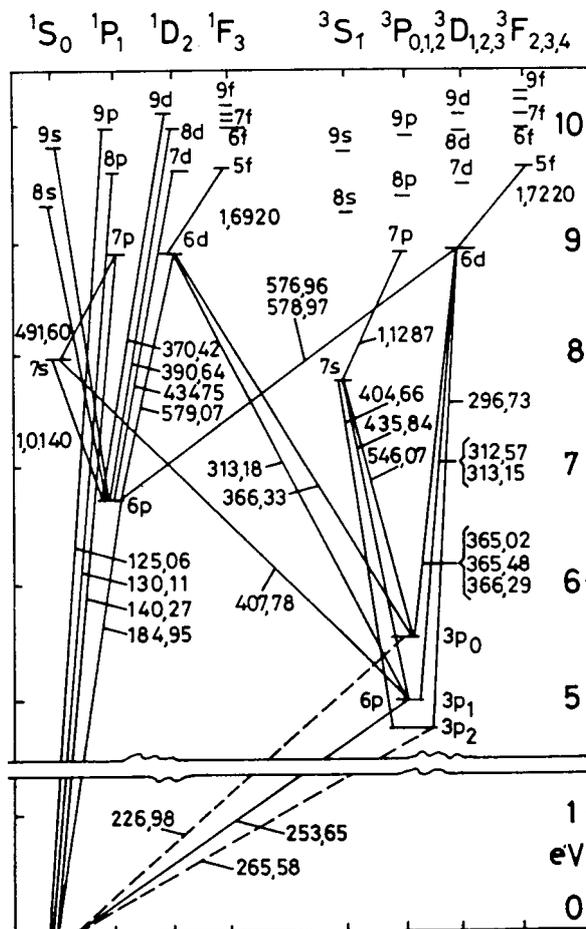


Fig. 4: Spectrum of mercury.



Hg, Cd and Zn are also two-electron systems and possess the structure of 2 series. The spin-orbit interaction, however, is relatively pronounced so that only the total angular momentum

$$J = L + S$$

Colour	λ/nm	Transition	Relative intensity
red	706.5	$3^3S \rightarrow 2^1P$	5
red	667.8	$3^1D \rightarrow 2^1P$	6
red	656.0	He II	4-6
yellow	587.6	$3^3D \rightarrow 2^3P$	10
green	504.8	$4^1S \rightarrow 2^1P$	2
green	492.2	$4^1D \rightarrow 2^1P$	4
blue	471.3	$4^3S \rightarrow 2^3P$	3
blue	447.1	$4^3D \rightarrow 2^3P$	6
blue	438.8	$5^1D \rightarrow 2^1P$	3
violet	414.4	$6^1D \rightarrow 2^1P$	2
violet	412.1	$5^3S \rightarrow 2^3P$	3
violet	402.6	$5^3D \rightarrow 2^3P$	5
violet	396.5	$4^1P \rightarrow 2^1S$	4
violet	388.9	$3^3P \rightarrow 2^3S$	10

Table 1: He-I spectrum.

is an energy conservation parameter. Splitting within a triplet is pronounced. Moreover, the selection rule

$$\Delta S = 0$$

is no longer valid since S is no longer a conservation parameter (transition from $L-S$ for the $j-j$ coupling).

Colour	λ/nm	Transition
red	690	$8^3P_2 \rightarrow 7^3S$
red	624	$9^1P \rightarrow 7^1S$
red	611	$8^1P \rightarrow 7^3S$
red	608	$8^1P \rightarrow 7^1S$
yellow	578	$\left\{ \begin{array}{l} 6^3D_2, 6^3D_1 \\ 6^1D_2 \rightarrow 6^1P_1 \end{array} \right.$
green	548	$7^3S \rightarrow 6^3P_1$
blue-green	496	Hg II
blue-green	492	$8^1D \rightarrow 6^1P_1$
blue	435	$7^1D \rightarrow 6^1P_1$
violet	408	$7^1S \rightarrow 6^3P_1$

Table 2: Measured Hg-1 spectrum.

Fig. 5: Spectrum of Cd.

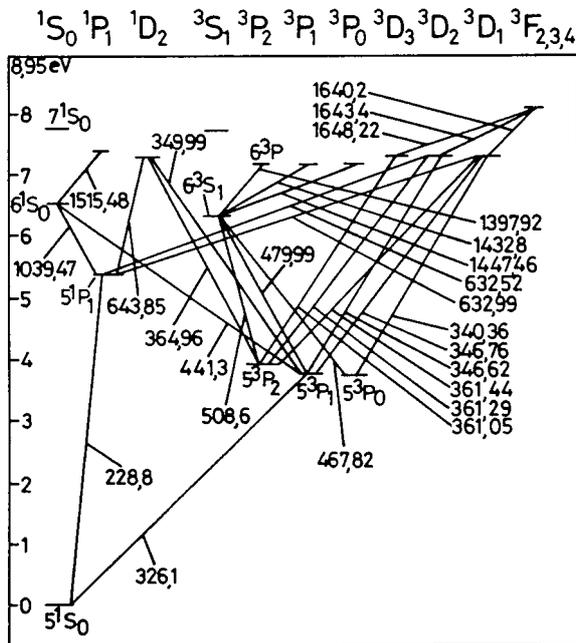


Table 3: Measured Cd spectrum.

Colour	λ /nm	Transition
red	645	$6^1D_2 \rightarrow 5^1P_1$
red	633	$5^3D_1 \rightarrow 5^1P_1$
green	517	$7^1S_0 \rightarrow 5^1P_1$
green	509	$6^3S_1 \rightarrow 5^3P_2$
blue	480	$6^3S_1 \rightarrow 5^3P_1$
blue	469	$6^3S_1 \rightarrow 5^3P_0$
violet	441	$6^1S_0 \rightarrow 5^3P_1$

Table 4: Measured /n spectrum.

Colour	λ /nm	Transition
red	636	$4^1P_1 \rightarrow 4^1D_1$
yellow	589	ZN II
yellow	579	$\left\{ \begin{array}{l} 5^3S_1 \rightarrow 7^3P_2 \\ 5^3S_1 \rightarrow 7^3P_1 \end{array} \right.$
green	534	$5^3S_1 \rightarrow 8^3P_0$
green	519	$4^1P_1 \rightarrow 6^1S_0$
green	508	$5^3S_1 \rightarrow 9^3P_1$
blue	481	$4^3P_2 \rightarrow 5^3S_1$
blue	472	$4^3P_1 \rightarrow 5^3S_1$
blue	468	$4^3P_0 \rightarrow 5^3S_1$
violet	463	$4^1P_1 \rightarrow f^1D_2$
violet	429	$\left\{ \begin{array}{l} 4^3P_1 \rightarrow 5^1S_0 \\ 4^1P_1 \rightarrow 7^1S_0 \end{array} \right.$