Resistance, phase shift and power in AC circuits

Related topics

Impedance, phase shift, phasor diagram, capacitance, self-inductance

Principle and task

Series circuits containing self-inductances or capacitances and ohmic resistances are investigated as a function of frequency. Measuring the electrical magnitudes with a work or power measurement instrument, real power or apparent power can be displayed directly.

Equipment

Work and power meter	13715.93	1
Power frequency generator 1 MHz	13650.93	1
Coil, 300 turns	06513.01	1
Connection box	06030.23	1
Electr.capaci. 47microF/63V bip	39105.45	1
PEK carbon resistor 1 W 5% 10 Ohm	39104.01	1
Connecting cord, 500 mm, black	07361.05	4

Problems

1. Series circuit of self-inductance and resistor (real coil)
- Investigation of impedance and phase shift as a function of frequency
- Investigation of the relation between real power and current intensity
- Determination of self-inductance and ohmic resistance

2. Series circuit of capacitor and resistor
- Investigation of impedance and phase shift as a function of frequency
- Investigation of the relation between real power and current intensity
- Determination of capacitance and ohmic resistance

Set-up and procedure

1. A 300 turn coil is connected to the 10 W output of the frequency power generator over the work and power measuring instrument. Frequency is varied within the range between 200 Hz and 20 Hz, e.g. in 20 Hz steps. The output voltage of the generator is set to about 2 V. Voltage, current intensity, phase shift, real and apparent power are measured as a function of frequency.

2. A series circuit consisting of a capacitor and an ohmic resistor is connected to the 10 W output of the frequency power generator over the work and power measuring instrument. Frequency is varied within the range between 100 Hz and 1000 Hz, e.g. till 20 Hz in 20 Hz steps, and then in steps of 100 Hz ore more. Voltage, current intensity, phase shift, real and apparent power are measured as a function of frequency.

The series of measurements should begin at a frequency of about 100 Hz and the output voltage of the generator should be selected so that measured current intensity is somewhat higher than 0.1 A, because this is the minimum current intensity required by the work and power measuring instrument to determine phase shift and real power. The resistor may be loaded up to 1 W, and up to 2 W for a short period of time.

Hint: if the phase angle display changes by ± 1°, the real power display also jumps. In this case, the displayed values should be averaged.

Theory and evaluation

Impedance is calculated from the measured pairs of values consisting of voltage U and current intensity *I*.

$$Z = \frac{U}{I} \qquad (1)$$

Fig. 1: experimental set-up: Resistance, phase shift and power in AC circuits.

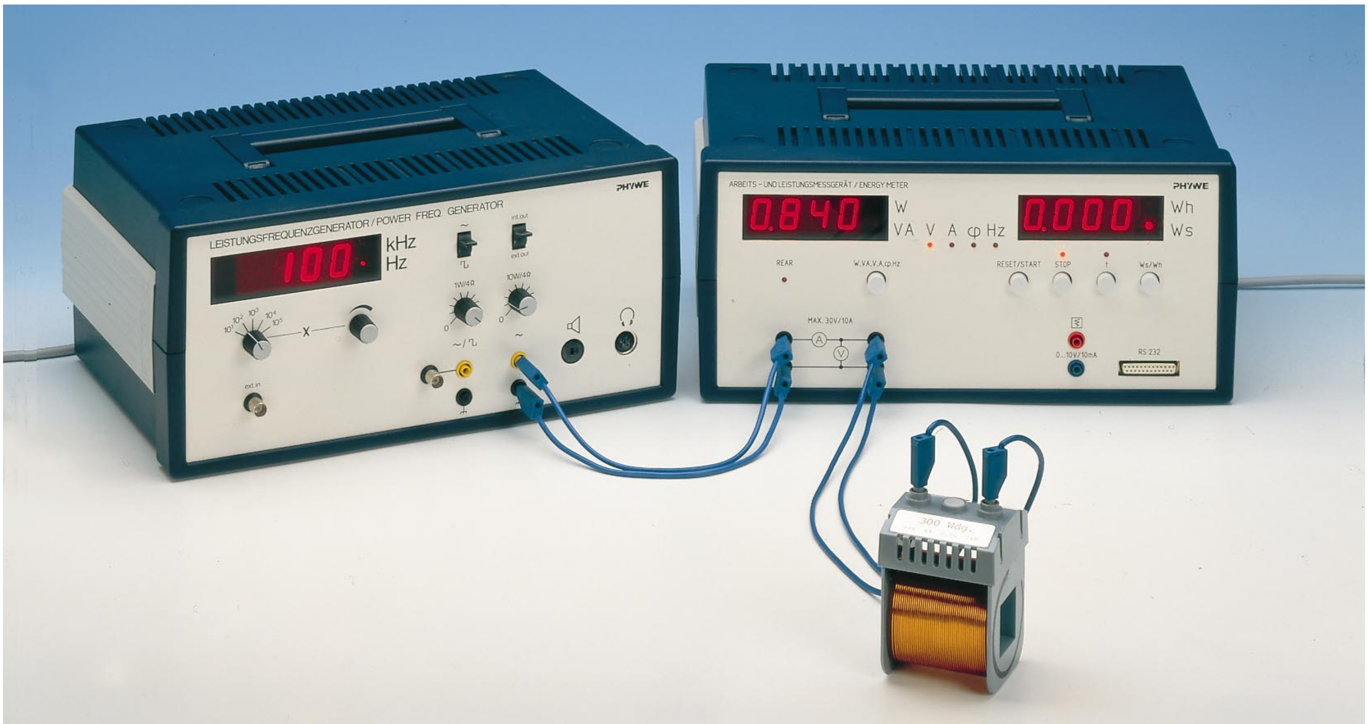
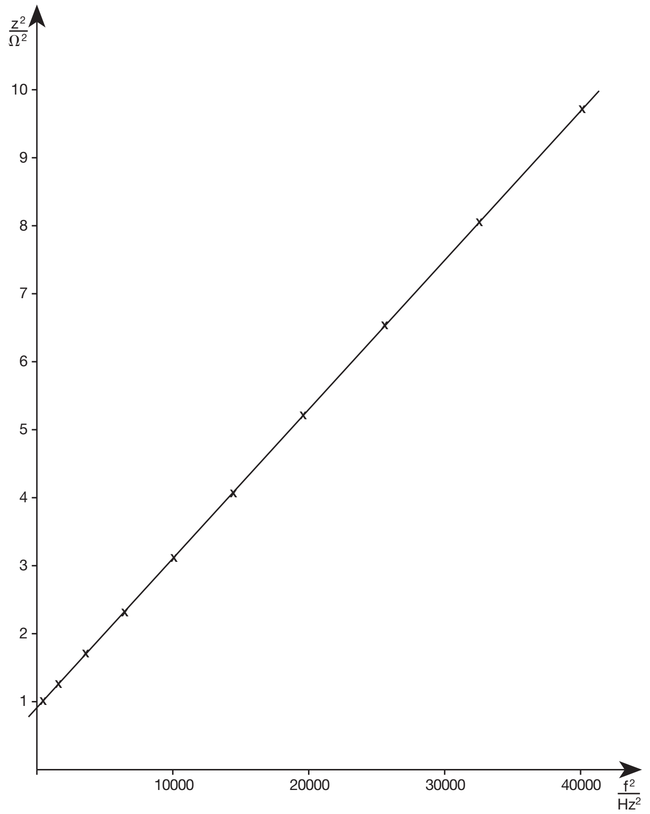


Fig. 2: self-inductance and resistor in series, Z^2 as a function of f^2.



Using a series circuit of inductive or respectively capacitive resistance X and ohmic resistance R, the impedance Z is obtained theoretically through vectorial addition of the inductive or the capacitive and the ohmic resistance:

$$Z = \sqrt{R^2 + X^2} \quad (2)$$

The phase shift φ between voltage and current intensity is given through:

$$tan\varphi = \frac{X}{R} \quad (3)$$

The real power P in the alternating current circuit is calculated from voltage, current intensity and phase shift:

$$P_w = U \cdot I \cdot cos\varphi \quad (4)$$

The angular relation

$$\cos\varphi = \sqrt{\frac{1}{1 + tan^2\,\alpha}}$$

allows for a further transformation of equation (4):

$$P_w = U \cdot I \cdot \sqrt{\frac{R^2}{R^2 + X^2}}$$

$$P_w = \frac{U \cdot I \cdot R}{Z}$$

$$P_w = R \cdot I^2 \quad (5)$$

1. Series circuit with self-inductance and resistor (real coil)
The inductive resistance of a coil depends on frequency:

$$X_L = 2\pi \cdot f \cdot L, \quad (6)$$

so that the impedance of a series circuit consisting of a self-inductance and a resistor displays the following dependence of frequency:

$$Z^2 = R^2 + (2\pi \cdot L)^2 \cdot f^2 \quad (7)$$

and the phase shift between voltage and current intensity is:

$$tan\varphi = \frac{2\pi \cdot L}{R} \cdot f \quad (8)$$

Table 1

$\frac{f}{Hz}$	$\frac{U}{V}$	$\frac{I}{A}$	$\frac{\varphi}{1^0}$	$\frac{P_w}{W}$	$\frac{P_s}{VA}$
200	2.055	0.657	72	0.417	1.348
180	2.039	0.716	70	0.499	1.459
160	2.020	0.788	68	0.596	1.590
140	1.995	0.874	65	0.736	1.743
120	1.962	0.975	62	0.898	1.910
100	1.918	1.09	57	1.138	2.089
80	1.869	1.23	52	1.447	2.300
60	1.798	1.38	43	1.814	2.481
40	1.710	1.53	32	2.217	2.614
20	1.630	1.65	17	2.571	2.690

Table 2

$\frac{f}{Hz}$	$\frac{Z}{\Omega}$	$\frac{Z^2}{\Omega^2}$	$\frac{f^2}{Hz^2}$	$\frac{I^2}{A^2}$	tan φ
200	3.13	9.78	40000	0.43	3.08
180	2.85	8.11	32400	0.51	2.75
160	2.56	6.57	25600	0.62	2.48
140	2.28	5.21	19600	0.76	2.14
120	2.01	4.04	14400	0.95	1.88
100	1.76	3.10	10000	1.19	1.54
80	1.52	2.31	6400	1.51	1.28
60	1.30	1.70	3600	1.90	0.93
40	1.12	1.25	1600	2.34	0.62
20	0.99	0.98	400	2.72	0.31

Tables 1 and 2 contain measurement values and calculated values for a series circuit consisting of a self-inductance and a resistor. For large angles, a change of the angular display φ by 1° also causes a large change of tanφ and cosφ (and thus a change of real power P_W). This explains why the measurement points deviate from the straight lines in figs. 3 and 4.
The ohmic resistance is obtained from the axis section in fig. 2 or from the slope of the straight line in fig. 4.

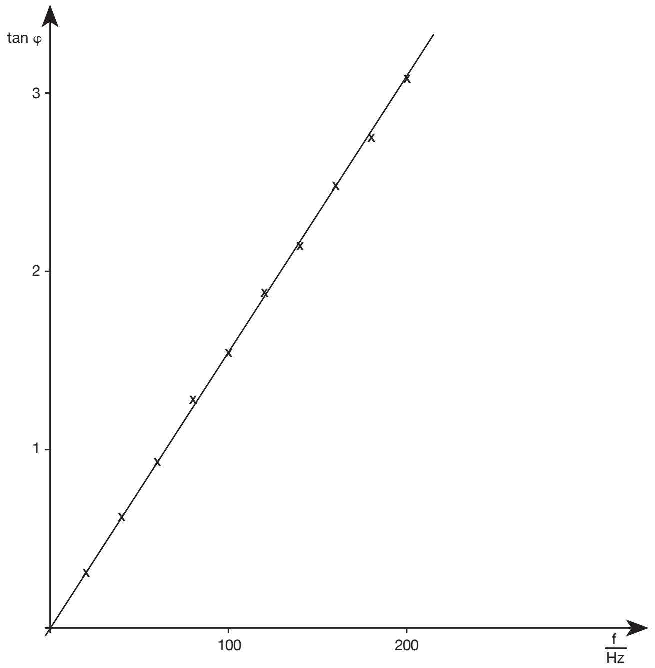
The axis section in fig. 2 and equation (7) yield:

$$R^2 = 0.92\ \Omega^2 \Rightarrow R = 0.96\ \Omega$$

The slope of the straight line in fig. 4 and equation (5) yield:

$$R = 0.96\ \Omega$$

Fig. 3: self-inductance and resistor in series, tan φ as a function of *f*.



The ohmic resistance of the coil is:

$R = 0.96\ \Omega$

The inductance L of the coil is obtained from the slope of the straight lines in fig. 2 or in fig. 3.

The slope of the straight line in fig. 2 and equation (7) yield:

$$4\pi^2 L^2 = 2.21 \cdot 10^{-4}\ \frac{V^2 s^2}{A^2} \Rightarrow L = 2.39\ \text{mH}$$

The slope of the straight line in fig. 3 and equation (8) yield:

$$\frac{2\pi \cdot L}{R} = 15.7\ \text{ms} \Rightarrow L = 2.37\ \text{mH}$$

The inductance of the coil is:

$L = 2.4\ \text{mH}.$

2. Series circuit with capacitor and resistor

The capacitive resistance is a function of frequency

$$X_c = \frac{1}{2\pi \cdot f \cdot C}, \qquad (9)$$

from which the following dependence of the impedance of a series circuit consisting of a capacitor and a resistor from frequency is obtained:

$$Z^2 = R^2 + \frac{1}{(2\pi \cdot C)^2} \cdot \frac{1}{f^2} \qquad (10)$$

and the phase shift between voltage and current intensity is:

$$tan\varphi = -\frac{1}{2\pi \cdot C \cdot R} \cdot \frac{1}{f} \qquad (11)$$

Table 3

f / Hz	U / V	I / A	φ / 1^0	P_w / W	P_2 / VA
100	3.48	0.104	-70	0.124	0.361
120	3.48	0.122	-67	0.164	0.419
140	3.46	0.137	-64	0.208	0.474
160	3.45	0.152	-61	0.254	0.524
180	3.44	0.166	-59	0.293	0.571
200	3.43	0.178	-56	0.341	0.610
300	3.40	0.226	-45	0.543	0.768
400	3.38	0.255	-37	0.688	0.861
500	3.37	0.272	-31	0.785	0.916
1000	3.34	0.298	-17	0.951	0.992

Table 4

f / Hz	Z / Ω	U^2 / Ω^2	$1/f$ / $1/10^{-3}$ Hz	$1/f^2$ / $1/10^{-6}$ Hz	I^2 / A^2	tanφ
100	33.5	1120	10.00	100.0	0.0108	-2.75
120	28.5	814	8.33	69.4	0.0149	-2.36
140	25.3	638	7.14	51.0	0.0188	-2.05
160	22.7	515	6.25	39.1	0.0231	-1.80
180	20.7	429	5.56	30.9	0.0276	-1.66
200	19.3	371	5.00	25.0	0.0317	-1.48
300	15.0	226	3.33	11.1	0.0510	-1.00
400	13.3	176	2.50	6.3	0.0650	-0.75
500	12.4	154	2.00	4.0	0.0740	-0.60
1000	11.2	126	1.00	1.0	0.0888	-0.31

Tables 3 and 4 contain measurement values and calculated values for a series circuit consisting of a capacitor and a resistor. For large angles, a change of the angular display φ by ±1°

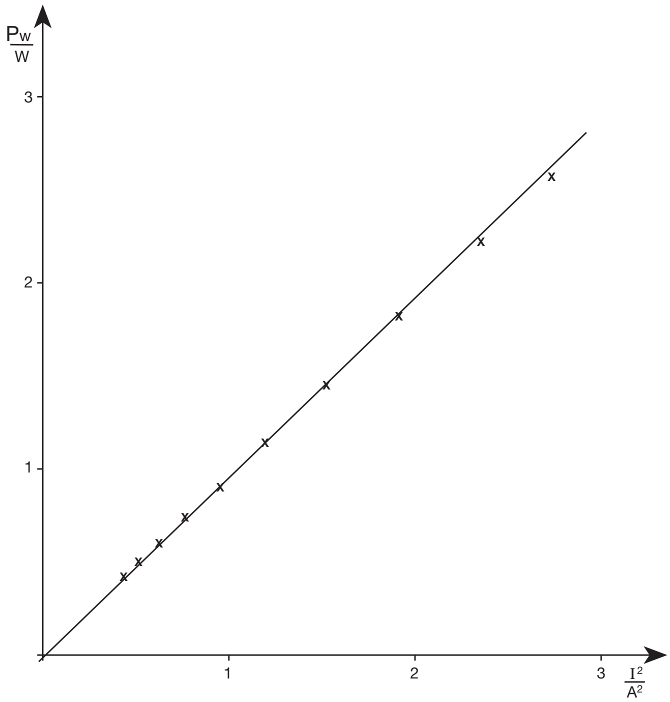
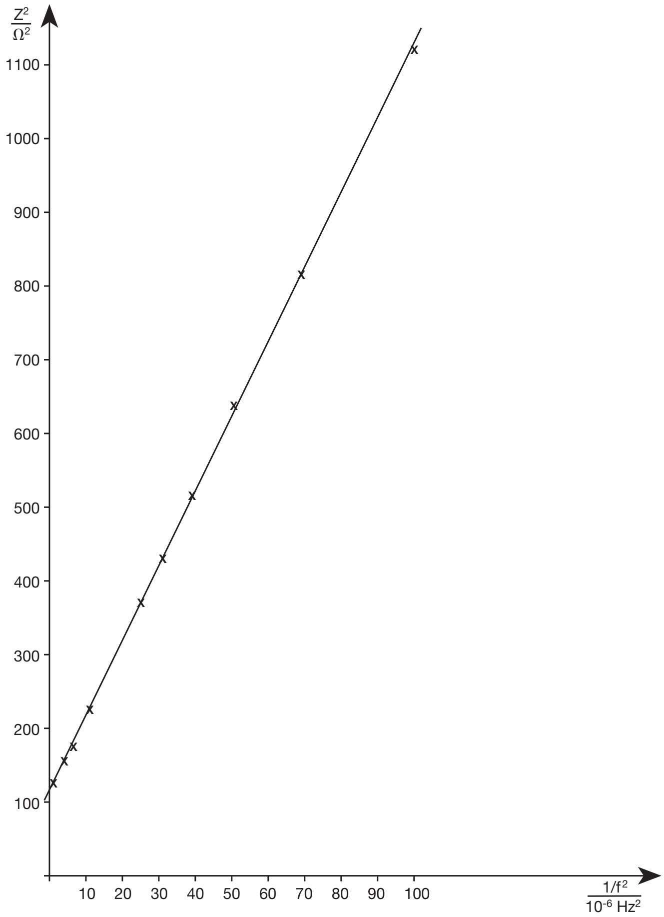


Fig. 4: self-inductance and resistor in series, P_W as a function of I^2.

Fig. 5: capacitor and resistor in series, Z^2 as a function of $1/f^2$.



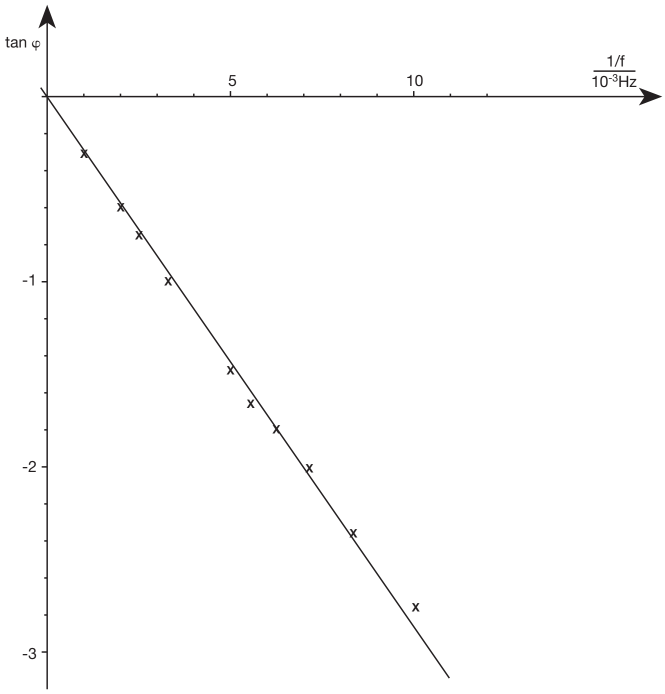
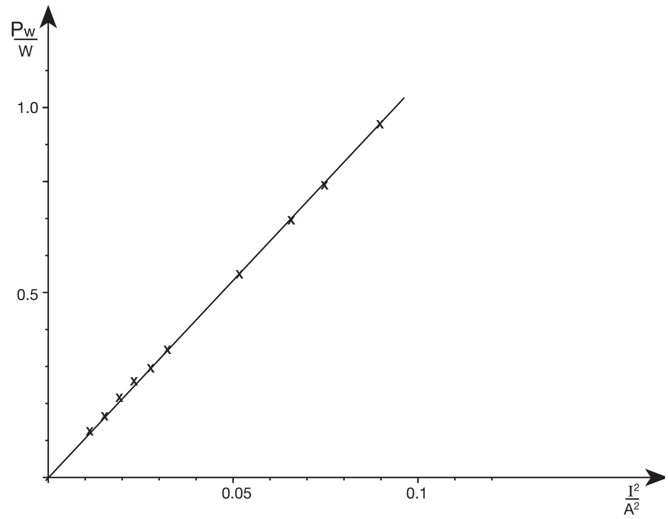


Fig. 6: capacitor and resistor in series, tan φ as a function of *1/f.*

Fig. 7: capacitor and resistor in series, P_W as a function of I^2.



also causes a large change of tanφ and cosφ (and thus a change of real power P_W). This explains why the measurement points deviate from the straight lines in figs. 6 and 7.

The axis section in fig. 5 and equation (10) yield:

$$R^2 = 115\ \Omega^2 \Rightarrow R = 10.7\ \Omega$$

The slope of the straight line in fig. 7 and equation (5) yield:

$$R = 10.7\ \Omega$$

The value of the ohmic resistance is:

$$R = 10.7\ \Omega$$

The capacitance *C* of the capacitor is given by the slope of the straight lines in fig. 5 or 6.

The slope of the straight line in fig. 5 and equation (10) yield:

$$\frac{1}{4\pi^2 C^2} = 10.15\ \frac{V^2}{A^2 s^2} \Rightarrow C = 50.0\ \mu F$$

The slope of the straight line in fig. 6 and equation (11) yield:

$$\frac{1}{2\pi \cdot R \cdot C} = 288.5\ s \Rightarrow C = 51.6\ \mu F$$

The capacitance of the capacitor is:

$$C = 51\ \mu F$$