RLC measuring bridge

Related topics
Wheatstone bridge, inductive and capacitive reactance, ohmic resistance, impedance, Kirchhoff's laws.

Principle and task
Ohmic resistances, inductanes and capacitances are determined in aWheatstone bridge circuit operated on AC. Balancing is done aurally through headphones, using the high sensitivity of the human ear.

Equipment

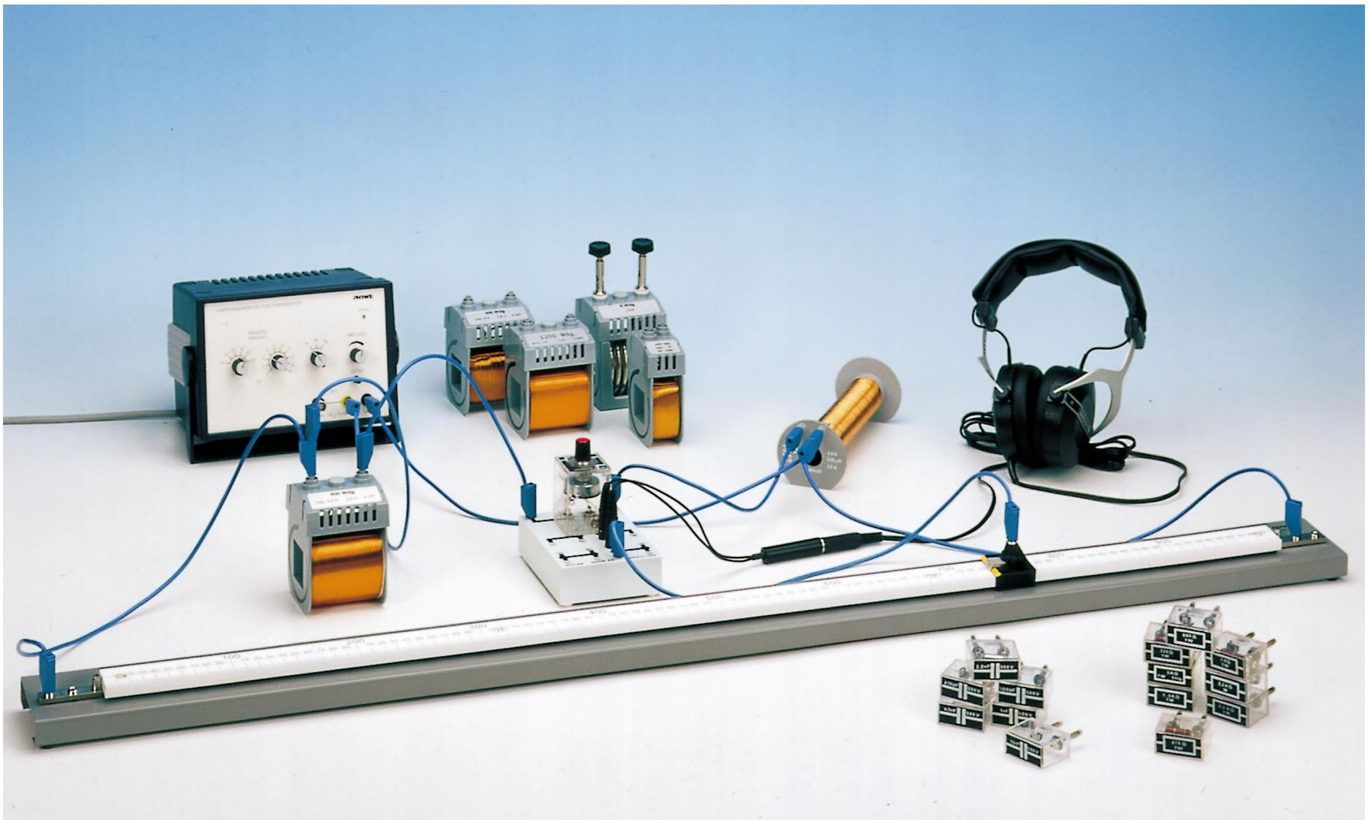
Slide wire meas. bridge, simple	07182.00	1
Headphone, stereo	65974.00	1
Function generator	13652.93	1
Coil, 6 turns	06510.00	1
Coil, 300 turns	06513.01	1
Coil, 600 turns	06514.01	1
Coil, 1200 turns	06515.01	1
Coil, 600 turns, short	06522.01	1
Induction coil, 300 turns, dia.40 mm	11006.01	1
PEK carbon resistor 1 W 5% 330 Ohm	39104.13	1
PEK carbon resistor 1 W 5% 470 Ohm	39104.15	1
PEK carbon resistor 1 W 5% 680 Ohm	39104.17	1
PEK carbon resistor 1 W 5% 1 kOhm	39104.19	1
PEK carbon resistor 1 W 5% 1.5 kOhm	39104.21	1
PEK carbon resistor 1 W 5% 2.2 kOhm	39104.23	1
PEK carbon resistor 1 W 5% 3.3 kOhm	39104.25	1
PEK potentiometer 100 Ohm lin 0.4 W	39103.01	1
PEK carbon resistor 1 W 5% 1 kOhm	39104.19	1
PEK capacitor /case 1/ 100 pF/500 V	39105.04	1
PEK capacitor /case 1/ 470 pF/500 V	39105.07	1
PEK capacitor /case 1/ 1 nF/ 500 V	39105.10	1
PEK capacitor/ case 1/ 2.2 nF/500 V	39105.11	1
PEK capacitor/ case 1/ 4.7 nF/500 V	39105.13	1
PEK capacitor/ case 1/ 10 nF/ 500 V	39105.14	1
Connection box	06030.23	1
Connecting cord, 250 mm, blue	07360.04	3
Connecting cord, 500 mm, blue	07361.04	2
Connecting cord, 750 mm, blue	07362.04	2

Problems
To determine
1. ohmic resistances
2. inductances
3. capacitances

with the Wheatstone bridge, using bridge balancing.

Set-up and procedure
Set up the experiment as shown in Fig. 1 or Figs. 2–4.
Tune the frequency of the audio oscillator to maximum aural sensitivty (500 Hz to 5 kHz). Balance the slide-wire measuring bridge so that no sound is audible on the headphones.

Fig.1: Experimental set up for deermining the filter characteristics of *R-C-L* networks.



1.

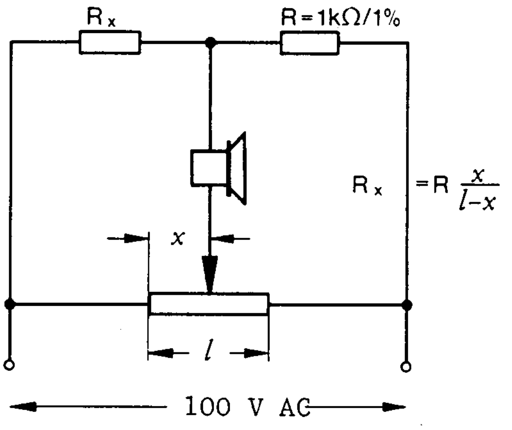


Fig. 2: Circuit for measuring resistance.

2. Potentiometer, P_1 is needed to compensate for the different d.c. resistances in the coils when determining inductive reactance.

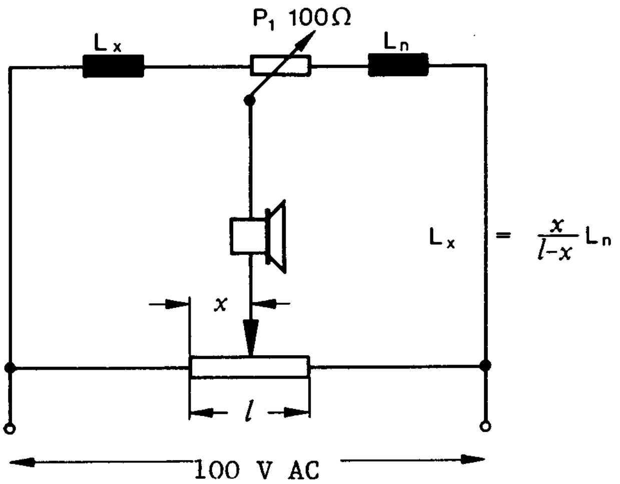


Fig. 3: Circuit for measuring inductances.

Now move both slides in each direction until you find a position at which the sound disappears.

3.

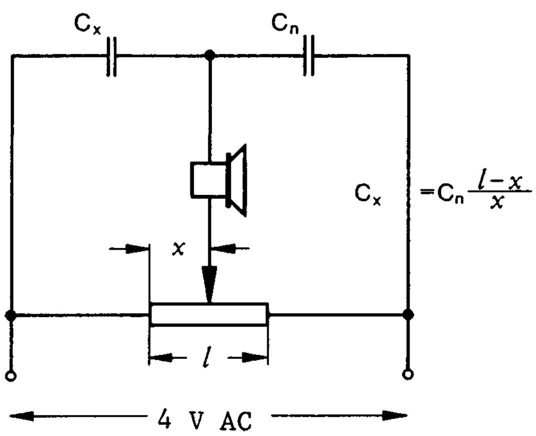
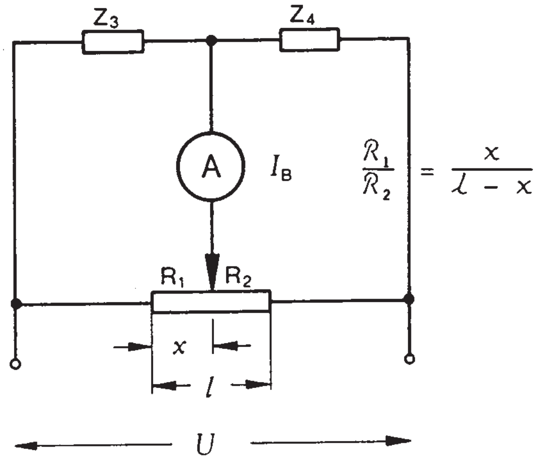


Fig. 4: Circuit for measuring capacitances.

Fig. 5: Wheatstone bridge.



The low resistance generator output is used here to obtain better matching.

Theory and evaluation

From Kirchhoff's laws we obtain the condition for brige balancing which relates to the general bridge construction in Fig. 5 with the unknown impedance Z_3 and the reference impedance Z_4.
The current I_B in the shunt branch is zero,

$$I_B = 0 \tag{1}$$

when

$$\frac{R_1}{R_2} = \frac{Z_3}{Z_4} \tag{2}$$

In complex notation, the general form of the impedance Z is

$$Z = R + \mathrm{i}\left(\omega L - \frac{1}{\omega C}\right) \tag{3}$$

where $\mathrm{i} = \sqrt{-1}$, $\omega = 2\pi f$, and f is the frequency of the applied voltage.

Substituting in (2) gives

$$\frac{R_1}{R_2} = \frac{R_3 + i\left(\omega L_3 - \frac{1}{\omega C_3}\right)}{R_4 + i\left(\omega L_4 - \frac{1}{\omega C_4}\right)} \tag{4}$$

The real and imaginary components must agree on both sides, so that we obtain two conditions:

Amplitude condition:

$$\frac{R_1}{R_2} = \frac{R_3 R_4 + \left(\omega L_3 - \frac{1}{\omega C_3}\right)\left(\omega L_4 - \frac{1}{\omega C_4}\right)}{R_4^2 + \left(\omega L_4 - \frac{1}{\omega C_4}\right)^2} \tag{5}$$

and phase condiction:

$$R_4\left(\omega L_3 - \frac{1}{\omega C_3}\right) = R_3\left(\omega L_4 - \frac{1}{\omega C_4}\right) \quad (6)$$

To fulfil (6), Z_3 and Z_4 must consist of like components.

1. Ohmic resistances

$$Z_3 = R_3,\ Z_4 = R_4$$

In this case, (2) simplifies to

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

The ratio R_1/R_2 is given by the length κ and the total length ι of the slide wire (Fig. 2). We thus obtain the unknown resistance

$$R_3 = R_4 \frac{x}{l-x} \quad (7)$$

The following average volues are obtained from the measurements when $R_4 = l$ kΩ and l = 100 cm:

Resistor (Cat. No.)	x / cm	R_3 / Ω
39104.12	21.3	271
39104.15	28.4	397
39104.17	45.05	820
39104.19	54.5	1200
39104.21	60.5	1530
39104.23	68.85	2210
39104.25	73.4	2760

2. Inductances

Besides an inductive resistance, coils also have an ohmic reasctance

$$Z_3 = R_3 + i\omega L_3$$

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From (6) we first obtain

$$\frac{R_1}{R_2} = \frac{L_3}{L_4} \quad (8)$$

In order to be able to fulfil this condiction we have also to connect a potentiometer into also to connect a potentiometer into the bridge branch with the coils (Fig. 3). R_3 or R_4 is then the sum of the potentiometer component and the ohmic resistance of the coil in the corresponding branch.

Substitutin (8) into (5) we obtain

$$\frac{R_1}{R_2} = \frac{L_3}{L_4}$$

so that

$$L_3 = L_4 \frac{x}{l-x} \quad (9)$$

The inductance of the reference coil (Cat. No. 11006.01) is determined from its dimensions (Kohlrausch, Praktische Physik [Practical Physics[, Vol. 2):

$$L_4 = 2.1 \cdot 10^{-6} \cdot N^2 \cdot r \cdot \left(\frac{r}{l}\right)^{3/4}$$

where L is in henrys, radius r and length ι are in metres, and N is the number of turns.

When N = 300, l = 0.16 m and r = 0.02 m we obtain

$$L_4 = 795\ \mu\text{H}$$

Coil (Cat. No.)	x / cm	L_3 / mH
06515.00	98.0	39.0
06512.00	96.6	22.6
06522.00	94.8	14.5
06514.00	92.4	9.7
06513.00	75.0	2.39

3. Capacitances

When

$$Z_3 = -\frac{i}{\omega C_3}$$

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(6) is fulfilled and we obtain

$$\frac{R_1}{R_2} = \frac{C_4}{C_3}$$

thus

$$C_3 = C_4 \frac{l-x}{x} \quad (10)$$

When C_4 = 100 pF we obtain the following measured values:

Capacitor (Cat. No.)	x / cm	C_3 / nF
39105.14	9.0	10
39105.13	17.9	4.6
39105.11	32.1	2.12
39105.10	50.1	0.98
39105.07	68.2	0.466
39105.04	91.1	0.097