High-pass and low-pass filters

Related topics

Circuit, resistance, capacitance, inductance, capacitor, coil, phase displacement, filter, Kirchhoff's laws, Bode diagram.

Principle and task

A coil, a capacitor, an ohmic resistance and combinations of these components are investigated for their filter characteristics as a function of frequency. The phase displacement of the filters is determined also as a function of frequency.

Equipment

Coil, 300 turns	06513.01	1
PEK carbon resistor 1 W 5% 1 kOhm	39104.19	2
PEK capacitor(case 2) 1 mmF/ 400 V	39113.01	1
PEK capacitor(case 2) 2.2 mmF/400 V	39113.02	1
Resistor in plug-in box 50 Ohms	06056.50	1
Connection box	06030.23	1
Difference amplifier	11444.93	1
Oscilloscope, 20 MHz, 2 channels	11454.93	1
Function generator	13652.93	1
Digital counter, 4 decades	13600.93	1
Connecting cord, 100 mm, blue	07359.04	1
Connecting cord, 500 mm, red	07361.01	4
Connecting cord, 500 mm, blue	07361.04	4
Screened cable, BNC, l 750 mm	07542.11	2

Problems

Determination of the ratio of output voltage to input voltage with the

1. *RC*/*CR* network,
2. *RL*/*LR* network,
3. *CL*/*LC* network,
4. 2 *CR* networks connected in series.
5. Determination of the phase displacement with the *RC*/*CR* network.
6. Determination of the phase displacement with 2 *CR* networks connected in series.

Set-up and procedure

The experimental set up is as shown in Fig. 1. Since normal voltmeters and ammeters generally measure only rms values and take no account to phase relationships, it is prefereable to use an oscilloscope. The experiments are carried out with sinusoidal voltages, so that, for rms values, the peak-to-peak values measured on the oscilloscope ($U_{\rho\text{-}\rho}$) must be divided by $2\sqrt{2}$. With the difference amplifier, the two oscilloscope inputs are non-grounded and are thus independent. If a half-wave (e.g. of the output voltage of the function generator on channel *A*) is brought to full screen width ($180° \triangleq 10$ cm) with the time-base switch of the oscilloscope – possibly with variable sweep rate – the phase displacement of the voltages on the resistance, the capacitor or the coil (channel *B*) can be read off directly in cm (18°/cm). The *Y*-positions of the two base-lines (*GND*) are made to coincide. After switching to other gain settings, the base-lines are readjusted. In order to achieve high reading accuracy, high gain settings are selected.

Fig.1: Experimental set up for deermining the filter characteristics of *R-C-L* networks.

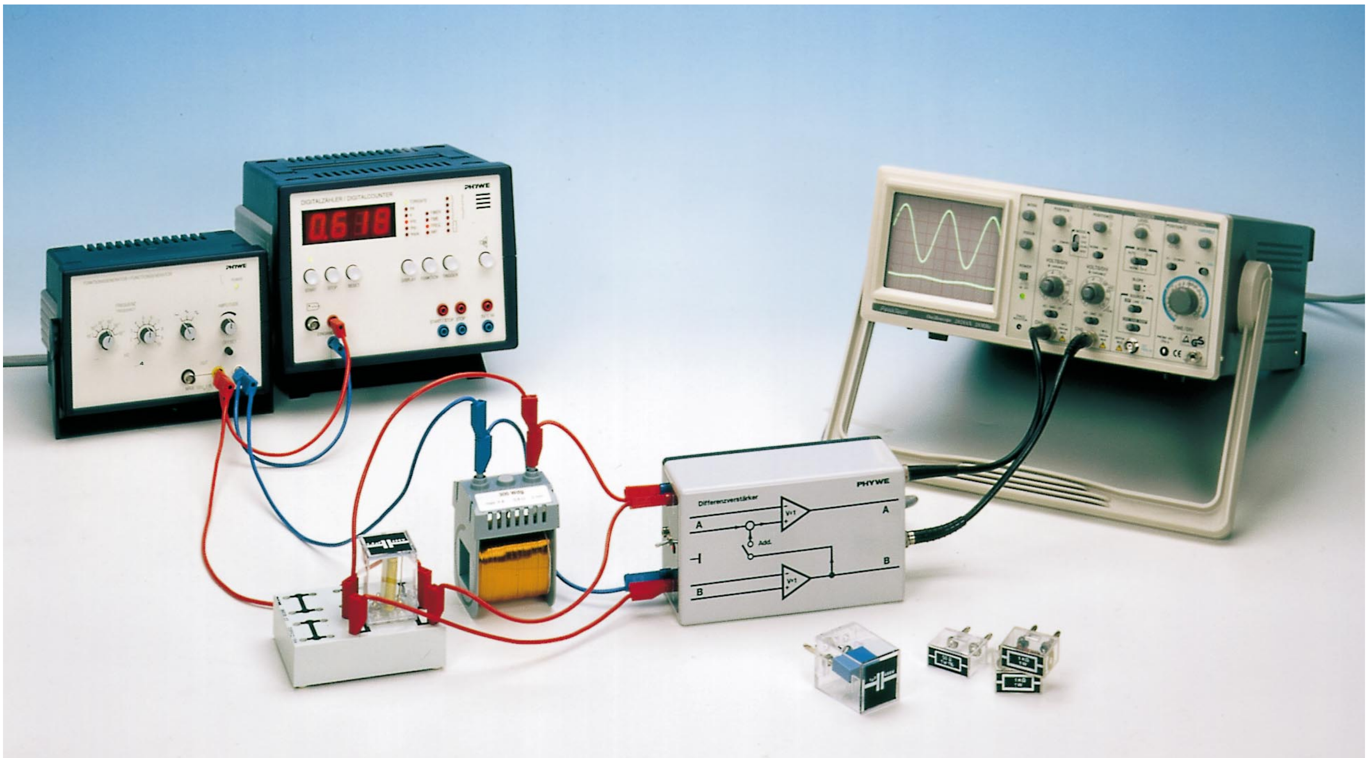
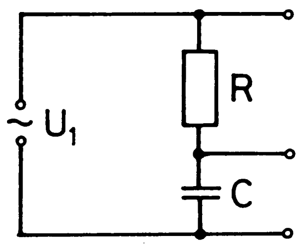


Fig. 2: *RC*/*CR* circuit.



Theory and evaluation

For a circuit with the supply voltage

$$U_1 = U_O \cos \omega t ,$$

in wich there are a capacitor of capacitance *C* and an ohmic resistor of resistance *R*, the network rule (see Fig. 2) reads:

$$U_1 = IR + \frac{Q}{C} , \qquad (1)$$

where I is the current, *Q* is the charge on the capacitor.If, taking account of the fact that

$$\frac{dQ}{dt} = I ,$$

(1) is differentiated and solved for I, one obtains

$$I = I_O \cos (\omega t\text{-}\alpha), \qquad (2)$$

where

$$I_O = \frac{U_O}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} .$$

The phase displacement α is given by

$$\tan \alpha = \frac{1}{\omega RC} . \qquad (3)$$

From (2), the voltage which can be tapped off across *R* is

$$U = RI = \frac{U_O}{\sqrt{1 + \frac{1}{\omega^2 R^2 C^2}}} \cos (\omega t\text{-}\alpha) \qquad (4)$$

(High-pass)

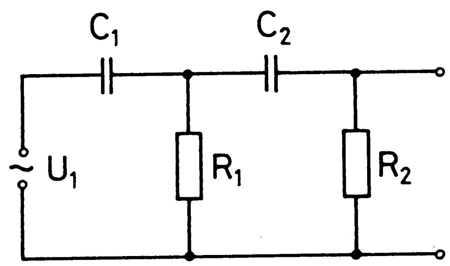
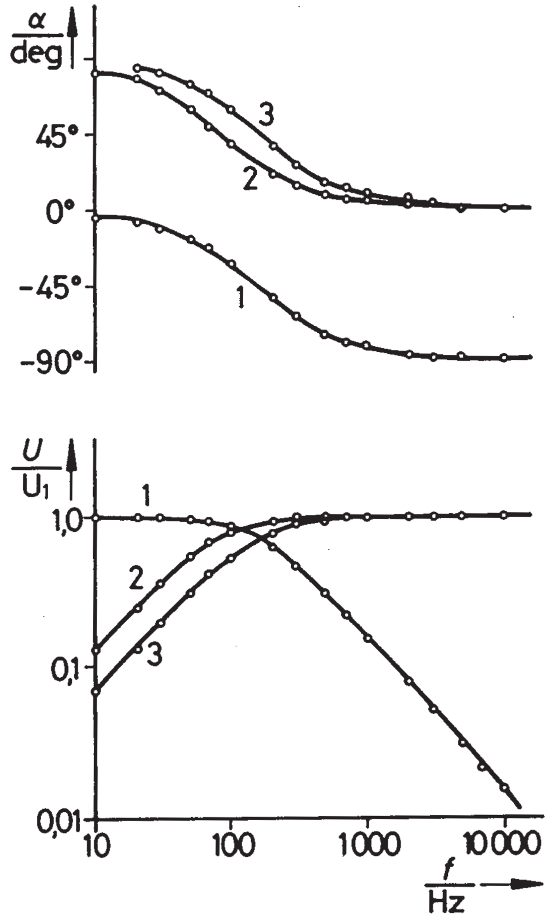


Fig. 3: Series connection of two *CR* networks.

Fig. 4: U/U_1 and phase displacement as a function of the frequency with the *RC*/*CR* network.



and, also from (2), the voltage which can be tapped off across *C* is

$$U = \frac{I}{\omega C} = \frac{U_o}{\sqrt{1 + \omega^2 R^2 C^2}} \cos (\omega t\text{-}\alpha)$$

(Low-pass)

If two *CR* networks are connected in series (see Fig. 3), the current source with the first *CR* network can be regarded as a new voltage source with the voltage U_{CR} as per (4).

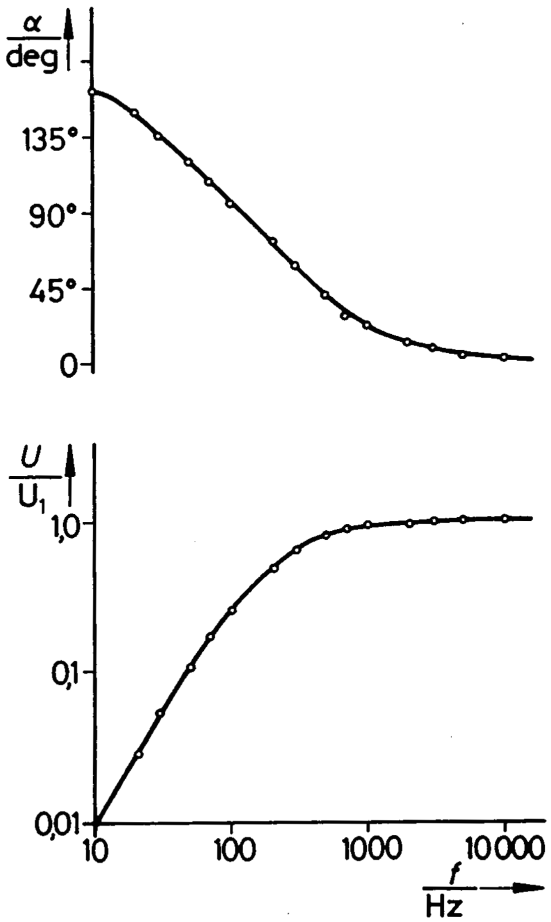
1. RC_1 : R = 1 kΩ, C = 1 µF
2. C_2R : R = 1 kΩ, C = 2 µF
3. C_1R : R = 1 kΩ, C = 1 µF

The voltage which can be tapped off across *R* is thus

$$U = \frac{U_O}{\sqrt{\left\{1 + \frac{1}{\omega^2 C_1^2 R_1^2}\right\}\left\{1 + \frac{1}{\omega^2 C_2^2 R_2^2}\right\}}} \cos (\omega t\text{-}\alpha_1\text{-}\alpha_2),$$

where α_1 and α_2 are the phase displacements of the individual netowrks in acordance with (3).

Fig. 5: U/U_1 and phase displacement as a function of the frequency with the C_1R/C_2R network.



If a coil of inductance L and an ohmic resistance R are in the circuit, the network rule reads

$$U_1 = IR + L\,\frac{dI}{dt}\,.$$

Solution for I gives

$$I = I_O \cos(\omega t - \alpha)\,,$$

where

$$I_O = \frac{U_O}{\sqrt{R^2 + \omega^2 L^2}}$$

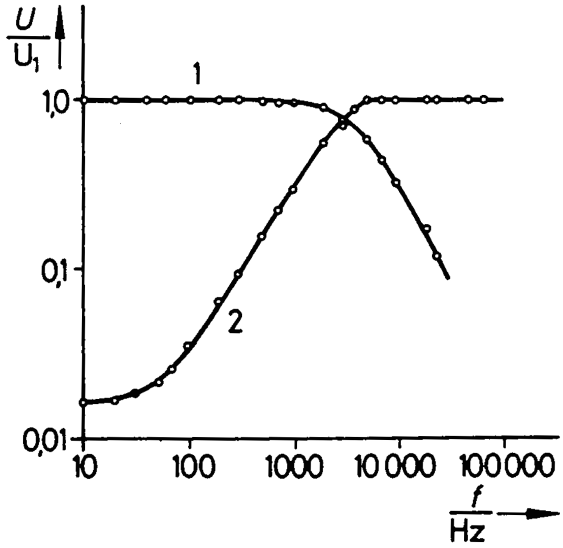
The phase displacement α is given by

$$\tan\alpha = \frac{\omega L}{R}\,.$$

From (4), the voltage which can be tapped off across R is:

$$U = R \cdot I = \frac{U_O}{\sqrt{1 + \frac{\omega^2 L^2}{R^2}}} \cos(\omega t - \alpha) \qquad \text{(Low-pass)}$$

Fig. 6: U/U_1 as a function of the frequency with the LR and RL network.



From (4), the voltage which can be tapped off across the coil is

$$U = \omega L I = \frac{U_O}{\sqrt{1 + \frac{R^2}{\omega^2 L^2}}} \cos(\omega t - \alpha) \qquad \text{(High-pass)}$$

1. LR : R = 50 Ω, L = 2 mH
2. RL : R = 50 Ω, L = 2 mH

If there are a coil of inductance L and a capacitor of capacitance C in a circuit, the network rule reads:

$$U_1 = L\,\frac{dI}{dt} + \frac{Q}{C}\,.$$

If (5) is differentiated and solved for I, one obtains

$$I = I_O \cos(\omega t - \alpha), \qquad (6)$$

where

$$I_O = \frac{U_O}{\omega L - \frac{1}{\omega C}}\,.$$

The phase displacement α is given by

$$\tan\alpha = \omega L - \frac{1}{\omega C}\,.$$

From (6), the voltage which can be tapped off across the coil is:

$$U = \omega L I = \frac{U_O}{1 - \frac{1}{\omega^2 LC}} \cos(\omega t - \alpha) \qquad (7)$$

(High-pass)

From (6), the voltage which can be tapped off across the capacitor is

$$U = \frac{I}{\omega C} = -\frac{U_O}{1 - \omega^2 LC} \cos(\omega t - \alpha) \qquad (8)$$

(Low-pass)

In (7) and (8), a singularity occurs when

$$\omega = \frac{1}{\sqrt{LC}}$$ (Thomson equation).

1. *LR* : *L* = 2 mH, *C* = 2 µF
2. *CL* : *L* = 2 mH, *C* = 2 µf.

Fig. 6: U/U_1 as a function of the frequency with the *CL/CL* network.