Capacitor in the AC circuit

Related topics

Capacitance, Kirchhoff's laws, Maxwell's equations, parallel connection, series connection, AC impedance, phase displacement.

Principle and task

The capacitor is connected in a circuit with a variable-frequency voltage source. The impedance and phase displacement are determined as a function of frequency and of capacitance. Parallel and series impedances are measured.

Equipment

Difference amplifier	11444.93	1
Function generator	13652.93	1
Digital counter, 4 decades	13600.93	1
Oscilloscope, 20 MHz, 2 channels	11454.93	1
Connection box	06030.23	1
PEK carbon resistor 1 W 5% 10 Ohm	39104.01	1
PEK carbon resistor 2 W 5% 100 Ohm	39104.63	1
Resistor in plug-in box 50 Ohms	06056.50	1
PEK capacitor (case 2) 1 mmF/400 V	39113.01	1
PEK capacitor(case 2) 2.2 mmF/400 V	39113.02	1
PEK capacitor(case 2) 4.7 mmF/400 V	39113.03	1
Screened cable, BNC, l 750 mm	07542.11	2
Connecting cord, 100 mm, red	07359.01	2
Connecting cord, 500 mm, red	07361.01	4
Connecting cord, 500 mm, blue	07361.04	2

Problems

1. Determination of the impedance of a capacitor as a function of capacitance and frequency;
2. Determination of the phase displacement between terminal voltage and total current as a function of the capacitance and the frequency in the circuit;
3. Determination of the total impedance of capacitors connected in parallel and in series.

Set-up and procedure

The experimental set up is as shown in Fig. 1. Since normal voltmeters and ammeters generally only measure rms values and take no account of the phase relationships, it is preferable to use an oscilloscope. The experiment will be carried out with sinusoidal voltages, so that to obtain rms values, the peak-to-peak values measured on the oscilloscope (U_{pp}) are to be divided by $2\sqrt{2}$.

In accordance with

$$I = U/R,$$

the current can be deduced by measurement of the voltage across the resistance. The circuit shown in Fig. 2 permits the simultaneous display of the total current and the capacitor voltage. If, by means of the time-base switch of the oscilloscope, one half-wave of the current (180°) is brought to the full screen width (10 cm) – possibly with variable sweep rate – the phase displacement of the voltage can be read off directly in cm (18°/cm). The Y-positions of the two base-lines (GND) are made to coincide.

After switching to other gain settings, the base-lines are readjusted. In order to achieve high reading accuracy, high gain settings are selected. The inputs to the difference amplifier are non-grounded.

Fig.1: Experimental set up for investigating the a.c impedance of the capacitor.

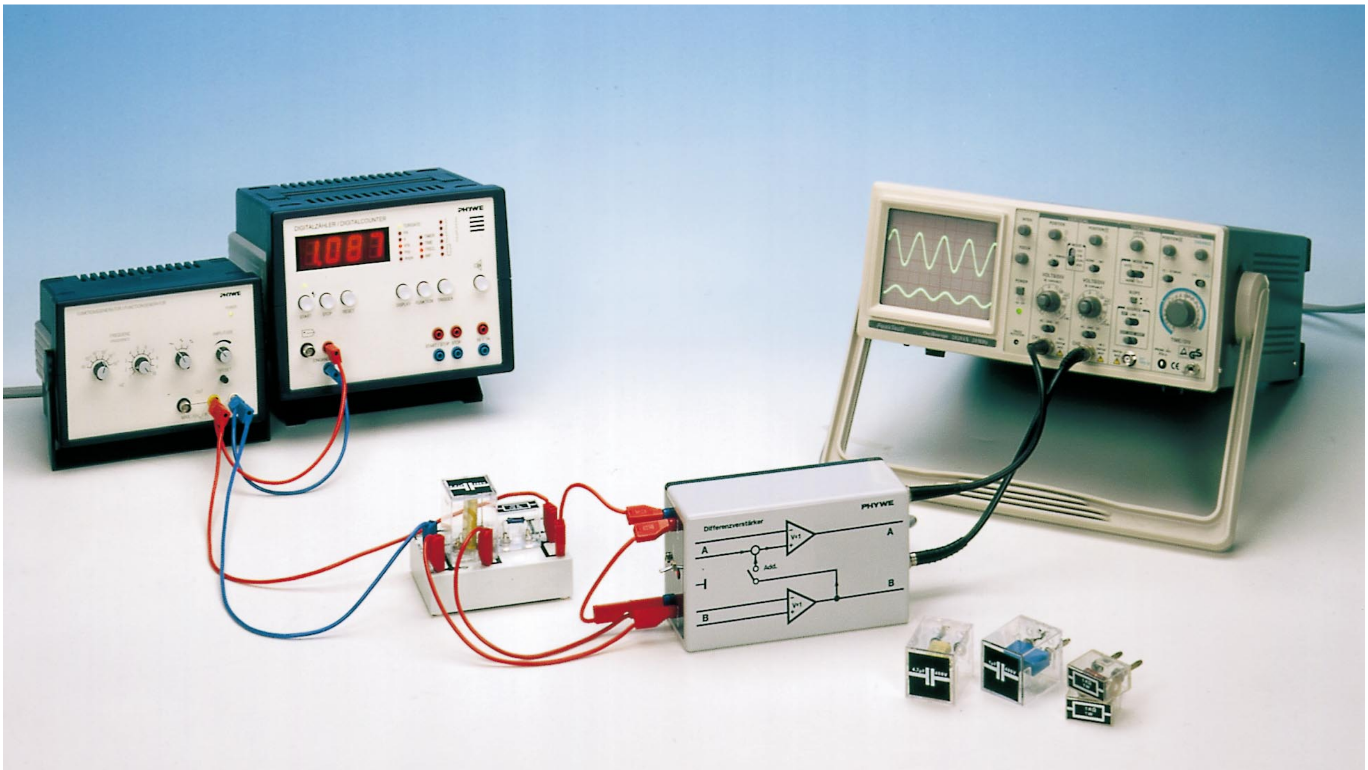
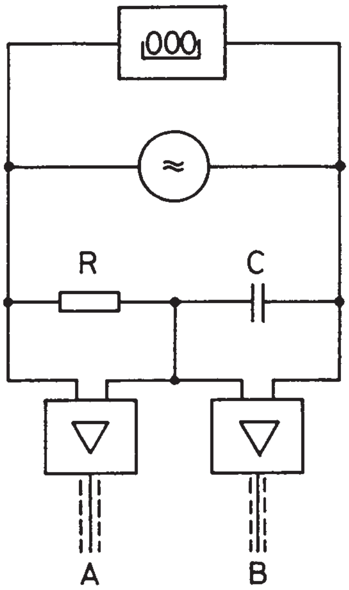


Fig. 2: Circuit for display of current and voltage with the oscilloscope.



To determine the impedance of a capacitor as a function of the frequency, the capacitor is connected in series with resistors of known value. The frequency is varied until there is the same voltage drop across the capacitor as across the resistor. The resitance and impedance values are then equal:

$$R_\Omega = \frac{1}{\omega C} \,. \qquad (1)$$

The phase displacement between the terminal voltage and the total current can be measured using a similar circuit to Fig. 2, but with channel *B* measuring the total voltage and not the voltage across the capacitor.

When coils are connected in parallel or in series, care should be taken to ensure that they are sufficiently far apart, since their magnetic fields influence one another.

Theory and evaluation

If a capacitor of capacitance *C* and an ohmic resistor *R* are connected in a circuit (see Fig. 2), the sum of the voltage drops on the individual elements is equal to the terminal voltage *U*

$$U = IR + \frac{Q}{C} \,, \qquad (2)$$

where *Q* is the quantity of charge on the plates of the capacitor and I is the current. Noting the fact that

$$I = \frac{dQ}{dt} \,,$$

it follows from (2) that

$$\frac{dU}{dt} = R\,\frac{dI}{dt} + \frac{I}{C} \qquad (3)$$

If the alternating voltage *U* has the frequency ω and the waveform

$$U = U_O \cos \omega t$$

then the solution of (3) is

$$I = I_O \cdot \cos(\omega t - \phi)$$

with the phase displacement ϕ given by

$$\tan \phi = -\frac{1}{\omega CR} \qquad (4)$$

and

$$I_O = \frac{U_O}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \qquad (5)$$

and the impedance by

$$\frac{U_O}{I_O} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

It is customary to threat complex impedance as operators $\hat{R}_i$:
Capacitor $R_C = -i/\omega C$, and
ohmic resistance $\hat{R} = R$.

With parallel connection,

$$\hat{R}_{tot}^{-1} = \sum_\iota \hat{R}_\iota^{-1}$$

and with series connection

$$\hat{R}_{tot} = \sum_\iota \hat{R}_\iota \,.$$

The real impedance of a curcuit is the absolute value of $\hat{R}_{tot}$ and the phase relationship, analogous to (4), is the ratio of the imaginary part to the real part of $\hat{R}_{tot}$.

From the regression line to the measured values of Fig. 3 and the exponential statement

$$Y = A \cdot X^B$$

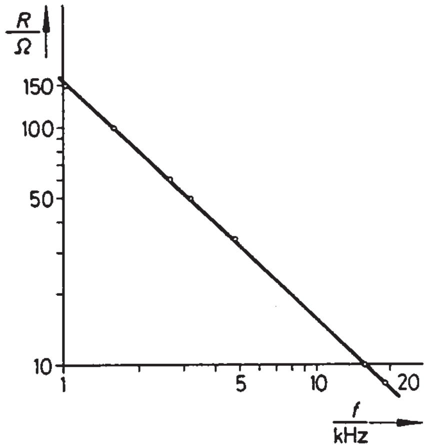
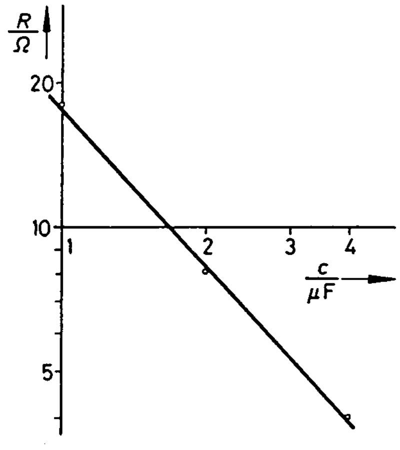


Fig. 3: Impedance of various capacitor as a function of the frequency (C = 1 μF).

Fig. 4: Impedance as a function of capacitance at constant frequency (f = 10 kHz).



there follows the exponent

B_1 = – 0.993 ± 0.001.

From the regression line to the measured values of Fig. 4 and the exponential statement

$Y = A \cdot X^B$

there follows the exponent

B = 0.99 ± 0.01 (see (1))

From the regression line to the measured values of Fig. 5 and the exponential statement

$Y = A \cdot X^B$

there follows the exponent

B = – 0.99 ± 0.01 (see (4))

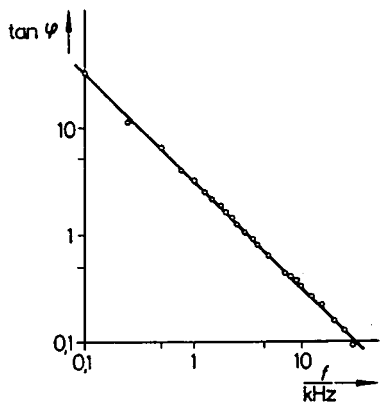
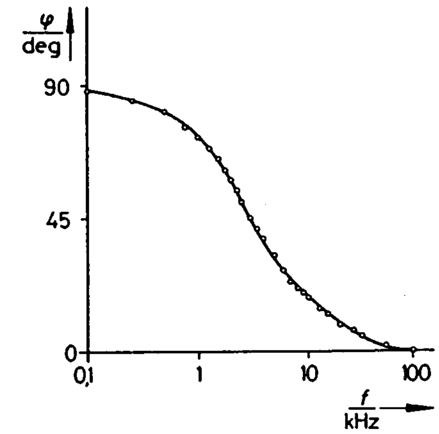


Fig. 5: Phase displacement (tan ϕ) between total current and total voltage as a function of frequency.

Fig. 6: Phase displacement (ϕ) between total current and total voltage as a function of frequency.



From the regression line to the measured values of Fig. 7 and the exponential statement

$Y = A \cdot X^B$

the exponent

B = – 0.997 ± 0.02 (see(4))

is obtained.

The frequency at which the total impedance of the capacitors was equal to the reference resistance 500 Ω was determined with capacitors connected in series and in parallel.

Table 1: Total capacitance of capacitors C_ι (1, 2 and 4 µF), connected in series (lines 1, 2) and in parallel (lines 3, 4).

	Capacitor	f (50 Ω)	C_{tot}
1	C_1 C_2	4.79 kHz	0.665 µF
2	C_1 C_3	3.98 kHz	2.800 µF
3	C_1 C_2	1.07 kHz	2.97 µF
4	C_1 C_3	0.644 kHz	4.94 µF

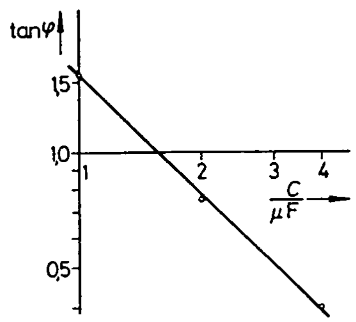


Fig. 7: Phase displacement (tan ϕ) between total current and total voltage as a function of the capacitance.