Magnetic field inside a conductor

Related topics
Maxwell's equations, magnetic flux, induction, current density, field strength.

Principle and task
A current which produces a magnetic field is passed through an electrolyte. This magnetic field inside the conductor is determined as a function of radius and current.

Equipment
Hollow cylinder, PLEXIGLAS 11003.10 1
Search coil, straight 11004.00 1
Power frequency generator 1 MHz 13650.93 1
LF amplifier, 220 V 13625.93 1
Digital multimeter 07134.00 2
Adapter, BNC-socket/4 mm plug pair 07542.27 1
Distributor 06024.00 1
Meter scale, demo, l = 1000mm 03001.00 1
Cursors, 1 pair 02201.00 1
Tripod base -PASS- 02002.55 1
Barrel base -PASS- 02006.55 1
Support rod -PASS-, square, l 400 mm 02026.55 1
Right angle clamp -PASS- 02040.55 1
Screened cable, BNC, l 1500 mm 07542.12 1
Connecting cord, 500 mm, yellow 07361.02 3
Connecting cord, 500 mm, blue 07361.04 3
Hydrochloric acid 1.19, 1000 ml 30214.70 1

Problems
Determination of the magnetic field inside a conductor as a function
1. of the current in the conductor,
2. of the distance from the axis of the conductor.

Set-up and procedure
The experimental set up is as shown in Fig. 1. The electrolyte (approx. 200 ml of 37% hydrochloric acid to 4 litres of water) is poured into the hollow cylinder after it has been thoroughly mixed. The aperture must not be tightly closed, so that gases released (H₂, O₂) can escape. The various connection sockets on the hollow cylinder permit separate measurements on the electrolyte and on the jacket (hollow cylinder). Account must be taken of the fact that the magnetic field strengths to be measured lie in the μT range, i.e. the cables carrying the current - especially the return lead – also produce a magnetic field which is of the same order of magnitude.

Fig. 1: Experimental set-up for determining the magnetic field inside a conductor.
For the field strength measurement in the electrolyte the return lead for the current is the grid, as a current in the wall of the hollow cylinder produces no magnetic field inside the cylinder.

With this connection, there is no resultant field in the space outside the cylinder. The induced voltage \( U_{\text{ind}} \) is

\[
U_{\text{ind}} = n \cdot A \frac{dB}{dt}
\]

with the number of turns \( n = 1200 \) and the effective area \( A = 74.3 \text{ mm}^2 \). Since the magnetic flux density \( B \) is produced by a sinusoidal current of frequency \( f \) or angular velocity \( \omega = 2\pi f \),

\[
B = B_0 \cdot \sin \omega t .
\]

Therefore, the induced voltage is

\[
U_{\text{ind}} = n \cdot A \cdot 2\pi \cdot f \cdot B_0 \sin (\omega t + \phi) .
\]

The phase displacement \( \phi \) is irrelevant for this measurement. Since, according to (4), the magnetic flux density is proportional to the current, the induced voltage is proportional to the current and the frequency. The current is limited by the formation of gas (electrolysis) and the frequency by the series-connected measuring instruments (\( f \leq 11 \text{ kHz} \)). The experiment was carried out at \( f = 5.5 \text{ kHz} \) and \( I < 1 \text{ A} \). The amplification was \( 1 \cdot 10^3 \); position 1 is calibrated on the 10 V output.

**Theory and evaluation**

Maxwell’s 1st equation

\[
\oint_C B \cdot d\mathbf{s} = \mu_0 \int_A \mathbf{j} \cdot d\mathbf{a} , \quad (1)
\]

together with Maxwell’s 4th equation

\[
\int_A \mathbf{B} \cdot d\mathbf{a} = 0 , \quad (2)
\]

gives the relationship between the steady electric current \( I \) flowing through the area \( A \)

\[
I = \int_A \mathbf{j} \cdot d\mathbf{a} \quad (3)
\]

and the magnetic field \( \mathbf{B} \) it produces.

\( C \) is the boundary of \( A \).

\( A' \) is any given enclosed area.

\( \mathbf{j} \) is the electrical current density.

\( \mu_0 \) is the magnetic field constant,

\[
\mu_0 = 1.26 \cdot 10^{-6} \text{ Vs} \cdot \text{Am} .
\]

From (1) and (2) one obtains

\[
B = \frac{\mu_0}{2\pi} \cdot I \quad (4)
\]

for a long straight conductor, where \( |r| \) is the distance of point \( P \), at which the magnetic flux density is measured, from the axis of the conductor.

Since the current density \( \mathbf{j} \) is uniform in the electrolyte, the current \( I \) flowing through the area \( A \) is expressed as a function of the current \( I_{\text{tot}} \) flowing through the whole cross-section of the electrolyte, from (3), as

\[
I = I_{\text{tot}} \cdot \frac{2}{R} ,
\]

so that (4) gives

\[
B = \frac{\mu_0}{2\pi} \cdot I_{\text{tot}} \cdot \frac{|r|}{R^2} . \quad (5)
\]

\( B \) is measured with an induction coil. The induced voltage \( U \) is

\[
U = B .
\]

From the regression line to the measured values of Fig. 2, and the exponential statement

\[
Y = A \cdot X^B
\]

the exponent follows as

\[
B = 0.989 \pm 0.003 \quad \text{(see (5))}
\]

From the regression lines to the measured values of Fig. 3 and the linear statement

\[
Y = A + B \cdot X
\]

the slope follows as

\[
B_1 = (-0.0545 \pm 0.0006) \text{ Vsm}^{-3} \quad \text{(see (5))}
\]

and

\[
B_2 = (+0.0548 \pm 0.0003) \text{ Vsm}^{-3}
\]
For the axis intercept, there follows

\[ A_1 = 16.02 \text{ mT} \]
\[ A_2 = -16.19 \text{ mT} \]

From these, the point at which the field strength disappears is obtained as

\[ A_1/B_1 = 294.0 \text{ mm} \]
\[ A_2/B_2 = 295.4 \text{ mm}. \]