PHYWE

Magnetic field outside a straight conductor

LEP 4.3.05

Related topics

Maxwell's equations, magnetic flux, induction, superimposition of magnetic fields.

Principle and task

A current which flows through one or two neighbouring straight conductors produces a magnetic field around them. The dependences of these magnetic fields on the distance from the conductor and on the current are determined.

Equipment

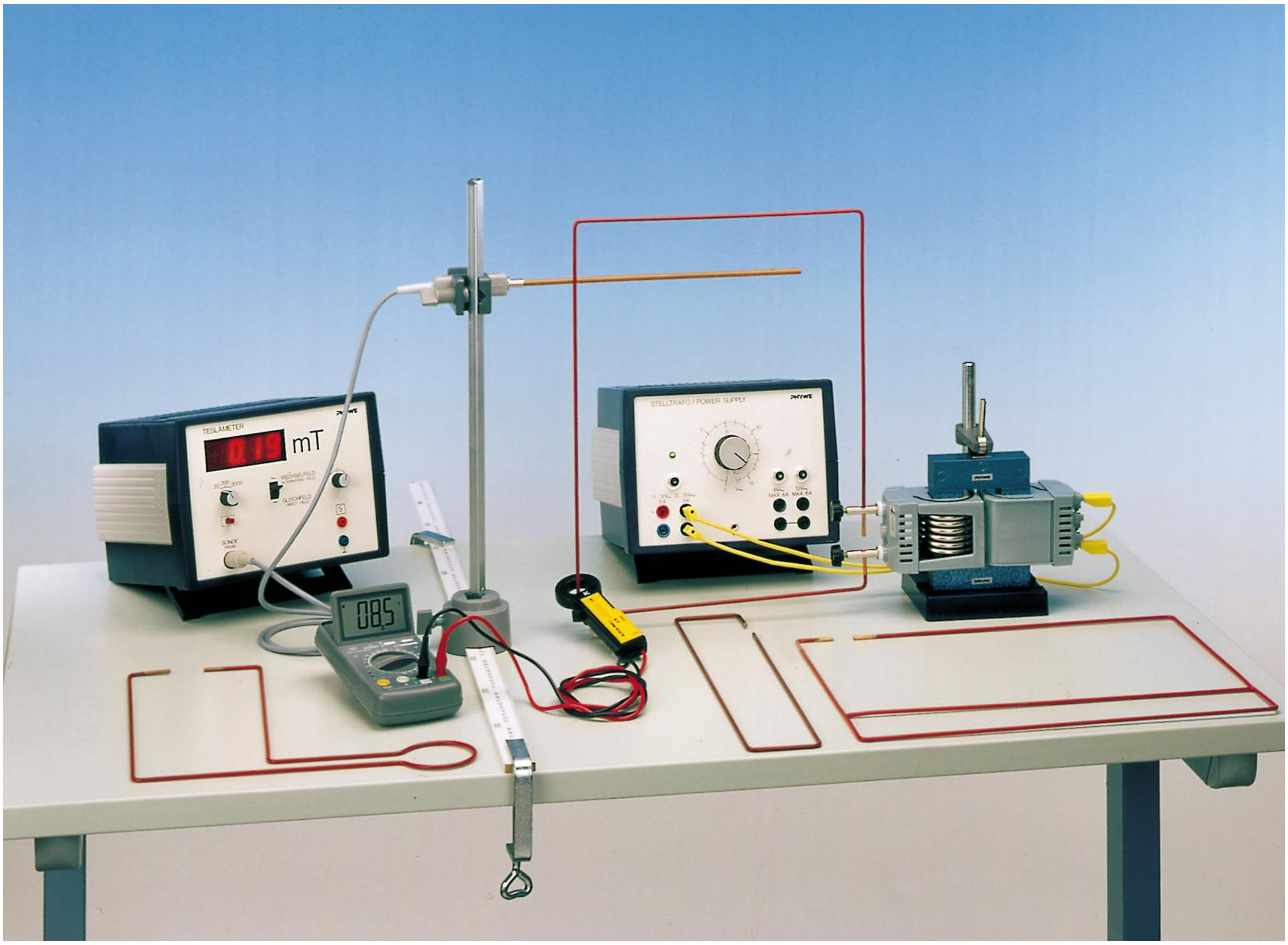
Current conductors, set of 4	06400.00	1
Coil, 6 turns	06510.00	1
Coil, 140 turns, 6 tappings	06526.01	1
Clamping device	06506.00	1
Iron core, short, laminated	06500.00	1
Iron core, U-shaped, laminated	06501.00	1
Power supply var. 15 VAC/12 VDC/5 A	13530.93	1
Teslameter, digital	13610.93	1
Hall probe, axial	13610.01	1
Current transformer	07091.00	1
Digital multimeter	07134.00	1
Meter scale, demo, l = 1000 mm	03001.00	1
Barrel base -PASS-	02006.55	1
Support rod -PASS-, square, l 400 mm	02026.55	1
Right angle clamp -PASS-	02040.55	1
G-clamp	02014.00	2
Connecting cord, 500 mm, yellow	07361.02	2

Problems

Determination of the magnetic field

1. of a straight conductor as a function of the current,
2. of a straight conductor as a function of the distance from the conductor,
3. of two parallel conductors, in which the current is flowing in the same direction, as a function of the distance from one conductor on the line joining the two conductors,
4. of two parallel conductors, in which the current is flowing in opposite directions, as a function of the distance from one conductor on the line joining the two conductors.

Fig. 1: Experimental set-up for determining the magnetic field in the space outside straight conductors.



Set-up and procedure

The experimental set-up is arranged as shown in Fig. 1. The current transformer is used to measure the secondary current (20 A - 120 A). Since the primary and secondary current have a linear relationship, the primary current can also be measured. However, a calibration curve for primary/secondary current should then be recorded for each conductor. Because of the heating of the conductors, the current must be readjusted of a “warm-up time” must be allowed to elapse. A phase displacement can occur between the “construction-kit” transformer and the magnetic field meter, giving the illusion of a “negative” magnetic field (minimum of the magnetic field indicator with increasing current). This can be eliminated by reversing the polarity of the primary of the transformer. Higher short-time secondary currents can be achieved by connecting the constant and variable voltage in series on the power unit. Attention should be paid to the correct phase angle.

Theory and evaluation

Maxwell’s 1st equation for the case when electric fields $\vec{E}$, variable with time, are absent,

$$\oint_C \vec{B} \cdot \vec{ds} = \mu_o \int_A \vec{j} \cdot \vec{da} \qquad (1)$$

together with Maxwell’s 4th equation

$$\int_A \vec{B} \cdot \vec{da} = o, \qquad (2)$$

provides the relationship between the steady electric current I flowing through the area A

$$I = \int_A \vec{j} \cdot \vec{da}$$

and the magnetic field B which it produces.

C is the boundary of A.
A’ is any given enclosed area.
$\vec{j}$ ist the electrical current density.
μ_o ist the magnetic field constant,

$$\mu_o = 1.26 \cdot 10^{-6} \frac{\text{Vs}}{\text{Am}}.$$

From (1) and (2) one obtains, for a long straight conductor

$$|\vec{B}| = \frac{\mu_o}{2\pi} \cdot \frac{I}{|\vec{r}|} \qquad (3)$$

where $\vec{r}$ is the distance of the conductor from the point at which the magnetic field is measured.
The direction of $\vec{B}$ is $\perp$ *both to* $\vec{r}$ and to $\vec{j}$·

For a finite conductor one obtains, with the notation of Fig. 2:

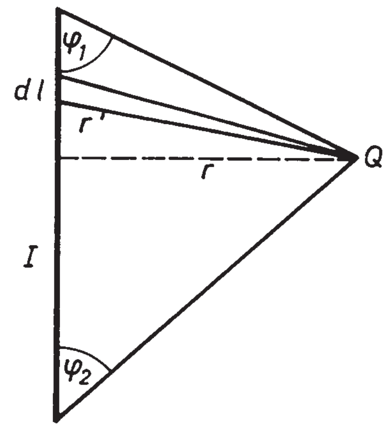
$$d\vec{B} = \frac{1}{4\pi} \mu_o \frac{I}{r^3} \vec{dl} \times \vec{r}\,'$$

(Biot-Savart)

and from this

$$|\vec{B}| = \frac{\mu_o}{4\pi} \frac{I}{r} (\cos \phi_1 - \cos \phi_2).$$

Fig. 2: Contribution of a conductor section dl to the magnetic field at point Q.



From the regression line to the measured values of Fig. 3 with the exponential statement

$$Y = A \cdot X^B$$

the exponent

$$B = 0.97 \pm 0.01 \qquad \text{(see (3))}$$

and the slope

$$A = 52.91 \pm 0.01 \text{ A/mT}$$

with (3) this gives

$$\mu_o = 1.3 \cdot 10^{-6} \frac{\text{Vs}}{\text{Am}}.$$

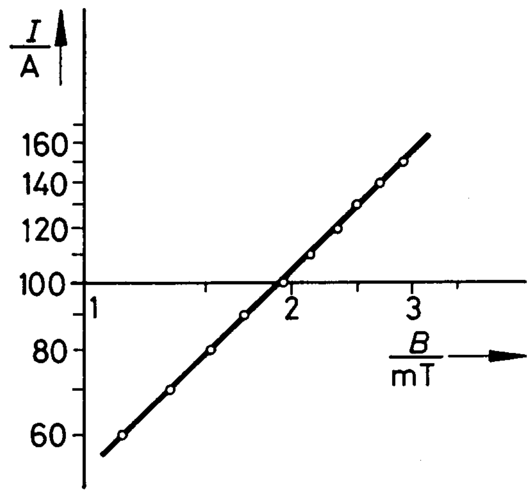
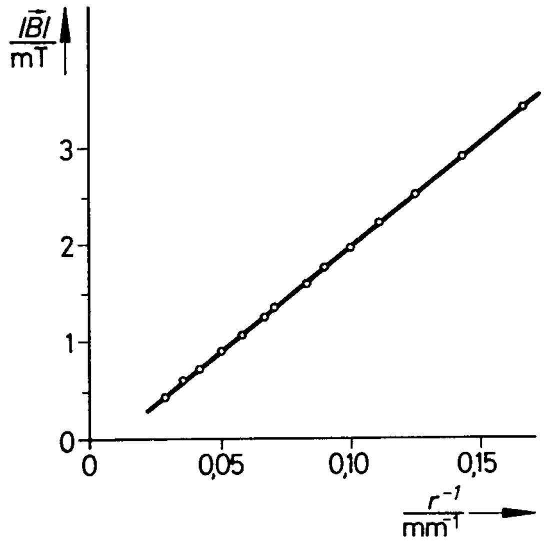


Fig. 3: Relationship between current value and magnetic field of a long conductor (distance between conductor and measuring point: 1.1 cm).

Fig. 4: Magnetic field of a long conductor as a function of distance (I = 100 A).



Because of the small zero-deflection due to the instrument and the effect of the other conductor and the "construction-kit" transformer, it is appropriate to carry out the measurement with small distances (up to approx. 3 cm) and with large currents (approx. 100 A).

For the case of two parallel conductors in the z-direction, both carrying the same current I in the same direction (p = 1) or in opposite directions (p = –1), the superimposition of the magnetic fields gives the components B_x and B_y of the magnetic field at point Q with the notation of Fig. 5.

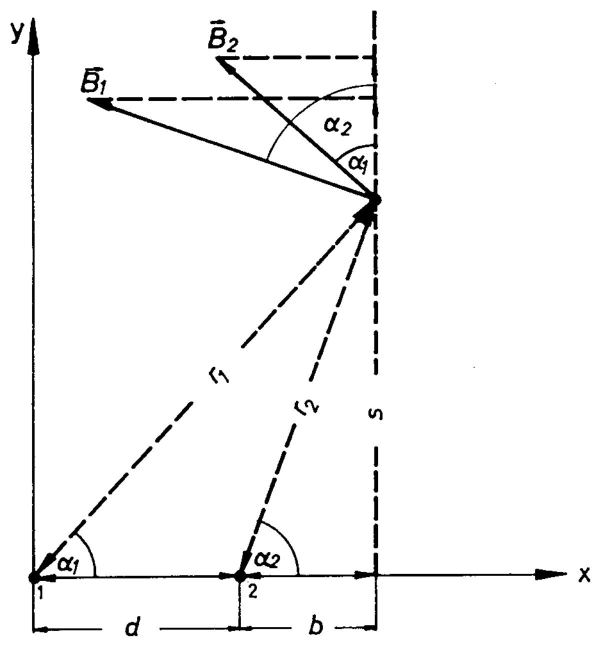
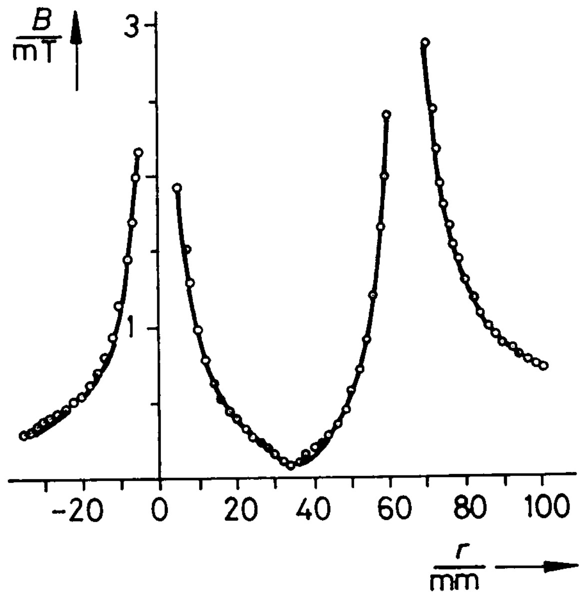


Fig. 5: Magnetic field of two conductors 1 and 2.

Fig. 6: Magnetic field component B_y of two parallel conductors on the x-axis as a function of the distance from one conductor, if the current in both conductors is in the same direction.



$$B_x = |\vec{B}_1| \sin\alpha_1 + p \cdot |\vec{B}_2| \sin\alpha_2 = \\ = \frac{\mu_o \; I}{2\pi \cdot s} (\sin^2\alpha_1 + p \cdot \sin^2\alpha_2)$$

$$B_y = |\vec{B}_1| \cos\alpha_1 + p \cdot |\vec{B}_2| \cos\alpha_2 = \\ = \frac{\mu_o \; I}{2\pi} (\frac{1}{b+d} \cdot \cos^2\alpha_1 + p \cdot \frac{1}{b} \cdot \cos^2\alpha_2)$$

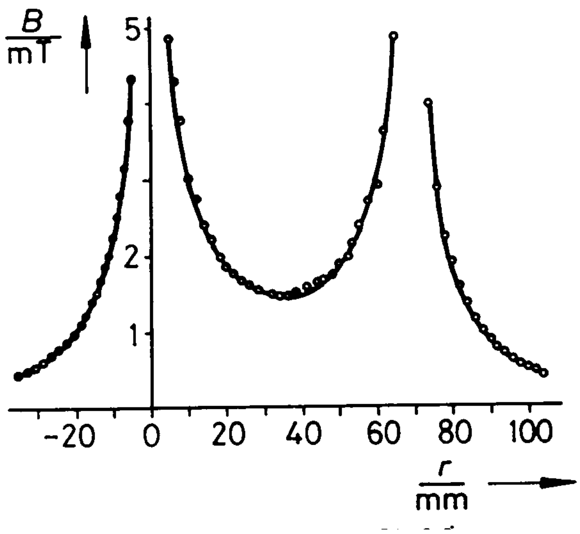
For Q on the x-axis, one obtains ($\alpha_1 = \alpha_2 = o$)

$$B_y = \frac{\mu_o \; I}{2\pi} (\frac{1}{b+d} + p \cdot \frac{1}{b}).$$

The peak at the minimum of the magnetic field originates from the reflection of the negative magnetic field as positive values, since the measuring instrument only indicates the absolute value of the magnetic field. The different values of the magnetic field at r = –5 mm and r = +5 mm occur because of the additive or subtractive superimposition of the magnetic fields of conductors 1 and 2.

The increase in the field at conductor 2 in comparison with conductor 1 at r = 65 mm as compared with r = 5 mm occurs because of the higher current density in conductor 2, which results from the resistance of the connecting piece between conductors 1 and 2. Finally, beyond conductor 2 (r = 75 mm), the effect of conductor 3 becomes noticeable. This is parallel to conductors 1 and 2, but the current in it flows in the opposite direction to that in conductors 1 and 2 and thus reinforces the magnetic field of 1 and 2 in this area.

Fig. 7: Magnetic field component B_y of two parallel conductors on the x-axis as a function of the distance from one conductor, if the current in the two conductors is in opposite directions (I = 107 A).



The strengthening of the fields can be clearly seen in the space between the two conductors, compared with the reduction in the area beyond the two conductors.