Magnetic field of single coils / Biot-Savart's law

LEP 4.3.02

Related topics
Wire loop, Biot-Savart's law, Hall effect, magnetic field, induction, magnetic flux density.

Principle and task
The magnetic field along the axis of wire loops and coils of different dimensions is measured with a teslameter (Hall probe). The relationship between the maximum field strength and the dimensions is investigated and a comparison is made between the measured and the theoretical effects of position.

Equipment

Induction coils, 1 set	11006.88	1
consisting of		
Induction coil, 300 turns, dia. 40 mm	11006.01	1
Induction coil, 300 turns, dia. 32 mm	11006.02	1
Induction coil, 300 turns, dia. 25 mm	11006.03	1
Induction coil, 200 turns, dia. 40 mm	11006.04	1
Induction coil, 100 turns, dia. 40 mm	11006.05	1
Induction coil, 150 turns, dia. 25 mm	11006.06	1
Induction coil, 75 turns, dia. 25 mm	11006.07	1
Conductors, circular, set	06404.00	1
Teslameter, digital	13610.93	1
Hall probe, axial	13610.01	1
Power supply, universal	13500.93	1
Distributor	06024.00	1
Meter scale, demo, l = 1000 mm	03001.00	1
Digital multimeter	07134.00	1
Barrel base -PASS-	02006.55	2
Support rod -PASS-, square, l 250 mm	02025.55	1
Right angle clamp -PASS-	02040.55	1
G-clamp	02014.00	2
Lab jack, 160×130 mm	02074.00	1
Reducing plug 4 mm/2 mm socket, 2	11620.27	1
Connecting cord, 500 mm, blue	07361.04	1
Connecting cord, 500 mm, red	07361.01	2

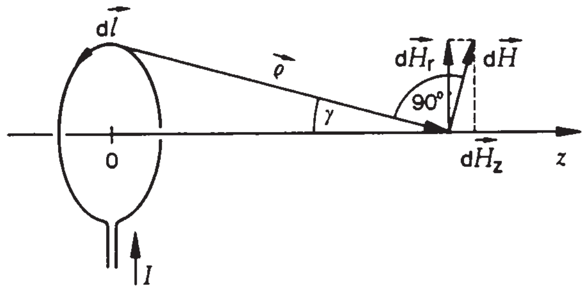
Problems
1. To measure the magnetic flux density in the middle of various wire loops with the Hall probe and to investigate its dependence on the radius and number of turns.
2. To determine the magnetic field constant μ_0.
3. To measure the magnetic flux density along the axis of long coils and compare it with theoretical values.

Set-up and procedure
Set up the experiment as shown in Fig. 1. Operate the power supply as a constant current source, setting the voltage to 18 V and the current to the desired value.

Measure the magnetic field strength of the coils (I = 1 A) along the z-axis with the Hall probe and plot the results on a graph. Make the measurements only at the centre of the circular conductors (I = 5 A). To eliminate interference fields and asymmetry in the experimental set-up, switch on the power and measure the relative change in the field. Reverse the current and measure the change again. The result is given by the average of the measured values.

Fig.1: Experimental set-up for measuring a magnetic field.

Fig. 2: Drawing for the calculation of the magnetic field along the axis of a wire loop.



Theroy and evaluation

From Maxwell's equation

$$\oint_K \vec{H} d\vec{s} = I + \int_F \dot{\vec{D}} d\vec{f} \tag{1}$$

wehre K is a closed curve around area F, H is the magnetic field strength, I is the current flowing through area F, and D is the electric flux density, we obtain for direct currents ($\dot{D} = 0$), the magnetic flux law:

$$\oint_K \vec{H} d\vec{s} = I \tag{2}$$

which, with the notations from Figs. 2, is written in the form of Biot-Savart's law:

$$d\vec{H} = \frac{I}{4\pi} \frac{d\vec{l} \times \vec{\rho}}{\rho^3} \tag{3}$$

The vector $d\vec{l}$ is perpendicular to, and $\vec{\rho}$ and $d\vec{H}$ lie in the plane of the drawing, so that

$$dH = \frac{I}{4\pi \rho^2} dl = \frac{I}{4\pi} \cdot \frac{dl}{R^2 + z^2} \tag{4}$$

$d\vec{H}$ can be resolved into a radial dH_r and an axial dH_z component.

The dH_z components have the same direction for all conductor elements $d\vec{l}$ and the quantities are added; the dH_r components cancel one another out, in pairs.

Therefore,

$$H_r(z) = 0 \tag{5}$$

and

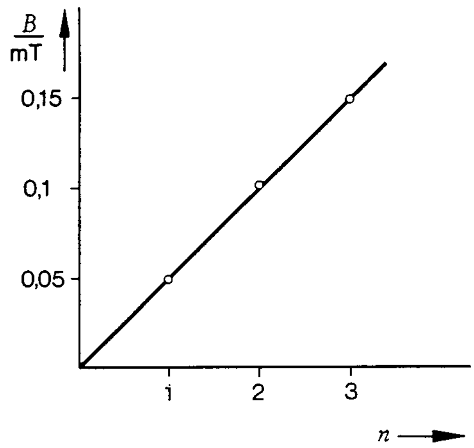
$$H(z) = H_z(z) = \frac{I}{2} \cdot \frac{R^2}{(R^2 + z^2)^{3/2}} \tag{6}$$

along the axis of the wire loop, while the magnetic flux density

$$B(z) = \frac{\mu_0 \cdot I}{2} \frac{R_2}{(R^2 + z^2)^{3/2}} \tag{7}$$

where $\mu_0 = 1.2566 \times 10^{-6}$ H/m is the magnetic field constant. If there is a small number of identical loops close together, the magnetic flux density is obtained by multiplying by the number of turns n.

Fig. 3: Magnetic flux density at the centre of a coil with n turns, as a function of the number of turns (radius 6 cm, current 5 A).



1. At the centre of the loop ($z = 0$) we obtain

$$B(0) = \frac{\mu_0 \cdot n \cdot I}{2R}. \tag{8}$$

Using the expression

$$B = A_1 \cdot n^{E_1}$$

and

$$B = A_2 \cdot R^{E_2}$$

the regression lines for the measured values in Figs. 3 and 4 give, for the number of turns, the following exponents E and standard errors:

$$E_1 = 0.96 \pm 0.04$$

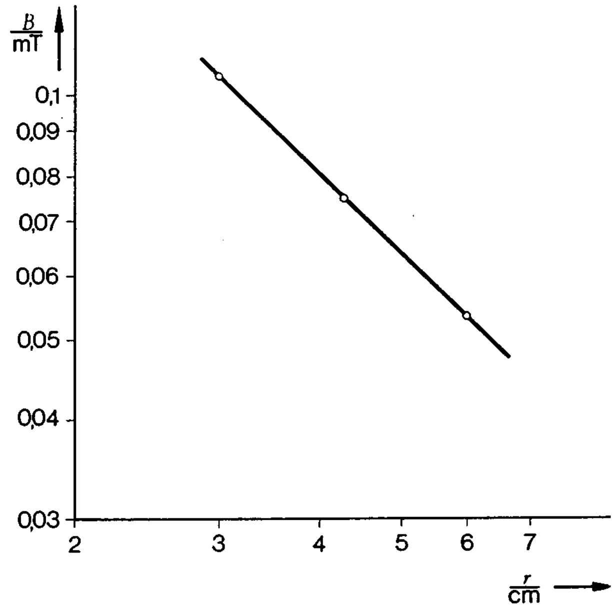
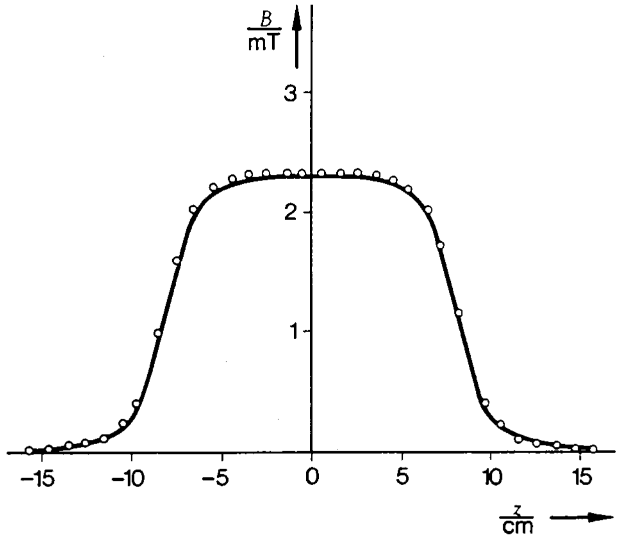


Fig. 4: Magnetic flux density at the centre of a single turn, as a function of the radius (current 5 A).

Fig. 5: Magnetic flux density along the axis of a coil of length l = 162 mm, radius R = 16 mm and n = 300 turns; measured values (0) and theoretical curve (continuous line) in accordance with equation (9).



and, for the radius (see equation (8))

$$E_2 = -0.97 \pm 0.02.$$

2. Using the measured values from Figs. 3 and 4, and equation (8), we obtain the following average value for the magnetic field constant:

$$\mu_0 = (1.28 \pm 0.01) \times 10^{-6} \text{ H/m}$$

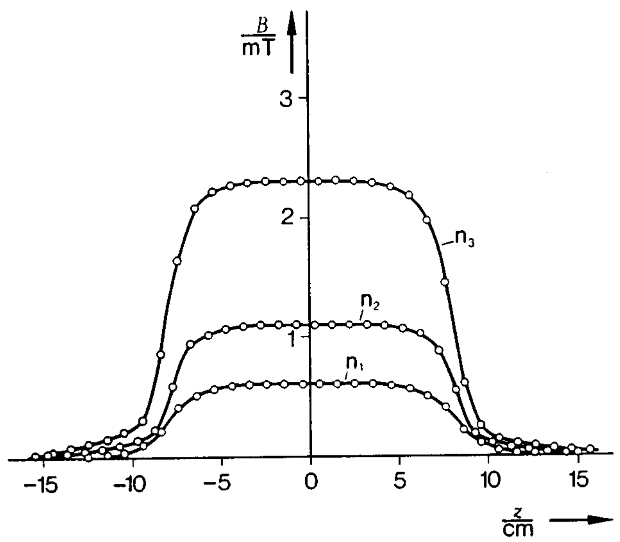


Fig. 6: Curve of magnetic flux density (measured values) along the axis of coil of length l = 160 mm, radius R = 13 mm and number of turns n_1= 75, n_2= 150 and n_3= 300 .

3. To calculate the magnetic flux density of a uniformly wound coil of length l and n turns, we multiply the magnetic flux density of one loop by the density of turns n/l and integrate over the coil length.

$$B(z) = \frac{\mu_0 \cdot I \cdot n}{2l} \cdot \left(\frac{\alpha}{\sqrt{R^2 + \alpha^2}} - \frac{b}{\sqrt{R^2 + b^2}} \right),$$

where

$$\alpha = z + l/2 \quad \text{and}$$
$$b = z - l/2.$$

The proportional relationship between magnetic flux density B and number of turns n at constant length and radius is shown in Fig. 6. The effect of the length of the coil at constant radius with the density of turns n/l also constant, is shown in Fig. 7.

Comparing the measured with the calculated values of the flux density at the centre of the coil,

$$B(0) = \frac{\mu_0 \cdot I \cdot n}{2} \cdot \left(R^2 + \frac{l}{2} \right)^{-\frac{1}{2}}$$

gives:

n	l mm	R mm	B (0) mT meas-ured	B (0) mT calcu-lated
75	160	13	0.59	0.58
150	160	13	1.10	1.16
300	160	13	2.30	2.32
100	53	20	1.81	1.89
200	105	20	2.23	2.24
300	160	20	2.23	2.29
300	160	16	2.31	2.31

Fig. 7: Curve of magnetic flux density (measured values) for coils with a constant density of turns n/l, coils radius R = 20 mm, lengths l_1 = 53 mm, l_2 = 105 mm and l_3 = 160 mm.