# Capacitance of metal spheres and of a spherical capacitor



## Related topics

Voltage, potential, charge, electric field, electrostatic induction, electrostatic induction constant, capacitance, capacitors, dielectrics.

## Principle and task

Metal spheres with different radii and a spherical capacitor are charged by means of a variable voltage. The induced charges are determined with a measuring amplifier. The corresponding capacitances are deduced from voltage and charge values.

## Equipment

| | | |
|---|---|---|
| Conductor ball, d 20 mm | 06236.00 | 2 |
| Conductor ball, d 40 mm | 06237.00 | 1 |
| Conductor ball, d 120 mm | 06238.00 | 1 |
| Hemispheres, Cavendish type | 06273.00 | 1 |
| Hollow plastics ball, w. eyelet | 06245.00 | 1 |
| Capillary tube, straight, l 250 mm | 36709.00 | 1 |
| Copper wire, d 0.5 mm, 50 m | 06106.03 | 1 |
| Insulating stem | 06021.00 | 2 |
| High-value resistor, 10 MOhm | 07160.00 | 1 |
| High voltage supply unit, 0-10 kV | 13670.93 | 1 |
| PEK capacitor/ case 1/ 10 nF/ 500 V | 39105.14 | 1 |
| Universal measuring amplifier | 13626.93 | 1 |
| Multi-range meter A | 07028.01 | 1 |
| Digital multimeter | 07134.00 | 1 |
| Connecting cord, 50 KV, 1000 mm | 07367.00 | 1 |
| Screened cable, BNC, l 750 mm | 07542.11 | 1 |
| Adapter, BNC socket - 4 mm plug | 07542.20 | 1 |
| Connector, T type, BNC | 07542.21 | 1 |
| Adapter, BNC-plug/socket 4 mm | 07542.26 | 1 |
| Vernier caliper, plastic | 03011.00 | 1 |
| Barrel base -PASS- | 02006.55 | 2 |
| Support base -PASS- | 02005.55 | 1 |
| Right angle clamp -PASS- | 02040.55 | 4 |
| Support rod -PASS-, square, l 630 mm | 02027.55 | 1 |
| Support rod -PASS-, square, l 400 mm | 02026.55 | 1 |
| Universal clamp with joint | 37716.00 | 1 |
| Croco.clip, insul., strong, 10 pcs | 29426.03 | 1 |
| Connecting cord, 100 mm, green-yell | 07359.15 | 1 |
| Connecting cord, 750 mm, green-yell | 07362.15 | 2 |
| Connecting cord, 500 mm, blue | 07361.04 | 2 |
| Connecting cord, 500 mm, red | 07361.01 | 2 |

## Problems

1. Determination of the capacitance of three metal spheres with different diameters.
2. Determination of the capacitance of a spherical capacitor.
3. Determination of the diameters of each test body and calculation of their capacitance values.

## Set-up and Procedure

**Part 1:**

The experimental set-up to determine the capacitance of spherical conductors is shown in Fig.1. Fig.2 only shows the part of the experimental set-up which must be modified in order to determine the capacitance of a spherical capacitor.
The spherical conductor ($d$ = 2 cm) held on a barrel base and insulated against the latter is connected by means of the high voltage cord over the 10 MΩ- protective resistor to the positive pole of the 10kV output of the high voltage power supply. The negative pole is earthed. This sphere is briefly brought into contact with the test spheres to charge it. High voltage always must be reset to zero after charging. After every measurement, the charging voltage is increased by 1kV. Before

Fig. 1 : Experimental set-up to determine the capacitance of conducting spheres.

being charged anew, the test spheres must be discharged through contact with the free earth connecting cable.
The charges of the test spheres are determined with a measuring amplifier. The high-resistance input of the electrometer is used for this. An auxiliary 10 nF capacitor is connected in parallel to the BNC test cord fitted with the adapter required to take over the charge.
The capacitances of the spherical conductors are determined from the voltage and charge values; this is done using the average calculated over a number of charge measurement values.

**Never apply high voltage to the amplifier input.**

**Part 2:**

To determine the capacitance of a spherical capacitor, the experimental set-up is altered as shown in Fig. 2. The Cavendish hemispheres are put together so as to form a complete sphere with a small circular orifice at the top.
The plastic sphere with conducting surface is suspended from a copper wire in the centre of the sphere. The copper wire is lead through a glass capillary tube which is wrapped in earthed aluminium foil to neutralise stray capacitances (Fig. 3). The aluminium foil may not touch the hemispheres.
The interior sphere must be connected to the central socket of the high voltage power supply. This is done by means of a crocodile clip over the high voltage cord, before which a 10 MΩ protective resistor is connected. The lower socket is earthed again.
Voltage is increased in steps of 100 V and may **not** increase above 1000 V for the safety of the digital multimeter. The corresponding mean values of the charges are read determined for the hemispheres similarly as described in Part. After every measurement, the hemispheres must be discharged with the free earthing cord. Whilst doing this, it must be assured that no high voltage is induced.

**Theory and evaluation**

**Part 1:**

The capacitance C of a sphere with radius R is given by:

$$C = 4\pi\varepsilon_0 R \quad \text{(electrostatic induction constant } \varepsilon_0 = 8{,}86 \cdot 10^{-12}\ \text{As/Vm)} \qquad (1)$$

Using (1), the capacitance of the conducting spheres may be calculated:

Sphere ($2R_1$ = 0,121 m) : C = 6,7 · $10^{-12}$ As/V = 6,70 pF
Sphere ($2R_2$ = 0,041 m) : C = 2,28 · $10^{-12}$ As/V = 2,28 pF
Sphere ($2R_3$ = 0,021 m) : C = 1,22 · $10^{-12}$ As/V = 1,22 pF

Fig. 2 : Part of the experimental set-up used to determine the capacitance of a spherical capacitor.

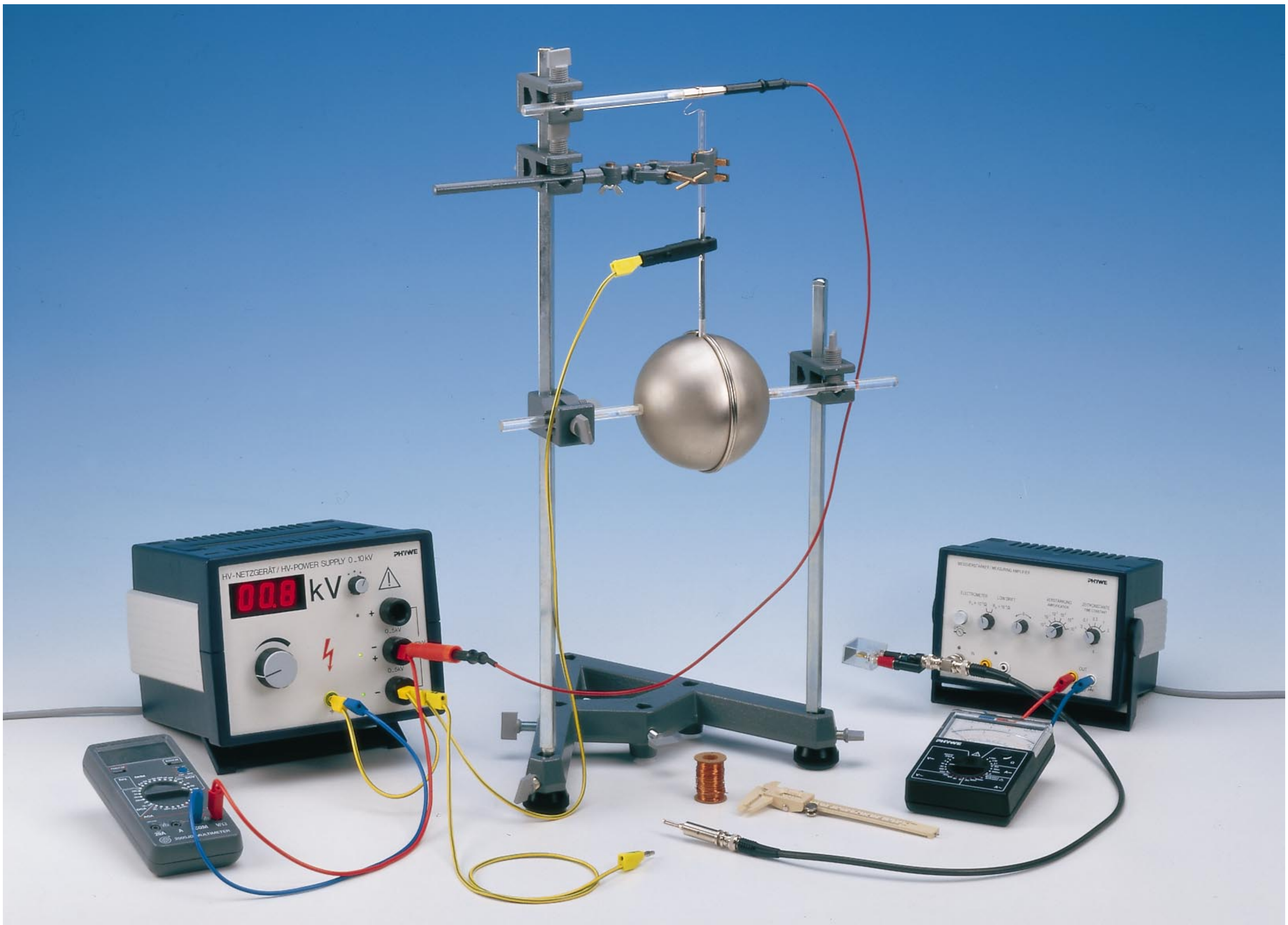
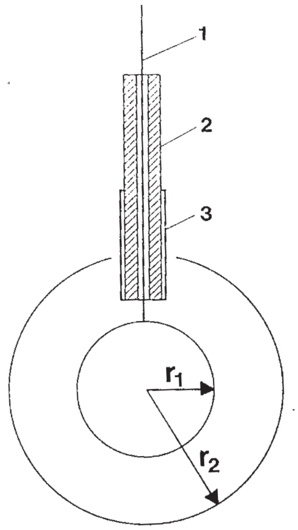


Fig. 3: Sketch showing set-up and the attachment of conducting spheres.
(1= Copper wire; 2 = capillary tube; 3 = aluminium foil)




Using (2), the voltage values $U_1$, which were determined by means of the measuring amplifier, allow to determine the corresponding charge value Q:

$$Q = (C_{co} + C_{ca})\, U_1 = (C_{co} + C_{ca}) \frac{U}{V};$$

$$\text{with } C_{co} << C_{ca};\; Q = C_{ca} U_1 = \frac{U}{V} \qquad (2)$$

($C_{co}$ = capacitance of the conductors;
$C_{ca}$ = capacitance of the parallel capacitor,
U = displayed voltage, V= amplification factor,
$U_1$ = measured voltage)

On the other hand, the charge Q of the conductor is:

$$Q = C_{co}\, U_2 \qquad (3)$$

Finally, charges may be calculated using (2) and (3).

$$U_1 / U_2 = C_{co} / C_{ca}$$

Fig. 4 shows charging voltage values $U_2$ as a function of measurement voltage $U_1$ for the different conducting spheres. With the slope of the lines and knowing $C_{ca}$, the capacitances of the conducting spheres are obtained:

$C(R_1)$ = 7,55 pF; $C(R_2)$ = 2,33 pF; $C(R_3)$ = 1,32 pF

(With the assistance of a measuring bridge, the capacitance $C_{ca}$ of the parallel capacitor was determined separately to be $C_{ca}$ = 9,1 nF. If exact capacitance determination is impossible, the nominal capacitance value of the capacitor must be used for calculation; this may, however, be affected by design tolerance deviations of ± 10%).

**Part 2:**
The capacitance C of a spherical capacitor is given by

$$C = 4\pi\varepsilon_0 \left(\frac{r_1 r_2}{r_2 - r_1}\right); \qquad (4)$$

($r_1$ = Radius of the interior sphere;
$r_2$ = Radius of the exterior sphere)

With $r_1$ = 0,019 m and $r_2$ = 0,062 m for the spherical capacitors, capacitance calculation yields C = 3,0 pF.
Fig. 5 once more represents measurement value pairs $U_1$ and $U_2$.
According to the assessment procedure described above and using the slope of the graph line, the empirically determined capacitance of the spherical capacitor is found to be C = 3,5 pF.
Capacitance values determined experimentally are always higher than the calculated values. This discrepancy is due to unavoidable dispersive capacitances.

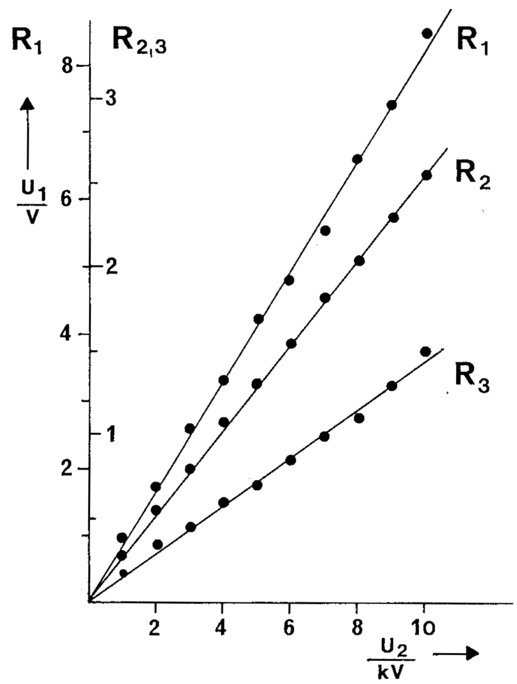



Fig. 4: $U_1$ as a function of $U_2$ measured on conducting spheres with three different diameters.

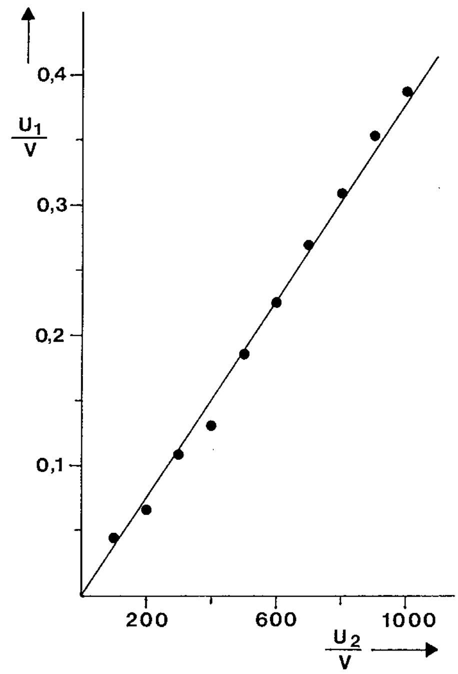



Fig. 5: $U_1$ as a function of $U_2$ measured on a spherical capacitor.