Related topics

First and second law of thermodynamics, reversible cycles, isochoric and isothermal changes, gas jaws, efficiency, Stirling engine, conversion of heat, thermal pump.

Principle and task

The Stirling engine is submitted to a load by means of an adjustable torque meter, or by a coupled generator. Rotation frequency and temperature changes of the Stirling engine are observed. Effective mechanical energy and power, as well as effective electrical power, are assessed as a function of rotation frequency. The amount of energy converted to work per cycle can be determined with the assistance of the pV diagram. The efficiency of the Stirling engine can be estimated.

Equipment

Stirling engine, transparent	04372.00	1
Motor/generator unit	04372.01	1
Torque meter	04372.02	1
Chimney for stirling engine	04372.04	1
Meter f. stirling engine, pVnT	04371.97	1
Sensor unit pVn for stirl.eng.	04371.00	1
Syringe 20ml, Luer, 10 pcs	02591.03	1
Rheostat, 330 Ohm , 1.0 A	06116.02	1
Digital multimeter	07134.00	2
Connecting cord, 500 mm, red	07361.01	2
Connecting cord, 500 mm, blue	07361.04	3
Screened cable, BNC, l 750 mm	07542.11	2
Oscilloscope, 20 MHz, 2 channels	11454.93	1
Thermocouple NiCr-Ni, sheathed	13615.01	2
Graduated cylinder, 50 ml, plastic	36628.01	1
Raw alcohol for burning, 1000 ml	31150.70	1

Optional accessories for solar motor work

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Accessories f. solar motor work	04372.03	1
Support base -PASS-	02005.55	1
Extension coupling, hinged	02045.00	1
Support rod, stainl. steel, 500 mm	02032.00	1

Problems

1. Determination of the burner's thermal efficiency

2. Calibration of the sensor unit

3. Calculation of the total energy produced by the engine through determination of the cycle area on the oscilloscope screen, using transparent paper and coordinate paper.

4. Assessment of the mechanical work per revolution, and calculation of the mechanical power output as a function of the rotation frequency, with the assistance of the torque meter.

5. Assessment of the electric power output as a function of the rotation frequency.

6. Efficiency assessment.

Set-up and procedure

Experimental set up should be carried out as shown in Fig. 1. The base plate (mounting plate) of the Stirling engine must be removed, so that the latter can be fixed on the corresponding mounting plate of the pVn sensor unit. The incremental transmitter of the pVn sensor unit is firmly connected to the axle of the Stirling engine. The latter is then fixed upon the large base plate.

Fig. 1: Experimental set-up: Stirling engine.
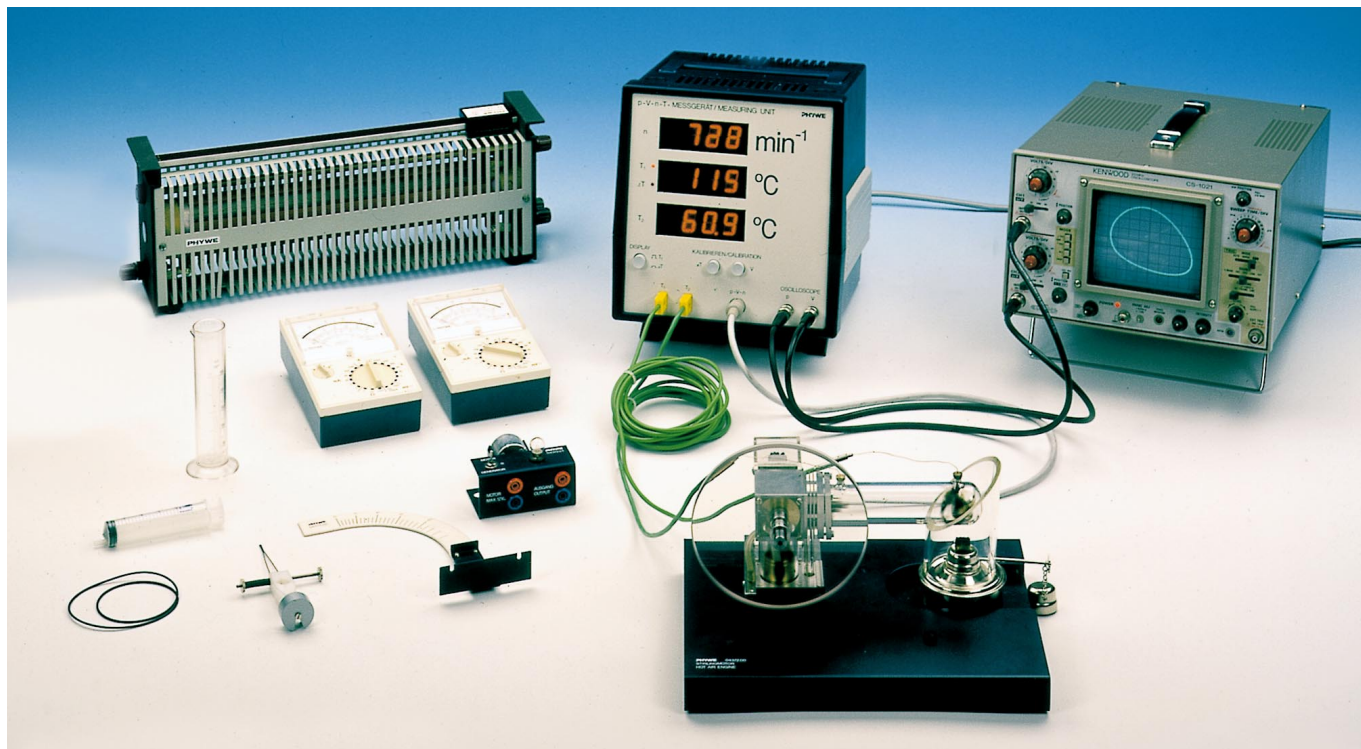
Before switching on the pVnT meter, make sure it is connected to the pVn sensor. Connect the p and V exits respectively to the Y and X oscilloscope channels.

After having been switched on, the pVnT meter display shows "cal". Both thermocouples must now be set to the same temperature, and the "Calibration ΔT"-button depressed. This calibration of the temperature sensors merely influences the temperature difference display, not the absolute temperature display.

The upper display now shows "OT", which means "upper dead centre point". At this point, the engine is at its minimum volume. Now bring the working piston down to its lowest position by turning the engine axle, and press the "calibration V" button. Wrong calibration will cause a phase shift in the volume output voltage, and thus lead to a distortion of the pV diagram. The three displays should now be on, showing 0 revs/min, and the actual temperatures for T_1 and T_2.

1. Thermal output of the burner.

The amount of alcohol in the burner is measured before and after the experiment with a measuring glass (or a scale). The corresponding duration of the experiment is recorded with a watch or clock.

2. Calibration of the pressure sensor

The pressure sensor must be calibrated so that the pV diagram can be evaluated quantitatively. This is carried out by means of a gas syringe.
The flexible tube is removed from the mounting plate, and the voltage corresponding to atmospheric pressure p_0 is determined with the oscilloscope. The latter should be operated in DC and Yt mode, with calibrated Y scale. The piston of the airtight gas syringe is drawn out (e.g. up to 15 or 20 ml), and the syringe is connected to the flexible tube. The pressure (voltage) display on the oscilloscope screen is varied through isothermal in- crease and decrease of the syringe volume. The actual pressure inside the syringe can be calculated.

3. Presentation and drawing of the pV diagram

The oscilloscope is now operated in the XY mode, with calibrated scales.
Place the lighted burner below the glass cylinder, and observe the temperature display. When the temperature difference has reached approximately 80 K, give the flywheel a slight clockwise push to start the engine. After a short time, it should reach approximately 900 revs/min, and a Stirling cycle ought to show on the oscilloscope screen.

Before carrying out measurements of any kind, wait until temperatures T_1 and T_2, as well as the rotation frequency, are approximately constant. The lower temperature should now be about 70 °C.
Rotation frequency and temperatures are recorded. Voltages corresponding to maximum and minimum pressures are read from the oscilloscope. The pV diagram is copied from the oscilloscope to a sheet of transparent paper. Make sure to look perpendicularly onto the screen when doing this. The Y axis ground line is drawn, too. Transfer the diagram to co-ordinate paper, in order to be able to determine the diagram surface.

4. Effective mechanical energy

In order to load the engine with a determined torque, the scale of the torque meter is fixed on the large base plate, and the inner metallic piece of the pointer is fixed on the axis before the flywheel. Friction between the pointer and the set-on metallic piece can be varied by means of the adjusting screw on the pointer. Adjustment must be done carefully, to make sure that the pointer will not begin to oscillate.
Start carrying out measurements with a low torque. After each adjustment, wait until torque, rotation frequency and temperatures remain constant. All values and the pV diagram are recorded.

5. Effective electric power

Replace the torque meter through the engine/generator unit. The small light bulb may not be inserted. The slide resistor is connected to the generator output, as shown in Fig. 2, and adjusted to the highest resistance value. Before starting to perform measurements, the Stirling engine without load should have approximately the same rotation frequency and temperatures as at the beginning of the previous series of measurements paragraph 3). The string is then wound around the Stirling engine flywheel and the large generator strap wheel. Voltage, current intensity, rotation frequency and temperatures are recorded, once rotation frequency and temperatures have steadied. Resistance is decreased stepwise, and further measurement values are recorded. Repeat the series of measurements using the small generator strap wheel.

Theory and evaluation

In 1816, Robert Stirling was granted a patent for a hot air engine, which is known today as the Stirling engine. In our times, the Stirling engine is used to study the principle of thermal engines because in this case the conversion process of thermal energy to mechanical energy is particularly clear and relatively easy to understand.

At present, the Stirling engine is undergoing a new phase of further development due to its many advantages.
Thus, for example, it constitutes a closed system, it runs very smoothly, and it can be operated with many different heat sources, which allows to take environmental aspects into consideration, too.
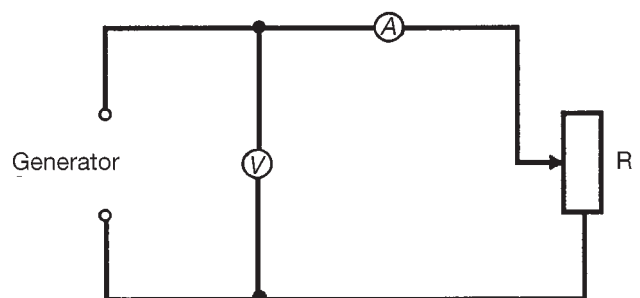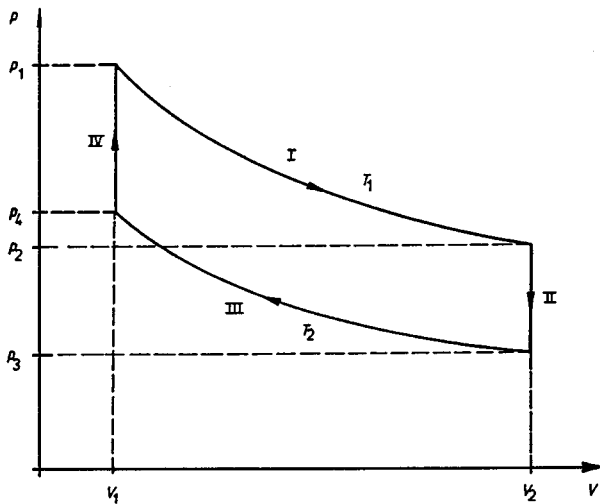

Fig. 2: Wiring diagram for the connection of the rheostat (slide resistor).

Fig. 3a: pV diagram for the ideal Stirling process.

Theoretically, there are four phases during each engine cycle (see. Fig. 3a and 3b):

1) An isothermal modification when heat is supplied and work produced

$$V_1 \rightarrow V_2 \qquad p_1 \rightarrow p_2 \qquad \text{and } T_1 = \text{const.}$$

2) An isochoric modification when the gas is cooled:

$$T_1 \rightarrow T_2 \qquad p_2 \rightarrow p_3 \qquad \text{and } V_2 = \text{const.}$$

3) An isothermal modification when heat is produced and work supplied:

$$V_2 \rightarrow V_1 \qquad p_3 \rightarrow p_4 \qquad \text{and } T_2 = \text{const.}$$

4) An isochoric modification when heat is supplied to the system:

$$T_2 \rightarrow T_1 \qquad p_4 \rightarrow p_1 \qquad \text{and } V_1 = \text{const.}$$

According to the first law of thermodynamics, when thermal energy is supplied to an isolated system, its amount is equal to the sum of the internal energy in- crease of the system and the mechanical work supplied by the latter:

$$dQ = dU + p dV$$

It is important for the Stirling cycle that the thermal energy produced during the isochoric cooling phase be stored until it can be used again during the isochoric heating phase (regeneration principle).
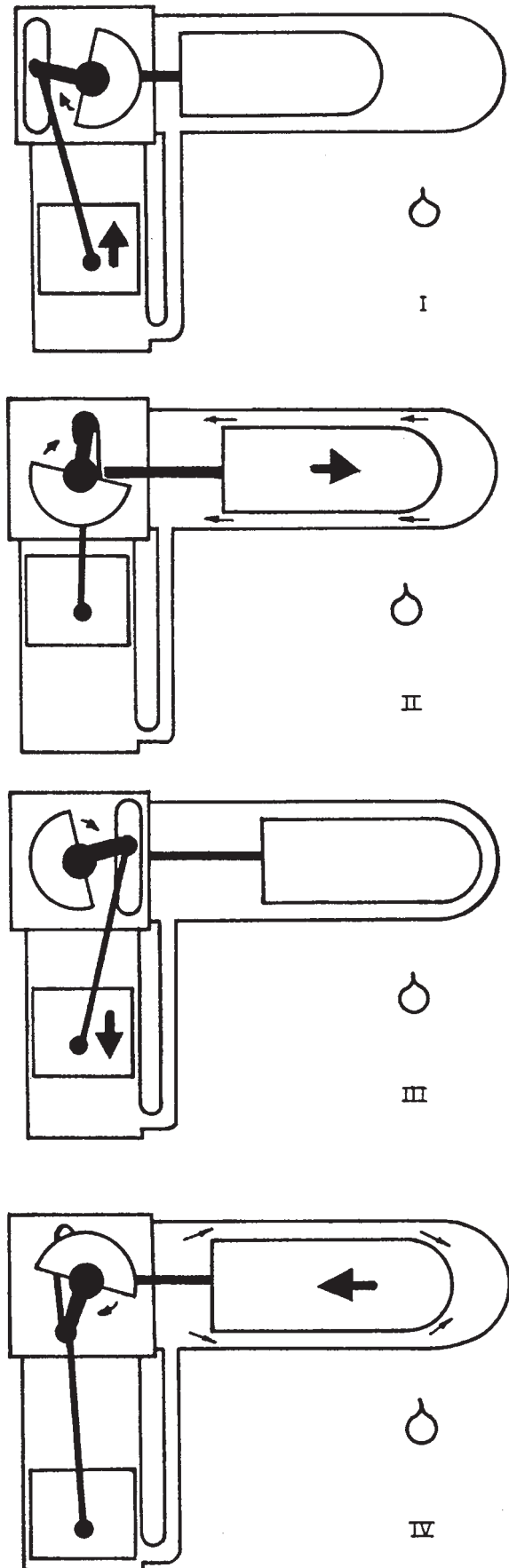

Fig. 3b: Functioning of the transparent Stirling engine.

Thus, during phase IV the amount of thermal energy released during phase II is regeneratively absorbed. This means that only an exchange of thermal energy takes place within the engine. Mechanical work is merely supplied during phases I and III. Due to the fact that internal energy is not modified during isothermal processes, work performed during these phases is respectively equal to the absorbed or released thermal energy.

Since $p \cdot V = \nu \cdot R \cdot T$,

where ν is the number of moles contained in the system, and R the general gas constant, the amount of work produced during phase I is:

$$W_1 = - \nu \cdot R \cdot T_1 \cdot \ln (V_2/V_1)$$

(it is negative, because this amount of work is supplied). Consequently, the amount of work supplied during phase III is

$$W_3 = + \nu \cdot R \cdot T_2 \cdot \ln (V_2/V_1)$$

$|W_1| > W_3$ because $T_1 > T_2$

The total amount of work is thus given by the sum of W_1 and W_3. This is equal to the area of the pV diagram:

$$W_t = W_1 + W_3$$
$$W_1 = - \nu \cdot R \cdot T_1 \cdot \ln (V_2/V_1) + \nu \cdot R \cdot T_2 \cdot \ln (V_2/V_1)$$
$$W_1 = - \nu \cdot R \cdot (T_1 - T_2) \cdot \ln (V_2/V_1)$$

Only part of this total effective energy W_t can be used as effective work W_m through exterior loads applied to the engine. The rest contains losses within the Stirling engine.

The maximum thermal efficiency of a reversible process within a thermal engine is equal to the ratio between the total amount of work $|W_t|$ and the amount of supplied thermal energy $Q_1 = - W_1$

$$\eta_{th} = W_t/W_1$$

$$\eta_{th} = \frac{\nu \cdot R \cdot (T_1 - T_2) \cdot \ln (V_2/V_1)}{\nu \cdot R \cdot T_1 \cdot \ln (V_2/V_1)}$$

$$\eta_{th} = \frac{T_1 - T_2}{T_1}$$

Carnot found this to be the maximum thermal efficiency for any thermal engine, which can only be reached theoretically. One sees that efficiency increases with increasing temperature differences.

1. Thermal power of the burner

Duration	Δt = 60 min
Amount of alc6hol burned	ΔV = 29 ml
Alcohol density	ρ = 0.83 g/ml
Specific thermal power	h = 25 kJ/g

This allows to determine the mass of alcohol burnt per second:

$$\frac{\Delta m}{\Delta t} = 6.69 \cdot 10^{-3} \text{ g/s}$$

as well as the thermal power of the burner: P_H = 167 W.

2. Calibration of the pressure sensor

The pressure sensor measures the relative pressure as compared to the atmospheric pressure p_0. The volume modification of the gas syringe allows to calculate the modification of pressure, assuming that the change of state is isothermal, with $p \cdot V$ = const.

At the initial volume V_0, pressure is equal to the atmospheric pressure p_0 Table 1 shows an example of measurement for which p_0 was assumed to be normal atmospheric pressure (1013 llPa). The volume of the small flexible connecting tube (0.2 ml) can be considered to be negligible.

Table 1

Compression				Expansion			
$\frac{V}{ml}$	$\frac{p}{hPa}$	$\frac{p - p_0}{hPa}$	$\frac{U}{V}$	$\frac{V}{ml}$	$\frac{p}{hPa}$	$\frac{p - p_0}{hPa}$	$\frac{U}{V}$
20	1013	0	2.35	15	1013	0	2.35
19	1066	53	2.51	16	950	− 63	2.15
18	1126	113	2.71	17	894	−119	1.99
17	1192	179	2.89	18	844	−169	1.85
16	1266	253	3.10	19	800	−213	1.71
15	1351	338	3.40	20	760	−253	1.59

Fig. 4 shows the output voltage of the pressure sensor as a function of pressure. The slope of the regression line is:

$$\frac{\Delta U}{\Delta p} = 3.04 \cdot 10^{-3} \frac{V}{hPa}$$

The voltage corresponding to atmospheric pressure p_0 is 2.35 V

Caution! Sensitivity of the pressure sensor may undergo large fluctuations. However, linearity between $\underline{U}$ and p is assured for all cases.
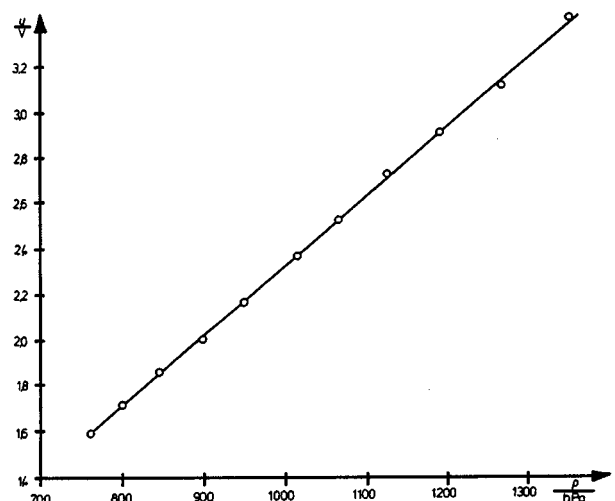

Fig. 4: Characteristic curve of the pressure sensor.

3. *pV* diagram surface

The oscilloscope's X measuring range is of 0.5 V/div. The *pVnT* measuring device displays the following voltages for the Stirling engine volumes (V_{min}, V_{max} are equipment constants):

$$V_{min} = 32 \text{ cm}^3 \rightarrow U_{min} = 0 \text{ V}$$

$$V_{max} = 44 \text{ cm}^3 \rightarrow U_{max} = 5 \text{ V}$$

$$\Delta V = 12 \text{ cm}^3 \rightarrow \Delta U = 5 \text{ V}$$

Thus, the scale factor for the X axis is 2.4 cm³/V or respectively 1.2 cm³/div.

With the used pressure sensor, the oscilloscope's Y measuring range was 0.2 V/div (with other pressure sensors it may be 0.5 V/div). Based upon the pressure calibration of Fig. 4, one finds a scale factor of 329 hPa/V or respectively 66 hPa/div for the Y axis.

Reading the voltages for maximum and minimum pressures with the oscilloscope being operated in the DC mode, the pressure values for the *pV* diagram can also be expressed in Pascal. In general, the ground line will be situated near p_0.

Fig. 5 shows two real *pV* diagrams for a Stirling engine with and without load (Fig. 5a: no load, Fig. 5b: with a load of $18.3 \cdot 10^{-3}$ Nm). Assessed surface values are given in table 2.

Fig. 5: Real pV diagrams (a) without, and (b) with exterior load.
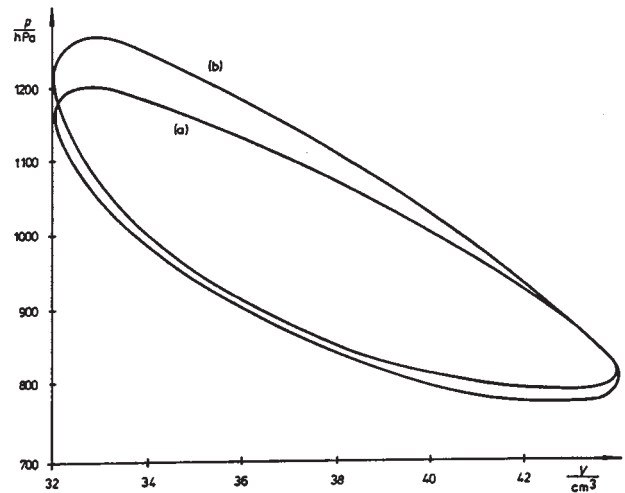
For other Stirling engines, the pV diagram may have a somewhat different shape. Thus, for example, the surface is a function of supplied thermal power and engine friction at equilibrium rotation frequency.
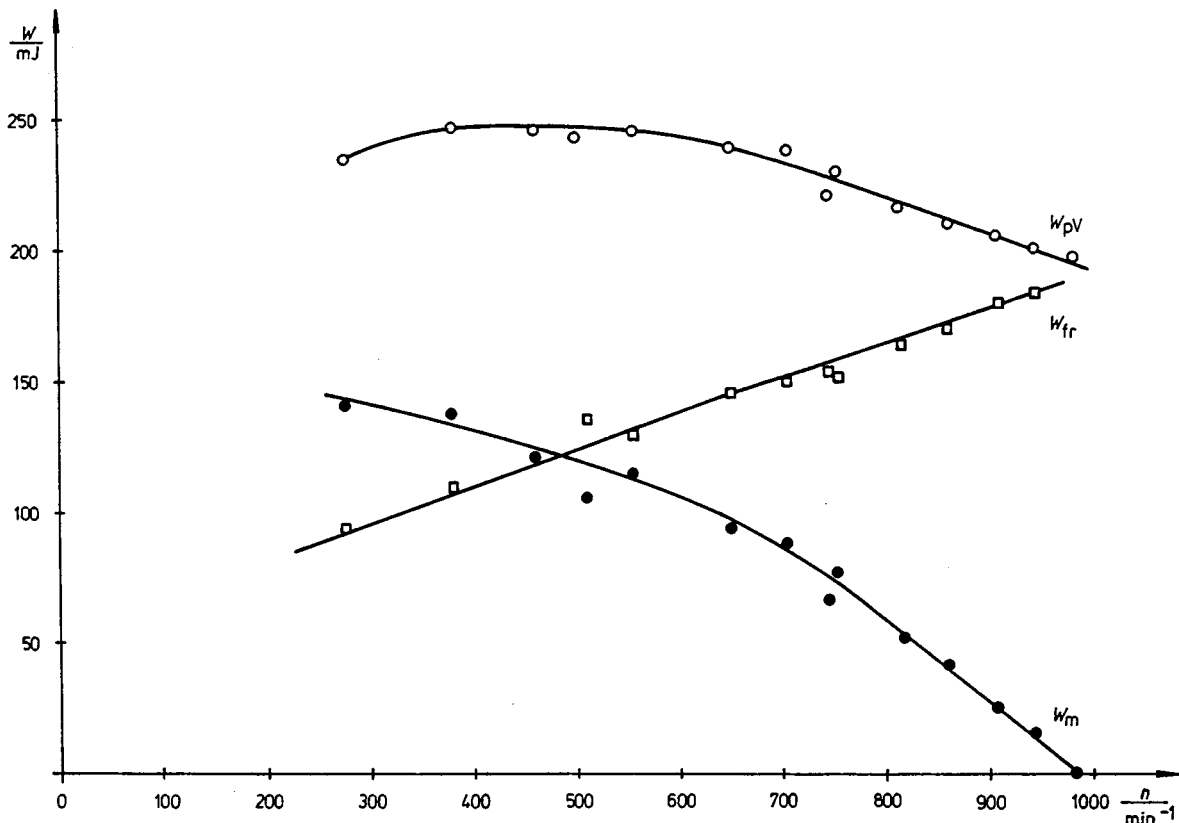

Fig. 6: Mechanical energy as a function of rotation frequency.

Comparison of the pV diagrams for an engine submitted or not to an exterior load shows that a higher pressure difference occurs for the load case, corresponding to the larger temperature difference measured at the Stirling engine. If the engine is submitted to a load, the surface of the *pV* diagram increases merely by 10 % – 20 %; it displays a maximum for medium loads (see Fig. 6).

4. Effective mechanical energy and power

Effective mechanical energy during a cycle is calculated with the assistance of the torque M displayed by the torque meter:

$$W_m = 2 \cdot \pi \cdot M$$

The displayed rotation speed n (revolutions per minute) is converted to the frequency f (revolutions per second). This allows to determine the mechanical power:

$$P_m = W_m \cdot f$$

Table 2 contains measured and calculated values. Fig. 6 displays the total effective energy W_{pV} assessed on the base of the *pV* diagram, effective mechanical energy W_m as well as friction energy per cycle W_{fr}, as a function of rotation frequency.

$$W_{fr} = W_{pV} - W_m$$

Table 2

M	n	T_1	T_2	W_m	f	P_m	W_{pV}	W_{fr}
10^{-3} Nm	min⁻¹	°C	°C	mJ	Hz	mW	mJ	mJ
0	982	163	74.8	0	16.4	0	198	198
2.5	945	169	77.7	16	15.8	248	201	185
4.0	908	168	78.7	25	15.1	379	205	180
6.5	860	177	77.5	41	14.3	583	210	169
8.2	817	177	77.1	52	13.6	675	216	164
10.5	745	178	76.5	66	12.4	818	221	155
12.2	752	179	76.3	77	12.5	959	230	153
14.0	705	185	76.7	88	11.8	1038	238	150
15.0	650	188	76.9	94	10.8	1017	239	145
16.8	519	190	76.3	106	8.7	919	243	137
18.3	555	192	75.5	115	9.3	1064	245	130
19.5	460	195	74.2	122	7.7	939	246	124
22.0	380	197	72.0	138	6.3	871	247	109
22.4	275	201	70.7	141	4.6	647	235	94

Rotation frequency reaches its maximum value when the engine is not submitted to exterior loads (here: 982 min⁻¹). It is a function of thermal input and friction; in general its values lie within the range 800…1000 min⁻¹. Rotation frequency decreases with increasing exterior loads, until the Stirling engine stops (in general between 150…300 min⁻¹). Temperature T_1 increases strongly with decreasing rotation frequencies; T_2 decreases a little due to the fact that the air in the regenerator (that ison the wall of the displacing piston) is pre heated or respectively cooled to a better extent when rotation frequency is low. Pressure within the Stirling engine also varies with temperatures. This is clearly visible on the *pV* diagram (see Fig. 5).

When adjusting a new torque, load fluctuations and shocks on the axle are unavoidable. Due to this, measurement values may display a large range of scattering. Friction energy per cycle increases with rotation frequencies.

Effective mechanical power displays a marked peak for rotation frequencies within a range of 500…600 min⁻¹ (see Fig. 7).

5. Effective electric power

Current intensity I and voltage U are measured at the load resistor. They allow to assess electric power:

$$P_e = U \cdot I$$

Table 3 contains measured and calculated values. Fig. 7 shows the effective mechanical and electric power of the Stirling engine as a function of the rotation frequency.
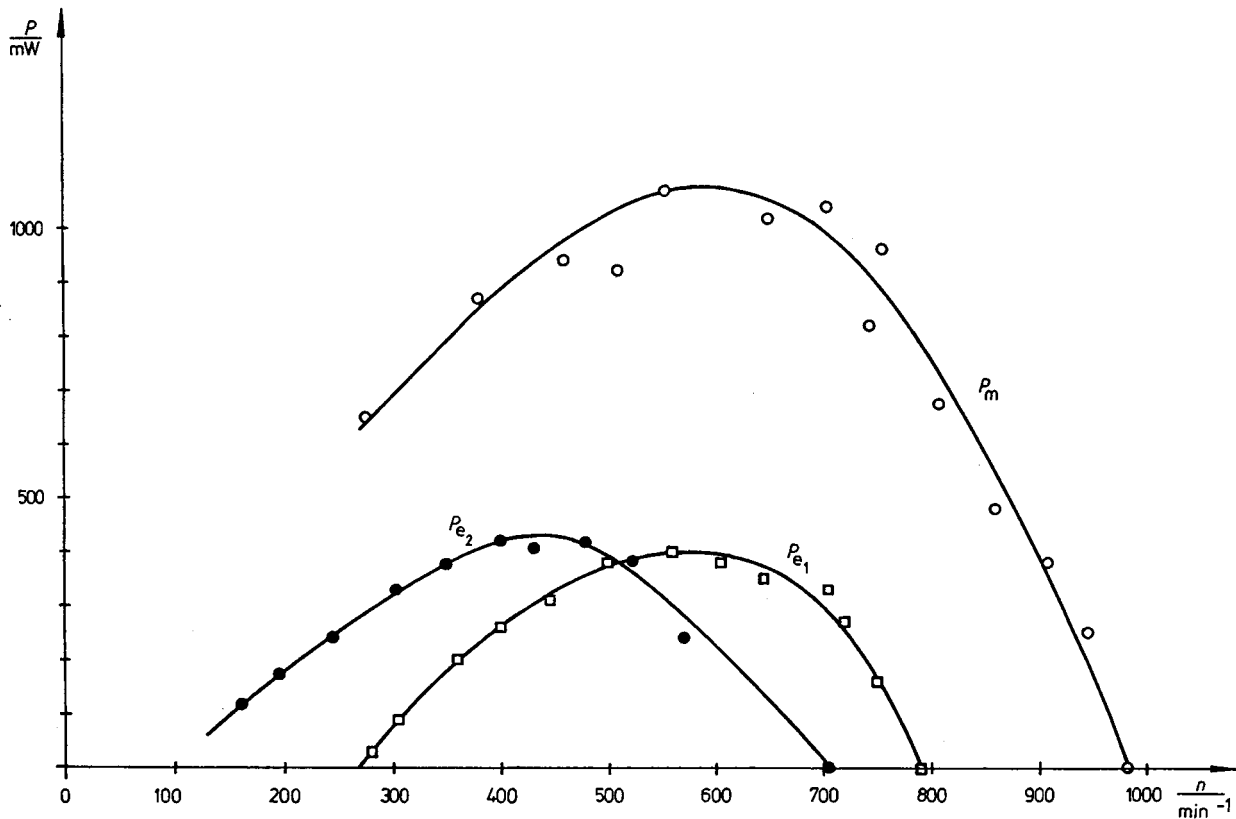
Table 3a: large strap wheel

n	T_1	T_2	I	U	P_{e_1}
min⁻¹	°C	°C	mA	V	mW
958	150	80.9	No-load operation without transmitting belt		
789	155	78.9	0.0	8.5	0
750	159	78.9	21.5	7.7	166
721	167	78.7	39.0	7.0	273
702	168	77.9	50.5	6.6	333
644	166	77.1	60.0	5.8	348
605	167	74.1	74.0	5.15	381
561	173	75.5	93.0	4.4	400
501	177	75.4	118	3.25	384
444	181	73.6	124	2.5	310
400	185	73.6	135	1.9	257
358	192	72.2	150	1.3	195
305	196	71.3	162	0.52	84
280	197	70.9	168	0.17	29

Table 3b: small strap wheel

n	T_1	T_2	I	U	P_{e_2}
min⁻¹	°C	°C	mA	V	mW
950	141	75.0	No-load operation without transmitting belt		
705	151	70.9	0.0	12.0	0
570	157	71.1	26.0	9.2	239
527	158	70.1	48.5	8.0	388
480	161	68.9	60.0	7.0	420
428	168	69.1	67.5	6.0	405
400	169	68.5	79.0	5.3	419
350	174	67.5	84.0	4.5	374
304	176	66.4	91.0	3.6	328
244	177	652	96.0	2.5	240
195	178	64.2	93.0	1.85	172
160	185	64.8	91.0	1.3	118

Fig. 7: Mechanical and electric power as a function of rotation frequency.

Larger voltages are obtained when the generator is coupled to the Stirling engine over the small strap wheel as when the large strap wheel is used. The power peak is shifted toward smaller rotation frequencies, but the peak sire remains roughly the same. Due to generator efficiency, the effective electric power is smaller than the effective mechanical power.

6. Real and ideal Stirling process, efficiency assessment

The idealised Stirling process runs along isochoric and isothermal lines (see Fig. 3a). The real process can stray considerably from this, due to several reasons:

a. Both pistons run with a constant phase shift of 900, which causes the diagram to have no sharp angles, as in the case of the idealised process.

b. Gas velocity is too high for an isothermal change of state in the case of an engine running at 1000 revs/ min.

c. The regenerator does not work at 100% efficiency. The air within the Stirling engine reaches the cold zone warmer, and the warm zone colder, as would be the case for the ideal process. Larger thermal input and cooling capacity are required.

d. During the ideal process, the total amount of working medium is forced from the cold zone into the warm zone. In the real process, there is a clearance volume, e.g. in the case of this Stirling engine the regenerator volume (that is the volume next to the displacing piston), and in the working cylinder.

e. There are large losses of pressure, as the working piston is not air tight.

f. Friction losses occur at all friction surfaces and within the streaming gas.

Isotherms can be adapted to a measured pV diagram with the assistance of the measured temperatures T_1 and T_2 This is carried out, using a measurement in the maximum power range of the Stirling engine as an example.

$$M = 18.3 \cdot 10^{-3} \text{ Nm}$$

$$T_1 = 192\,°C = 465 \text{ K}$$

$$T_2 = 75.5\,°C = 349 \text{ K}$$

The following relation is valid for an ideal gas:

$$p \cdot V = \nu \cdot R \cdot T$$

Due to the fact that the working piston of the Stirling engine is not air tight, the number of moles ν contained within the engine during operation must be evaluated with the assistance of the pV diagram (see Fig. 5). One or two points are selected in the middle of the diagram surface. They are allocated to the isotherm at the average temperature $T_m = \frac{T_1 + T_2}{2} = 407$ K.

Example:

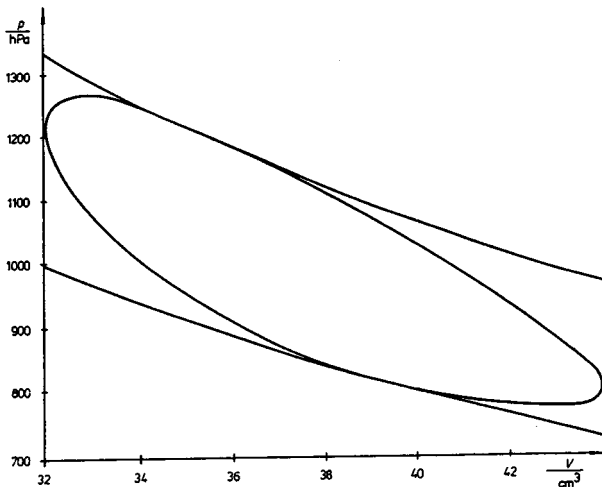1st point: $V = 38.0$ cm^3 p = 969 hPa corresponds to this

2nd point: $p = 1017$ hPa $V = 36.8$ cm^3 corresponds to this

Fig. 8: *pV* diagram and isotherms.

With $R = 8.31$ J/(mole K), one obtains, as an average of both assessments:

$$\nu = 1.10 \cdot 10^{-3} \text{ moles}$$

The isotherms for temperatures T_1 and T_2, calculated with the assistance of this value, are represented in Fig. 8, together with the *pV* diagram. When comparing measured and theoretical curves, it must be taken into account that the displayed temperatures oniy can be average values. In the vicimty of the flame, temperature is higher than T_1 and lower than T_2 within the working cylinder. Volume increase only takes place within the cold working cylinder; for this reason average temperatures are shifted towards lower values than those displayed for a large volume, and the curve of the *pV* diagram is somewhat steeper than the isotherms. Overlapping may also occur when comparing various *pV* diagrams with theoretical curves.

Efficiency assessment for this maximum power example:

The effective energy per cycle is (see Table 2):

$$W_m = 115 \text{ mJ}$$

During one cycle, the burner supplies the following thermal energy:

$$W_H = P_H/f$$
$$W_H = 18.0 \text{ J}$$

This yields a total efficiency of:

$$\eta = W_m/W_H$$
$$\eta = 115 \text{ mJ}/18.0 \text{ J}$$
$$\eta = 0.6\,\%$$

The efficiency of the Stirling engine is constituted by several components:

Efficiency of the heater:

$$\eta_H = |W_1|/W_H$$
$$\eta_H = \nu \cdot R \cdot T_1 \ln (V_2/V_1)/W_H$$
$$\eta_H = 1.35 \text{ J}/18.0 \text{ J}$$
$$\eta_H = 7.5\,\%$$

Thermal efficiency (Carnot):

$$\eta_{th} = W_t/W_1$$
$$\eta_{th} = (T_1 - T_2)/T_1$$
$$\eta_{th} = (465 \text{ K} - 349 \text{ K})/465 \text{ K} = 25\,\%$$

Interior efficiency:

$$\eta_i = W_{pV}/|W_t|$$
$$\eta_i = W_{pV}/(\nu \cdot R \, (T_1 - T_2) \ln (V_2/V_1)$$
$$\eta_i = 245 \text{ mJ}/339 \text{ mJ} = 72\,\%$$

Mechanical efficiency:

$$\eta_m = W_m/W_{pV}$$
$$\eta_m = 115 \text{ mJ}/245 \text{ mJ} = 47\,\%$$

Note:

The experiments can also be performed with the help of the sun as heating source. Therefore you need the accessories for solar motor work. The setup is shown in Fig. 9.

Fig. 9: Stirling engine with accessories for heating by the sun.

Clicking on the picture shows our stirling engine in action....