Related topics

Fourier transform, lenses, Fraunhofer diffraction, index of refraction, Huygens' principle, fog technique.

Principle and task

The electric field distribution of light in a specific plane (object plane) is Fourier transformed into the 4f configuration by 2 lenses and optically filtered by appropriate diaphragms.

Equipment

Optical base plate w. rubber ft.	08700.00	1
Laser, He-Ne 0.2/1.0 mW, 220 V AC*	08180.93	1
Adjusting support 35×35 mm	08711.00	2
Surface mirror 30×30 mm	08711.01	2
Magnetic foot f. opt. base plt.	08710.00	12
Holder f. diaphr./beam splitter	08719.00	2
Lens, mounted, $f = +100$ mm	08021.01	3
Lensholder f. optical base plate	08723.00	3
Screen, white, 150×15 mm	09826.00	1
Slide -Emperor Maximilian-	82140.00	1
Screen, with arrow slit	08133.01	1
Diffraction grating, 4 lines/mm	08532.00	1
Diffraction grating, 50 lines/mm	08543.00	1
Diaphragms, d 1, 2, 3, 5 mm	09815.00	1
Screen, with diffracting elements	08577.02	1
Objective 25× N.A. 0.45	62470.00	1
Sliding device, horizontal	08713.00	1
xy shifting device	08714.00	2
Adapter ring device	08714.01	1
Pin hole 30 micron	08743.00	1

Ruler, plastic, 200 mm	09937.01	1
Ultrasonic generator	11744.93	1
Glass cell, 150×55×100 mm	03504.00	1
Table with stem	09824.00	2
Support rod, stainl.steel, 250 mm	02031.00	1
Bosshead	02043.00	1
Universal clamp	37715.00	1

*Alternative

He/Ne Laser, 5 mW with holder	08701.00	1
Power supply f. laser head 5 mW	08702.93	1

Problems

1. Optical Filtration of diffraction objects in 4f set-up.

2. Reconstruction of a filtered image.

Set-up and procedure

– In the following, the pairs of numbers in brackets refer to the coordinates on the optical base plate in accordance with Fig. 1b. These coordinates are only intended to help with coarse adjustment.

– Perform the experimental set-up according to Fig. 1a and 1b. The recommended set-up height (beam path height) is 130 mm.

– The **E25x** beam expansion system (magnetic foot at [1,5.5]) and the lens **L₀** [1,3] are not to be used for the first beam adjustment.

Fig. 1a: Experimental set-up for Fourier optics (4f set-up).
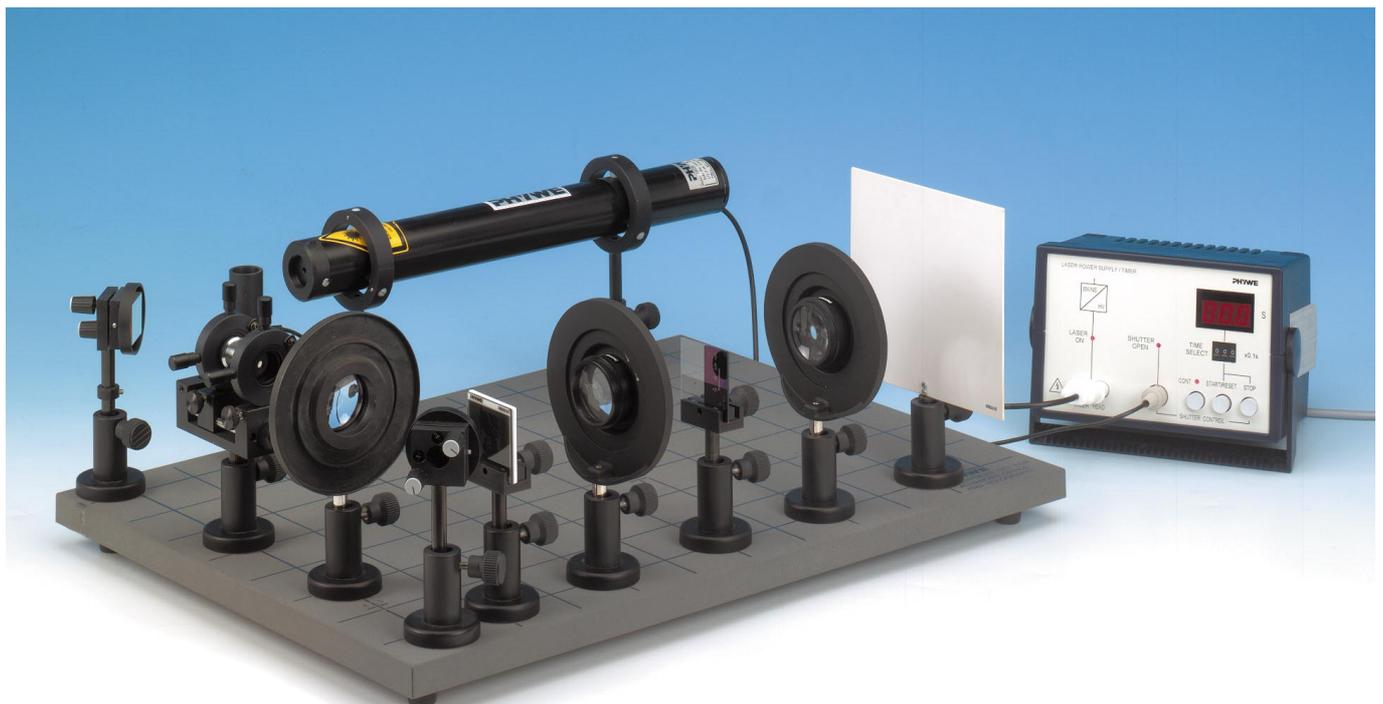
Fig. 1b: Experimental set-up for Fourier optics, 4f set-up, filtering and reconstruction. * only required for the 5 mW laser!
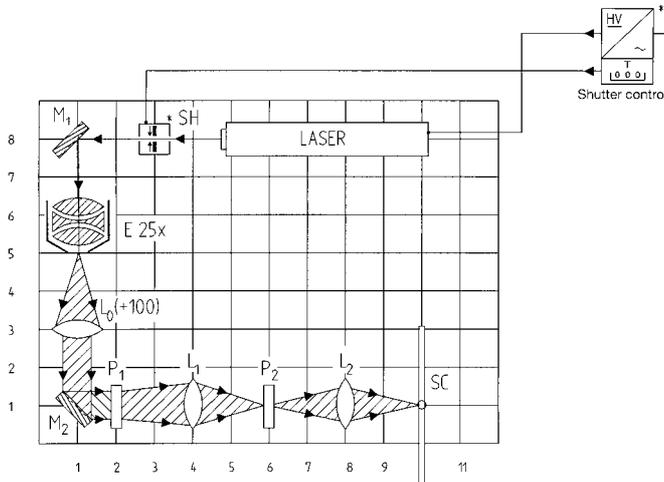
– When adjusting the beam path with the adjustable mirrors **M₁** [1,8] and **M₂** [1,1], the beam is set along the 1st x and 1st y coordinates of the base plate.

– Now, place the **E25x** [1,5.5] beam expansion system without its objective and pinhole, but equipped instead only with the adjustment diaphragms, in the beam path. Orient it such that the beam passes through the circular stops without obstruction. Now, replace these diaphragms with the objective and the pinhole diaphragm. Move the pinhole diaphragm toward the focus of the objective. In the process, first ensure that a maximum of diffuse light strikes the pinhole diaphragm and later the expanded beam. Successively adjust the lateral positions of the objective and the pinhole diaphragm while approaching the focus in order to ultimately provide an expanded beam without diffraction phenomena. The **L₀** [1,3] (f = +100mm) is now positioned at a distance exactly equal to the focal length behind the pinhole diaphragm such that parallel light now emerges from the lens. No divergence of the light spot should occur with increasing separation. (test for parallelism via the light spot's diameter with a ruler at various distances behind the lens **Lₒ** in a range of approximately 1 m).

— Now set-up the additional optical components.

4-f-set-up:

– Place a plate holder **P₁** [2,1] in the object plane. Position the lens **L₁** [4,1] at the focus (f = 100 mm) and the second plate holder **P₂** [6,1] at the same distance behind the lens. Additionally, place another lens **L₂** [8,1] at a distance equal to the focus f = 100 mm and at the same distance of 10 cm set up the screen **Sc** [10,1]. The parallel light beam that strikes the lens **L₁** must appear on the screen **Sc** at the same height and with the same extension (Check with the ruler!).

To begin with, observations without optical filtration:

– (a)Clamp the *diaphragm with the arrow* (arrow pointing upwards) as the first diffracting structure in plate holder **P₁** [2,1] in the object plane and shifted laterally in such a manner that the light from the mirror **M₂** strikes the arrow head. An arrow also appears on the observation screen (compare with theory!). The arrow is now turned 90° so that it points in a horizontal direction (shift the diaphragm laterally until the arrow head is will illuminated). In this case also, compare the image on the screen with the theory.

– (b)The photographic slide of Emperor Maximilian serves as the next diffracting structure. It is placed in plate holder **P₁** and laterally shifted until the light beam illuminates significant contours of the face (e.g. the nose) (in a recognisable form!). Observe the image in the observation plane (what is different than the original?).

– (c)Place the grid (4 lines/mm) in the object plane **P₁** as a further diffracting structure. In the process, observe the Fourier-transformed image in the Fourier plane **P₂** [6,1] with the screen and subsequently examine it in the observation plane, **Sc** at [10,1]. One ascertains that the lines of the grid cannot be resolved in the image (but they cannot be resolved in the real grid either!). By turning the screen around its vertical axis, the image can be expanded in a distorted manner such that the grid lines can be discerned.

Observations with optical filtration:

– (a) Clamp the grid (50 lines/mm) in the object plane **P₁** in a plate holder. Now perform a **low-pass filtration** in the Fourier plane **P₂**, by positioning a pinhole diaphragm (diameter: 1–2 mm) in such a manner that only a single, arbitrary diffraction maximum passes through Observe the image on the screen **Sc**.

– (b)Clamp the slide (Emperor Maximilian) and the grid (4 lines/mm in vertical direction) together in the optical plane in the plate holder **P₁**. To begin with, observe the Fourier spectrum in **P₂** [6,1] on the screen **Sc** and subsequently the image on the screen **Sc** at position [10,1].
In the Fourier plane, one sees sharply focused diffraction structures. This can be checked by moving a black diaphragm (blackened part of a pinhole diaphragm) in the direction of light propagation. The filter plane **P₂** (intended position of the plate holder) should correspond with the thus-determined Fourier plane (most sharply focused diffraction structures): If the Fourier plane (or the focal length of the lens) has shifted, remove the grid and the slide in order to readjust the second lens **L₂**, since this lens also should have the same focal length f_B as the first lens.

– Move the lens **L₂** in the direction of the optical axis along the 1st y co-ordinate of the base plate such that behind this lens (according to Fig. 1b to the right of lens **L₂**) parallel light (constant diameter of the light spot) is present (check with a ruler).

– The grid with slide can then again be placed in the object plane and the screen should be set up at the same distance from the 2nd lens **L₂** as from the 2nd lens to the Fourier plane **P₂**.

Fig. 2: Experimental set-up for the fog method. Making the phase grid visible with an ultrasonic wave.

Fig. 3: Principle of the set-up for coherent optical filtration.
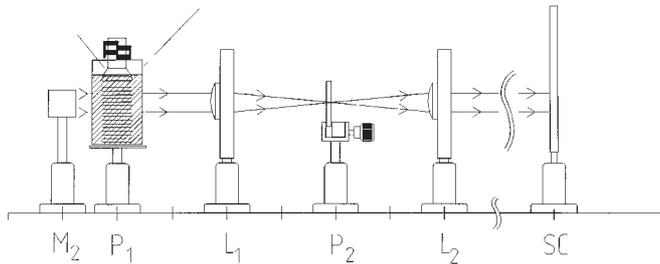
Observation:

In the Fourier plane, one sees primarily the grid spectrum with discrete diffraction maxima. On the observation screen **Sc** [10,1], the slide (Emperor Maximilian) can be seen. It is possible that the fine grid cannot be seen due to the marked speckle formation. To still be able to resolve this grid, the screen can be turned around its vertical axis to expand the image. It is necessary to turn the screen to such an extent that it is nearly parallel to the direction of light propagation. In this manner, the fine structures will be enlarged and readily visible. Now perform a low-pass filtration with a pinhole diaphragm (diaphragm with diffraction objects) in the Fourier plane **P₂** [6,1] by selectively filtering out all but a single arbitrary diffraction maximum. It is advisable to use the central zero diffraction maximum, as the greatest light intensity is located there. And accordingly, the image on the screen is the brightest.

On using the pinhole diaphragm with a diameter of 0.25 mm, the grid structure disappears. The image of the slide (Emperor Maximilian) is not affected when the screen is in a perpendicular position (no distortion). If the screen is turned nearly parallel to the direction of light propagation, the grid structure cannot be seen.

However, as soon as the pinhole diaphragm has a diameter equal to 0.5 mm, it is impossible to filter out the grid structure. When the screen is turned to the horizontal position, the image of the slide always has superimposed grid lines.

– (c) **Fog technique:** This procedure makes phase gradients visible, such as those, for example, which occur in the flow of gases having different densities (air flow of a candle). Marked phase changes also occur at the edge of a pane of glass.

– To observe this, place the glass pane (as phase object), diaphragm with phase objects, in the object plane **P₁** [2,1] in such a manner that the left or right edge (without blackening!) is reproduced on the screen **Sc** [10,1] with only slight light reflections.
In the Fourier plane [6,1] a half-plane is now covered with a black diaphragm (e.g. arrow diaphragm). To achieve this, the diaphragm is fixed in a plate holder and then the total unit with magnetic base is shifted in such a manner that one diaphragm edge, which is parallel to the glass plate's edge, is cautiously moved into the region of the main diffraction maximum (brightest point in the Fourier spectrum). Half of the diffraction maximum must be covered so that the elevat-

ed intensity at the margin of the pane of glass can be seen in the image on the screen **Sc** (The edge appears as a black stripe!).
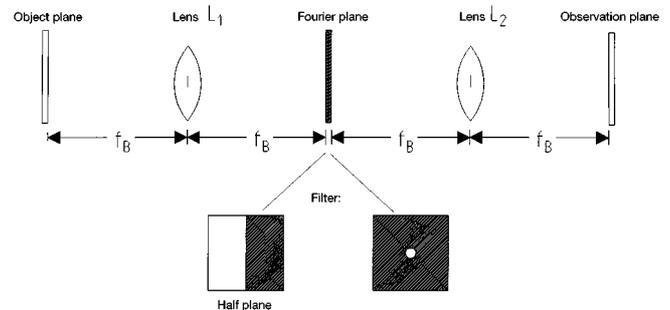
– Density oscillations in water as a phase object: The experimental set-up remains the same as in (c) (see Fig. 2). The only difference is that in the object plane [2,1] the plate holder with the diaphragm is removed. Instead of it, place two magnetic bases in the positions [2,1] and [2,2], which are each provided with a small table with rod. Place the cell which has been filled with water on this small tables such that the light spot remains visible on the screen **Sc** [10,1] without additional reflections.
Clamp the handle of the sonic transducer in a universal clamp and fix it onto the optical plate using a right-angle clamp to connect it to a support rod clamped into a magnetic base.
Now position the sonic transducer with its surface plane parallel to the water's surface such that it is immersed in the liquid by about 5 mm without bubble formation. (Caution: Do not immerse the transducer too deeply!)
Connect the sonic transducer with the ultrasonic generator. In its switched-off state, one does not observe any changes on the screen **Sc**.
The transducer is driven by a sinusoidal excitation, i.e. depress the right pushbutton on the ultrasound device (~). When the device is switched on and the excitation level is appropriately elevated (Do not select an excessively strong excitation, as otherwise there is danger of bubble formation and of evaporation of the liquid!) – intermediate level adjustment – a standing sound field is generated in the glass cell. It serves as a phase grid for the light wave which is incident perpendicular to it (see theory). In this state also, one notices no change of any kind on the observation screen **Sc**.
Now, cover a half-plane of the Fourier spectrum in the Fourier plane [6,1] with the black diaphragm (arrow diaphragm). To do this, fix the diaphragm in a plate holder and then shift the entire unit with the plate holder in a magnetic base vertically in such a manner that one edge of the diaphragm, which is parallel to the direction of sound propagation, is cautiously moved into the region of the main diffraction maximum (brightest point in the Fourier spectrum). Half of the diffraction maximum must be covered, so that intensity elevation in the region of the ultrasonic wave can be seen in the image on the screen **Sc** (parallel, horizontal stripes). If the sonic excitation is reduced, these stripes disappear.

Fig. 4: Composite spectrum, e.g. raster image.
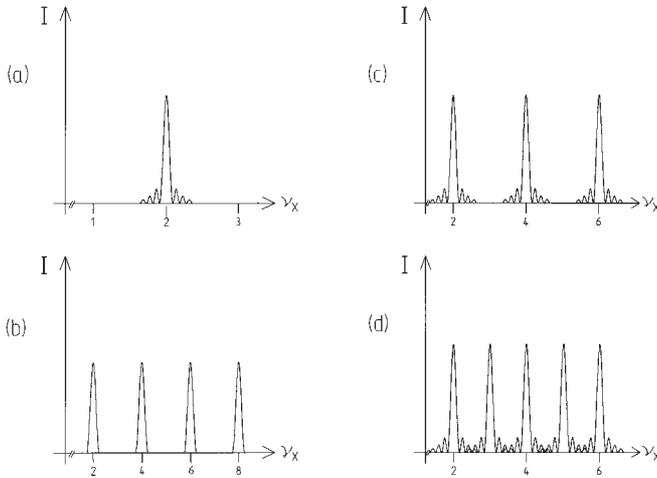
Theory and Evaluation

For information on the fundamentals of Fourier optics and the Fourier transformation by a lens, see the "Fourier optics – 2f Arrangement" experiment.

Coherent optical filtration

By intervening in the Fourier spectrum, optical filtration can be performed which can result in image improvement, etc. The appropriate operation for making the original image visible again, ins the inverse Fourier transform, which however cannot be used due to diffraction. The Fourier transform is again used; this leads to the **4f set-up** (see Fig.3).

Using the 1st lens **(L_1)**, the spectrum with the appropriate spatial frequencies is generated in the Fourier plane from the original diffracting structure τ (x,y) (see the "Fourier optics – 2f Arrangement" experiment). In this plane, the spectrum can be altered by fading out specific spatial frequency fractions. A modified spectrum is created, which is again Fourier transformed by the 2nd lens **(L_2)**. If the spectrum is not altered, one obtains the original image in the inverse direction in the image plane (right focal plane of the 2nd lens) (partial experiment (a) with the arrow diaphragm). This follows from the calculation of the twofold Fourier transformation:

$$\tilde{F}\,[\tilde{F}\,[f(x,y)]] = f(-x,-y) \qquad (1)$$

The simplest applications for optical filtration are the high- and low-pass filtration.

Low-pass raster elimination

In the experiment, the photographic slide was provided with a raster by superimposing grid lines on it in one direction. The scanning theory states that an non-raster image (in this case:

Emperor Maximilian) can be exactly reconstructed if the image is band-limited in its spectrum, i.e. if it only contains spatial frequencies in the Fourier plane up to an upper limiting frequencies.

The raster image can be described mathematically as follows:

$$b\,(x,y) = \text{comb}\,\frac{y}{b} \cdot g\,(x,y) \qquad (2)$$

with the comb function

$$\text{comb}\,\frac{y}{b} \;=\; \sum_{-\infty}^{+\infty} \delta\,(y-nb) \qquad (3)$$

This describes the grid lines (the grid) and $g\,(x,y)$ the non-raster image (in this case: the slide). The slit separation of the grid is b. The Fourier spectrum $B(\nu_x,\nu_y)$ of the entire image becomes the following with the convolution law:

$$\tilde{F}\,[b\,(x,y)] = B(\nu_x,\nu_y) = b \cdot \text{comb}\,(b \cdot \nu_y) \cdot G(\nu_x,\nu_y) \qquad (4)$$

where $G(\nu_x,\nu_y)$ is the Fourier transformation of the non-raster image. In addition, the fact that the Fourier transformation of an comb function is also a comb function. This means that the Fourier spectrum once again a grid which is formed by the reiteration of the spectrum of the non-raster image (see Fig. 4). Each grid point with its immediate surroundings contains the total information of the non-raster image $g(x,y)$.

It is important that the grid points in the Fourier plane are sufficiently far apart that the spectra of the unlined image do not overlap. Only in this case is it possible to filter out a single image point with a pinhole diaphragm. This spatial frequency filtration can be considered as multiplication of the spectrum by an aperture function $A(\nu_x,\nu_y)$ (pinhole diaphragm in the Fourier spectrum). In this case, an appropriate measuring dimension would be a diameter of $\approx 1/b$. Therefore:

$$\begin{aligned} B_{\text{filtered}}\,(\nu_x,\nu_y) &= B\,(\nu_x,\nu_y) \cdot A\,(\nu_x,\nu_y) \\ &= b\,(\text{-}x,\text{-}y) * \tilde{F}\,[A\,(\nu_x,\nu_y)] \end{aligned} \qquad (5)$$

Fog procedure

The fog procedure makes it possible to see phase objects (e.g. phase grids through a standing ultrasonic wave). In this case, the Fourier spectrum is filtered with a half-plane filter. This amplitude filter should fade out exactly one half-plane of the spatial frequency spectrum including half of the zero order! The intensity of the image in the observation plane (in this case at **Sc** [10,1]) is then proportional to the gradient of the phase, where the direction of the observed gradient is a function of the position of the half-plane:

$$I(\text{-}x,\text{-}y) \sim \left| \frac{\partial \varphi\,(x,y)}{\partial x} \right| \qquad (6)$$

with φ: phase of light.
The principle can be described as follows:
The total image information is contained in a half-plane of the spatial frequency spectrum. In order to obtain the original intensity distribution of the diffracting structure by repeated Fourier transform, the superimposition of the two spatial frequency half-planes is necessary. If this superimposition is prevented, the phase information of the diffracting structure is made visible.

Fig 5: Schematic diagram of a standing ultrasonic wave based on a pressure change (or density change).
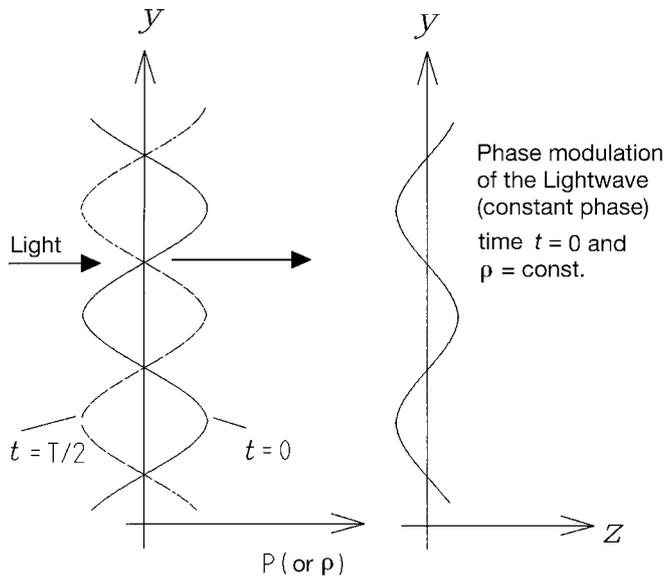
Information on the ultrasonic wave:

The ultrasonic wave forms a standing sound wave between the ultrasonic transducer and the cell's bottom. This is a periodically oscillating pressure (and density) variation of the water with spatially fixed pressure nodes. Since the optical refraction index is proportional to the density of the medium, the propagation velocities of the light which is perpendicularly incident to the direction of sonic propagation are different in the various pressure regions. This results in a phase modulation of the light behind the ultrasonic wave (see Fig. 5)

Since the ultrasonic wave is a standing wave, the location of the phase gradients does not change (it disappears only in the pressure maxima and minima of the sonic field), but the intensity changes periodically. Due to the sluggishness of perception, a temporal average is taken here as the sound frequency is approximate 800 kHz:

Application areas of optical filtration:

Low-pass filtration can be used as a spatial frequency filter to eliminate the disturbances of the wave front, which result from soiling of the lenses, in a beam expansion by a microscope's objective (as **E25x** in this experiment). Another direct application of low-pass filtration is the elimination of raster lines in composite images. For example, in astronomy, composite satellite images are freed from their raster.

In Fourier optics, numerous procedures exist for making phase objects or modulations visible:

(1.) The **phase-contrast procedure** effects the transformation of a phase modulation on a diffraction object into an amplitude modulation by inserting a λ/4 wafer into the zero diffraction order. This modulation can then be observed in the observation plane.
The phase-contrast microscope (Frits Zernicke, Nobel Prize in 1953) is a wide-spread application of this procedure.

(2.) The **fog technique** in which a half-plane filter is used in the Fourier plane makes the phase gradients visible (as described in this experiment).

(3.) The **dark-field technique** is a *high-pass filtration* in the Fourier plane, i.e. the zero diffraction order is filtered out by a small disk of the appropriate size. Using this filtration method, thin sections, organic preparation, currents of air and pressure waves in fluid dynamics research are made visible.

Additional fields of application result from the use of **holographic filters**. Images of Fourier spectra of certain diffracting structures on holographic photo material can be used as filters in the Fourier plane of the 4f set-up. These then are used *for pattern recognition*, i.e. for the repeated recognition of fundamental diffracting structure.