Related topics

Electromagnetic theory of light, reflection coefficient, reflection factor, Brewster's law, law of refraction, polarization, polarization level.

Principle and task

Plane-polarized light is reflected at a glas surface. Both the rotation of the plane of polarization and the intensity of the reflected light are to be determined and compared with Frewsnel's formulae for reflection.

Equipment

Laser, He-Ne 1.0 mW, 220 V AC	08181.93	1
Polarising filter, on stem	08610.00	2
Prism, 60 degrees, h 36.4 mm, flint	08237.00	1
Prism table with holder	08254.00	1
Photoelement f. opt. base plt.	08734.00	1
Protractor scale with pointer	08218.00	1
Articulated radial holder	02053.01	1
Stand tube	02060.00	1
Tripod base -PASS-	02002.55	1
H-base -PASS-	02009.55	2
Right angle clamp -PASS-	02040.55	4
Support rod -PASS-, square, l 400 mm	02026.55	1
Support rod -PASS-, square, l 250 mm	02025.55	1
Support rod -PASS-, square, l 630 mm	02027.55	1
Multirange meter with amplifier	07034.00	1
Dry cell, 1.5 V	11620.34	6

Problems

1. The reflection coefficients for light polarized perpendicular and parallel to the plane of incidence are to be determined as a function of the angle of incidence and poltted graphically.
2. The refractive index of the flint glass prism is to be found.
3. The reflection coefficients are to be calculated using Fresnel's formulae and compared with the measured curves.
4. The reflection factor for the flint glass prism is to be calculated.
5. The rotation of the polarization plane for plane polarized light when reflected is to be determined as a function of the angle of incidence and presented graphically. It is then to be compared with values calculated using Fresnel's formulae.

Set-up and procedure

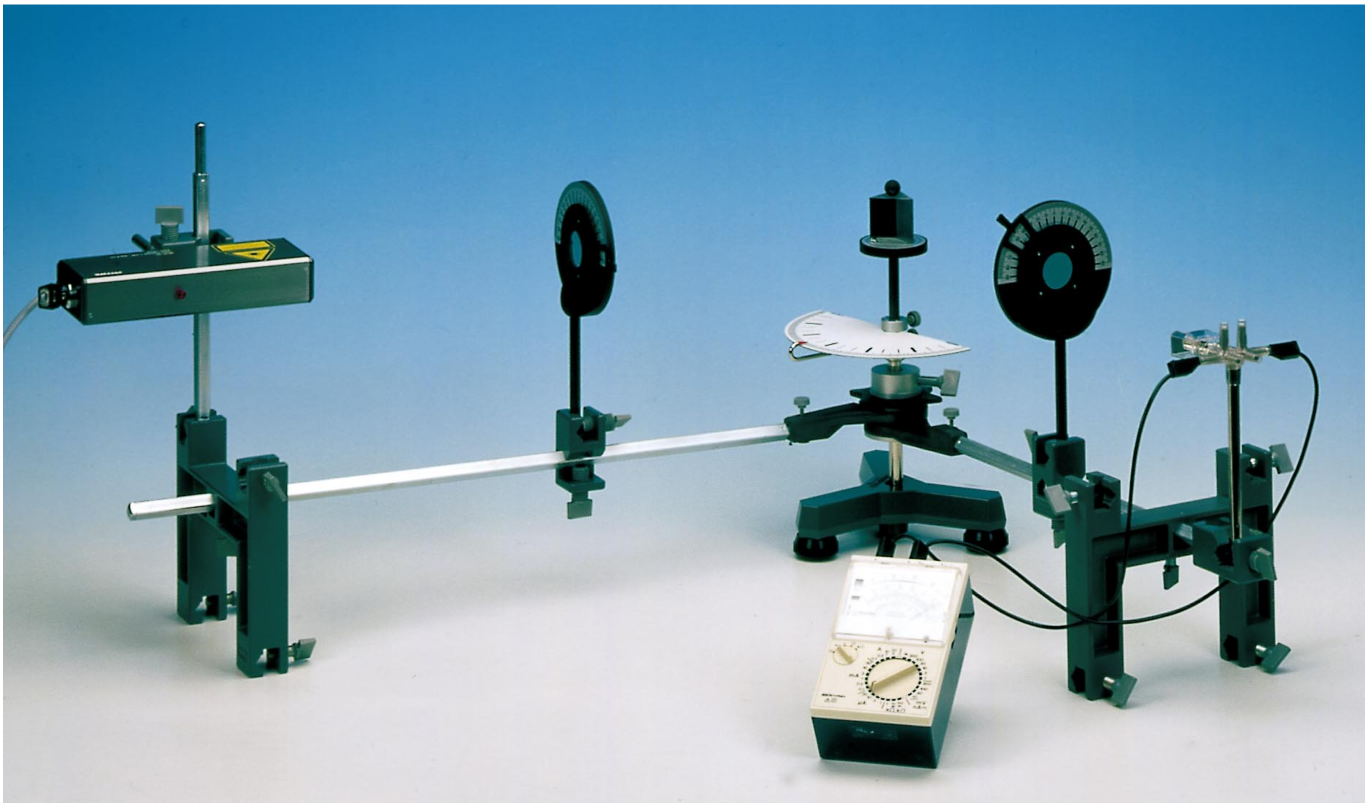
The experimental set-up is as shown in Fig. 1. In this respect, the following information should be noted:

A Setting up the swivel device

1. The pointer must first be screwed into the stand tube with its red tip pointing upwards. The stand tube is then inserted into the articulated radial holder until it reaches the stop and then tightened. The lower end of the stand tube is fastened in the tripod base.

Caution: Never look directly into a non attenuated laser beam

Fig. 1: Experimental set-up for the determination of the rotation of the plane of polarization by reflection.



2. The prism table, at which the protractor scale has been fixed, is fastened in the stand tube so that it can easily rotate but cannot wobble.
3. The support rods are fastened in the H-feet with the edge pointing upwards and joined to the articulated radial holder. The pointer and the longitudinal edge of the protractor scale should line up with the upper edge of the support rods (see Fig. 2). This can be easily carried out by turning the tripod base.
4. Finally, the clamping screw on the articulated radial holder is tightened slightly.

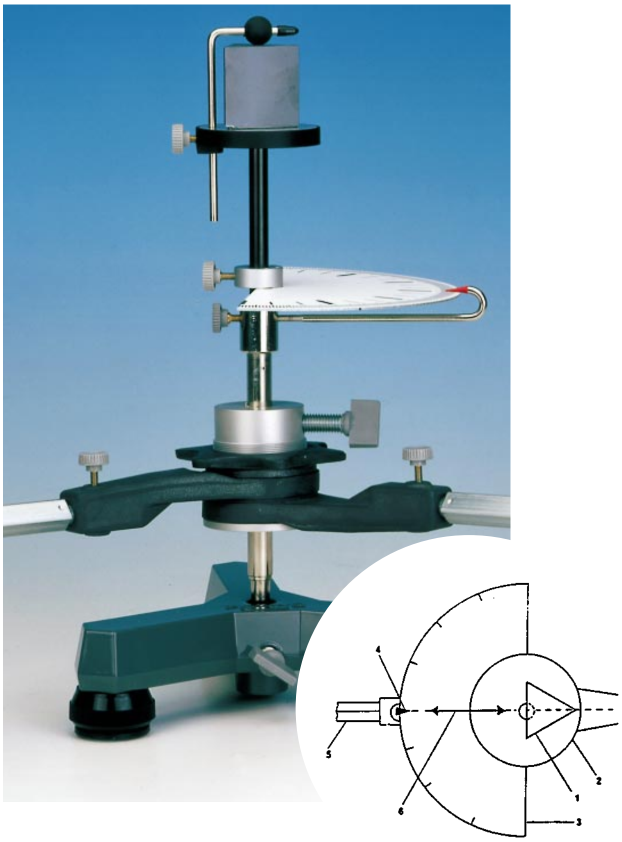


Fig. 2: Articulated radial holder with prism arrangement for finding the reflection at $\alpha = 0°$
(1 darkened side of prism, 2 prism table, 3 protractor scale, 4 pointer, 5 support rod, 6 light beam).

B Beam path adjustment

1. The laser beam must be located over the centre of the prism table for finding the zero position.
2. Then the photocell, which is lined up with the support rod and switched to the maximum current range (300 μA), is swivelled into the beam. In this position the support rods are lined up with the longitudinal edge of the protractor scale.
3. The protractor scale is turned to zero to define the setting of the angle of incidence $\alpha = 0$. Then the prism should be placed on the table with its reflecting surface centrally positioned so that the incident beam is reflected back along its own path (Fig. 2).

C Carrying out the measurements

1. After the laser has warmed up for about 15 minutes, the primary intensity i_o'' of the beam polarized parallel to the plane of incidence is found. The laser is located in the normal position. Then the prism should be placed in position as in section B 3. After that, the angle of incidence is changed in steps of 5° from $\alpha \leq 10°$ and a step angle of 1° should be selected in the region of the Brewster angle. The photocell is swivelled to obtain the maximum current for the determination of the intensity i_r''.
   Then the laser is turned through 90° and fixed on one of the legs on the H-base using the short support rod. The laser light is now oscillating normally polarized to the prism's plane of incidence. First, the primary intensity $i_o^{\perp}$ must again be determined. Then the angle of incidence is varied in 5° steps and the corresponding intensity of the reflected beam is found.
2. The laser is set up again in the normal position for the determination of the degree of rotation of the polarization plane by reflection. The photocell is lined up to the direction of the beam without the prism in place. Using a polarization filter mounted in front of the laser for a precise determination of the plane of oscillation, the filter is turned until the registered intensity is at a minimum. Then the filter is turned through 45° and the prism placed in position using the familiar method. The degree of rotation of the plane of polarization for the reflected beam is found with a second polarization filter located between the prism and the detector. The angle of incidence is changed in 5° steps. The angle of rotation for the plane of polarization is the average of a number of measurements.

Theory and evaluation

In a light wave the electrical field vector E and the magnetic vector B oscillate perpendicular to one another and in phase. Their magnitudes are governed by Maxwell's equation:

$$|B| = n\,|E| \qquad (1)$$

where n is the refractive index of the medium through which the light passes. The transported wave engergy in the direction of propagation is represented by the Poynting vector. The following relationship applies for this:

$$S \sim E \times B \quad \text{and} \quad |S| \sim |E|^2 \qquad (2)$$

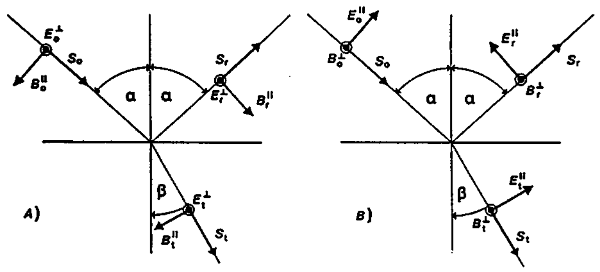
If light is incident at the angle α on the boundary surface of an isotropic medium with a refractive index n, then a fraction of the intensity is reflected and the remainder penetrates the medium at the refraction angle β.

The following indices are used in the theory below:

$x^{\perp}$, x'' = the direction of oscillation of the electric or the magnetic field vectors located normally or parallel to the angle of incidence.

x_o, x_r, x_t = incident, reflected and refracted vector components.

Fig. 3: Direction of oscillation of the electric vector A) normal, B) parallel to the direction of incidence.



In Fig. 3 A) the electric vector $E_0^{\perp}$ of the incident light wave oscillated perpendicular to the plane of incidence; the magnetic vector $B_0^{\parallel}$ oscillates parallel to this according to (2). According to the law of the continuity of tangential components (components that oscillate parallel to the surface of the object) and taking the direction of the beam into account, the following applies:

$$E_0^{\perp} + E_r^{\perp} = E_t^{\perp}; \qquad (B_0^{\parallel} - B_r^{\parallel}) \cos\alpha = B_t^{\parallel} \cos\beta \quad (3)$$

Using (1), (3) then gives

$$(E_0^{\perp} - E_r^{\perp}) \cos\alpha = n\,(E_0^{\perp} + E_r^{\perp}) \cos\beta \quad (4)$$

The ratio of the field strengths, taking into account the law of refraction, is

$$\zeta^{\perp} = \frac{E_r^{\perp}}{E_0^{\perp}} = \frac{\cos\alpha - n\cos\beta}{\cos\alpha + n\cos\beta} = -\frac{\sin(\alpha-\beta)}{\sin(\alpha+\beta)} \quad (5)$$

ζ is defined as the reflection coefficient.

Fig. 3 B) shows an incident light wave with a vector $E_0^{\parallel}$ oscillating parallel to the plane of incidence.

The following applies analogous to (3):

$$B_0^{\perp} + B_r^{\perp} = B_t^{\perp}; \; (E_0^{\parallel} - E_r^{\parallel}) \cos\alpha = E_t^{\parallel} \cos\beta$$

using (1), (6) gives the following:

$$(E_0^{\parallel} - E_r^{\parallel}) \cos\alpha = \frac{1}{n}\,(E_0^{\parallel} - E_r^{\parallel}) \cos\beta$$

The following is obtained, similar to (5):

$$\zeta^{\parallel} = \frac{E_r^{\parallel}}{E_0^{\parallel}} = \frac{n\cos\alpha - \cos\beta}{n\cos\alpha + \cos\beta} = -\frac{\tan(\alpha-\beta)}{\tan(\alpha+\beta)} \quad (8)$$

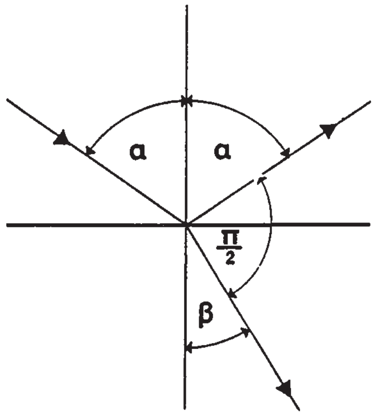
Fresnel's formulae (5) and (8) can be written in another form by eliminating the refraction angle β using Snell's law of refraction:

$$\zeta^{\perp} = \frac{E_r^{\perp}}{E_0^{\perp}} = -\frac{(\sqrt{n^2 - \sin^2\alpha} - \cos\alpha)^2}{n^2 - 1} \quad (9a)$$

$$\zeta^{\parallel} = \frac{E_r^{\parallel}}{E_0^{\parallel}} = \frac{n^2\cos\alpha - \sqrt{n^2 - \sin^2\alpha}}{n^2\cos\alpha + \sqrt{n^2 - \sin^2\alpha}} \quad (9b)$$

$\zeta^{\perp} \geq \zeta^{\parallel}$ applies for all angles of incidence α between zero and $\pi/2$.

Fig. 4: Brewster's law.



Special cases

A: For normal angles of incidence ($\alpha = \beta = 0$)

$$\zeta^{\perp} = \zeta^{\parallel} = \left|\frac{n-1}{n+1}\right| \quad (10)$$

B: For the glancing case of incidence ($\alpha = \pi/2$)

$$\zeta^{\perp} = \zeta^{\parallel} = 1 \quad (11)$$

C: If the reflected and refracted beams are perpendicular to one another ($\alpha + \beta = \pi/2$) (see Fig. 4), then it follows from (8) that

$$\zeta^{\parallel} = 0 \quad (12)$$

i.e. the reflected light beam is fully polarized. The electric vector oscillates in this case only normal to the plane of incidence. Since according to Snell's law of refraction

$$\sin\alpha = n\sin\beta = n\sin\left(\frac{\pi}{2} - \alpha\right) = n\cos\alpha$$

then for this special case we get for the angle of incidence

$$\tan\alpha_p = n$$

(α_p = polarization or Brewster angle).

Tab. 1: Current values $i_r^{\shortparallel}$ and $i_r^{\perp}$ as functions of angle α.

	$i_0^{\shortparallel}$ = 235 µA		$i_0^{\perp}$ = 230 µA	
$\frac{\alpha}{Degr.}$	$\frac{i_r^{\shortparallel}}{\mu A}$	$\zeta^{\shortparallel} = \frac{E_r^{\shortparallel}}{E_0^{\shortparallel}} = \sqrt{\frac{i_r^{\shortparallel}}{i_0^{\shortparallel}}}$	$\frac{i_r^{\perp}}{\mu A}$	$\zeta^{\perp} = \frac{E_r^{\perp}}{E_0^{\perp}} = \sqrt{\frac{i_r^{\perp}}{i_0^{\perp}}}$
10	11.5	0.221	13.5	0.243
15	11.0	0.216	15.0	0.255
20	10.5	0.211	16.5	0.268
25	9.4	0.200	18.0	0.280
30	8.2	0.187	20.0	0.292
40	5.2	0.149	27.0	0.343
45	3.6	0.124	31.0	0.367
50	2.2	0.097	35.0	0.390
52.5	1.6	0.083		
55	1.1	0.068	41.0	0.422
56	0.77	0.057		
57	0.71	0.055		
58	0.65	0.052		
59	0.67	0.053		
60	0.73	0.056	49.0	0.462
62.5	1.6	0.082		
70	7.3	0.176	72.0	0.560
75	20.0	0.292	110.0	0.692
80	43.0	0.428	140.0	0.780
85	87.0	0.608	180.0	0.885

Tab. 1 contains the current values i_r measured with the photocell at the light intensities refelcted at the angle α. The current level is directly proportional to the light intensity which is in turn proportional to the square of the field strength according to (2).

Fig. 5 shows the experimentally determined curves for $\zeta^{\perp}$ and $\zeta^{\shortparallel}$ as function of the angle of incidence α. The curve for $\zeta^{\shortparallel}$ exhibits a singificant minimum at $\alpha_p = 58.5°$.

Using this value and the crossing point of the ζ curves with the ordinate axis obtained by extrapolation, a value of n = 1.63 is obtained for the refractive index according to (13) and (10). The curves calculated theoretically acording to (9) using n = 1.63 exhibit good agreement with the experiment. The flatter shape of the $\zeta^{\shortparallel}$ curve at α_p is caused by the laser light having a degree of polarisation <1.

If the reflecting components from (9a) and (9b) are squared and added, then the following is obtained for the reflection factor R at normal incidence:

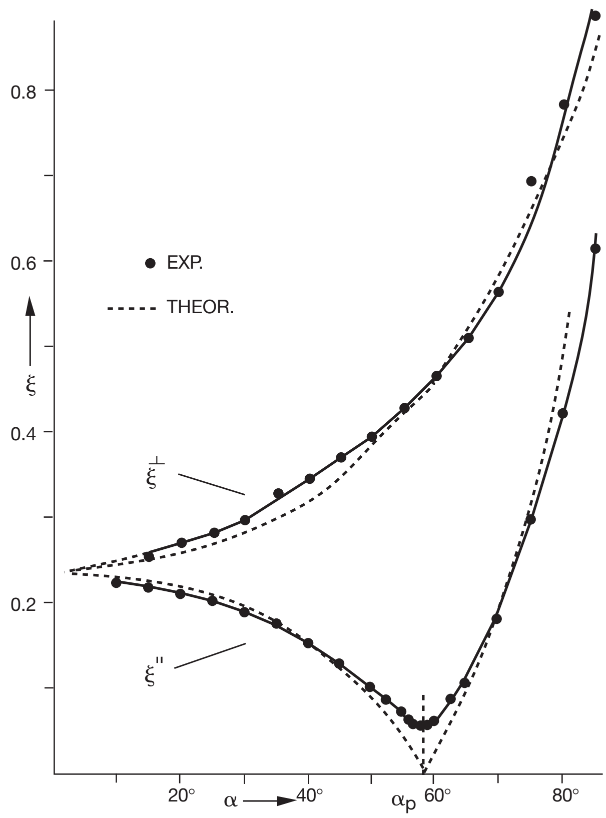
$$R = \frac{(E_r^{\perp})^2 + (E_r^{\shortparallel})^2}{(E_0^{\perp})^2 + (E_0^{\shortparallel})^2} = \left(\frac{n-1}{n+1}\right)^2 \qquad (14)$$

The reflection factor R for the flint glass prism (n = 1.63) used in the experiment is found to be approximately 0.06.

Another method of checking Fresnel's formulae consists of the following:

Plane-polarized light with an electric vector, which is rotated by a fixed azimuth angle δ with respect to the plane of incidence, impinges on a glass reflector. The rotation of the plane of polarization for the reflected beam is then determined as a function of the angle of incidence. In Fig. 6 the plane of the paper represents the reflecting surface.

Fig. 5: Measured and calculated curves for $\zeta_r^{\shortparallel}$ and $\zeta_r^{\perp}$ as a function of the angle of incidence.



If after reflection the electric vector oscillates at the azimuth angle ω, then a rotation of the plane of polarization through the angle $\psi = \delta - \omega$ is produced. For the comonents parallel and normal to the plane of incidence:

$$E_r^{\shortparallel} = E_r \cos\omega;\ E_r^{\perp} = E_r \sin\omega \qquad (15)$$

or

$$\tan\omega = \frac{E_r^{\perp}}{E_r^{\shortparallel}} = \frac{E_r^{\perp} \cdot E_0^{\shortparallel} \cdot E_0^{\perp}}{E_0^{\perp} \cdot E_r^{\shortparallel} \cdot E_0^{\shortparallel}} \qquad (16)$$

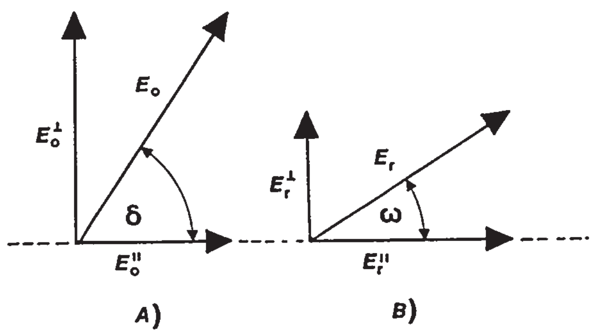


Fig. 6: Rotation of the direction of oscillation by reflection.

Using (5) and (8), it follows from (16):

$$\tan \omega = -\frac{\sin(\alpha-\beta)}{\sin(\alpha-\beta)} \cdot \frac{\tan(\alpha-\beta)}{\tan(\alpha+\beta)} \cdot \tan \delta \qquad (17)$$

For the special case where $\delta = \pi/4$:

$$\tan \psi = \tan\left(\frac{\pi}{4} - \omega\right) = \frac{1 - \tan \omega}{1 + \tan \omega}$$

If tan ω from (17) is substituted into (18), then after rearrangement one obtains:

$$\tan \psi = \frac{\cos(\alpha+\beta) + \cos(\alpha-\beta)}{\cos(\alpha+\beta) - \cos(\alpha-\beta)} \qquad (19)$$

$$= -\frac{\cos \alpha \sqrt{1 - \sin^2 \beta}}{\sin \alpha \cdot \sin \beta}$$

The elimination of the refraction angle β using the law of refraction gives the final form from (19):

$$\psi = \arctan\left(-\frac{\cos \alpha \sqrt{n^2 - \sin^2 \alpha}}{\sin^2 \alpha}\right) \qquad (20)$$

If the plane of polarization is rotated by $\psi = \pi/4$, then (20) produces Brewster's law:

$$\tan \alpha_p = n$$

Fig. 7 shows the measured rotation of the plane of polarization as a function of the angle of incidence in good agreement with the values calculatd according to (20).

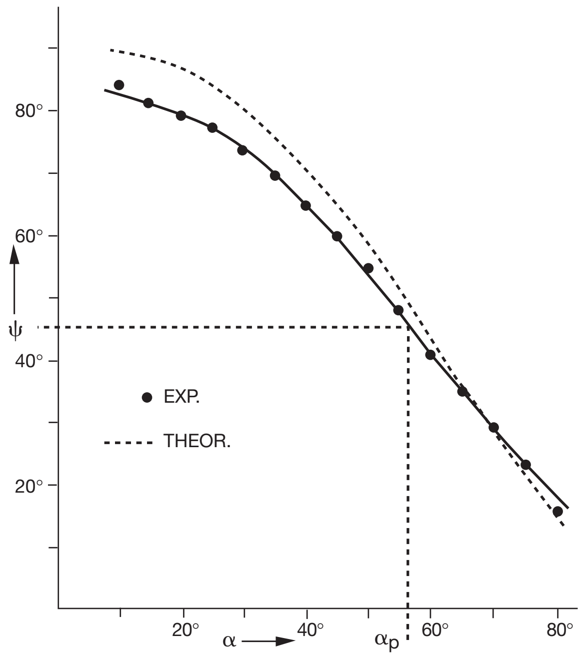


Fig. 7: Measured and calculated curves for the rotation of the direction of oscillation as function of the angle of incidence (azimuth of the incident beam is 45°).