Diffraction intensity through a slit and a wire – Babinet's theorem

LEP
2.3.06

Related topics

Huygens' principle, interference, Fraunhofer and Fresnel diffraction, Babinet's theorem, Poissons' spot, coherence, laser

Principle and task

An aperture consisting of a single slit and a complementary strip (wire) is illuminated with a laser beam. The corresponding diffraction patterns are measured according to position and intensity with a photocell which can be shifted.

Equipment

Laser, He-Ne 1.0 mw, 220 V AC	08181.93	1
Universal measuring amplifier	13626.93	1
Optical profile bench l = 60 cm	08283.00	1
Base f. opt. profile-bench, adjust.	08284.00	2
Slide mount f. opt. pr.-bench, h 80 mm	08286.02	2
Slide mount, lateral.adjust., cal.	08082.03	1
Object holder, 5×5 cm	08041.00	1
Photoelement f. opt. base plt.	08734.00	1
Screen, with diffracting elements	08577.02	1
PEK carbon resistor 1 W 5 % 2.2 kOhm	39104.23	1
Multi-range meter A	07028.01	1
Connecting cord, 750 mm, red	07362.01	1
Connecting cord, 750 mm, blue	07362.04	1

Problems

1. Determination of the intensity distribution of the diffraction patterns due to a slit and complementary strip (wire).
2. Determination of the intensity relations of the diffraction pattern peaks for the single slit.
3. Babinet's theorem is discussed using the diffraction patterns of the slit and the complimentary strip.

Set-up and procedure

The experimental set-up is shown in fig. 1. The laser beam should impinge in the centre of the diffracting object, so the diffraction pattern will have a symmetrical distribution of intensities. The photocell with its slit aperture is positioned in the centre of the shifting range.

The laser and the measuring amplifier should warm up for about 15 minutes before starting measurements, in order to avoid undesirable intensity fluctuations. The photocell is connected to the 10^4-Ω input of the measuring amplifier. The 2.2 kΩ resistor is connected in parallel to the photocell. When the amplification factor is changed, the zero point of the measuring amplifier must be checked.

Caution: Never look directly into a non attenuated laser beam

Fig. 1: Experimental set-up for the investigation of the diffraction intensity of slit and strip.
(The positions of the slide mounts and of the components on the optical bench are: laser and measurement slide mount each at an end of the bench; object holder = 19 cm).

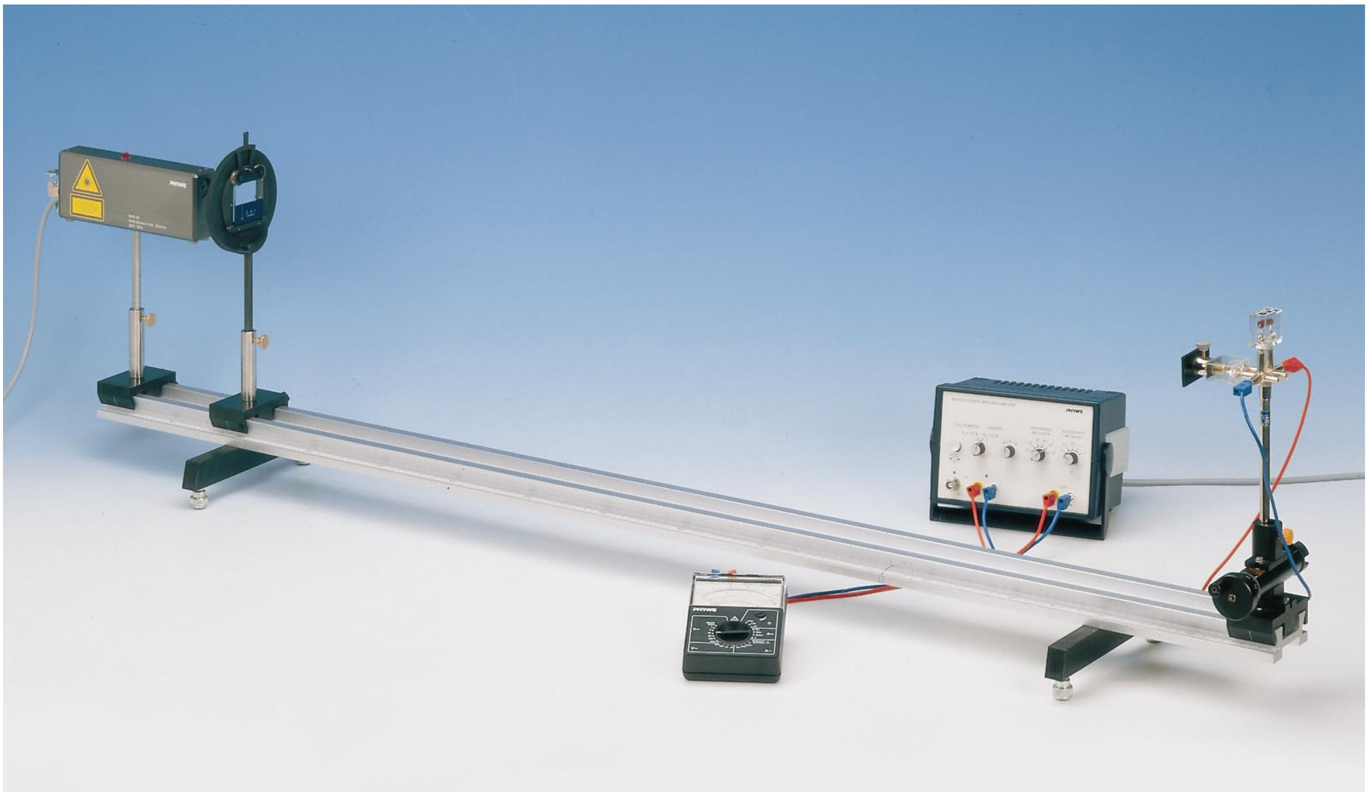
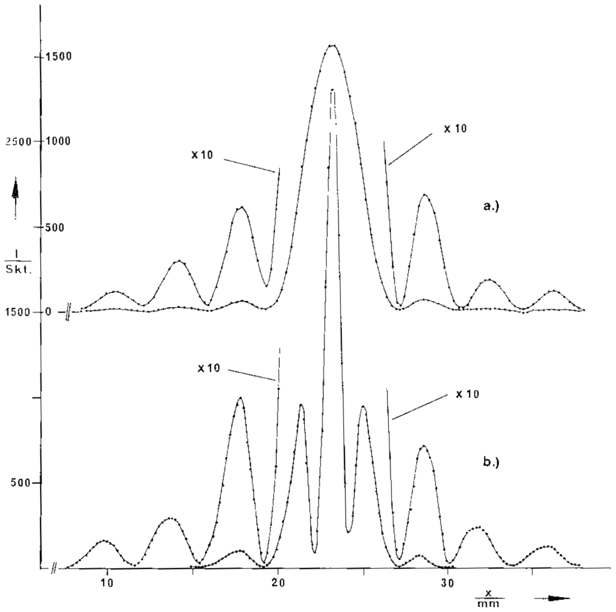


Fig. 2: Diffraction intensity I as a function of the position *x* for single slit a) and strip b).
Width of the diffracting object $b = 0.2$ mm.
The intensities in the areas next to the central peak are represented extended by a factor of 10.
(Distance between diffracting object and photocell $L = 120$ cm; Wavelength of the laser light $\lambda = 632.8$ nm)



Theory and evaluation

If monochromatic light of wavelength λ impinges on a slit with the width b, the following will be true for the light intensity I of beams deflected by an angle φ:

$$I(\varphi) \propto b^2 \frac{\sin^2\left(\frac{\pi}{\lambda} \cdot b \cdot \sin\varphi\right)}{\left(\frac{\pi}{\lambda} \cdot b \cdot \sin\varphi\right)^2} \qquad (1)$$

The zero points of the numerator give the angular positions of intensity minima:

$$\sin\varphi_k = \frac{k \cdot \lambda}{b}\ ;\ (k = 1,2,3,\ldots) \qquad (2)$$

The following is approximately valid for the intensity peaks

$$\sin\varphi_k{}^* = \frac{2k^* + 1}{2} \cdot \frac{\lambda}{b}\ ;\ (k^* = 1,2,3,\ldots) \qquad (3)$$

The particular case of the central peak for $\varphi = 0$ is not detected by equation (3).

Using (1) and (3), one obtains the following for the intensity peaks:

for $\sin\varphi = 0$ one has: $I_0 \propto b^2$ (principal maximum) (4)
for $\sin\varphi = 3/2\ \lambda/b$ one has: $I_1 \propto 0.045\ I_0$ (1st order peak)
for $\sin\varphi = 5/2\ \lambda/b$ one has: $I_2 \propto 0.016\ I_0$ (2nd order peak)
for $\sin\varphi = 7/2\ \lambda/b$ one has: $I_3 \propto 0.0083\ I_0$ (3rd order peak)

Fig. 2a shows the intensity distribution for diffraction through a single slit with the width $b = 0.2$ mm.
Table 1 gives the width of the slit calculated by means of equations (2) and (3), as well as the intensity relations to be compared to (4). Due to the somewhat different intensities of the secondary peaks of the same order, their arithmetic average is used every time for further evaluations.
The distance of the diffracting objects from the photocell is $L = 120$ cm; the wavelength of the laser light is $\lambda = 632.8$ nm

Table 1

b = 0.203 mm; $\Delta b/b = \pm 3\,\%$
I_1/I_0 = 65.5 Skt./1550 Skt. = 0.042
I_2/I_0 = 24.5 Skt./1550 Skt. = 0.016
I_3/I_0 = 12.5 Skt./1550 Skt. = 0.0081

The intensity relations determined empirically coincide very well with the values expected according to (4).
According to Babinet's theorem, the diffraction patterns of complementary diffracting objects outside the central diffraction pattern are identical. Fig. 2b shows the intensity distribution for diffraction due to the complementary strip with the width $b^* = 0.2$ mm.
Comparison with fig. 2a shows that the positions of the peaks and of the secondary peaks coincide exactly with those of the complementary slit. Only the central peak of the single slit is crossed symmetrically by two supplementary minima. In the case of Fraunhofer diffraction, one always finds brightness in the area of the geometrical shadow of the strip.