Diffraction at a slit and Heisenberg’s uncertainty principle

LEP 2.3.01

Related topics

Diffraction, diffraction uncertainty, Kirchhoff’s diffraction formula, measurement accuracy, uncertainty of location, uncertainty of momentum, wave-particle dualism, de Broglie equation.

Principle and task

The distribution of intensity in the Fraunhofer diffraction pattern of a slit is measured. The results are evaluated both from the wave pattern viewpoint, by comparison with Kirchhoff’s diffraction formula, and from the quantum mechanics standpoint to confirm Heisenberg’s uncertainty principle.

Equipment

Laser, He-Ne 1.0 mW, 220 V AC	08181.93	1
Diaphragm, 3 single slits	08522.00	1
Diaphragm, 4 double slits	08523.00	1
Diaphragm, 4 multiple slits	08526.00	1
Diaphragm holder	08040.00	1
Photoelement f. opt. base plt.	08734.00	1
Slide mount, lateral. adjust., cal.	08082.03	1
PEK carbon resistor 1 W 5% 2.2 kOhm	39104.23	1
Multi-range meter A	07028.01	1
Universal measuring amplifier	13626.93	1
Optical profile-bench, l 1500 mm	08281.00	1
Base f. opt. profile-bench, adjust.	08284.00	2
Slide mount f. opt. pr.-bench, h 80 mm	08286.02	2
Connecting cord, 500 mm, red	07361.01	1
Connecting cord, 500 mm, blue	07361.04	1

Problems

1. To measure the intensity distribution of the Fraunhofer diffraction pattern of a single slit (e.g. 0.1 mm).
   The heights of the maxima and the positions of the maxima and minima are calculated according to Kirchhoff’s diffraction formula and compared with the measured values.

2. To calculate the uncertainty of momentum from the diffraction patterns of single slits of differing widths and to confirm Heisenberg’s uncertainty principle.

Set-up and procedure

Different screens with slits (0.1 mm, 0.2 mm and 0.05 mm) are placed in the laser beam one after the other. The distribution of the intesnity in the diffraction pattern is measured with the photo-cell as far behind the slit as possible. A slit (0.3 mm wide) is fitted in front of the photocell. The voltage drop at the resistor attached parallel to the imput of the universal measuring amplifier is measured and is approximately proportional to the intensity of the incident light.

Important: In order to ensure that the intensity of the light from the laser is constant, the laser should be switched on about half an hour before the experiment is due to start. The measurements should be taken in a darkened room or in constant natural light. If this is not possible, a longish tube about 4 cm in diameter and blackened on the inside (such as a cardboard tube used to protect postal packages) can be placed in front of the photcell.

Caution: Never look directly into a non attenuated laser beam

Fig. 1: Experimental set-up for measuring the distribution of intensity in diffraction patterns.

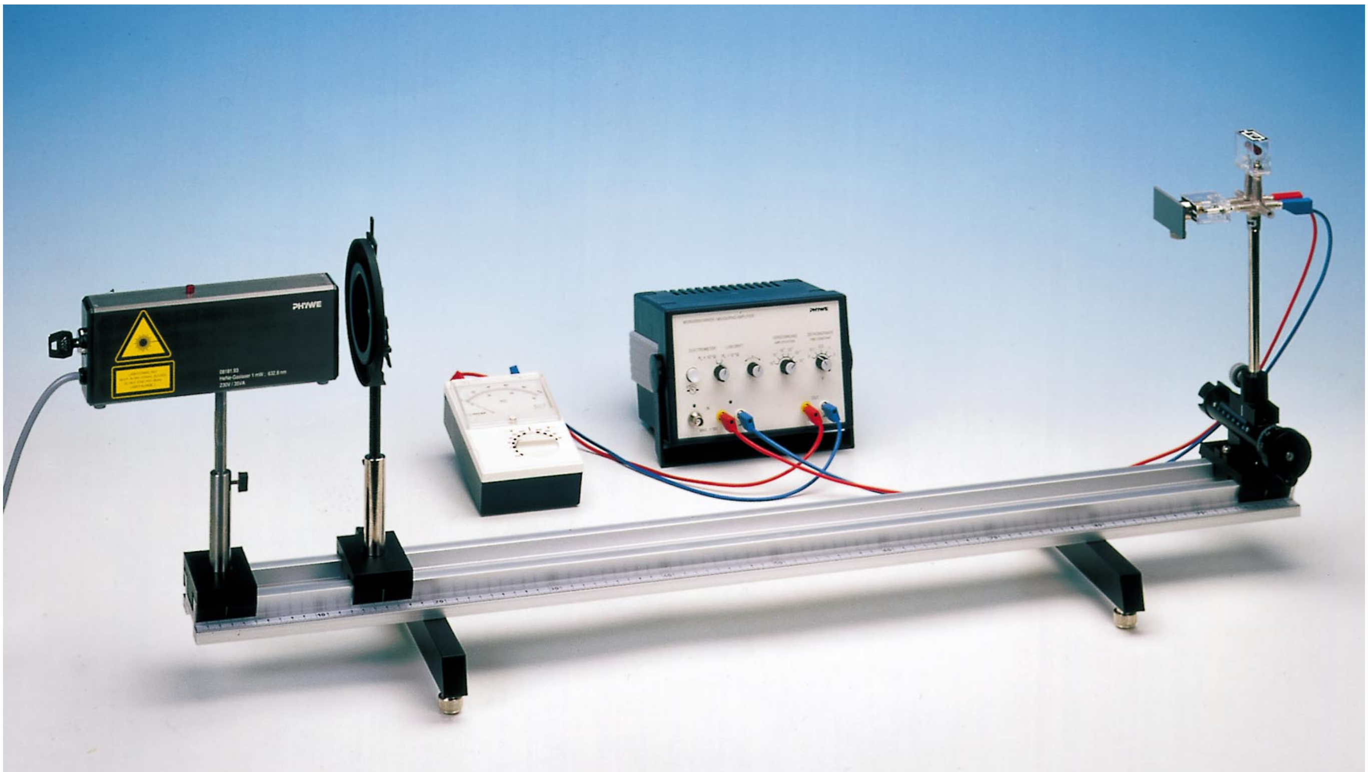
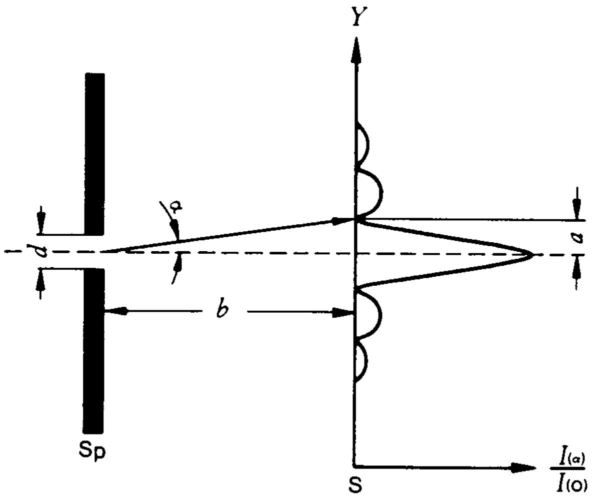


Fig. 2: Diffraction (Fraunhofer) at great distance (Sp = aperture or slit, S = screen).



The principal maximum, and the first secondary maximum on one side, of the symmetrical diffraction pattern of a slit 0.1 mm wide (for example) are recorded. For the other slits, it is sufficient to record the two minima to the right and left of the principal maximum, in order to determine α (Fig. 2).

Theory and evaluation

1. Ovservation from the wave pattern viewpoint
When a parallel, monochromatic and coherent light beam of wave-length λ passes through a single slit of width *d*, a diffraction pattern with a principal maximum and several secondary maxima appears on the screen (Fig. 2).

The intensity, as a function of the angle of deviation α, in accordance with Kirchhoff's diffraction formula, is

$$I(\alpha) = I(0) \cdot \left(\frac{\sin \beta}{\beta}\right)^2 \qquad (1)$$

where

$$\beta = \frac{\pi d}{\lambda} \cdot \sin \alpha$$

The intensity minima are at

$$\alpha_n = \arcsin n \cdot \frac{\lambda}{d}$$

where $n = 1, 2, 3 \ldots$

The angle for the intensity maxima are

$$\alpha'_0 = 0$$

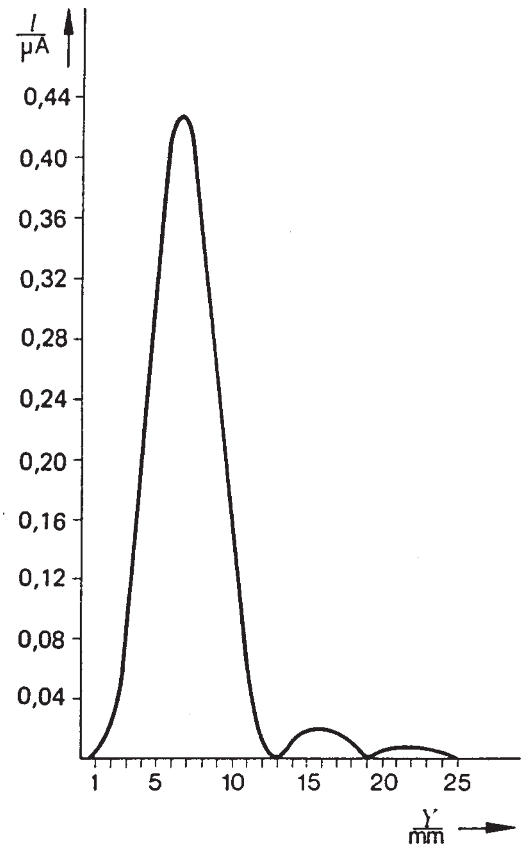
$$\alpha'_1 = \arcsin 1.430 \cdot \frac{\lambda}{d}$$

$$\alpha'_2 = \arcsin 2.459 \cdot \frac{\lambda}{d}$$

The relative heights of the secondary maxima are:

$$I(\alpha_1) = 0.0472 \cdot I(0)$$
$$I(\alpha_2) = 0.0165 \cdot I(0)$$

Fig. 3: Intensity in the diffraction pattern of a 0.1 mm wide slit at a distance of 1140 mm. The photocurrent is plotted as a function of the position.



The measured values (Fig. 3) are compared with those calculated.

Minima

Measurement	Calculation
$\alpha_1 = 0.36°$	$\alpha_1 = 0.36°$
$\alpha_2 = 0.72°$	$\alpha_2 = 0.72°$
$\alpha_3 = 1.04°$	$\alpha_3 = 1.07°$

Maxima

$\alpha_1 = 0.52°$	$\alpha_1 = 0.51°$
$\alpha_2 = 0.88°$	$\alpha_2 = 0.88°$
$\frac{I(\alpha_1)}{I(0)} = 0.044;$	$\frac{I(\alpha_1)}{I(0)} = 0.047$
$\frac{I(\alpha_2)}{I(0)} = 0.014;$	$\frac{I(\alpha_2)}{I(0)} = 0.017$

Kirchhoff's diffraction formula is thus confirmed within the limits of error.

2. Quantum mechanics treatment

The Heisenberg uncertainty principle states that two canonically conjugate quantities such as position and momentum cannot be determined accurately at the same time.

Let us consider, for example, a totality of photons whose residence probability is described by the function f_y and whose momentum by the function f_p. The uncertainty of location y and of momentum p are defined by the standard deviations as follows:

$$\Delta y \cdot \Delta p \geqq \frac{h}{4\pi} \qquad (2)$$

where $h = 6.6262 \times 10^{-34}$ Js, Planck's constant ("constant of action"), the equals sign applying to variables with a Gaussian distribution.

For a photon train passing through a slit of width d, the expression is

$$\Delta y = d \qquad (3)$$

Whereas the photons in front of the slit move only in the direction perpendicular to the plane of the slit (x-direction), after passing through the slit they have also a component in the y-direction.

The probability density for the velocity component ν_y is given by the intensity distribution in the diffraction pattern. We use the first minimum to define the uncertainty of velocity (Figs. 2 and 4).

$$\Delta \nu_y = c \cdot \sin \alpha_1 \qquad (4)$$

where α_1 = angle of the first minimum.

The uncertainty of momentum is therefore

$$\Delta p_y = m \cdot c \cdot \sin \alpha_1 \qquad (5)$$

where m is the mass of the photon and c is the velocity of light.

The momentum and wavelength of a particle are linked through the *de Broglie relationship:*

$$\frac{h}{\lambda} = p = m \cdot c \qquad (6)$$

Thus,

$$\Delta p_y = \frac{h}{\lambda} \sin \alpha_1 \qquad (7)$$

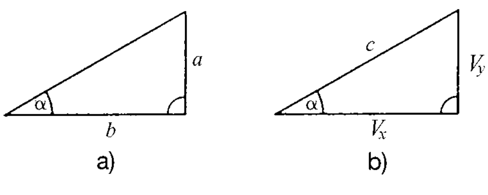


Fig. 4: Geometry of diffraction at a single slit
a) path covered
b) velocity compoment of a photon

The angle α_1 of the first minimum is thus

$$\sin \alpha_1 = \frac{\lambda}{d} \qquad (8)$$

according to (1).

If we substitute (8) in (7) and (3) we obtain the uncertainty relationship

$$\Delta y = \Delta p_y = h \qquad (9)$$

If the slit width Δy is smaller, the first minimum of the diffraction pattern occurs at larger angles α_1.

In our experiment the angle α_1 is obtained from the position of the first minimum (Fig. 4a):

$$\tan \alpha_1 = \frac{a}{b} \qquad (10)$$

If we substitute (10) in (7) we obtain

$$\Delta p_y = \frac{h}{\lambda} \sin \left(\arctan \frac{a}{b}\right) \qquad (11)$$

Substituting (3) and (11) in (9) gives

$$\frac{d}{\lambda} \sin \left(\arctan \frac{a}{b}\right) = 1 \qquad (12)$$

after dividing by h.

The results of the measurements confirm (12) within the limits of error.

Width of slit* d/mm	First minim α/mm	b/mm	$\frac{d}{\lambda} \sin \left(\arctan \frac{\alpha}{b}\right)$
0.101	7.25	1140	1.01
0.202	3.25	1031	1.01
0.051	10.8	830	1.05

* The widths of the slits were measured under the microscope.