Related topics

Interference, wavelength, phase, refraction index, light velocity, virtual light source.

Principle and task

A measurement cuvette set in the beam path of a Michelson interferometer can be evacuated or filled with CO_2. The refraction indexes of air or CO_2 are determined through the assessed modification of the interference pattern.

Equipment

Michelson interferometer	08557.00	1
Laser, He-Ne 1.0 mw, 220 V AC	08181.93	1
Glass cell for Faraday effect	08625.00	1
Manual vacuum pump	08745.10	1
Optical profile bench l = 60 cm	08283.00	1
Base f. opt. profile-bench, adjust.	08284.00	2
Slide mount f. opt. pr.-bench, h 30 mm	08286.01	3
Slide mount f. opt. pr.-bench, h 80 mm	08286.02	1
Swinging arm	08256.00	1
Lens holder	08012.00	1
Lens, mounted, f +20 mm	08018.01	1
Digital manometer	03106.00	1
Compressed gas, CO_2, 21 g	41772.06	1
Fine control valve	33499.00	1
Right angle clamp -PASS-	02040.55	1
Universal clamp	37715.00	1
Barrel base -PASS-	02006.55	1
Screen, metal, 300×300 mm	08062.00	1
Tubing adaptor, ID 3-6/7-11 mm	47517.01	1
Tubing connect., Y-shape, ID 8-9 mm	47518.03	1
PVC tubing, i.d. 7 mm	03985.00	1
Silicon tube int. diam. 3 mm	39292.00	1

Set-up and procedure

The experimental set-up is shown in fig. 1. The cuvette is carefully fastened in the support previously installed. The f = 20 mm lens situated roughly in the middle between the laser and the interferometer is used to broaden the laser beam. The manual pump and the CO_2 pressure bottle are connected to the cuvette over the Y-shaped connector, which is fixed by a double clamp to the stem of the lens support, using flexible PVC tubes (∅ = 7 mm) of adequate length. The reducing piece is used to connect the pressure probe with the cuvette; it is fastened with the universal clamp. In order to assure adaptation to the diameter of the PVC flexible tube, a small piece of flexible silicone tube (∅ = 3 mm) is previously slid over the measuring sleeve of the pressure probe.

The light spots observed on the screen are brought to coverage using the two adjustment screws at the back of one of the mirrors. Continue adjusting until concentric interference rings appear.

At the beginning of measurement, the second mirror is set with the screw micrometer so that the centre of the interference rings is dark. To determine the refraction index of air,

Caution: Never look directly into a non attenuated laser beam

Fig. 1: Experimental set-up: Refraction index of air and CO_2 with Michelson interferometer.

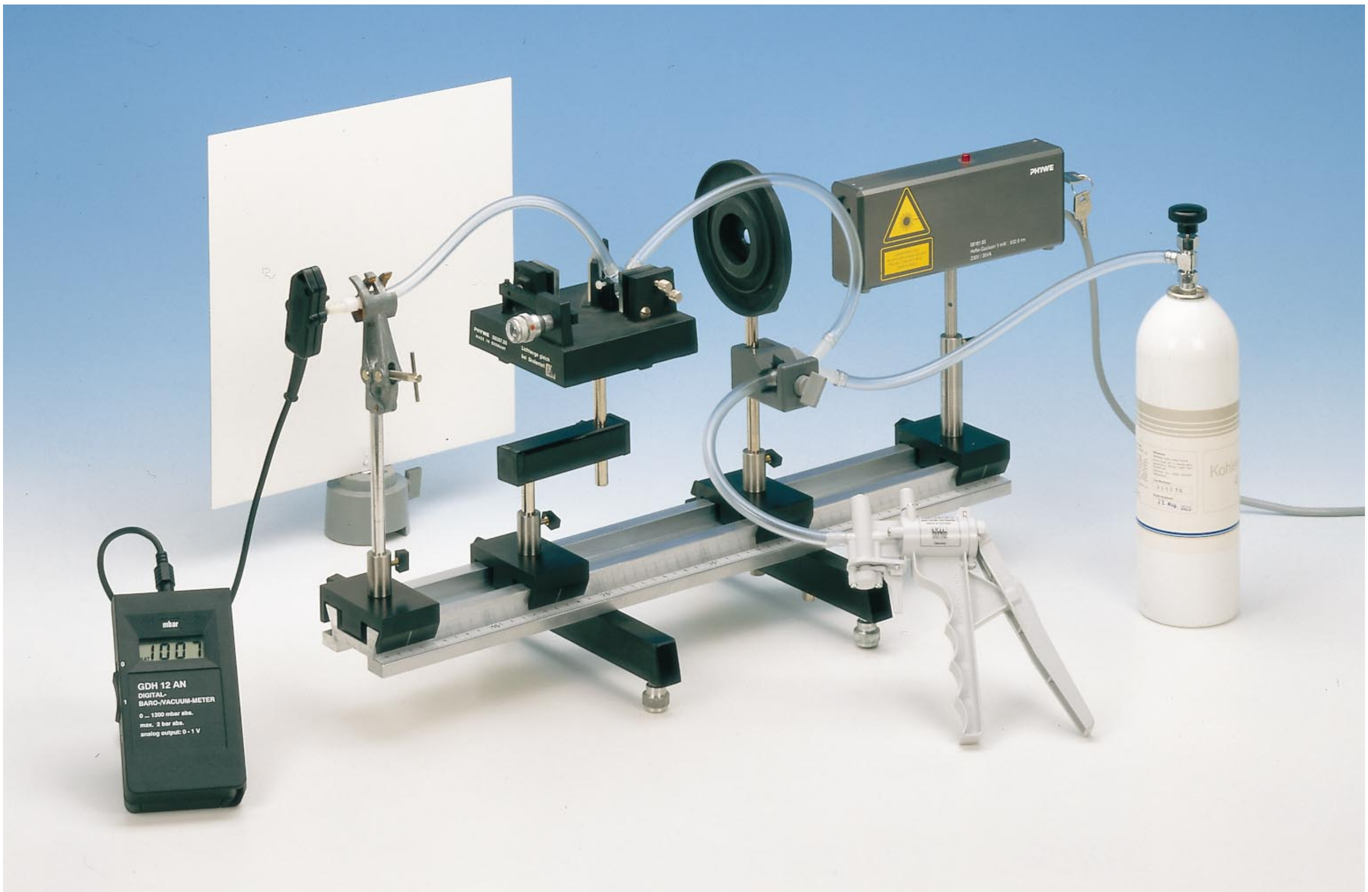
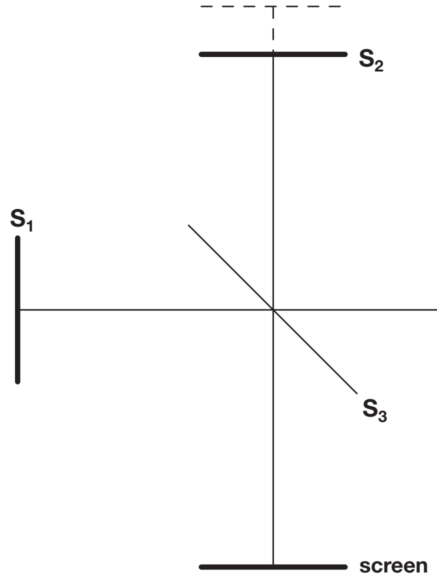


Fig. 2: Schematic representation of Michelson's set-up.



pressure in the cuvette is gradually reduced with the manual pump. During this process, the interference rings draw together, and alternating darkness and light is observed at the centre. The number N of appearing minima is plotted against the corresponding values of air pressure. The measurement is repeated several times, and the cuvette is vented over the manual pump between every measurement series.
To determine the refraction index of CO_2, the cuvette is carefully filled with this gas over the reducing valve. During this process, the number of interference rings coming out of the centre of the interference system must be determined.
Before measuring again, the CO_2 gas must be removed from the cuvette. For this, evacuate the cuvette with the manual pump and fill it again with air. Repeat this procedure several times.
If the change of the interference pattern is too fast to permit reliable counting, measurement can be carried out alternatively as follows:
the cuvette, filled with CO_2 at atmospheric pressure, is evacuated again; the change of interference patterns can be easily followed during this process. The value for CO_2 gas is determined from the corresponding values for air.
Attention: to avoid damages to the cuvette due to overpressure, it is imperative to remove the pressure probe during this measurement, so the outlet channel of the cuvette will be free.

Theory and evaluation

In Michelsonís interferometer, a light beam is split into two partial beams of equal amplitude by means of a semi transparent glass plate S_3 (fig. 2). The light beams are reflected on mirrors S_1 and S_2, and after having travelled different paths, they are reunited after the glass plate. Depending on the path difference and on the resulting phase shift between the partial beams, constructive or destructive interference is observed.
If two waves of the same frequency ω (but different amplitudes a and with different phases α impinge at the same place, they will overlap to a wave

$$y = a_1 \sin(\omega t - \alpha_1) + a_2 \sin(\omega t - \alpha_2) \quad (1)$$

The resulting wave can be expressed as:

$$y = A \sin(\omega t - \delta) \quad (2)$$

with the following amplitude

$$A^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos\delta \quad (3)$$

and phase relation

$$\delta = \alpha_1 - \alpha_2 \quad (4)$$

Now imagine one mirror is turned towards the other one, so that they are both parallel and separated by a distance d (fig. 3). When observed under an angle Θ, the two reflected images of the light source are separated by a distance $2d$. Constructive interference will appear when

$$2d \cos\Theta = m\lambda \;; m = 1,2,3,\ldots \quad (5)$$

(λ = wavelength of the used light)

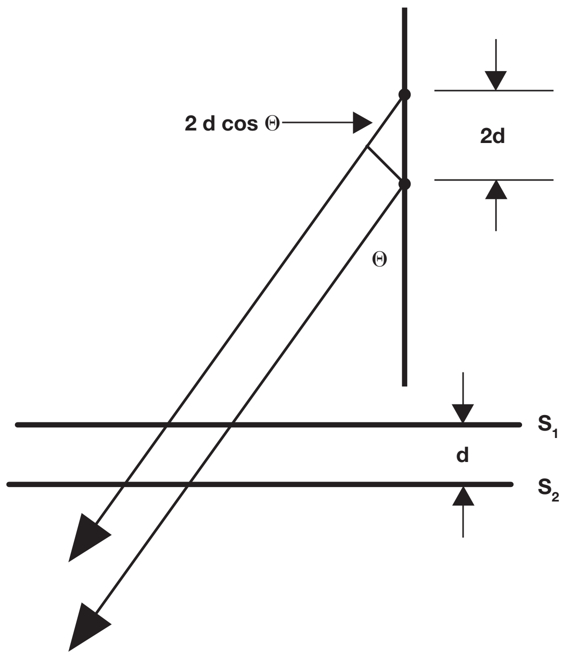


Fig. 3: Schematic representation of the determination of the phase position of the interfering partial beams.

As Θ remains constant, circular rings appear for determined values of m and d.
If both partial beams have the same amplitude $a = a_1 = a_2$, using (3), the following holds for the intensity distribution:

$$I \propto A^2 = 4a^2 \cos^2\frac{\delta}{2} \text{ with } \delta = \frac{2\pi}{\lambda} 2d \cos\Theta \quad (6)$$

Peaks will appear whenever Θ is a multiple of 2π
If the movable mirror is shifted by $\Delta d = N \lambda/2$, N new interference rings will appear.

Determination of the refraction index of air

The refraction index n of a gas is linearly dependent on pressure p:

$$n(p) = n(p = 0) + \frac{\Delta n}{\Delta p} \cdot p \qquad (7)$$

Theoretically, if $p = 0$, absolute vacuum prevails and $n = 1$.
To start with, the difference quotient $\Delta n/\Delta p$ is determined from the measurement data:

$$\frac{\Delta n}{\Delta p} = \frac{n(p+\Delta p)-n(p)}{\Delta p} \qquad (8)$$

The following is valid for the optical path x:

$$x = n(p) \cdot s \qquad (9)$$

where s is the geometrical length of the evacuated cuvette and $n(p)$ the refraction index of the gas contained in the cuvette. Varying pressure in the cuvette by Δp, the optical wavelength is changed by Δx:

$$\Delta x = n(p + \Delta p) \cdot s - n(p) \cdot s \qquad (10)$$

Starting off at ambient pressure p_0 and decreasing pressure, one observes N times the restoration of the initial position of the interference pattern (e. g. an intensity minimum at the centre of a ring) up to a given pressure p. A change from minimum to minimum corresponds to a change of the optical path by a wavelength λ. Thus, between pressures p and Δp, the optical path changes by:

$$\Delta x = \{n(p) - n(p + \Delta p)\} \cdot \lambda \qquad (11)$$

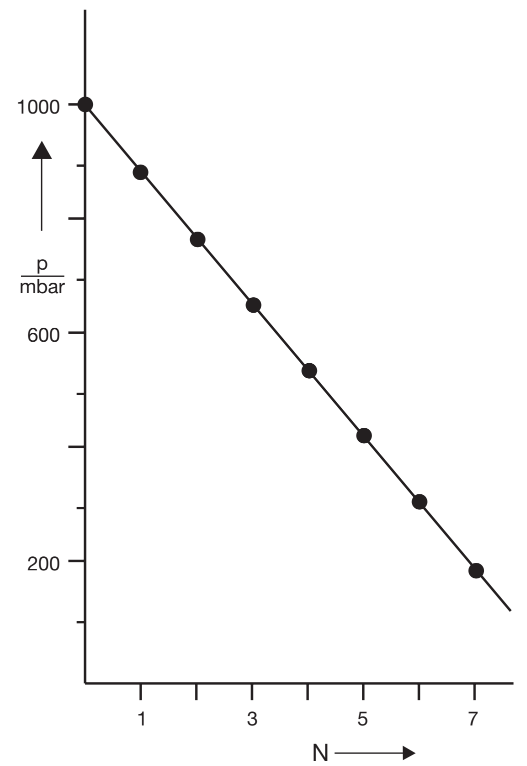


Fig. 4: Number N of minima changes as a function of air pressure in the measuring cuvette.

Taking into account the fact that light travels twice through the cuvette, (10) and (11) yield:

$$n(p+\Delta p) - n(p) = \{N(p) - N(p+\Delta p)\} \cdot \frac{\lambda}{2s} \qquad (12)$$

and with (8) one obtains:

$$\frac{\Delta n}{\Delta p} = -\frac{\Delta N}{\Delta p} \cdot \frac{\lambda}{2s} \qquad (13)$$

In fig. 4, the number of changes of minima is plotted against the corresponding pressures p, up to $N = 7$.

Using (13) and (7) (with $s = 10$ mm, $\lambda = 632.8$ nm and n (p = 0) = 1), the slope of the straight line, $\Delta N/\Delta p = 8{,}45 \times 10^{-3}$ mbar^{-1}, yields a value of $n = 1.000268$. The ambient atmospheric pressure was 1004 mbar.

The corresponding value found in literature for normal pressure ($p = 1013$ mbar) at a temperature of 22 °C and for a wavelength of $\lambda = 632.8$ nm, is $n = 1.00269$.

Determination of the refraction index of CO_2

Air and CO_2 have different refraction indexes, so the change of refraction index is proportional to the amount of CO_2 introduced into the cuvette. For the different optical paths x in the cuvette, one thus finds:

$x_1 = n_1 \cdot 2s$ for air with a refraction index n_1 (14)

$x_2 = n_2 \cdot 2s$ for CO_2 with a refraction index n_2 (15)

The change of optical path Δx is determined by counting the changes between peaks and minima. A change from a peak to a minimum corresponds to a change by $\lambda/2$. If N changes occur, the total change of the optical path is:

$$\Delta x = N \cdot \frac{\lambda}{2} \qquad (16)$$

and thus, the following is valid:

$$x_2 = x_1 + \Delta x \; ; \; n_2 = n_1 + \Delta n \qquad (17)$$

From (14), (15) and (17), one obtains:

$$\Delta x = 2s \cdot \Delta n \qquad (18)$$

Measurements show that when the cuvette is filled with CO_2 gas, 9 changes between peaks and minima are observed. This corresponds to a change of the optical path by $\Delta x = 4{,}5\ \lambda$.
One thus obtains from (18):

$$\Delta n = \frac{4{,}5\lambda}{2s} = 1{,}42 \cdot 10^{-4} \qquad (19)$$

When the refraction index of air decreases during the evacuation of the cuvette, one observes that the interference rings draw together towards the centre. The opposite behaviour is observed during the filling process with CO_2 gas, that is, the interference rings come out of the centre. This allows to conclude that the refraction index of CO_2 is larger than that of air. With the value given in (19) and the result found for air, one finally obtains that $n(CO_2) = 1.000410$. The corresponding value found in literature for normal pressure and at 20 °C is $n_{(Lit.)} = 1{,}000416$.