Fresnel's zone construction / Zone plate

Related topics

Huygens-Fresnel principle, Fresnel and Fraunhofer diffraction, interference, coherence, Fresnel zone construction, zone plates.

Principle and task

A zone plate is illuminated with parallel laser light. The focal points of several orders of the zone plate are projected on a ground glass screen.

Equipment

Laser, He-Ne 1.0 mW, 220 V AC	08181.93	1
Zone plate, after fresnel	08577.03	1
Lens holder	08012.00	4
Lens, mounted, f + 20 mm	08018.01	1
Lens, mounted, f + 50 mm	08020.01	1
Lens, mounted, f +100 mm	08021.01	1
Lens, mounted, f – 50 mm	08026.01	1
Object holder, 5×5 cm	08041.00	2
Ground glass screen, 50×50×2 mm	08136.01	1
Polarising filter, 50 × 50 mm	08613.00	1
Optical profile-bench, l 1000 mm	08282.00	1
Base f. opt. profile-bench, adjust.	08284.00	2
Slide mount f. opt. pr.-bench, h 30 mm	08286.01	7

Problems

1. The laser beam must be widened so that the zone plate is well illuminated. It must be assured that the laser light beam runs parallel over several meters.
2. The focal points of several orders of the zone plate are projected on a ground glass screen. The focal lengths to be determined are plotted against the reciprocal value of their order.
3. The radii of the zone plate are calculated.

Set-up and procedure

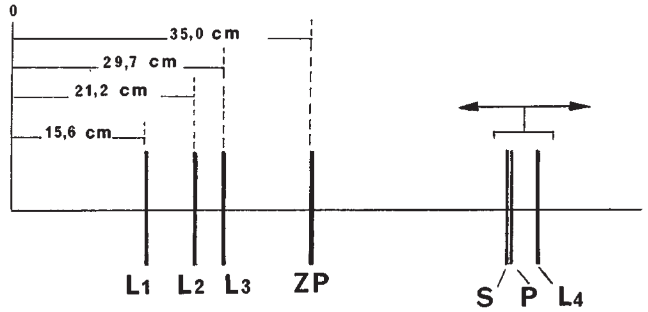
Fig.1 shows the complete experimental set-up. The slide mount for the laser is placed at the head of the optical bench. To start with, the laser beam is widened with lenses L_1, L_2 and L_3 to a diameter of approx. 5 mm (cf. Fig. 2). Careful shifting of lenses L_2 and L_3 allows to make the laser beam parallel over a length of several meters (maximum 10 m). The correct values for the different focal lengths of the zone plate can only be obtained under this condition. For this purpose, a piece of black cardboard, into which a hole is punched with a desk punch and which is used as a test diaphragm, proves useful. The other components should then be mounted, making sure the zone plate is well illuminated. The image of the zone plate is observed on the ground glass plate, which is located nearly at the end of the optical bench at the beginning, with magnifying lens L_4. Moving the ground glass screen and L_4 in the direction of the zone plate simultaneously, the different focal points of the zone plate are searched for and the corresponding focal lengths are determined. The polarising filter, which is used to reduce the brightness of the image, is set together with the ground glass screen in the same mounting frame.

Caution: Never look directly into a non attenuated laser beam

Fig. 1: Experimental set-up to determine the different focal lengths of a zone plate.

Fig. 2: Position of the optical components.

f (L_1) = + 20mm	ZP = zone plate
f (L_2) = − 50mm	S = ground glass screen
f (L_3) = + 100mm	P = Polarisation filter
f (L_4) = + 50mm	



Theory and evaluation

According to Fresnel, interference of waves diffracted by obstacles may be treated simply by splitting the primary wave front into so called zones. The optical path difference from the common boundaries of a zone pair up to a point of observation P is always λ/2. Secondary waves originating from neighbouring zones impinge in P with opposed phases, thus extinguishing each other except for a part coming from the first zone.

Using a so called zone plate, which consists of alternating transparent and opaque circles, it is possible to let either the odd or the even zones exert an influence at a point of observation P.

If the number of zones is 2k, the amplitude A at point P is (under the justified assumption that the secondary waves have the same amplitude, due to the fact that the areas of the single zones are equal):

$$A = A_1 + A_3 + A_5 + ... + A_{2k-1};\ A \approx kA_1 \qquad (1)$$

At the point of observation P, the amplitude A without zone plate is $^1/_2$ A1 (contribution of half of the first zone). Using a zone plate, it is thus possible to increase light intensity at P by a factor of $4k^2$. This means that the zone plate acts as a focusing lens.

In Fig.3, the first rings of a zone plate illuminated by a plane wave (parallel beam) are shown.

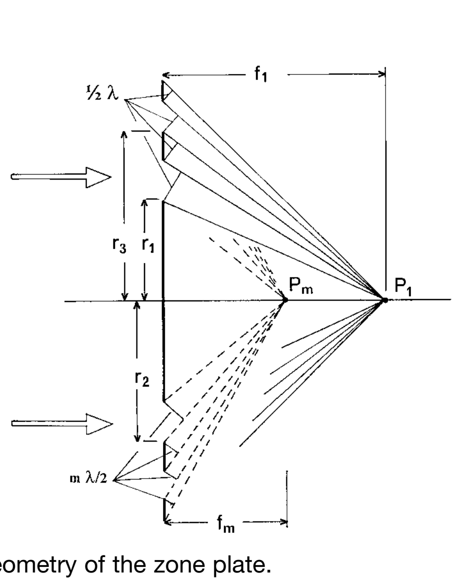


Fig. 3: Geometry of the zone plate.

Assuming the distance between P and the centre of the zone plate to be f_1, in case of constructive interference at P, the following holds for radii r_n (n=1,2,3 ...):

$$r^2_n = \left(f + n\frac{\lambda}{2}\right)^2 - f^2 = f^2 + nf\lambda + n^2\frac{\lambda^2}{4} = nf\lambda;$$

$$\text{with } nf\lambda >> n^2\frac{\lambda^2}{4} \qquad (2)$$

For the radii r_n of the zone plate and the focal length *f* we thus have:

$$r_n = (nf\lambda)^{1/2};\ f = r_n^2 \cdot \frac{1}{n\lambda} \qquad (3)$$

If the point of observation P is shifted along OP towards the zone plate, alternating brightness and darkness are observed, which means that the zone plate has several focal points.

$$f_m = f_1/m \ (m = 1, 3, 5, 7, ...) \qquad (4)$$

The existence of these focal points of higher order is due to the difference in the optical path of the zone rays of 3/2λ, 5/2λ, 7/2λ, 9/2λ ...

The zone plate used for the experiment has 20 zones, the radius of the first dark central circle is r_1 = 0.6 mm. The following radii are found to be:

$$r_n = n^{1/2} \cdot 0.6 \text{ mm} \qquad (5)$$

In Table 1, the averaged experimental values are compared to the values calculated according to (3), (4) and (5) and with λ = 632.8 nm. Fig 4 shows the empirical focal lengths as a function of the inverse value of the order of the focal points.

m	f(theor.)/cm	f(exp.)/cm	n	r(theor.)/mm	r(exp.)/mm
1	56.9	57.8	1	0.60	0.61
3	19.0	19.3	2	0.85	0.86
5	11.4	11.7	3	1.04	1.05
7	8.1	8.5	4	1.20	1.21
9	6.3	6.6	5	1.34	1.35

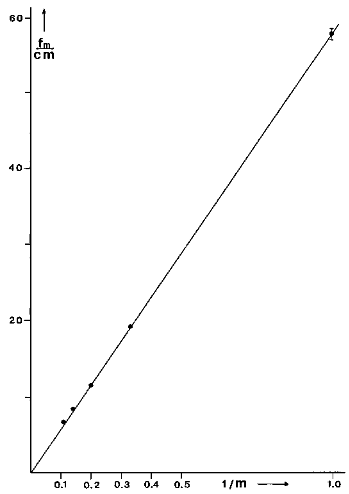


Fig. 4: Focal length of first and higher order of the zone plate as a function of the reciprocal value of the order.