## Related topics

Interference of equal inclination, interference of thin layers, plane parallel plate, refraction, reflection, optical path difference.

## Principle and task

Monochromatic light falls on a plane parallel mica plate. The light rays, reflected at the front surface as well as at the rear surface, will interfere to form a pattern of concentric rings. The radii of the rings depend on the geometry of the experimental set-up, the thickness of the mica plate and the wavelength of the light.

## Equipment

## Mica plate

Colour filter, 440 nm
08558.00

Colour filter, 525 nm
08411.00

Colour filter, 580 nm
Spectral lamp Hg 100, pico 9 base
08414.00
08415.00
08120.14

Spectral lamp Na , pico 9 base
Lamp holder, pico 9, f. spectr.lamps
Swinging arm
Plate holder with tension spring
Screen, metal, $300 \times 300 \mathrm{~mm}$
08120.07
08119.00
$08256.00-2$
08288.00
08062.00
02060.00
$08283.00 \quad 1$
08284.002
08286.022
02010.002
$09936.00 \quad 1$

## Problems

The experiment will be performed with the light of a Na-lamp and with the light of different wavelengths of a Hg -vapour tube.

1. The thickness of the mica plate is determined from the radii of the interference rings and the wavelength of the Na lamp.
2. The different wavelengths of the Hg -vapour tube are determined from the radii of the interference rings and the thickness of the mica plate.

## Set-up and procedure

The set-up of the experiment is done in accordance with Fig. 1.

Using the Hg-vapour lamp, a colour filter mounted on the swinging arms is placed directly in front of the lamp.

The metal screens are put next to each other and fixed at the edge of the table. The optical bench is arranged vertically to the metal screens.

After switching on the power supply the spectral lamp needs about one minute to warm up and to reach its full brightness. We observe the interference rings and, in their center, the shadow of the lamp-housing on the screens. The optical bench is now shifted laterally until most of the interference rings can be seen at one side of the lamp-housing. Still, a few

Fig. 1: Experimental set-up for the observation of the interferenc ephenomena at a mica plate.

2.2.03

Fig. 2: Appearance of interference rings at a plane parallel plate.

rings must remain visible on the other side of the lamp-housing to be able to determine the center of the ring system.

The radii $r_{\mathrm{m}}$ of the bright and dark interference rings were measured for the yellow, green and blue light of the Hg -vapour lamp.

## Theory and evaluation

A light ray falling under an angle of incidence $\alpha$ on a plane parallel plate of thickness $d$ and refractive index $n$ is partially reflected at the front surface (ray a) partially refracted into the plate (angle of refraction $\beta$ ). The refracted ray is partially reflected at the lower (rear) surface and, that way, again reaches the upper surface where a part of it leaves the plate as ray $b$ parallel to ray $\alpha$. Many parallel light rays will interfere with each other in the refelcted light as well as in the transmitted light. The light-source $L$ can be as extended as disired. All rays having the same angle of incidence will interfere with each other. The contrast of the interference rings is better within the refelcted pattern than within the transmitted one. The ratio of the light-intensities of two neighbouring rays is approximately 10:9 within the reflected pattern, i.e. the amplitudes of the waves are approximately equal.
From Fig. 2 we learn for which angles $\alpha$ we obtain constructive or destructive interference respectively.

The optical path difference between the rays $\alpha$ and $b$ is

$$
\begin{equation*}
\Delta=2 n S_{2}+\frac{\lambda}{2}-S_{1} \tag{1}
\end{equation*}
$$

with

$$
\begin{equation*}
S_{2}=\frac{d}{\cos \beta} \tag{2}
\end{equation*}
$$

and

$$
\begin{equation*}
S_{1}=2 d \tan \beta \sin \alpha \tag{3}
\end{equation*}
$$

The phase step of $\lambda / 2$ is due to the fact that the ray $\alpha$ is reflected at a medium which is optically "denser".

Using the general law of refraction

$$
\begin{equation*}
n=\frac{\sin \alpha}{\sin \beta} \tag{4}
\end{equation*}
$$

and substituting (2) and (3) into (1) leads to

$$
\Delta=\frac{2 n d}{\cos \beta}-2 n d \frac{\sin \beta}{\cos \beta} \sin \beta+\frac{\lambda}{2}
$$

respectively

$$
\Delta=2 n d \cos \beta+\frac{\lambda}{2}
$$

and

$$
\begin{equation*}
\Delta=2 d \sqrt{n^{2}-\sin ^{2} \alpha}+\frac{\lambda}{2} \tag{5}
\end{equation*}
$$

Bright interference rings can be observed if

$$
\begin{equation*}
\Delta=m \cdot \lambda \quad m=\text { integer } \tag{6}
\end{equation*}
$$

Dark interference rings are received for

$$
\begin{equation*}
\Delta=m \lambda+\frac{\lambda}{2} \quad m=\text { integer } \tag{7}
\end{equation*}
$$

The square root of equation (5) is now expanded in series after factorizing $n^{2}$ :

$$
2 d n \cdot \sqrt{1-\left(\frac{\sin \alpha}{n}\right)^{2}} \simeq 2 d n\left(1-\frac{1}{2} \frac{\sin ^{2} \alpha}{n^{2}}\right)
$$

for

$$
\frac{\sin ^{2} \alpha}{n^{2}} \ll 1
$$

higher order terms can be neglected.
That way we get an approximation for the dark interference rings using equations (7) and (5)

$$
\begin{equation*}
m \lambda=2 n d-\frac{d}{n} \sin ^{2} \alpha \tag{8}
\end{equation*}
$$

That interference order $m$ of the rings plotted versus $\sin ^{2} \alpha$ results in a straight line whose slope is $d / n \cdot \lambda$.

The interference order increases with decreasing radii ( $\Delta$ increases with decreasing values of $\alpha!$ ). Since the absolute value of $m$ for a particular ring is not known, the initial value $m_{0}$ is arbitrarily attributed to the innermost dark ring, in agreement with equation (8). This way, the following dark rings will have


Fig. 3: Geometry of the experimental set-up.

Fig. 4: Interference order $m$ as a function of $\sin ^{2} \alpha$ for Na -light.

the interference orders $m_{0}-1, m_{0}-2$ etc. Consequently, all bright interference rings which are inbetween the dark rings will have the uneven interference orders

$$
m_{0}-\frac{1}{2}, m_{0}-\frac{3}{2} \text { etc. }
$$

The angle $\alpha$ can be calculated from the radius $r_{\mathrm{m}}$ of the ring under consideration and the light-path I (distance: spectral lamp, mica plate, screen, see Fig. 3)

$$
\sin \alpha=\frac{r_{m}}{\sqrt{r_{m}^{2}+r^{2}}}
$$

1. Fig. 4 shows the interference order $m$ of the interference rings as a function of $\sin ^{2} \alpha$ for the Na-light. For the slope of the regression line through the measured values we find:

$$
\frac{d}{n \lambda}=55.5
$$

with $n=1.6$ and $\lambda=589 \mathrm{~nm}$ we obtain for the thickness of the mica plate

$$
d=0.052 \mathrm{~nm} .
$$

Fig. 5: Inteference order $m$ as a function of $\sin ^{2} \alpha$ for the blue, green, and yellow spectral lines of the Hg-lamp.

2. Fig. 5 shows the interference order as a function of $\sin ^{2} \alpha$ for the blue, green, and yellow spectral lines of the Hg-lamp. For the slopes of the regression lines through the measured values we obtain:

| blue: | 73.1 |
| :--- | :--- |
| green: | 58.9 |
| yellow: | 56.4 |

with $n=1.6$ and $d=0.052 \mathrm{~mm}$ we obtain for the wavelengths of the Hg -spectral line

|  |  | Literature value |
| :--- | :--- | :---: |
| blue: | $\lambda=445 \mathrm{~nm}$ | $(436 \mathrm{~nm})$ |
| green: | $\lambda=552 \mathrm{~nm}$ | $(546 \mathrm{~nm})$ |
| yellow: | $\lambda=576 \mathrm{~nm}$ | $(578 \mathrm{~nm})$ |

