Related topics

Fraunhofer diffraction, grating spectroscope, angular dispersion of a diffraction grating, resolving power, Rayleigh criterion, spectral lines.

Principle and task

The angle of diffraction of the spectral lines of a mercury vapour lamp is determined with a grating spectroscope and the grating constant is calculated. The resolving power necessary to separate certain spectral lines is determined.

Equipment

Spectrometer/goniom. w. vernier	35635.02	1
Power supply for spectral lamps	13662.93	1
Spectral lamp Hg 100, pico 9 base	08120.14	1
Lamp holder, pico 9, f. spectr.lamps	08119.00	1
Diffraction grating, 600 lines/mm	08546.00	1
Diffraction grating, 4 lines/mm	08532.00	1
Diffraction grating, 8 lines/mm	08534.00	1
Diffraction grating, 10 lines/mm	08540.00	1
Diffraction grating, 50 lines/mm	08543.00	1
Vernier caliper	03010.00	1
Tripod base -PASS-	02002.55	1
Barrel base -PASS-	02006.55	1
Right angle clamp -PASS-	02040.55	1
Support rod -PASS-, square, l 250 mm	02025.55	1

Problems

1. Determination of the grating constant of a Rowland grating based on the diffraction angle (up to the third order) of the high intensity spectral lines of mercury.
2. Determination of the angular dispersion of a grating.
3. Determination of the resolving power required to separate the different Hg-Lines. Comparison with theory.

Set-up and procedure

The experimental set-up is shown in Fig. 1. To start with, the telescope is adjusted to infinite distance. Then both tubes are adjusted horizontally with the adjusting screws and finally they are adjusted so that the directions of their axes coincide. The Hg-lamp is placed directly before the slit and must illuminate it completely. A sharp image of the slit is formed in the plane of the eyepiece scale and is observed using the eyepiece lens as a magnifying lens. The slit should be selected as narrow as possible.

To start with, the grating constant of the high resolution Rowland grating is determined. For this, the grating is set perpendicular to the collimator axis and the grating table is fixed. The diffraction angles of the 6 high intensity Hg spectral lines are determined for the first and second order. Furthermore, recognisable third order lines should also be evaluated. The angle 2φ of a spectral line of the same order of diffraction is measured to the right and to the left of the zero order. Two measurement readings are taken for every angle (two verniers).

Usually, the eyepiece scale is difficult to see, due to reduced brightness for higher orders diffraction. In these cases, better visibility may be obtained by lighting the grating askew from the direction of the telescope with a torch light.

The number of illuminated grating slits is reduced to determine the resolving power of the grating. For this purpose, a slide caliper is placed as an auxiliary slit in front of the collimator lens in such a way, that no light reaches the grating when the

Fig. 1: Experimental set-up with vernier used as auxiliary slit to determine the resolving power.

caliper is closed (cf. Fig. 1).The auxiliary slit is then opened so that for example the yellow and green lines of Hg can be observed as clearly separate lines. The width x of the auxiliary slit is then reduced until the two lines merely appear separated. The average width of the auxiliary slit is determined over several experimental runs. Gratings with up to 50 lines/mm are used to determine the resolution required for the yellow-green lines. The Rowland grating is used to separate the pair of yellow Hg lines.

(Cf. Table 3 for grating type and useful order of diffraction)

Theory and evaluation

If monochromatic light with the wavelength λ impinges on a diffraction grating, the intensity diffracted according to the angle φ is given by:

$$I(\varphi) = I(0) \left(\frac{\sin u}{u}\right)^2 \left(\frac{\sin Nv}{\sin v}\right)^2 ;$$

$$\text{with } v = \pi \frac{g}{\lambda} \sin\varphi \text{ and } u = \pi \frac{s}{\lambda} \sin\varphi \qquad (1)$$

(s = width of the slit; g = distance between two slits = grating constant; N = number of slits)

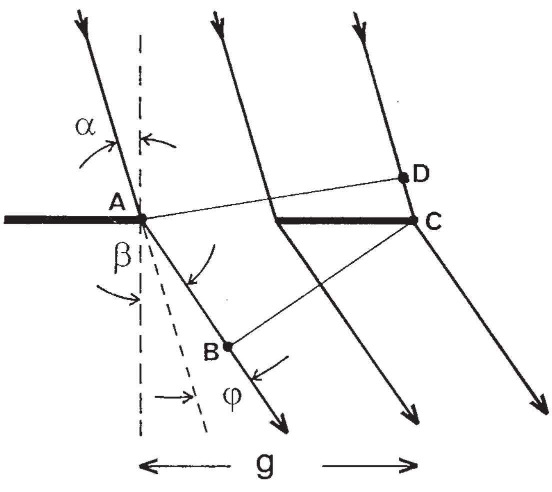
The first bracket describes the distribution of intensities due to diffraction by a single slit, whereas the combined effect of all the slits is described by the second bracket. If one bracket is zero, then total intensity $I(\varphi) = 0$. This means however, that minima due to a single slit continue to exist when N slits act together. On the other hand, peaks due to a single slit can be interspersed by further secondary minima if the second bracket is zero.

The diffraction direction φ of maximum z for a given grating fulfils the following relation:

$$v_z = \pi \frac{g}{\lambda} \sin\varphi_z = z\pi \text{ or } \sin\varphi_z = z\frac{\lambda}{g} ;$$

$$z = \text{order of diffraction} = \pm (0,1,2,\ldots) \qquad (2)$$

There are (N-1) secondary minima between every two peaks.



$$AB - CD = g\,(\sin\beta - \sin\alpha) = z\,\lambda$$

Fig. 2: Diffraction at the grating.

If light impinges at an angle α against the perpendicular to the grating, the following is valid (cf. Fig. 2):

$$\sin\varphi + \sin\beta = \frac{z\lambda}{g} = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = 2\sin\frac{\varphi}{2}\cos\frac{\alpha-\beta}{2} ;$$

$$\text{with } \beta - \alpha = \varphi \qquad (3)$$

The angle β is considered to be positive when the diffracted and incident beams are on the same side of the perpendicular to the grating. If each angle is on another side of the perpendicular, then β is negative. In case of perpendicular incidence ($\alpha = 0$), the following applies:

$$\sin\varphi = z\,\lambda/g \qquad (4)$$

By differentiation of (3) one obtains the angle of dispersion $d\varphi/d\lambda$ of the grating:

$$\frac{d\varphi}{d\lambda} = \frac{z}{g\cos\beta} = \frac{z}{g\cos\varphi} \quad (\text{with } \beta = \varphi, \text{ if } \alpha = 0) \qquad (5)$$

Angular dispersion is independent of the angle of incidence and compared to prisms, it remains nearly constant for small diffraction angles.

Two spectral lines λ_1 and λ_2 only can be separated if they are so far apart from each other that the peak of λ_1 coincides with the minimum of λ_2 (Rayleigh criterion). The quotient of the average wavelength and the difference between the wavelengths of lines which merely appear separated is called the spectral resolving power.

$$A = \frac{\frac{1}{2}(\lambda_1 + \lambda_2)}{\lambda_2 - \lambda_1} = \frac{\bar{\lambda}}{\Delta\lambda} \qquad (6)$$

The following is valid for the spectral resolving power of a diffraction grating:

$$A = zN \qquad (7)$$

(z = order of diffraction; N = effective (illuminated) number of slits)

Table 1 shows the results (for z = ± 1) of a typical measurement for the determination of grating constant g according to (4). The diffraction angle φ of a spectral line is calculated from

No.	Colour	λ/nm	φ(1)	φ(2)	φ(1)	φ(2)	$\bar{\varphi}$	g/µm
			z = +1		z = -1			
1	viol.	404.656	253° 56′	73° 56′	225° 40′	45° 43′	14.12°	1.6586
2	blue	435.405	255° 05′	75° 06′	224° 35′	44° 38′	15.24°	1.6562
3	bl.-green	491.604	257° 06′	77° 07′	222° 34′	42° 39′	17.25°	1.6578
4	green	546.074	259° 06′	79° 08′	220° 37′	40° 41′	19.23°	1.6577
5	yellow	576.960	260° 15′	80° 16′	219° 28′	39° 34′	20.37°	1.6575
6	yellow	578.966	260° 20′	80° 20′	219° 23′	39° 27′	20.46°	1.6565

Table 1: Typical measurement values (diffraction of the first order) to determine the grating constant of the Rowland grating.

the half angular difference of the corresponding diffraction order ± z. Finally, as two values φ (1) and φ (2) are determined for every angle due to the two verniers, the average value is:

$$\overline{\varphi} = 1/4\,[\,(\varphi\,(1)_{+z} - \varphi\,(1)_{-z}) - (\varphi\,(2)_{+z} - \varphi\,(2)_{-z})\,]$$

If one also considers the refraction of lines No.1-No. 6 in the second order (z = ± 2), as well as those third order lines (z = ± 3) which still can be recognised, the grating constant is found to be:

g = (1.6567 ± 0.0016) μm ; Δg/g = ± 0,1 %; (603.6 ± 0.6)/mm

Table 2 shows the angular dispersion values determined according to (5) (z = ± 1; dφ must be converted to radian for assessment. The values dφ and dλ are obtained from the difference between the corresponding values of neighbouring lines).

Table 2: Evaluation of the angular dispersion

No.	λ/nm	φ/°	dλ/nm	dφ/rad	$(d\varphi/d\lambda) m^{-1}$	$(g \cos\varphi)^{-1}/m^{-1}$
1	404.656	14.12				
2	435.405	15.24	30,75	0.01955	6.36×10^5	6.22×10^5
3	491.604	17.25	56,20	0.03490	6.21×10^5	6.28×10^5
4	546.074	19.23	54,50	0.03490	6.40×10^5	6.35×10^5
5	576.960	20.37	30,90	0.01989	6.44×10^5	6.41×10^5
6	578.966	20.46	2,00	0.00152	7.56×10^5	6.44×10^5

According to (6), a theoretical resolving power A = 562 nm/ 32 nm = ~17.6 is required to separate the green Hg-line- λ_1 ~ 546 nm from the pair of yellow lines λ_2 ~ 578 nm. To separate the two yellow Hg-lines λ_1 = 576.960 nm and λ_2 = 578.966 nm, A must at least be 289.
Table 3 gives the averages of the auxiliary slit width *x* obtained over several measurements, which are necessary to separate the lines of different diffraction orders *z* for various gratings. The last three columns give the values for the corresponding resolving power A.

Table 3: Evaluation of the resolving power (x = average value of the auxiliary slit width)

Separation of the green Hg-line from the pair of yellow lines.						
lattice	z = 1	z = 2	z = 3	A (z=1)	A (z=2)	A (z=3)
4/mm	—	x = 2.32 mm	—	—	18.6	—
8/mm	x = 2.27 mm	x = 1.09 mm	—	18.2	17.4	—
10/mm	x = 1.80 mm	x = 0.84 mm	x = 0.58 mm	18.0	16.8	17.4
50/mm	x = 0.35 mm	—	—	17.5	—	—
Separation of the yellow Hg-lines						
603.6/mm	x = 0.475 mm	—	—	287	—	—

The average values of A required to separate the yellow-green as well as the yellow-yellow lines, which are determined experimentally to be 17.7 and. 287, concur satisfactorily with the theoretical values.