

Related topics

Wave length, phase velocity, group velocity, wave equation, harmonic wave.

Principle and task

A quadrangular rubber rope is inserted through the demonstration motor and a linear polarised fixed wave is generated. With the help of a stroboscope, the frequency and the wave length are determined. Then the phase velocity of rope waves with a fixed tensile stress is ascertained. Subsequently, the mathematical relationship between the phase velocity of the rope and the tensile on the rope is examined.

Equipment

Grooved wheel, after Hoffmann	02860.00	1
Square section rubber strip, l 10 m	03989.00	1
Laboratory motor, 220 V AC	11030.93	1
Gearing 10/1, for 11030.93	11028.00	1
Cotton cord, l. 10m	02091.00	1
Support base -PASS-	02005.55	1
Support rod -PASS-, square, l 250 mm	02025.55	1
Support rod -PASS-, square, l = 1000 mm	02028.55	1
Right angle clamp -PASS-	02040.55	3
Rod with hook	02051.00	2
Pulley, fixed, on rod, dia. 65 mm	02260.00	1
Spring balance 10 N	03060.03	1
Bench clamp, -PASS-	02010.00	1
Silk thread, 200 m	02412.00	1
Measuring tape, l = 2 m	09936.00	1
Digital stroboscope	21809.93	1

Problems

1. With constant tensile stress, the frequency f , which depends on the wavelength λ of the wave that propagates itself along the rope. The frequency is plotted as a function of $1/\lambda$. From this graph, the phase velocity c is determined.
2. The phase velocity c of the rope waves, which depends on the tensile stress on the rope is to be measured. The quadrant of the phase velocity is plotted as a function of tensile stress.

Set-up and procedure

The experimental set-up is done as per Fig. 1. A 3 to 6 meters long rubber rope is tied to one end of a cotton cord of approx. 40 cm length. The joint is reinforced by wrapping a silk thread around it and knotting it. A loop is formed of the other end of the cotton cord and hooked on to the rod. Similarly, a cotton cord of approx. 50–60 cm length is tied to the other end of the rubber strip. The rope is then slanted slightly, so that it can be observed better, and attached to the 1 m long support rod, through the coupling, for measuring the wave velocity. The rope tension is to be adjusted such that, the rope neither sags nor is too taut. The rope is placed over notch 4 of the plastic cylinder, which is fastened to the base plate through the bore hole 2 (linear oscillation initiation; please see operating instructions). While adjusting the hook on the rod, care is to be taken that the hook lies in the groove plane and is fastened properly. In order to see, that the rope tension does not change, the direction of rotation of the motor is set in such a manner that the frictional force of the rotating notch only affects the inelastic cotton cord.

Fig. 1: Experimental set-up for the examination of rope waves.

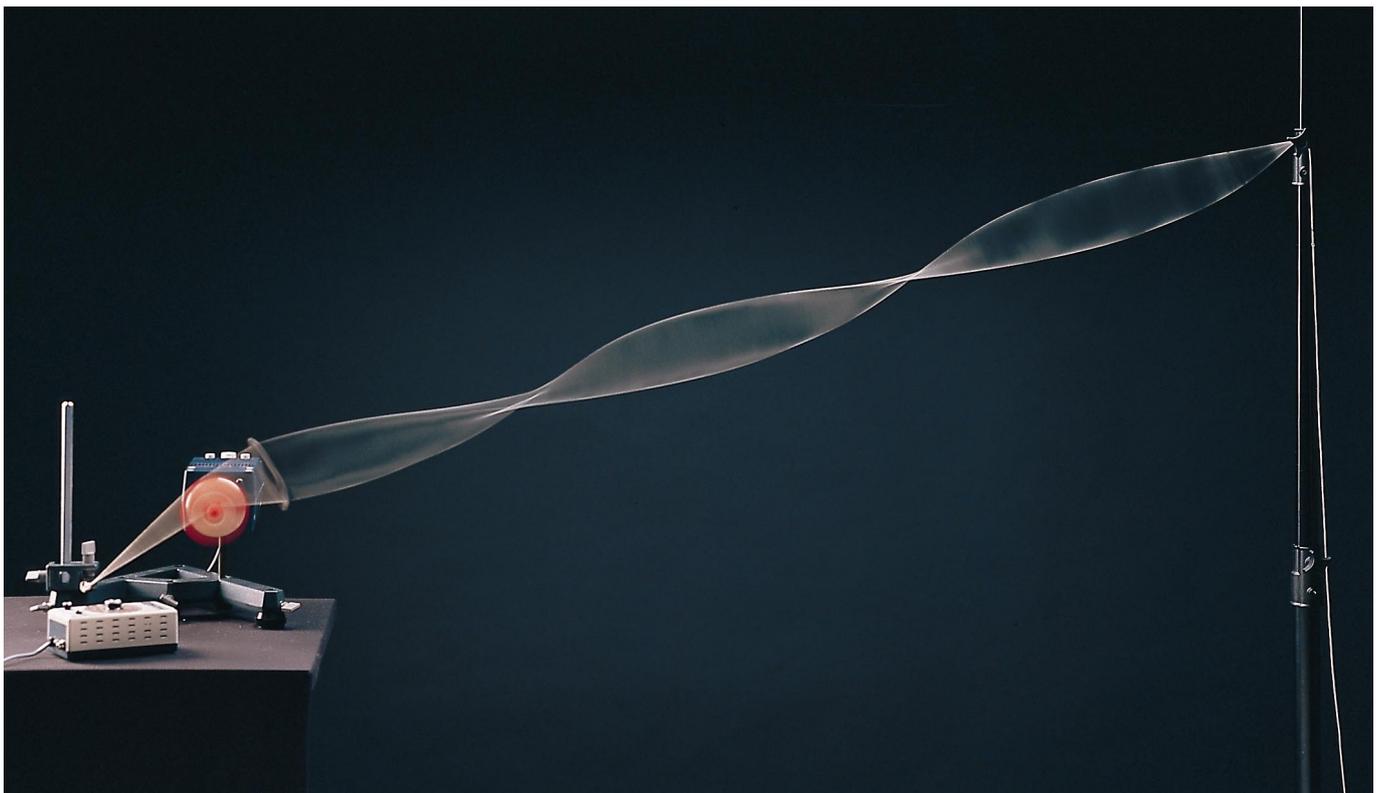
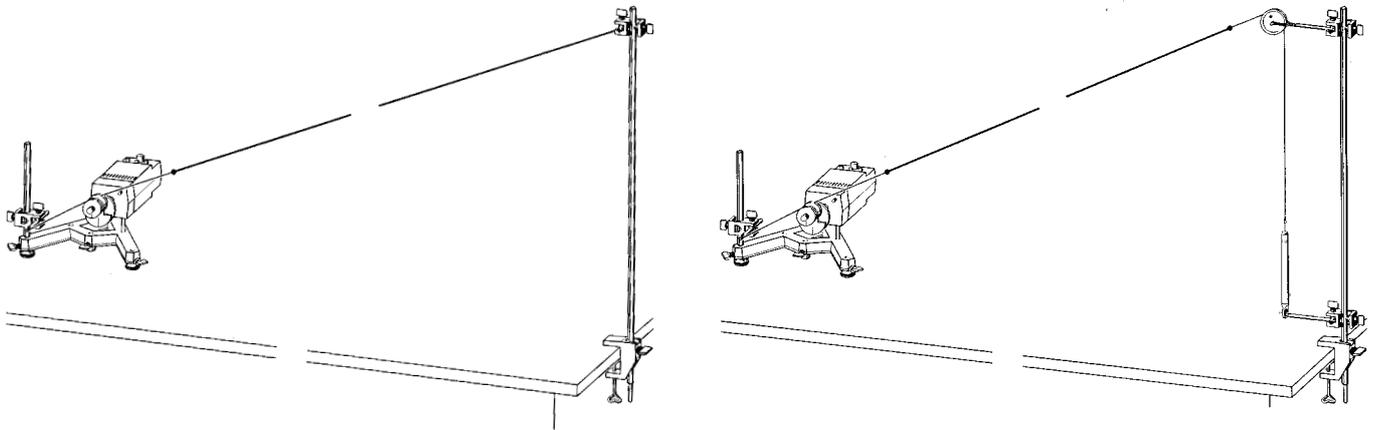


Fig. 1a: Experimental set-up for the examination of rope waves.



In the first section of the experiment, the dependency of the wave length λ on the excitation frequency f with constant tensile stress on the quadrangular rubber strip is determined. The frequencies of the rubber strip, varying with the electric motor (regulated operating mode) are calculated with the help of a stroboscope. It is suggested to work in a darkened room, with a bright background. The wave length is determined by doubling the distance between two wave knots.

In the second part of the experiment, the dependency of wave propagation velocity c on the tensile stress σ of the rope is observed. For this purpose, the cotton cord is removed from the right angle clamp and is fastened to a pulley around the lower end of the support rod. The tensile stress of the rope is determined with the help of the Dynamometer, for which the loop of the cotton cord is hooked to the wire eye of the Dynamometer. A second right angle clamp is fastened to the support rod, in which a rod with a hook is clamped to mount the Dynamometer. It is to be ensured, that during the operation the rope and its entire length is in one plane. The joint of the rubber strip and the cotton cord should always lie at a minimum distance of 10 cm from the grooved wheel. To take into account the weight of the Dynamometer, the zero-point calibration should be carried out in this position.

The tensile stress has to be varied by shifting the support base on the table, and also through the shifting of the right angle clamp on the support rod holding the thread. With constant excitation frequency (measured by a stroboscope approx. 10 Hz) one can determine the quadrant of the wave propagation velocity c of the rope wave, as the distance between two wave nodes and compared with the tensile stress σ acting upon the rope.

Theory

A wave is an oscillation which propagates itself in space and time and usually periodically through matter and space. One can differentiate between transverse and longitudinal waves. In the case of transverse waves, the oscillation is perpendicular to the direction of the propagation of the wave. In the case of longitudinal waves, the oscillation and the propagation are in the same direction. A simple example of a wave is a harmonic wave, as is represented in Fig. 2. We consider only a section of the wave which oscillates with frequency f around the rest position. Due to the acting forces, the position of the oscillation moves further. In case of a harmonic wave, which originates from a harmonic oscillation, it moves in a spatial

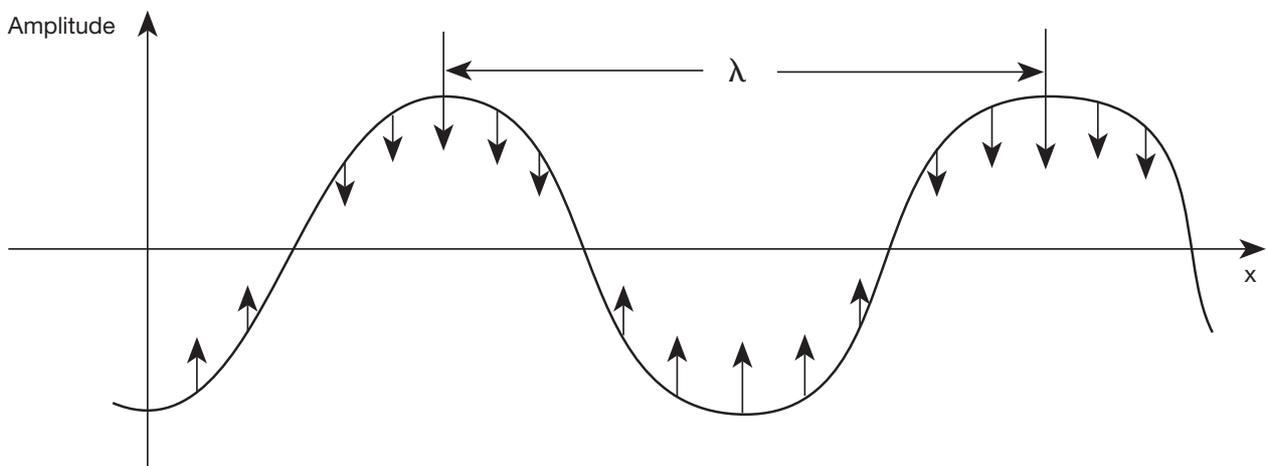
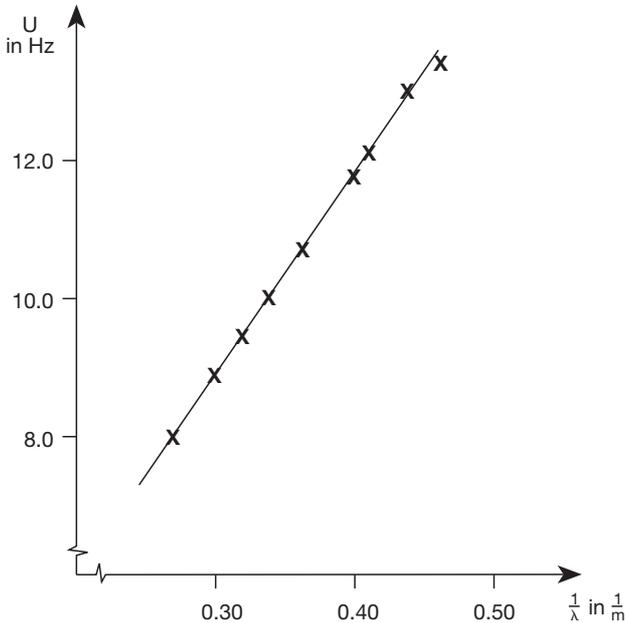


Fig. 2: To define the wave length of a harmonic wave.

Fig. 3: The frequency of a rope wave depending on the reciprocal wave length.



distance of λ to be found again in an identical position of oscillation (phase). The wave length of the harmonic wave is called λ . For traversing this distance, the wave requires the same amount of time as the time required for the oscillation T in a single section. The velocity (phase velocity $c(\lambda)$) determined on this basis amounts to

$$c = \frac{\lambda}{T} = \lambda f \quad (1)$$

This equation is valid for all harmonic waves with the above mentioned fundamental principles.

In this experiment, a periodic wave is produced in the cotton thread through the experimental motor, which propagates along the rubber rope and is reflected back at the support rod. The wave travelling in both directions, superimpose on each other, to form a standing wave with wave nodes (area of the smaller vibrational amplitudes) and antinodes (area of the larger vibrational amplitudes). It can be shown that the distance between wave nodes, equals half the wave length $\lambda/2$ (please see Experiment Literature Physics, Transverse waves 1; Cat. No. 16050.41) A calibration example is represented in Fig. 3, by which the linear relationship between frequency and the reciprocal wave length (also called wave number) is evident, as per the equation 1:

$$f = \frac{c}{\lambda} \quad (2)$$

The slope of the straight line is also the phase velocity of the wave.

The non-harmonic waves also give the harmonic waves of different wavelengths the additive together (Addition theorem). As the phase velocity of a harmonic wave is dependent on the frequency as per rule, the individual harmonic wave quickly run away (dispersion). The velocity of propagation of a non harmonic wave (group velocity) is not equated with phase velocity!

But the phase velocity also depends upon the momentary position of the medium, here the rope e. g., on the applied rope tension:

The applied forces at the section can act tangentially on the rope. As the rope is twisted, an effective perpendicular component \vec{F}_D is produced for that section from the sum of the applied tension force \vec{F}_a and \vec{F}_R (s. Fig. 4). It amounts to, that force and counteracting force are equal, but rotated against each other about the angular difference $d\theta$. The resulting accelerating force:

$$|\vec{F}_0| = |\vec{F}_a - \vec{F}_r| = 2 |\vec{F}_a| \sin(d\theta/2) \approx |\vec{F}_a| d\theta \quad (3)$$

The angular difference is replaced through second derivation (three dimensional) of the deviation to x co-ordinates $y''(x)$ multiplied by the length of the rope dx when the deviation is not too much:

$$|\vec{F}_D| = |\vec{F}_a| y'' dx \quad (4)$$

The section of the rope of mass

$$m = A \cdot dx \cdot \rho \quad (5)$$

(A is the cross sectional area of the rope, ρ the density of mass of the material of the rope) is accelerated through the force given in Equation (4) With the smaller deviation the force acts pre-dominantly towards the Y-axis so that

$$|\vec{F}_D| \approx F_y = m \ddot{y} = |\vec{F}_a| y'' dx$$

follows from the second derivation of time. With the rope tension $\sigma = F/A$ the so-called wave equation results from (4) and (5):

$$\ddot{y} = \frac{\sigma}{\rho} y'' \quad (6)$$

This equation contains the general solution

$$y = f(x \pm ct)$$

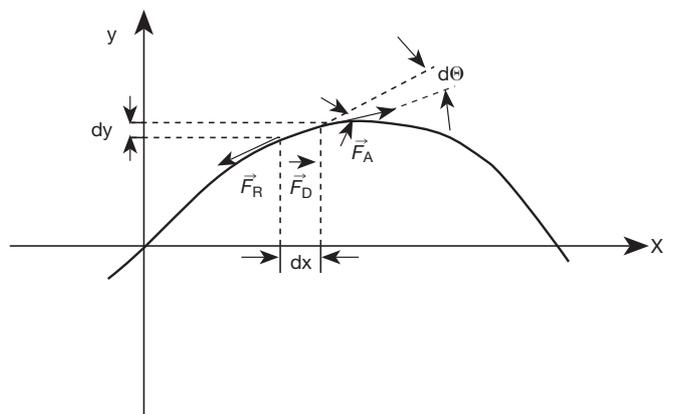
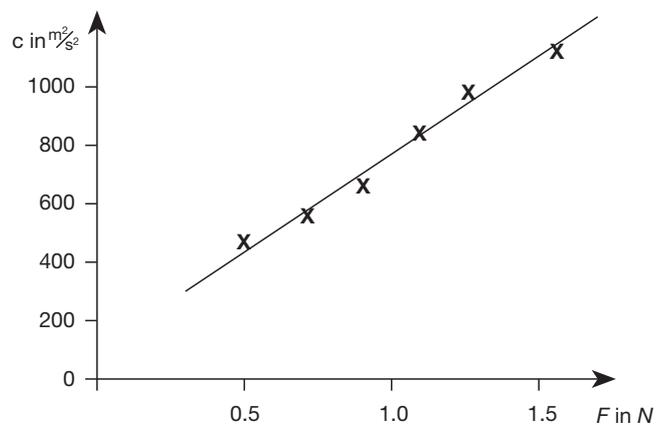


Fig. 4: The relationship of force in a rope wave.

Fig. 5: The square of phase velocity depending upon the force F applied on the rope.



where

$$c = \sqrt{\frac{\sigma}{\rho}} \quad (7)$$

as velocity of propagation of the rope wave.

The square of the phase velocity c^2 is inversely proportional to the mass density ρ of the rope medium and proportional to the applied rope tension σ .

The facts of the latter case, are elucidated from Fig. 5: The square of the velocity of propagation c and the value of the tension F are proportional to each other. The decrease in the cross sectional area A of the rubber strip, on account of the elongation of the strip is neglected, and the rope tension σ of the applied force F and c are proportional to it. This phenomena is used with tension of chords of pluck instruments, where through the variation of chord tension over the velocity of propagation the resonance frequency of the cord is determined.