The propagation of a periodically excited continuous transverse wave

LEP 1.3.32

Related topics

Periodic motion, frequency, wavelength, phase velocity, standing waves, natural frequency, free and fixed end, damping of waves.

Principle and task

The periodicity of connected stationary oscillators is demonstrated on the example of a continuous, harmonic transverse wave generated by a wave machine. The number of oscillations carried out by different oscillators within a certain time is determined and the velocity of propagation is measured. A relation between frequency, wavelength and phase velocity is established. The formation of standing waves is demonstrated and studied.

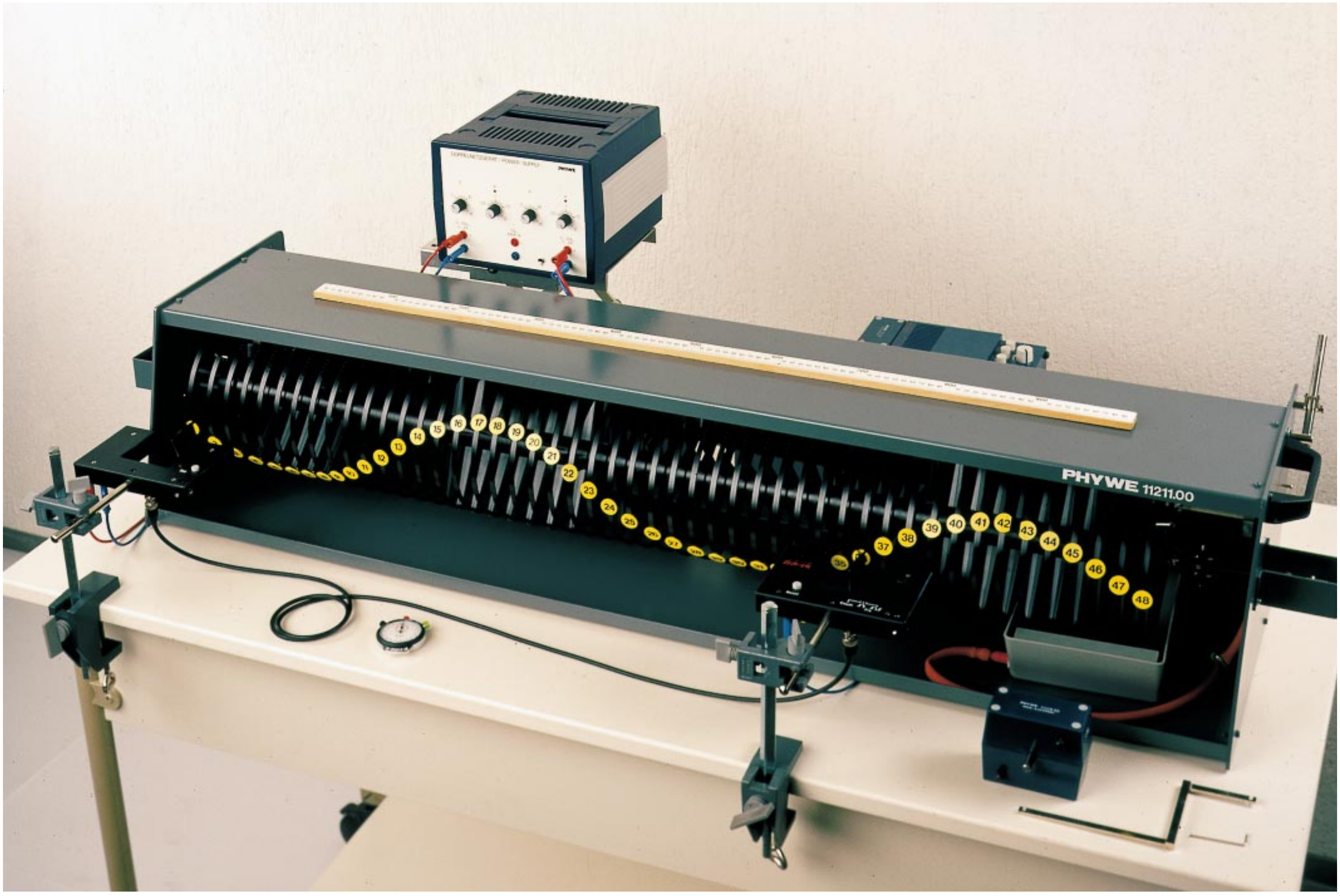
Equipment

Wave machine	11211.00	1
Power supply -2op-, 2×15 V/2 A	13520.93	1
Light barrier with Counter	11207.08	1
Light barrier	11207.02	1
Laboratory motor, 220 V AC	11030.93	1
Gearing 30/1, for 11030.93	11029.00	1
Gearing 100/1, for 11030.93	11027.00	1
Stop watch, interruption type	03076.01	1
Screened cable, BNC, l 1500 mm	07542.12	1
Bench clamp -PASS-	02010.00	3
Meter scale, demo, l = 1000 mm	03001.00	1
Right angle clamp -PASS-	02040.55	2
Support rod -PASS-, square, l 400 mm	02026.55	2
Connecting cord, 2000 mm, red	07365.01	2
Connecting cord, 2000 mm, blue	07365.04	2

Problems

1. The frequency of the oscillators 1, 10, 20, 30 and 40 is to be determined with the electronic counter of the light barrier and the stop-watch for a particular frequency of excitation.
2. By means of a path-time measurement the phase velocity of a transverse wave is to be determined.
3. For three different frequencies the corresponding wavelengths are to be measured and it is to be shown that the product of frequency and wavelength is a constant.
4. The four lowest natural frequencies with two ends of the oscillator system fixed are to be detected.
5. The four lowest natural frequencies with one end of the oscillator system fixed and the other one free are to be detected.

Fig. 1: Experimental set-up of the wave machine.



Set-up and procedure

The wave machine is to be set up as shown in Fig. 1. The laboratory motor with 30:1 gearing is connected to the drive eccentric of the wave machine for problems 1 to 3. The 100:1 gearing is used for problems 4 and 5. The connecting rod is used to couple the control shaft to the first oscillator of the system. For further details see the instruction manual of the wave machine.

The damping system must be used to dampen reflections at the free end of the oscillating system. To do this, the attenuation bath is placed beneath the last oscillator and filled with water.
A light stop is attached to the oscillator whose frequency is to be measured. To measure the frequency, the light barrier with digital counter should be arranged such that its optical path is crossed by the light stop at each zero assage of the oscillator. The digital counter is working as a pulse counter. For a particular number of pulses the measuring time At is determined with the stopwatch.

The determination of the phase velocity by means of a path-time-measurement in the wave machine is demonstrated on a continuous, harmonic transverse wave of length 3/4 λ. Two light stops are attached to oscillators 1 and 36 and the light barriers which are interconnected by a BNC cable are adjusted at a height such that, at maximum deflection of the oscillator the light stop just crosses the barrier. The first light barrier triggers the chronometer incorporated in the second light barrier. Finally the distance Δx between oscillator 1 and 36 is measured.

For the wavelength measurements the speed of the laboratory motor is set such that a continuous transverse wave of 3 to 4 λ is generated. After the frequency has been measured as described above, the brake system of the wave machine is activated and the wave is "frozen". From the "frozen" wave wavelength λ can easily be found by using the meter scale.

For the detection of the natural frequencies the right end of the oscillator system is fixed, that means oscillator 48 is fixed to the wave machine case with the second connecting rod. Proceeding from a frequency of excitation f = 0 Hz, one looks for those frequencies where standing waves with large amplitudes form. This is done by gradually increasing the speed of rotation of the motor and measuring the frequencies for which large amplitudes are observed.

Finally the right end of the oscillator system is left free. A small amplitude is chosen and the damping system removed. For the fundamental frequency we can identify length L of the oscillating system with $\lambda/4$. The lowest frequency can thus be determined from a path-time measurement. Then, proceeding from the highest frequency of excitation, one looks for those frequencies which allow the formation of standing waves with large amplitudes by gradually decreasing the rate of rotation of the motor. These frequencies are determined experimentally with the aid of the digital counter and the stopwatch.

Theory and evaluation

A harmonic transverse wave results from the propagation of a harmonic excitation along the system of coupled oscillators.
If one observes the oscillator system at a fixed position x ($x = 0, x_1, x_2 \ldots$), then each oscillator carries out a periodic movement relative to time with the same frequency f respectively with the same period of oscillation T at its equilibrium position. Because $\varphi_n = A\, f\,(t, x_n)$ holds, one obtains for the harmonic excitation:

$$\varphi_n = A \sin 2\pi f \left(t - \frac{X_n}{C_p}\right)$$

$$= A \sin 2\pi \left(f \cdot t - \frac{X_n}{\lambda}\right) \text{ with } C_p = f \cdot \lambda$$

A = amplitude of oscillation

λ = wavelength

C_p = phase velocity

This equation includes both the spatial and the temporal periodicity of the wave, as at x = 0, the amplitude of the oscillation is given by

$$\varphi_1 = A \sin 2\pi f t$$

or $$\varphi_1 = A \sin wt \qquad \text{with } w = 2\pi f = \frac{2\pi}{T}$$

By superimposing of two waves it can be shown:

a) If both ends of the system are fixed, the condition

$$k \cdot \frac{\lambda}{2} = L$$

is valid for the occurence of standing waves (internodal distance $\lambda/2$), whereby L is the system length and k can assume the values 1, 2, 3.... For k = 1 this yields the fundamental frequency with a wavelength of $\lambda = 2 \cdot L$.

b) If one end of the system is fixed and the other is free, the condition

$$(2k - 1)\,\frac{\lambda}{4} = L$$

holds for standing waves (internodal distance $\lambda/2$). For k=1 this yields the fundamental frequency with a wavelength of $\lambda = 4\,L$.

1. Place the light stop onto the first oscillator and align the light barrier accordingly. Then switch the laboratory motor on and select a speed of rotation which will produce a continuous transverse wave with a total length of 4λ to 5λ. Once a stable transverse wave has formed, start the digital counter and the stopwatch simultaneously and stop them when 280 pulses have been counted. Record measuring time At and the number of pulses.

   Now repeat this measurement on different oscillators, whereby it is practical to convert the experimental arrangement with the oscillator system "frozen in place" and the drive still running. This ensures that all of the measurements are carried out at one speed of rotation of the motor (frequency of excitation).

The results obtained in one experiment are compiled in the following table:

Oscillator-No.	1	10	20	30	40
Pulses	280	280	280	280	280
n	140	140	140	140	140
$\frac{\Delta t}{s}$	73	73.4	72	73.2	73.6
$\frac{n/\Delta t}{s^{-1}} = f$	1.9	1.9	1.9	1.9	1.9

whereby: Pulses = number of pulses within the measuring period.
n = number of oscillations
t = measuring time.

All the oscillators of the system move at the same frequency f = n/Δt = const. = 1.9 Hz respectively with the same period of oscillation

$$T = \frac{1}{f} = \frac{1}{1.9} \text{ s} = 0.5 \text{ s}$$

2. One determines phase velocity Cp of the wave from measured values Δx and Δt using the relation

$$C_p = \frac{\Delta x}{\Delta t}$$

When the first oscillator starts to move upwards from its maximum lower deflection, the reset key of the timer is activated. When the oscillator reaches its maximum upper deflection, the light gate triggers the timer. It stops when oscillator 36 assumes this state of oscillation.

Note the indicated measured value and verify by repeating the measurement.

Finally, determine the path Ax travelled by the phase by measuring the distance between the two oscillators.

The table contains test examples for 5 individual measurements:

	1	2	3	4	5	
$\frac{\Delta t}{s}$	1.04	1.02	1.03	1.04	1.03	$\overline{\Delta t}$ = 1.03 s
$\frac{\Delta x}{cm}$	86.0	86.0	86.0	86.0	86.0	$\overline{\Delta x}$ = 86.0 cm

As mean value for C_p we find

$$C_p = 83.5 \frac{\text{cm}}{\text{s}}$$

3. Switch the laboratory motor on and set the speed of rotation such that a continuous transverse wave of length 3λ to 4λ is generated.

Once the system has reached a steady state of Oscillation start both the digital counter and the stopwatch simultaneously and stop them both when measuring time Δt = 60 s is completed.

Record the values for measuring time At and number of zero passages *N* of oscillator 1.

Now activate the breaking system "freeze" the wave. Determine the characteristic wavelength λ for the frequency of the "frozen" waveform by linear measurement.

Release the breaking system and repeat the measurement at a low frequency of excitation (lower speed of rotation of the motor).

The following table gives an experimental example for 3 different frequencies:

	N	n	$\frac{\Delta t}{s}$	$\frac{f}{s^{-1}}$	$\frac{\lambda}{cm}$	$\frac{C_p}{cm/s}$
1	308	154	60	2.57	33.0	84.8
2	258	129	60	2.15	40.0	86.0
3	158	79	60	1.32	65.0	85.8

whereby

N = number of pulses within measuring period Δt.

n = *N*/2, number of oscillations within measuring time Δt.

The mean value from the three individual measurements for phase velocity is

$$C_p = \frac{1}{3} \sum_{i=1}^{3} \lambda_i \cdot f_i$$

$$C_p = 85.5 \frac{\text{cm}}{\text{s}}$$

This is in rather good agreement with the previous result from path-time measurement.

4. For both ends of the fixed oscillator system it can be observed that with an increasing speed of rotation of the laboratory motor, standing waves are first observed for a frequency of 0.38 Hz. This is the lowest natural frequency. The oscillation associated with this natural frequency is called the fundamental frequency. Its wavelength is λ = 2L.

All higher natural frequencies f_k are supposed to be integral multiples k of the lowest natural frequency f_g.

$$f_k = k \cdot f_g$$

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The resonance frequencies f_k are called “harmonics”.The resonance frequencies measured with increasing speed of rotation are listed in the following table:

$\frac{f_k}{Hz}$	k	$\frac{f_k}{k}$	λ
0.38	1	0.38	2L/1
0.74	2	0.37	2L/2
0.94	3	0.31	2L/3
1.43	4	0.36	2L/4

From the frequency ratio k and the number Z of nodes, one easily 6btains the relation

$$Z = \frac{f_k}{f_g} + 1$$

5. From the relation (2k–1) $^{\lambda}/_{4}$ = L we obtain for k = 1 a wavelength of λ = 4 L. The corresponding frequency is the fundamental frequency. It can be found from a path-time measurement. If t is the time which a perturbation needs to pass through the oscillator system of length L then 4 t is the time which corresponds to the period T and the fundamental frequency is thus given by the expression

$$f_g = \frac{1}{T} = \frac{1}{4t}$$

From a path-time measurement t was found to be 1.42 s.

Hence, f_g = 0.18 Hz

The laboratory motor was then switched on and set to its highest speed of rotation. While slowly diminishing its speed those frequencies of excitation, which are the natural frequencies of the system and thus make the formation of standing waves at resonance possible were measured.

The following table contains the results:

$\frac{f_k}{Hz}$	k	$\frac{f_k}{k}$	λ
1.56	9	0.17	4L/9
1.18	7	0.17	4L/7
0.83	5	0.17	4L/5
0.55	3	0.18	4L/3
0.18	1	0.18	4L/1

From the frequency ratio k one obtains the number of nodes Z from the relation

$$Z = \frac{1}{2} \cdot \frac{f_k}{f_g} + \frac{1}{2} = \frac{1}{2}\ (k + 1)$$

Note

Please see instruction manual for complete discription of the wave machine. The manual contains additional experiments specially in the field of demonstration of wave propagation.