Related topics

Moment of inertia, torque, angular momentum, nutation, precession.

Principle and task

If the axis of rotation of the forcefree gyroscope is displaced slightly, a nutation is produced. The relationship between precession frequency or nutation frequency and gyro-frequency is examined for different moments of inertia.
Additional weights are applied to a gyroscope mounted on gimbals, so causing a precession.

Equipment

Gyro, Magnus type, incl. Handb.	02550.00	1
Stopwatch, digital, 1/100 sec.	03071.01	1
Digital stroboscope	21809.93	1

Problems

1. To determine the precession frequency as a function of the torque and the angular velocity of the gyroscope.
2. To determine the nutational frequency as a function of the angular velocity and the moment of inertia.

Note

A detailed handbook (128 pages) containing additional experiments is included, free of change, in the equipment.

Set-up and procedure

The gyroscope is set-up as shown in Fig. 1. There must be no additional masses on the axes or in the disc (rotor). The disc should be at a state of neutral equilibrium in all spatial directions and therefore remain stationary in every position without swinging to and from. Small corrections can be made by moving the two slotted compensating weights. To determine the precession frequency the additional masses are placed on the gyroscope axis mounting and gently screwed on. Using two different additional weights, the three combinations (m_1, o); (m_2, m_1) and (m_2, o) can be obtained.

The gyroscope is carefully set in motion with the starting handle. The angular velocity is measured with the stroboscope, and the precession frequency is determined with the stop watch. It is best to measure double the frequency with the stroboscope and half the precession period with the stop watch.

The additional masses are removed to measure the nutation. The gyroscope is set in motion as before. A sharp tap with the hand on the gyro mounting – movement about the inner gyroscopic axis – produces a nutation.

The frequency of this nutation is measured with the stroboscope. The nutation frequencies are determined for different gyroscope velocities and for different additional weights (symmetrically equal).

Fig.1: Gyroscope in cardanic mounting.

Fig. 2: Moving frame of reference for the gyroscope.

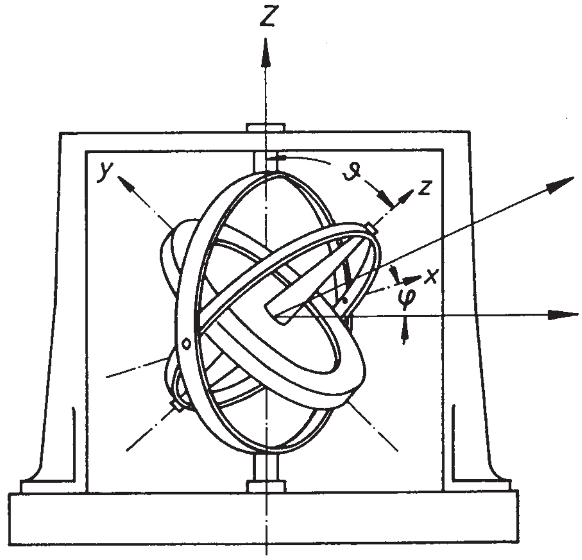
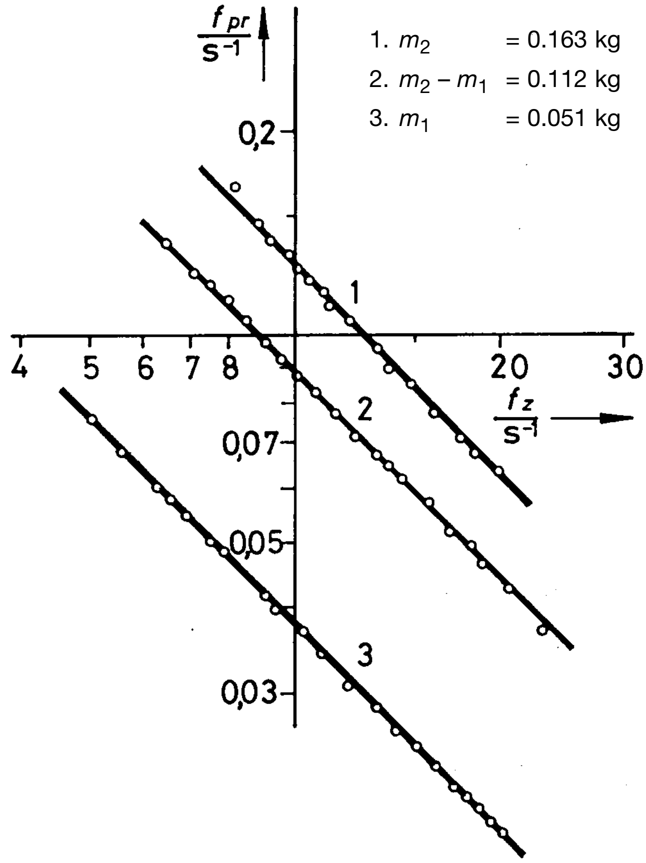


Fig. 3: Precession frequency as a function of the gyro frequency for different additional masses.



Theory and evaluation

The equation of motion for a rigid body with an angular momentum $\vec{L}$ to which a torque T is applied, in the inertial system XYZ, is:

$$\frac{d\vec{L}}{dt} = \vec{T}.$$

The angular momentum of a gyroscope can be divided up into one part in the direction of the figure axis $\vec{L}$, which stems only from the rotor, and a remainder $\vec{L}_{ns}$:

$$\vec{L} = \vec{L}_s + \vec{L}_{ns}.$$

If the equation of motion is transformed to a moved reference system xyz which rotates at an angular velocity Ω, with the origin at the centre of gravity, and whose axes are defined by the inner gimbal frame and the figure axis (see Fig. 2) we obtain:

$$\frac{d'L_s}{dt} + \frac{d'}{dt}\vec{L}_{ns} + \vec{\Omega} \times \vec{L}_s \times \vec{L}_{ns} = \vec{T}$$

The derivative against time must be formed in the rotating (dashed symbol) system. If the angular momentum of the rotor in the direction of the figure axis is consant:

$$\vec{L}_s = \hat{I}_s \cdot \vec{\omega} = I_s^z \cdot \omega_z,$$

with the moment of inertia I_s^z of the rotor along the figure axis, and the angular velocity ω_z, and also if

$$|\vec{L}_s| >> |\vec{L}_{ns}|,$$

then we obtain the fundamental equation for gyroscope theory:

$$\frac{d'}{dt}\vec{L}_{ns} + \vec{\Omega} \times \vec{L}_s = \vec{T}. \qquad (1)$$

The uncoupled equations for the components of (1), essential for precession and nutation, are obtained from (1):

$$\left(\frac{I_x I_y}{(I_s^z\ \omega_z)^2}\ \frac{d'^2}{dt^2} + 1\right)\Omega_x$$

$$= \frac{I_y}{(I_s^z\ \omega_z)^2} \cdot \frac{d'T_x}{dt}$$

$$\left(\frac{I_x I_y}{(I_s^z\ \omega_z)^2}\ \frac{d'^2}{dt^2} + 1\right)\Omega_y = \frac{T_x}{I_s^z\ \omega_z}.$$

(T_y disappears for weights placed on the rotor axis.)
Introducing the Euler angles θ and ϕ we obtain:

$$\Omega_x = \frac{d'\theta}{dt}$$

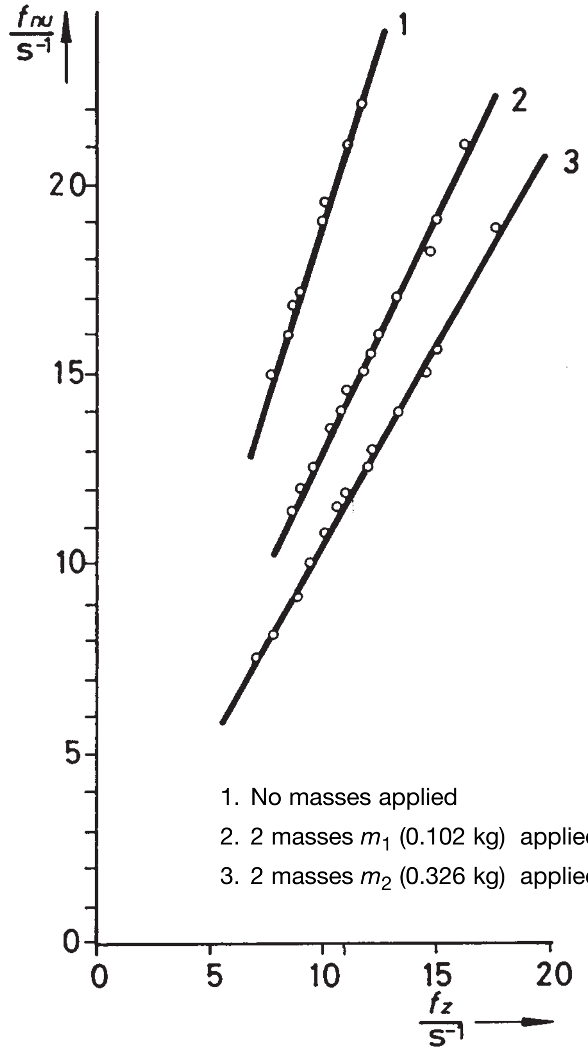
$$\Omega_y = \sin\theta \cdot \frac{d'}{dt}\phi$$

$$T_x = mgr \cdot \sin\theta,$$

where:

m is the mass at a distance r from the origin which produces the torque due to gravitational acceleration.

Fig. 4: Nutation frequency f_{nu} as a function of the gyro frequency f_z for different additional masses (I_x, ${}^zI_y$).



If the precession frequency is small compared with the rotor frequency ω_z, then:

$$\frac{I_y}{(I_s^z\ \omega_z)^2}\ mgr\cos\theta << 1.$$

This term can be neglected and we obtain:

$$\left(\frac{I_x I_y}{(I_s^z\ \omega_z)^2}\ \frac{d'^2}{dt^2}+1\right)\frac{d'\theta}{dt}=0$$

$$\left(\frac{I_x I_y}{(I_s^z\ \omega_z)^2}\ \frac{d'^2}{dt^2}+1\right)\frac{d'\phi}{dt}=\frac{mgr}{I_s^z\ \omega_z}.$$

For the initial conditions where:

$$\left.\frac{d'\theta}{dt}\right|_{t=0}=0$$

$$\left.\frac{d'\phi}{dt}\right|_{t=0}=\phi_o$$

we have:

$$\frac{d'\theta}{dt}=\sqrt{\frac{I_y}{I_x}}\left(\frac{mgr}{I_s^z\ \omega_z}-\phi_o\right)\sin\theta_o\ \cdot$$

$$\cdot\ \sin\frac{I_s^z\ \omega_z}{\sqrt{I_x I_y}}\,t$$

$$\frac{d'\phi}{dt}=\frac{mgr}{I_s^z\ \omega_z}+\left(\phi_o-\frac{mgr}{I_s^z\ \omega_z}\right)\ .\ \cos\frac{I_s^z\ \omega_z}{\sqrt{I_x I_y}}\,t.$$

For the initial condition where:

$$\phi_o=\frac{mgr}{I_s^z\ \omega_z}$$

the precession frequency is:

$$\omega_{pr}\ \frac{d'\phi}{dt}=\frac{mgr}{I_s^z}\cdot\frac{1}{\omega_z}$$

and θ = constant.

From the regression line of the data of Fig. 3, using

$$Y = A \cdot X^B$$

we obtain the exponents:

$B_1 = 1.02 \pm 0.01$
$B_2 = 1.01 \pm 0.005$ (see (2))
$B_3 = 1.01 \pm 0.005$

If the figure axis of the rotating gyroscope is pushed with the hand, and therefore

$$\frac{d'\theta}{dt} \neq 0,$$

the figure axis performs a periodic movement with the nutation frequency

$$\omega_{nu}=\frac{I_z}{\sqrt{I_x I_y}}\cdot\omega_z \qquad (3)$$

From the regression line of the data of Fig. 4, using the linear

$$Y = A + B \cdot X$$

the gradients

$B_1 = 0.95 \pm 0.02$
$B_2 = 0.82 \pm 0.02$ (see 3))
$B_3 = 0.57 \pm 0.02$

are obtained.