Laws of gyroscopes / 3-axis gyroscope

Related topics

Moments of inertia, torque, angular momentum, precession, nutation

Principle and task

The momentum of inertia of the gyroscope is investigated by measuring the angular acceleration caused by torques of different known values. In this experiment, two of the axes of the gyroscope are fixed.
The relationship between the precession frequency and the gyro-frequency of the gyroscope with 3 free axes is examined for torques of different values applied to the axis of rotation.
If the axis of rotation of the force-free gyroscope is slightly displaced, a nutation is induced. The nutation frequency will be investigated as a function of gyro-frequency.

Equipment

Gyroscope with 3 axes	02555.00	1
Light barrier with Counter	11207.08	1
Power supply 5 V DC/0.3 A	11076.93	1
Additional gyro-disc w. counter-weight	02556.00	1
Stopwatch, digital, 1/100 sec.	03071.01	1
Barrel base -PASS-	02006.55	1
Slotted weight,10 g, black	02205.01	4

Problems

1. Determination of the momentum of inertia of the gyroscope by measurement of the angular acceleration.
2. Determination of the momentum of inertia by measurement of the gyro-frequency and precession frequency.
3. Investigation of the relationship between precession and gyro-frequency and its dependence from torque.
4. Investigation of the relationship between nutation frequency and gyro-frequency.

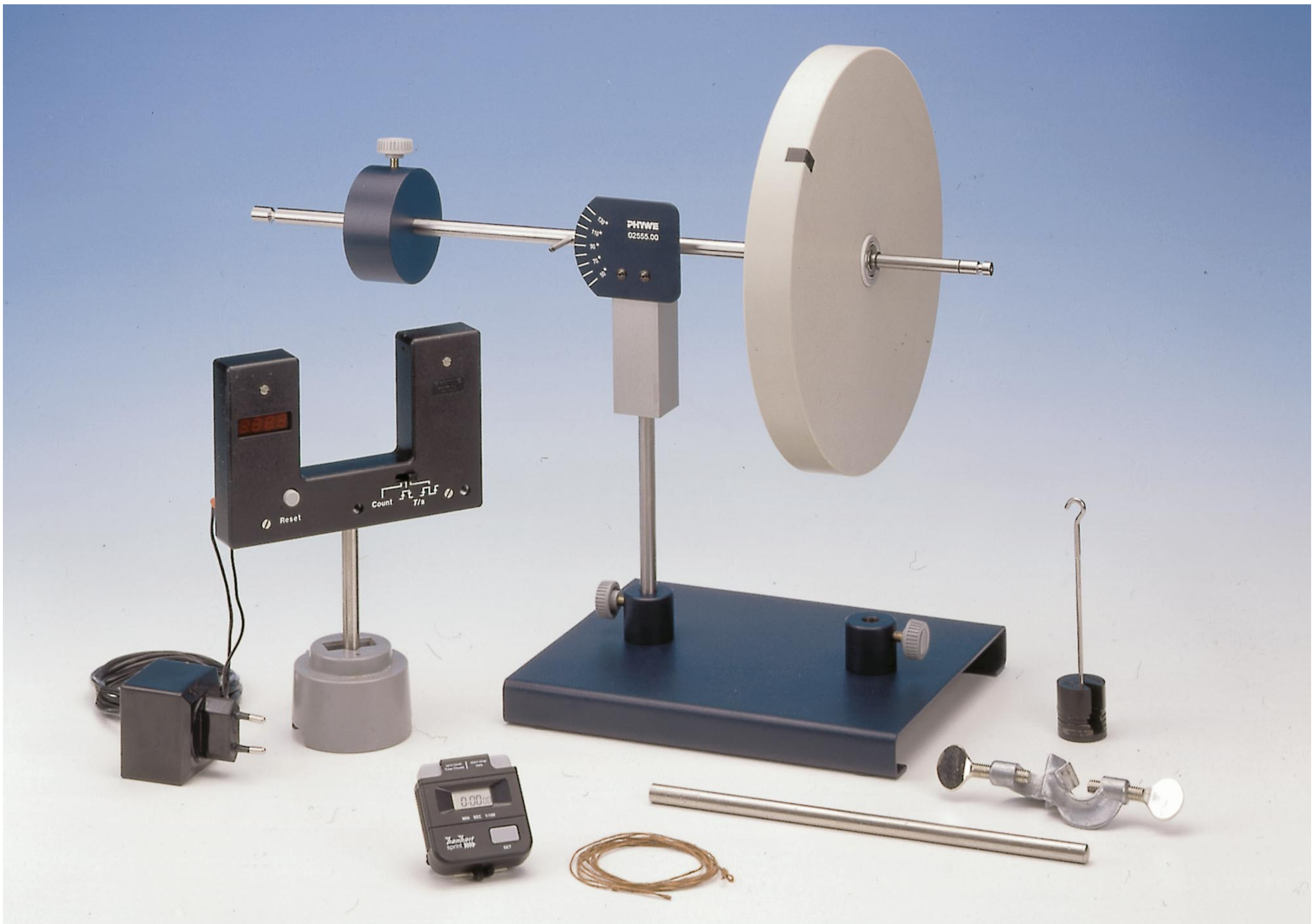
Set-up and procedure

1. To start with, the polar momentum of inertia I_p of the gyroscope disk must be determined.

For this, the gyroscope is fixed with its axis directed horizontally and positioned on the experimenting table in such a way that the thread drum projects over the edge of the table (fig. 2). The thread is wound around the drum and the accelerating mass m (m = 60 g; plate with 5 slotted weights) is fastened to the free end of the thread.

Several experiments are carried out for different drop heights h of the accelerating mass, from which the corresponding average falling time t_F from the moment the gyroscope disk is released until the mass touches the floor is determined. The diagram of t_F^2 versus h is plotted and the moment of inertia of

Fig. 1: 3 axis gyroscope.



the gyroscope disk is determined from the slope of the straight line.

2. The gyroscope, on which no forces act, and which can move freely around its 3 axes, is wound up and the duration t_R of one revolution (rotation frequency) is determined by means of the forked light barrier, with the axis of the gyroscope lying horizontally. Immediately after this, a mass m^* = 50 g is hung at a distance r^* = 27 cm into the groove at the longer end of the gyroscope axis. The duration of half a precession rotation t_P /2 must now be determined with a manual stopwatch (this value must be multiplied by two for the evaluation). The mass is then removed, so the gyroscope axis can regain immobility, and tR can be determined again. The inverse of the average value from both measurements of t_R is entered into a diagram above precession time t_p. In the same way, the other measurement points are recorded for decreasing number of gyroscope rotations. The slope of the resulting straight line allows to calculate the momentum of inertia of the gyroscope disk.

3. If a slight lateral blow is given against the axis of the rotating gyroscope on which no forces are acting, the gyroscope starts describing a nutation movement. The duration of one nutation t_N is determined with the manual stopwatch and this is plotted against the duration of one revolution, which is again determined by means of the forked light barrier.

Theory and evaluation

1. Determination of the momentum of inertia of the gyroscope disk.

If the gyroscope disk is set to rotate by means of a falling mass m (fig. 2), the following relation is valid for the angular acceleration:

$$\frac{d\omega_R}{dt} = \alpha = \frac{M}{I_P} \qquad (1)$$

(ω_R = angular velocity; α = angular acceleration; I_P = polar momentum of inertia; $M = F \cdot r$ = torque).

According to the law of action and reaction, the force which causes the torque is given by the following relation:

$$F = m\,(g - a) \qquad (2)$$

(g = terrestrial gravitational acceleration; a = trajectory acceleration)

The following relations are true for the trajectory acceleration a and the angular acceleration a:

$$a = \frac{2h}{t_F^2}\,;\ \alpha = \frac{a}{r} \qquad (3)$$

(h = dropping height of the accelerating mass, t_F = falling time; r = radius of the thread drum).

Introducing (2) and (3) into (1), one obtains:

$$t_F^2 = \frac{2\,I_P + 2\,mr^2}{mgr^2}\,h \qquad (4)$$

From the slope of the straight line t_F^2 = f (h) from fig. 3, one obtains the following value for the momentum of inertia of the gyroscope disk:

$I_P = (8{,}83 \pm 0{,}15).10^{-3}$ kg m^2

In general, the following is valid for the momentum of inertia of a disk:

$$I_P = \frac{1}{2}MR^2 = \frac{\pi}{2}R^4 d\rho \qquad (5)$$

Taking the corresponding values for the radius R and the thickness d of the circular disk, and the specific weight of plastic ρ = 0,9 g/cm^3, one obtains from (4):

$I_P = 8{,}91.10^{-3}$ kg m^2.

2. Determination of the precession frequency

Let the symmetrical gyroscope G in fig. 4, which is suspended so as to be able to rotate around the 3 main axes, be in equilibrium in horizontal position with counterweight C. If the gyroscope is set to rotate around the x-axis, with an angular

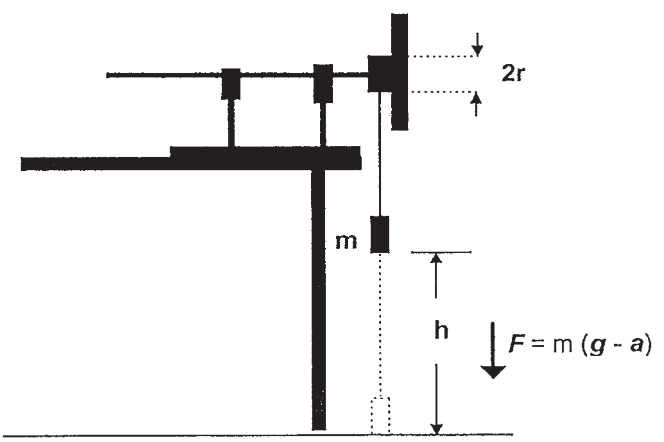


Fig. 2: schematic representation of the experimental set-up to determine the momentum of inertia of the gyroscope disk.

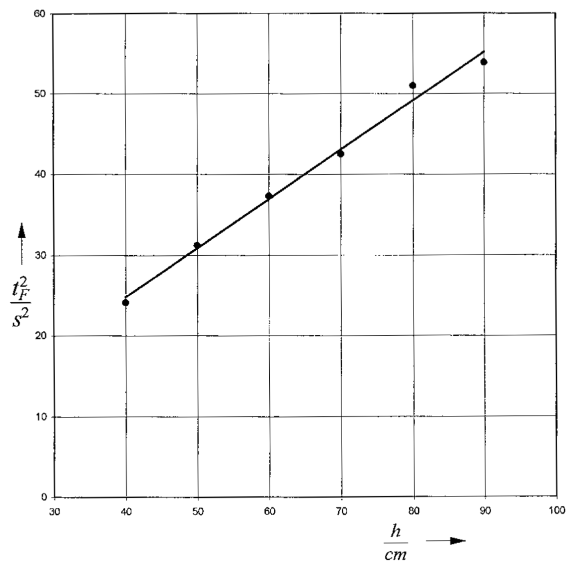
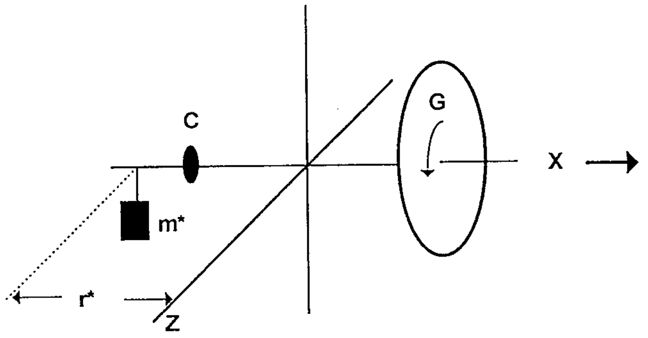


Fig. 3: Determination of the momentum of inertia from the slope of straight line t_F^2 = f(h).

Fig. 4: schematic representation of the gyroscope submitted to forces.



velocity ω, the following is valid for the angular impulse L, which is constant in space and in time:

$$L = I_p \cdot \omega_R \quad (6)$$

Adding a supplementary mass m^* at the distance r^* from the support point induces a supplementary torque M^*, which is equal to the variation in time of the angular impulse and parallel to it.

$$M^* = m^* \, gr^* = \frac{dL}{dt} \quad (7)$$

Due to the influence of the supplementary torque (which acts perpendicularly in this particular case), after a lapse of time dt, the angular impulse L will rotate by an angle $d\varphi$ from its initial position (fig. 5).

$$dL = L \, d\varphi \quad (8)$$

The gyroscope does not topple under the influence of the supplementary torque, but reacts perpendicularly to the force generated by this torque. The gyroscope, which now is submitted to gravitation, describes a so-called precession movement. The angular velocity φ_P of the precession fulfils the relation:

$$\omega_p = \frac{d\varphi}{dt} = \frac{1}{L}\frac{dL}{dt} = \frac{1}{I_p\omega_R}\frac{dL}{dt} = \frac{m^* gr^*}{I_p\omega_R} \quad (9)$$

Taking $\omega_P = 2\pi / t_P$ and $\omega_R = 2\pi / t_R$ one obtains:

$$\frac{1}{t_R} = \frac{m^* gr^*}{4\pi^2}\frac{1}{I_p}t_P \quad (10)$$

According to (10), fig. 6 shows the linear relation between the inverse of the duration of a revolution t_R of the gyroscope disk and the duration of a precession revolution t_P for two different masses m^*. The slopes of the straight lines allow to calculate

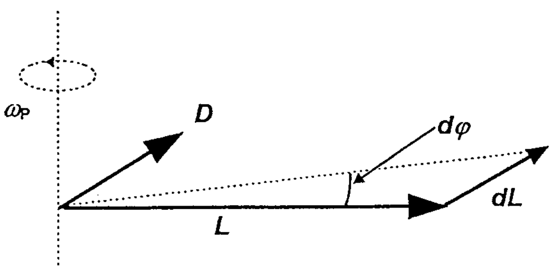
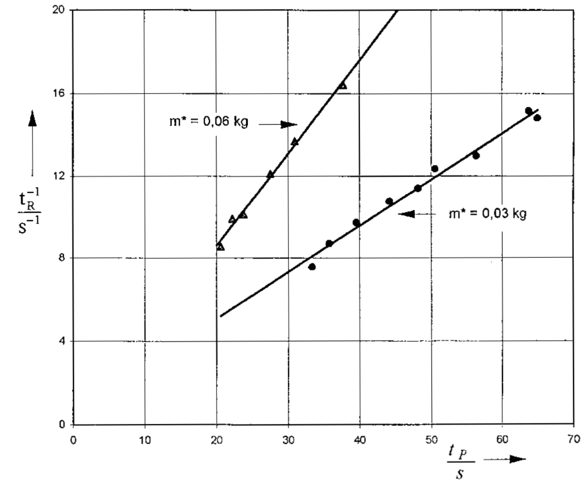


Fig. 5: precession of the horizontal axis of the gyroscope.

Fig. 6: determination of the momentum of inertia from the slope of straight line $t_R^{-1} = f(t_P)$.



the values of the momentum of inertia, for which one obtains: $I_P = (8{,}89 \pm 0{,}15)$ kgm^2 for $m^* = 0{,}03$ kg and $I_P = (9{,}29 \pm 0{,}17)$ kgm^2 for $m^* = 0{,}06$ kg.

The double value of the torque (double value of m^*) causes the doubling of the precession frequency.
If m^* is hung into the forward groove of the gyroscope axis, or if the direction of rotation of the disk is inverted, the direction of rotation of the precession is also inverted.
If the supplementary disk identical to the gyroscope disk is used too, and both are caused to rotate in opposite directions, no precession will occur when a torque is applied.

3. Determination of the nutation frequency
Fig. 7 represents the relation

$$\omega_N = k\omega_R \; ; \; t_R = kt_N \quad (11)$$

between the nutation frequency ω_N and rotation frequency ω_R. The constant k depends on the different moments of inertia relative to the principal axes of rotation.

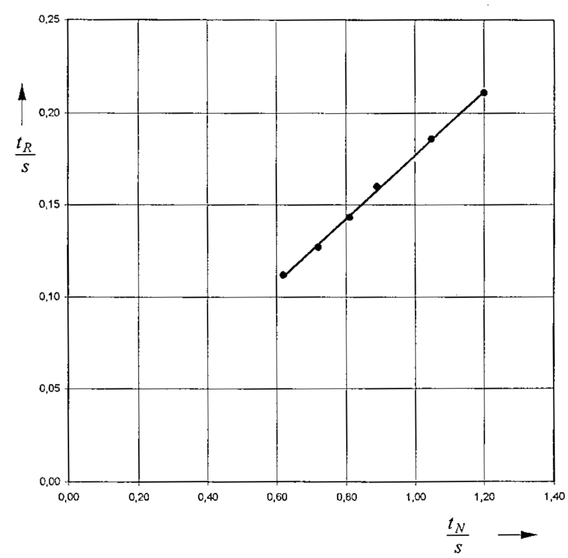


Fig. 7: nutation time t_N as a function of time for one revolution t_R.