Related topics

Maxwell disk, energy of translation, energy of rotation, potential energy, moment of inertia, angular velocity, angular acceleration, instantaneous velocity, gyroscope.

Principle and task

A disk, which can unroll with its axis on two cords, moves in the gravitational field. Potential energy, energy of translation and energy of rotation are converted into one another and are determined as a function of time.

Equipment

Support base -PASS-	02005.55	1
Support rod-PASS-, square, l = 1000 mm	02028.55	3
Right angle clamp -PASS-	02040.55	4
Meter scale, demo, l = 1000 mm	03001.00	1
Cursors, 1 pair	02201.00	1
Maxwell wheel	02425.00	1
Connecting cord, 1000 mm, red	07363.01	1
Connecting cord, 1000 mm, blue	07363.04	1
Light barrier with Counter	11207.08	1
Holding device w. cable release	02417.04	1
Plate holder	02062.00	1
Adapter, BNC-plug/socket 4 mm	07542.26	1
PEK capacitor/case 1/0,1 mm F/500 V	39105.18	1
Power supply 5V DC/0,3 A	11076.93	1
Connecting cord, 1500 mm, red	07364.01	1
Connecting cord, 1500 mm, blue	07364.04	1

Fig.1: Experimental set up for investigating the conservation of energy, using the Maxwell disk.

Problems

The moment of inertia of the Maxwell disk is determined.

Using the Maxwell disk,
1. the potential energy,
2. the energy of translation,
3. the energy of rotation,

are determined as a function of time.

Set-up and procedure

The experimental set up is as shown in Fig. 1 and 2. Using the adjusting screw on the support rod, the axis of the Maxwell disk, in the unwound condition, is aligned horizontally. When winding up, the windings must run inwards.

The winding density should be approximately equal on both sides. It is essential to watch the first up and down movements of the disk, since incorrect winding (outwards, crossed over) will cause the "gyroscope" to break free.

The release switch, the pin of which engages in a hole in the circumference of the disk, is used to release the disk mechanically and to start the counter when determining distance and time. The release switch chould be soadjusted that the disk does not oscillate or roll after the start. Furthermore, the cord should always be wound in the same direction for starting.

Measurement of the time *t* required by the wheel from *s* Start to reach the light barrier

- Press the wire release and lock in place
- Place the selection key of the fork type light barrier on
- Press the "Reset" button of the light barrer.
- Loosening the wire release stopper, sets the wheel into motion and the counter of the light barrier starts.
- **After the wheel has past the needle of the holder, the wire release is pressed again and locked before the light barrier is interrupted.**
- The counter is stopped as soon as the axis of rotation enters the path of light of the fork type light barrier.

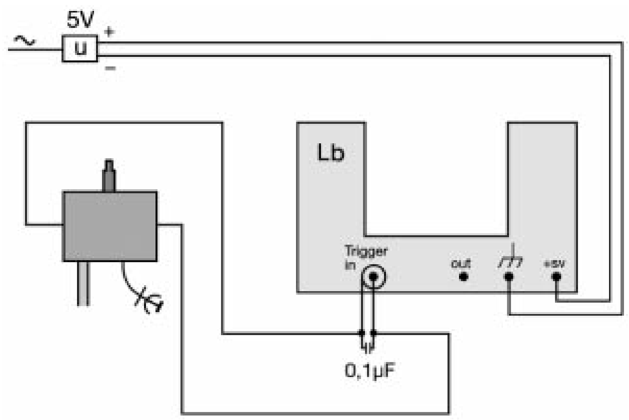
Note

If the counter stops with loosening, namely when pressing the wire relase, a capacitor with a high capacitance is connected parallel to the relase.

Measurement of (Δt to ascertain the translational velocity v

- Fix the wheel in the start position by means of the holder.
- Place the switch on of the fork type light barrier.
- Loosening the wire relase stopper sets the wheel into motion, the counter of the light barrier does not start yet.

Fig. 2: Connection of the light barrier (Lb).



- As soon as the axis of rotation enters the fork type light barrier, the counter starts and stops when it moves past the light ray.
- The velocity at the time $t + \frac{\Delta t}{2}$ is ascertained from the measured time Δt by

$$v\left(t + \frac{\Delta t}{2}\right) = \frac{\Delta s}{\Delta t}$$

Since distance s and time t can by measured relatively accurately, independently of one another, equation (1) below is most suitable for determining the moment of inertia. The times Δt generally have less accuracy. It is not therefore apposite to derive furhter values (e. g. I_z from equation (2)) from these data. They are, however, useful for checking the energy values obtained and calculated from the distance-time measurement.

Theory and evaluation

The total energy E of the Maxwell disk, of mass m and moment of inertia I_z about the axis of rotation, is composed of the potential energy E_p, the energy of translation E_T and the energy of rotation E_R:

$$E = m \cdot \vec{g} \cdot \vec{s} + \frac{m}{2} \vec{v}^2 + \frac{I_z}{2} \vec{\omega}^2 .$$

Here, $\vec{\omega}$ denotes the angular velocity, $\vec{v}$ the translational velocity, $\vec{g}$ the acceleration due to gravity and $\vec{s}$ the (negative) height.

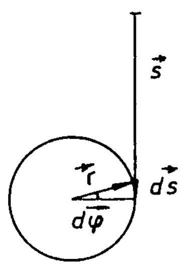
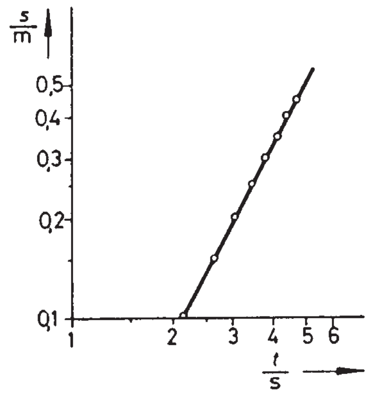


Fig. 3: Relationship between the increase in angle $d\varphi$ and the decrease in height $d\vec{s}$ in the Maxwell disk.

Fig. 4: Distance travelled by the centre of gravity of the Maxwell disk as a function of time.



With the notation of Fig. 3,

$$d\vec{s} = d\vec{\varphi} \times \vec{r}$$

and

$$\vec{v} \equiv \frac{d\vec{s}}{dt} = \frac{d\vec{\varphi}}{dt} \times \vec{r} \equiv \vec{\omega} \times \vec{r},$$

where $\vec{r}$ is the radius of the spindle.

In the present case, $\vec{g}$ is parallel to $\vec{s}$ and $\vec{\omega}$ is perpendicular to $\vec{r}$, so that

$$E = -m \cdot g \cdot s(t) + \frac{1}{2} \cdot (m + I_z/r^2)\,(v(t))^2.$$

Since the total energy E is constant over time, differentiation gives

$$\frac{dE}{dt} = 0 = -m \cdot g \cdot v(t) + (m + I_z/r^2)\, v(t) \cdot \dot{v}(t).$$

For $s(t=0) = 0$ and $v(t=0) = 0$, one obtains

$$s(t) = \frac{1}{2}\,\frac{m \cdot g}{m + I_z/r^2} \cdot t^2 \qquad (1)$$

and

$$v(t) \equiv \frac{ds}{dt} = \frac{m \cdot g}{m + I_z/r^2} \cdot t \qquad (2)$$

The mass m here is m = 0.436 kg. The radius r of the spindle is r = 2.5 mm.

From the regression line to the measured values of Fig. 4, with the expontial statement

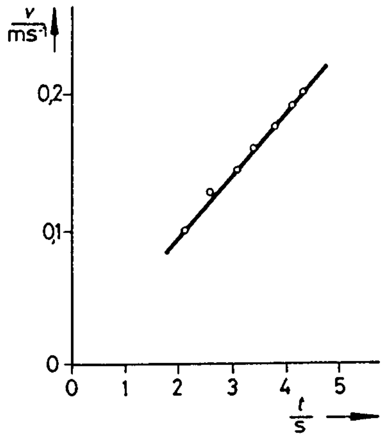
$$Y = A \cdot X^B$$

we obtain:

$$B = 1.99 \pm 0.01 \text{ and}$$

$$A = 0.0196 \pm 0.0015\ \text{m/s}^2.$$

Fig. 5: Velocity of the centre of gravity of the Maxwell disk as a function of time.



With (1), there follows a moment of inertia

$$I_z = 9.84 \cdot 10^{-4}\ \text{kgm}^2.$$

From the regression line to the measured values of Fig. 5, with the exponential statement

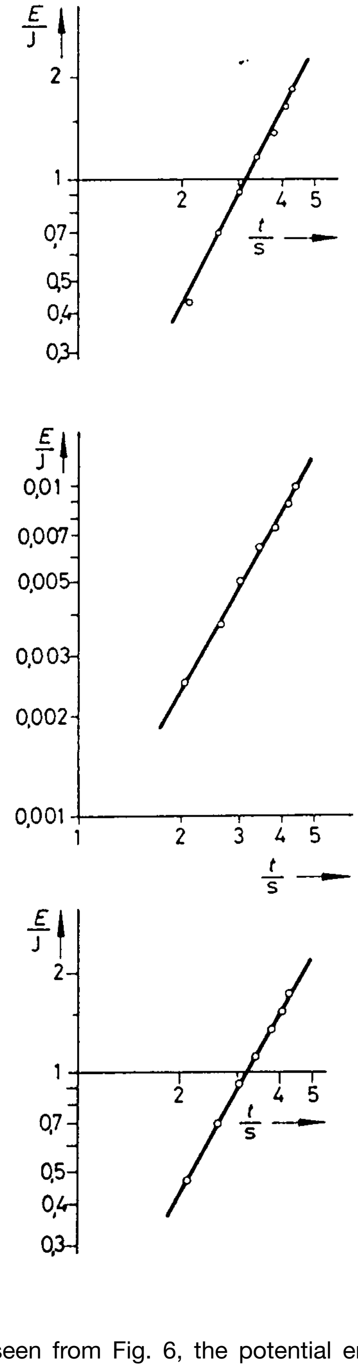
$$Y = A \cdot X^B$$

we obtain for

$$B = 1.03 \pm 0.015 \qquad \text{(see (2))}$$

Fig. 6: Energy of the Maxwell disk as a function of time.

1. Negative potential energy
2. Energy of translation
3. Energy of rotation



As can be seen from Fig. 6, the potential energy is almost completely converted into rotation energy.