Projectile motion

Related topics
Trajectory parabola, motion involving uniform acceleration, ballistics.

Principle and task
A steel ball is fired by a spring at different velocities and at different angles to the horizontal. The relationships between the range, the height of projection, the angle of inclination, and the firing veolocity are determined.

Equipment

Ballistic pendulum	11229.00	1
Recording paper, 1 roll, 25 m	11221.01	1
Steel ball, *d* 19 mm	02502.01	2
Two-tier platform support	02076.03	1
Meter scale, demo, *l* = 1000 mm	03001.00	1
Barrel base	02006.10	1
Timer 4-4	13605.99	1
Speed measuring attachement	11229.30	1
Connecting cord, 750 mm, red	07362.01	1
Connecting cord, 750 mm, yellow	07362.02	2
Connecting cord, 750 mm, blue	07362.04	1

Problems
1. To determine the range as a function of the angle of inclination.
2. To determine the maximum height of projection as a function of the angle of inclination.
3. To determine the (maximum) range as a function of the initial velocity.

Set-up and procedure
Tthe ballistics units is adjusted. The scale is set to read 90° and a ball is fired upwards (setting 3) and is caught in the hand. The support base adjusting screws are turned until a vertical projection is obtained.

The initial velocities of the ball corresponding to the three tension stages of the firing spring can be determined using the speed measuring attachment and timer (resolution 0.1 ms), or from the maximum height for a vertical projection from the expression $v_0 = \sqrt{2gh}$. The inital velocities may vary greatly from unit to unit.

Fig. 1: Experimental set-up for measuring the meximum range of a projectile with additional equipment to measure the initial velocity.

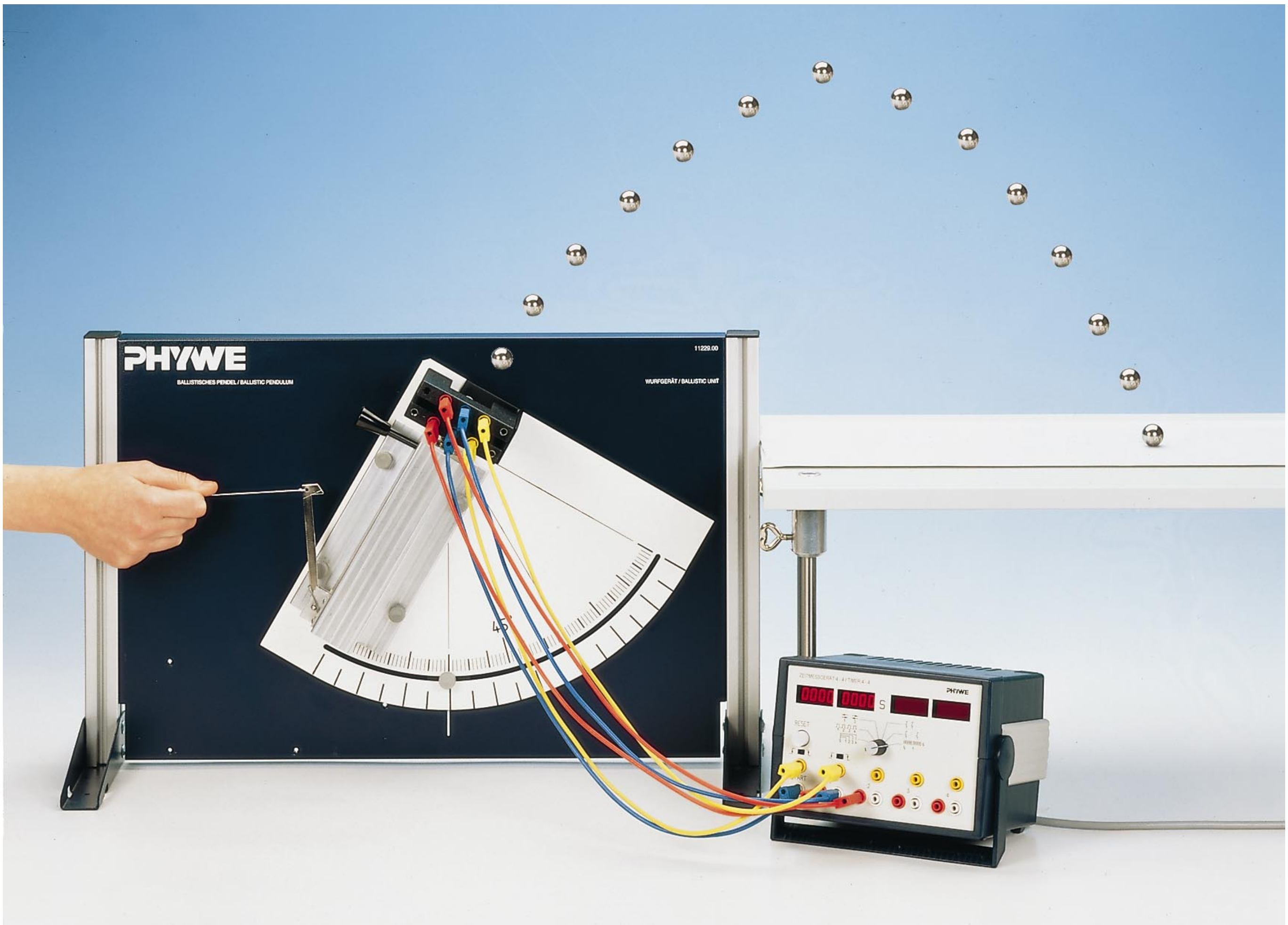
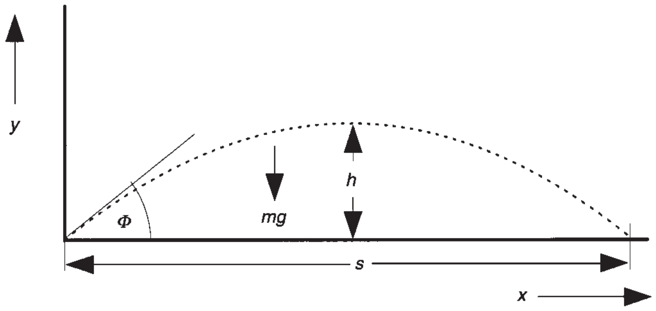


Fig. 2: Movement of a mass point under the effect of gravitational force.



The 2-tier platform support (02076.01) is used for determining th range. To mark the points of impact, the recording strip is secured to the bench with adhesive tape. It is best to measure the long ranges before the short ones (secondary impact points!) and to mark the primary impact points with a felt pen. The distance from the ballistics unit is frequently checked with the metre scale during the test. An empty box can be placed behind the bench to catch the balls.

To measure the height of projection the metre scale is clamped in the barrel base and moved parallel to the plane of projection. The empty box is again used to catch the balls. The heights of projection can be determined ballistically quite well by eye.

Theory and evaluation

If a body of mass m moves in a constant gravitational field (gravitational force $m\vec{g}$), the motion lies in a plane.

If the coordinate system is laid in this plane (x, y plane – see fig. 2) and the equation of motion:

$$m \frac{d^2}{dt^2} \vec{r}(t) = m\vec{g}$$

where:

$$\vec{r} = (x,y) \; ; \; \vec{g} = (o, -g)$$

is solved, then, with the initial conditions

$$r(o) = 0$$

$$\vec{v}(o) = (v_o \cos\phi, v_o \sin\phi)$$

we obtain the coordinates as a function of time t:

$$x(t) = v_o \cdot \cos\phi \cdot t$$

$$y(t) = v_o \cdot \sin\phi \cdot t - \frac{g}{2} t^2 :$$

From this, the maximum height of projection h is obtained as a function of the angle of projection ϕ:

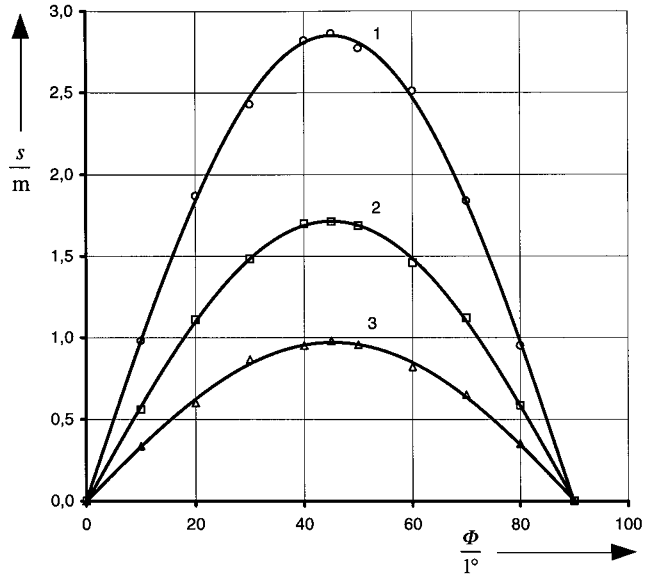
$$h = \frac{v_o^2}{2\,g} \sin^2\phi$$

and the maximum range s is:

$$s = \frac{v_o^2}{g} \sin 2\phi$$

Fig. 3: Maximum range as a function of the angle of inclination ϕ for different initial velocity v_o:

Curve 1 v_o 5.3 m/s
Curve 2 v_o 4.1 m/s
Curve 3 v_o 3.1 m/s



From the regression line of the data of fig. 5, using the expression:

$$Y = A \cdot X^B$$

we obtain the exponent

$$B = 2.01 = 0.001 \qquad \text{(see (1))}$$

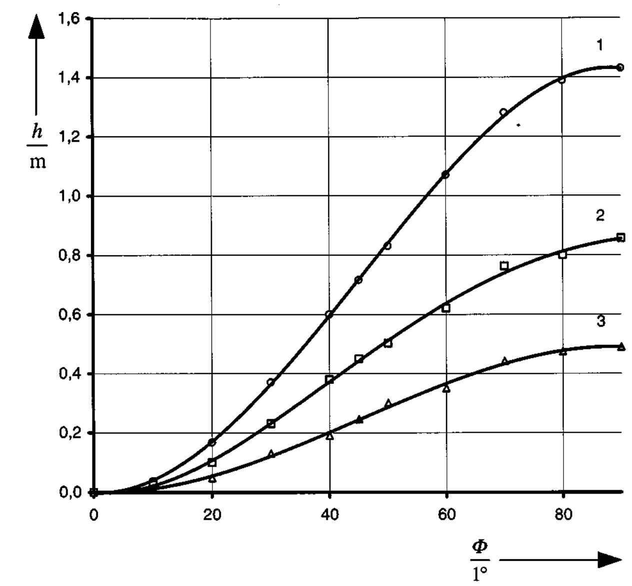
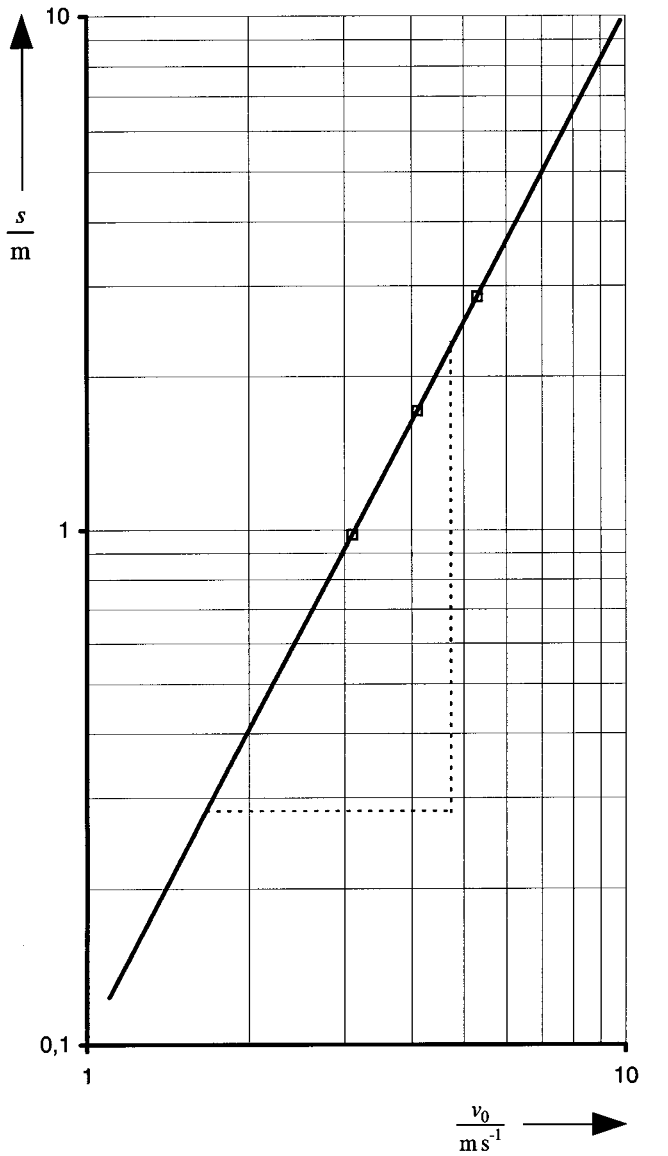


Fig. 4: Maximum height of projection h as a function of the angle of inclination ϕ for the initial velocities as in Fig. 1:

Fig. 5: Maximum range s as a function of the inital velocity v_o with a fixed angle of inclinination $\phi = 45°$.



Note

To ensure an accurate determination of the initial velocity with the timer, the time taken for the ball to cover the measuring distance must be taken into account. If v_{exp} is the experimentally determined inital velocity we obtain

$$v_o = \sqrt{v_{exp}^2 + 2g\,d\,\sin\phi}$$

where d is the distance between the point of rotation and the centre between the light barriers.