Related topics
Conservation of momentum, conservation of energy, linear motion, velocity, elastic loss.

Principle and task
The velocities of two gliders, moving without friction on an air-cushion track, are measured before and after collision, for both elastic and inelastic collision.

Equipment
Air track rail 11202.17 1
Blower 13770.93 1
Pressure tube, l 1.5 m 11205.01 1
Glider f. air track 11202.02 2
Screen with plug, l 100 mm 11202.03 2
Tube with plug 11202.05 2
Needle with plug 11202.06 2
Fork with plug 11202.08 1
Rubber bands f. fork w/plug, 10 pcs 11202.09 1
Plate with plug 11202.10 1
Starter system 11202.13 1
Magnet w. plug f. starter system 11202.14 1
Endholder for air track rail 11202.15 1
Slotted weight, 10 g, black 02205.01 10
Slotted weight, 50 g, black 02206.01 6
Light barrier, compact 11207.20 2
Timer 4-4 13605.99 1
Portable balance Mod. LS2000 46002.93 1
Barrel base 02006.10 2
Support rod -PASS-, square, l 400 mm 02026.55 2
Right angle clamp -PASS- 02040.55 2
Connecting cord, 1000 mm, red 07363.01 2
Connecting cord, 1000 mm, yellow 07363.02 2
Connecting cord, 1000 mm, blue 07363.04 2

Problems
1. Elastic collision
A glider whose mass always remains unchanged collides with a second resting glider at a constant velocity. A measurement series, in which the velocities of the first glider before the collision and the velocities of both gliders after it are to be measured, is conducted by varying mass of the resting glider.

Plot the following parameters as functions of the mass ratio of the gliders:

1.1 The impulses of the two gliders as well as their sum after the collision. For comparison the mean value of the impulses of the first glider is entered as a horizontal line in the graph.

1.2 Their energies, in a manner analogous to Problem 1.1

1.3 In accordance with the mean value of the measured impulse of the first glider before the collision, theoretical values of the impulses for the two gliders are entered for a range of mass ratios from 0 to 3. For purposes of comparison the measuring points (see 1.1) are plotted in the graph.
1.4 In accordance with the mean value of the measured energy of the first glider before the collision, the theoretical values of the energy after the collision are plotted analogously to Problem 1.3. In the process, the measured values are compared with the theoretical curves.

2. Inelastic collision
A glider, whose mass always remains unchanged, collides with a constant velocity with a second resting glider. A measurement series with different masses of the resting glider is performed: the velocities of the first glider before the collision and those of both gliders, which have equal velocities, after it are to be measured.

2.1 The impulse values are plotted as in Problem 1.1.

2.2 The energy values are plotted as in Problem 1.1.

2.3 The theoretical and measured impulse values are compared as in Problem 1.3.

2.4 As in Problem 1.4, the theoretical and measured energy values are compared. In order to clearly illustrate the energy loss and its dependence on the mass ratios, the theoretical functions of the total energy of both gliders and the energy loss after the collision are plotted.

Set up and procedure
The experimental set-up is performed as shown in Fig. 1. The starting device serves to start the glider; three defined and reproducible initial energies can be selected with the various latch positions. It is recommended that the second position be used for all measurements.

The momentum is determined by measuring the velocity of the glider. For this purpose, the time during which the screen fitted on the glider impinges on the light barrier is used, in accordance with:

\[ v = \frac{\Delta s}{\Delta t} \]

(\(\Delta s = \) length of screen, \(\Delta t = \) shading time).

Connect the light barriers with input jacks 1 and 3 on the timer [connect jacks having the same colours (red and yellow) and the two earth (ground) jacks to each other]. Select the “Collision experiments” operating mode (2 double arrows printed on the front panel). In this mode, up to two shading periods are measured and displayed for each light barrier. When varying the mass ratios, ensure that the additional masses are added symmetrically in each case. Before initiating the measurements, check the track’s adjustment.
Laws of collision / Air track

Theory and evaluation

In the elastic collision of two bodies having masses \( m_1 \) and \( m_2 \), kinetic energy and momentum are conserved:

\[
\frac{\Delta p_1^2}{2m_1} + \frac{\Delta p_2^2}{2m_2} = \frac{\Delta p_1'^2}{2m_1} + \frac{\Delta p_2'^2}{2m_2}
\]

\( \Delta p_1 + \Delta p_2 = \Delta p_1' + \Delta p_2' \),

where \( \Delta p_1, \Delta p_2 \) are the moments before the collision and \( \Delta p_1', \Delta p_2' \) those after the collision.

Due to the unidimensional sequence of movement, we will dispense with the vectorial notation. For a central elastic with \( \rho_2 = 0 \):

\[
\rho_1 = \frac{m_1 - m_2}{m_1 + m_2} \cdot \rho_1 = \frac{1 - \frac{m_1}{m_2}}{1 + \frac{m_1}{m_2}} \cdot \rho_1
\]

\[
\rho_2 = \frac{2m_2}{m_1 + m_2} \cdot \rho_1 = \frac{2}{1 + \frac{m_1}{m_2}} \cdot \rho_1
\]

Fig. 5: Elastic collision: calculated energies after the collision as functions of the mass ratio of the gliders.

Fig. 6: Inelastic collision: momenta after the collision as functions of the mass ratio of the gliders.

Fig. 7: Inelastic collision: energy after the collision as functions of the mass ratio of the gliders.

Fig. 8: Inelastic collision: calculated momenta after the collision as functions of the mass ratio of the gliders.
From the contribution of the impulse $p$, the energies $E$ can be calculated according to $E = \frac{p^2}{2m}$:

$$E_1' = -\frac{1 - \frac{m_1}{m_2}^2}{1 + \frac{m_1}{m_2}} \cdot E_1$$

$$E_1' = \frac{4}{\left(1 + \frac{m_1}{m_2}\right)^2} \cdot \frac{m_1}{m_2} \cdot E_1$$

Fig. 2 and Fig. 3 show the results for a sample measurement. In particular, one can see that the total impulse and the total energy before and after the collision, except for a slight loss, are equal. The comparison of the measured values with the theoretical values according to the formulas given above, can be seen in Fig. 4 and Fig. 5.

In an inelastic collision, only the momentum is conserved. In addition, the velocities after the collision are equal:

$$p_1' = \frac{m_1}{m_2} \cdot p_2$$

Therefore,

$$p_1' = \frac{1}{1 + \frac{m_2}{m_1}} \cdot p_1$$

$$p_2' = \frac{1}{1 + \frac{m_1}{m_2}} \cdot p_1$$

The following is obtained for the energies of the two gliders after the collision:

$$E_1' = \frac{1}{\left(1 + \frac{m_2}{m_1}\right)^2} \cdot E_1$$

$$E_2' = \frac{1}{\left(1 + \frac{m_1}{m_2}\right)^2} \cdot \frac{m_1}{m_2} \cdot E_1$$

The evaluation of a sample measurement (Fig. 6 and Fig. 7) shows that also for an inelastic collision, the total impulse is conserved; whereas, depending on $m_1/m_2$, a substantial energy loss occurs.

The theoretical curves are compared with the measured values in Fig. 8 and Fig. 9. In Fig. 9, the energy loss is additionally plotted [energy loss $= E_1 - (E_1' + E_2')$]. One sees that for a mass ratio of 1, the kinetic energy is reduced by exactly 50%.