

### Related topics

Spring constant, limit of elasticity, extension and compression, use of an interface system.

### Principle and task

The validity of Hooke's Law is proven using various helical springs with different spring constants. In comparison, the behaviour of a stretched rubber band is examined, for which there is no proportionality between acting force and resulting extension.

### Equipment

COBRA-interface 2	12100.93	1
PC COBRA data cable RS232, 2 m	12100.01	1
Measuring module Newton	12110.00	1
Newton sensor	12110.01	1
Movement sensor with cable	12004.10	1
Softw. COBRA Force (Win)	14290.61	1
Basic Softw. f. PHYWE Windows prog	14099.61	1
Adapter, BNC-socket/4 mm plug pair	07542.27	1
Adapter, BNC socket - 4 mm plug	07542.20	1
Helical spring, 3 N/m	02220.00	1
Helical spring, 20 N/m	02222.00	1
Helical spring, 30 N/m	02224.00	1

Right angle clamp -PASS-	02040.55	2
Support rod -PASS-, square, l = 1000 mm	02028.55	1
Stand tube	02060.00	1
Barrel base -PASS-	02006.55	1
Bench clamp -PASS-	02010.00	1
Plate holder	02062.00	1
Meter scale, demo, l = 1000 mm	03001.00	1
Nylon thread, l 50 m	02095.00	1

The PHYWE WINDOWS® Basic Software (14099.61) must be installed on the computer being used in order to run the software.

### Problems

1. Calibration of the recorder of movements.
2. Measurement of the tensile force as a function of the path.
3. Measurement of a hysteresis curve.
4. Verification of Hooke's law.

### Set-up

— As shown in Fig. 1 plug the force measuring module into the MODUL 1 input of the interface. Connect the cable of the recorder of movements to COBRA as shown in Fig. 2.

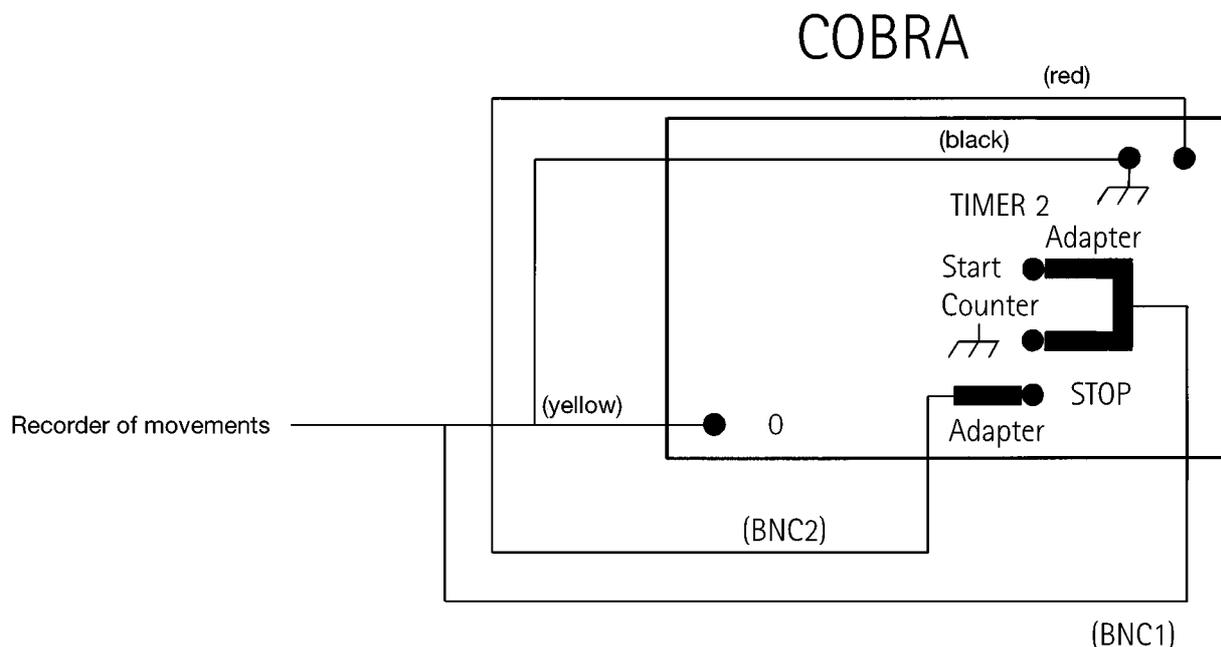
Fig. 1: Experimental set-up: Hooke's law with PC interface.



**Procedure**

- COBRA: Depending on whether COBRA is connected to COM1 or COM2 of the computer, double click on the icon to start the program F(x)\_COM1 or F(x)\_COM2.
- First, click on the <Start> button and then on the <Tara> button. The system performs a tare which takes about 5 seconds. The force meter must not be moved or jarred during taring. If it is jarred, the system may briefly display the message "Overflow" accompanied by a beep. After incorrect taring, the message "Tara Error ?" is displayed on the monitor. In this case, you can restart taring by pressing the <Tara> button once the yellow highlighting has gone out. Should the "Tara Error?" message not disappear even after several taring attempts, then either there are too many vibrations in the room or the set sample time <Delta t/ms> is too small. However, the yellow "Tara Error?" message bears no effect on the measurement procedure described here, since the message only indicates that the measuring ranges  $\pm 40$  mN and/or  $\pm 4$  mN probably could not be tared correctly due to heavy vibrations. The measurements taken here, though, only require the  $\pm 4$  N range.
- You can set the sample rate by adjusting the scroll bar <Delta t/ms>. Each connected computer has its own maximum sample rate. If the sample rate is too short, there may be errors in the data communications. Make sure when <Digit> is pressed that the number of measurement values displayed corresponds to the set sample rate. In other words, with a sample rate of <Delta t/ms> = 300 ms, the force indicator should change about 3 times a second. If the display changes less than this in spite of the fact that the spring is in place and in motion, then you must increase the sample time <Delta t/ms>.
- The measuring range <4 N> is recommended to verify Hooke's Law: this range has a resolution of 3.5 mN. If the measuring interval ( $\pm 4$  N) is exceeded, the message "Overflow" appears accompanied by a warning tone.
- If there are large deviations in the measured force values during measurement, you can reduce the effect of interference by adjusting the <Average> scroll bar to take the average of the displayed number of measurements. The greater the number is for <Average>, the "steadier" the force display will be. However, this does lead to a slow response-time, i.e. if there is a sudden change in force, it takes a few seconds before the correct new value is attained. It is recommended that you use an <Average> value between 1 and 25 for taring. An <Average> value of 1 is recommended for the measurements for Hooke's Law.
- Calibration of the recorder of movements:  
Only use the shorter of the two cord grooves for measuring the path with the recorder of movements. Since the relationship between the linear path and the number of revolutions of the cord groove depends on the thread and the sample time <Delta t/ms> used, a one-time calibration of the path-measuring system is required. To do this, set up the equipment as shown in Fig. 1. An edge of the barrel base is located directly at a dm subdivision on the scale (e.g. 200 mm). Now, enter a path length (e.g. 0.1 m) in the input window "x0" and then press the "Return" key on the keyboard. If the <Stop> button is displayed, click on it first and then click on the <Start> button which then appears. If the <Start> button is already displayed from the start, just click on it once. In any case, the measurement recording must be turned off and then back on one time. Turning it off and on "resets" the path probe to the value 0.000 m. The

Fig. 2: Connecting the recorder of movements to COBRA.

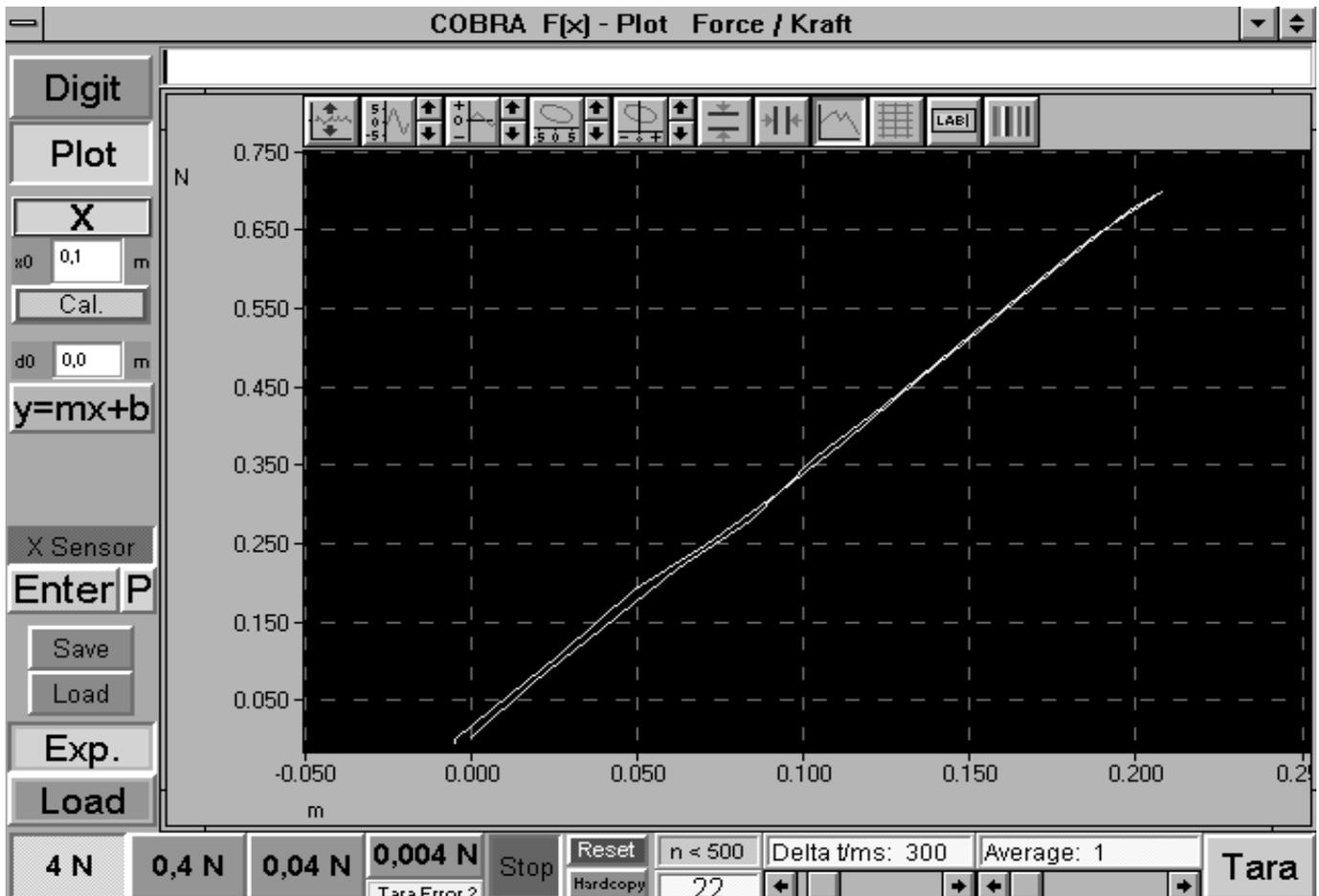


value 0.0 must be displayed in the input field "d0". Click on the <Digit> button and the <X Sensor> button. Now, slowly and steadily move the barrel base along the scale and away from the recorder of movements by the set length "x0". The length "x0" refers to the length on the scale and not the value displayed on the monitor! Next, click on the <Cal.> button. Calibration is now complete. This calibration is set automatically each time the measuring program is restarted and does not need to be repeated. You only need to repeat calibration if you use another thread or set another sample time <Delta t/ms>. If the path sensor is used in another experiment set-up and turned one axis vertically to the axis of the cord groove, then the sign of the path measurements changes. In this case, you should enter the corresponding value under "x0" with a "-" sign and recalibrate.

- Measurement of tensile force as a function of the path:
- 1. Measurement with path probe and manual confirmation of measured values:
  - Place barrel base in starting position and click on <X Sensor>, the spring should be in a nearly relaxed position.

- Click on <Tara>, turn off and on once with <Start> <Stop>, the red <Stop> button should now be visible. If you click on <Digit>, the force  $F$  will oscillate around 0.000 N and the path  $x$  will display 0.000 m. When recording measurements, you can switch back and forth at anytime between <Digit> and <Plot>. With the <Plot> function, there is a yellow measurement point visible in the middle of the graph when the measurements are started: this point moves up and down due to vibrations on the force meter.
- Click on <Reset> to set the number of measurement points to zero and delete the graph.
- Click on the <Enter> button on the screen to accept the current pair of values ( $x;F$ ) into computer memory and display them graphically.
- Slowly and steadily move the barrel base about 2 cm away from the recorder of movements. When the spring has settled, click on the <Enter> button once again. Repeat this procedure for the rest of the measurement points.
- When the spring is stretched about 20 cm to 30 cm, reverse the direction and relax the spring step by step. The result should be as shown in Figure 3.

Fig. 3: Measurement example of the extension and compression of a helical spring. This series of measurements was taken with a path probe and manual confirmation of measurement values.



- You can also click on the  $\langle y=mx+b \rangle$  button at any time. This function draws a regression line through the measured values. The calculated parameters  $m$  and  $b$  are displayed at all times. With the function, make sure that " $x / m$ " is displayed on the x-axis and the number  $n$  of measurement points to be considered is same as the total number of measurement points (Fig. 4).
- 2. Measurement with path probe and permanent, automatic recording of measurements:
  - Place barrel base in starting position and click on  $\langle X$  Sensor  $\rangle$  and  $\langle P \rangle$ .
  - Click on  $\langle \text{Tara} \rangle$ , turn off and on once with  $\langle \text{Start} \rangle$   $\langle \text{Stop} \rangle$ , the red  $\langle \text{Stop} \rangle$  button should now be visible. If you click on  $\langle \text{Digit} \rangle$ , the force  $F$  should oscillate around 0.000 N, and the path  $x$  should display 0.000 m. When recording measurements, you can switch back and forth at any time between  $\langle \text{Digit} \rangle$  and  $\langle \text{Plot} \rangle$ . With the  $\langle \text{Plot} \rangle$  function, there is a yellow measurement point visible in the middle of the graph when the measurements are started: this point moves up and down due to vibrations on the force meter.
  - Click on  $\langle \text{Reset} \rangle$  to set the number of measurement points to zero and delete the graph.
  - Slowly and steadily move the barrel base away from the recorder of movements. The program registers a measurement curve continuously. Measurement points are only

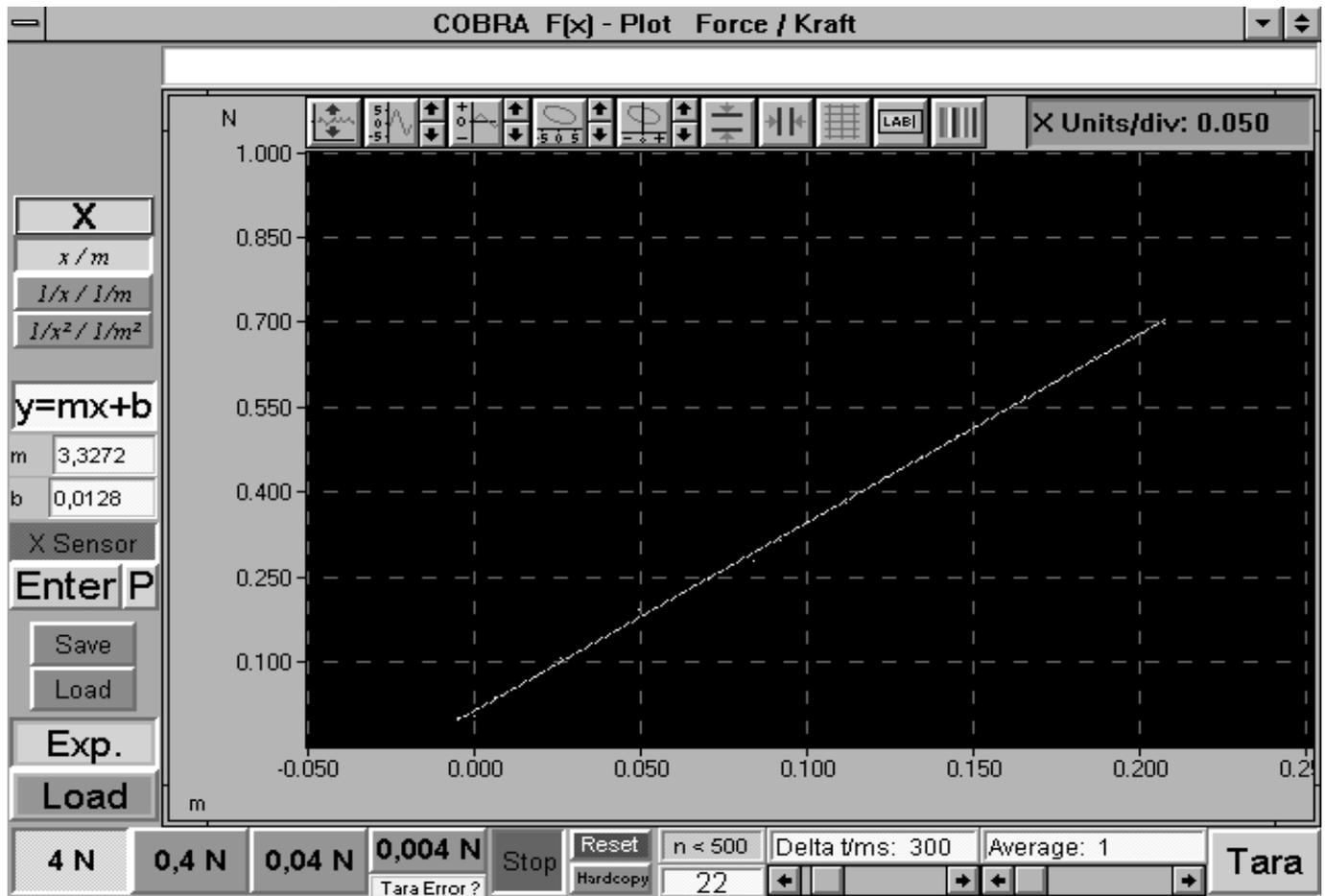
entered into the graph if the cord groove on the recorder of movements is moved. No new measurement points appear on the graph if the barrel base is stopped. After about 20 cm reverse the direction and slide the barrel base back to its starting position (See Fig. 5).

**NOTE:**

Figure 5 shows a "hysteresis curve". This of the spring is not representative of Hooke's Law but rather a sign of the time lag from the force probe and the force modulation caused by the clearly visible vibration of the spring during movement. The "hysteresis" decreases the slower the barrel base is moved and the less the spring vibrates. Since both branches of the curve have symmetrical positions, though, the regression  $\langle y=mx+b \rangle$  also provides very useful results for the spring constant  $D$ . (For this function, make sure that the number  $n$  of the measurement points to be considered is the same as the total number of measurement points).

- 3. Measurements without a path probe:
  - Place barrel base in starting position. Do not click on the button  $\langle X$  Sensor  $\rangle$ .
  - Click on  $\langle \text{Tara} \rangle$ , turn off and on once with  $\langle \text{Start} \rangle$   $\langle \text{Stop} \rangle$ , the red  $\langle \text{Stop} \rangle$  button should now be visible. If you click on  $\langle \text{Digit} \rangle$ , the force  $F$  should oscillate around

Fig. 4: Adaptation of a linear regression line to the measurement data.



0.000 N and the path  $x$  should display 0.000 m. When recording measurements, you can switch back and forth at any time between <Digit> and <Plot>. With the <Plot> function, there is a yellow measurement point visible in the middle of the graph when the measurements are started: this point moves up and down due to vibrations effecting the force meter.

- Click on <Reset> to set the number of measurement points to zero and delete the graph.
- Enter the value 0.0 in the input window “d0” and hit the “Return” key on the keyboard. The first measurement point is recorded after the <Enter> button is clicked.
- Slide the barrel base to the next position and enter the distance of the path into the input window “d0” (do not forget to hit “Return”). Click on <Enter> to register the new pair of measurements ( $x; F$ ).

### Results

When forces act on a solid object, the resulting deformation of the solid object (disregarding translational and rotational movements) depends both on the material and the size and direction of the force. If the solid object reassumes its original form when the external force is removed from it, i.e. the internal restoring forces of the material can bring the solid object

back to its original state of equilibrium, then this object is called elastic. A helical spring is a typical example of an elastic body.

Figures 3 and 4 show that, as long as the excursion  $\Delta l$  of the spring from its neutral position is not too large, the restoring force  $F_r$  is proportional to the degree of extension or compression of the spring.

$$\vec{F}_r \sim -\Delta \vec{l}$$

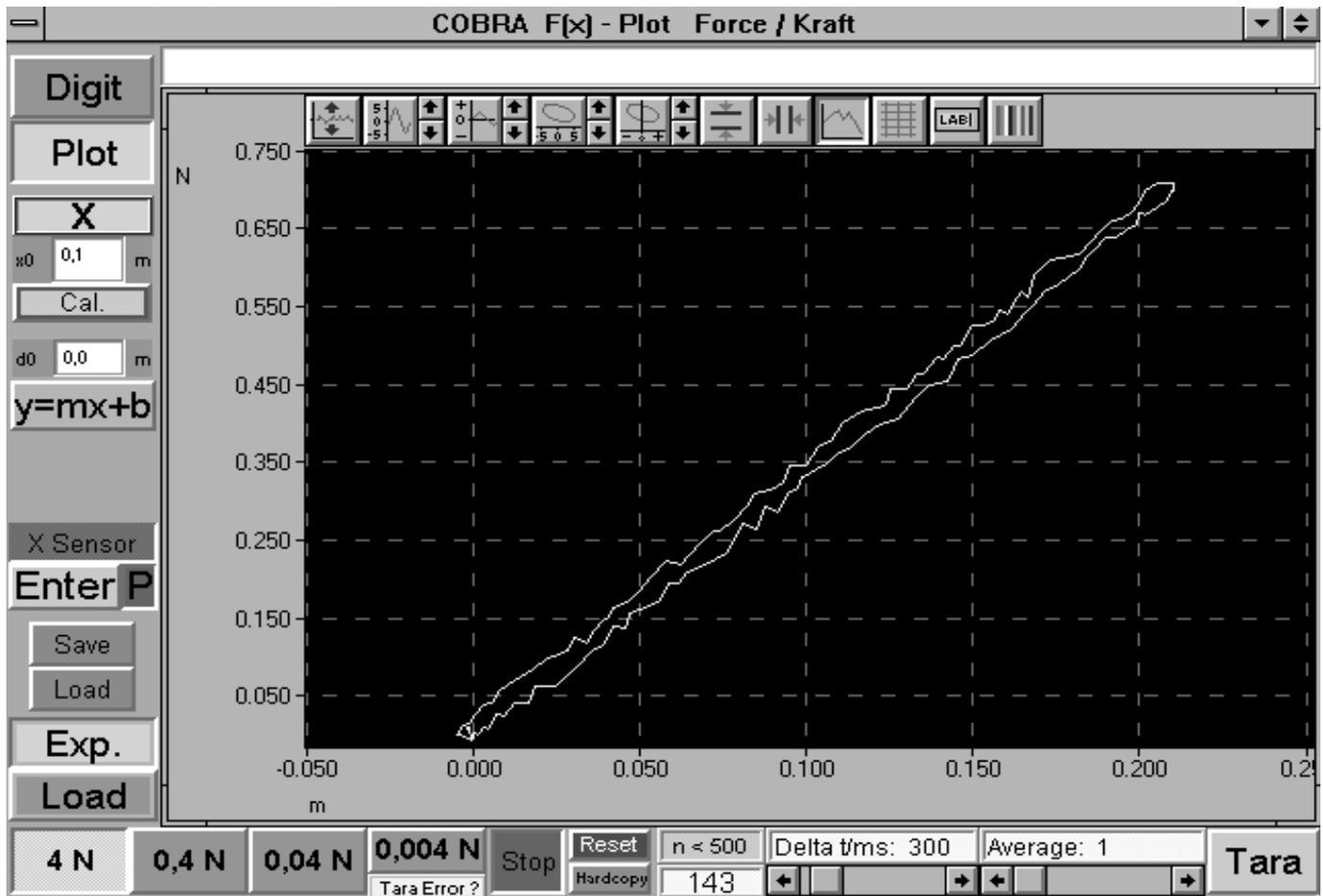
or

$$\vec{F}_r = -D \Delta \vec{l}$$

This is Hooke's Law (linear force law). The proportionality factor  $D$  is referred to as the spring constant in the case of helical springs. If the <y=mx+b> button is activated, then the program automatically displays a regression line with the parameters  $m$  and  $b$ . A coefficient comparison with Hooke's Law shows that the slope  $m$  of the regression line is precisely the same as the spring constant  $D$ . In the measurement example shown in Fig. 4, the spring has a constant of  $D = 3.3272 \text{ N/m}$ .

The proportionality between force  $F_r$  and extension  $\Delta l$ , however, only applies up to a characteristic limit of tension. Beyond this limit, the shape of the solid object is permanent-

Fig. 5: Measurement example of the extension and compression of a helical spring. This series of measurements was taken with a path probe and permanent, automatic recording of measurement values.



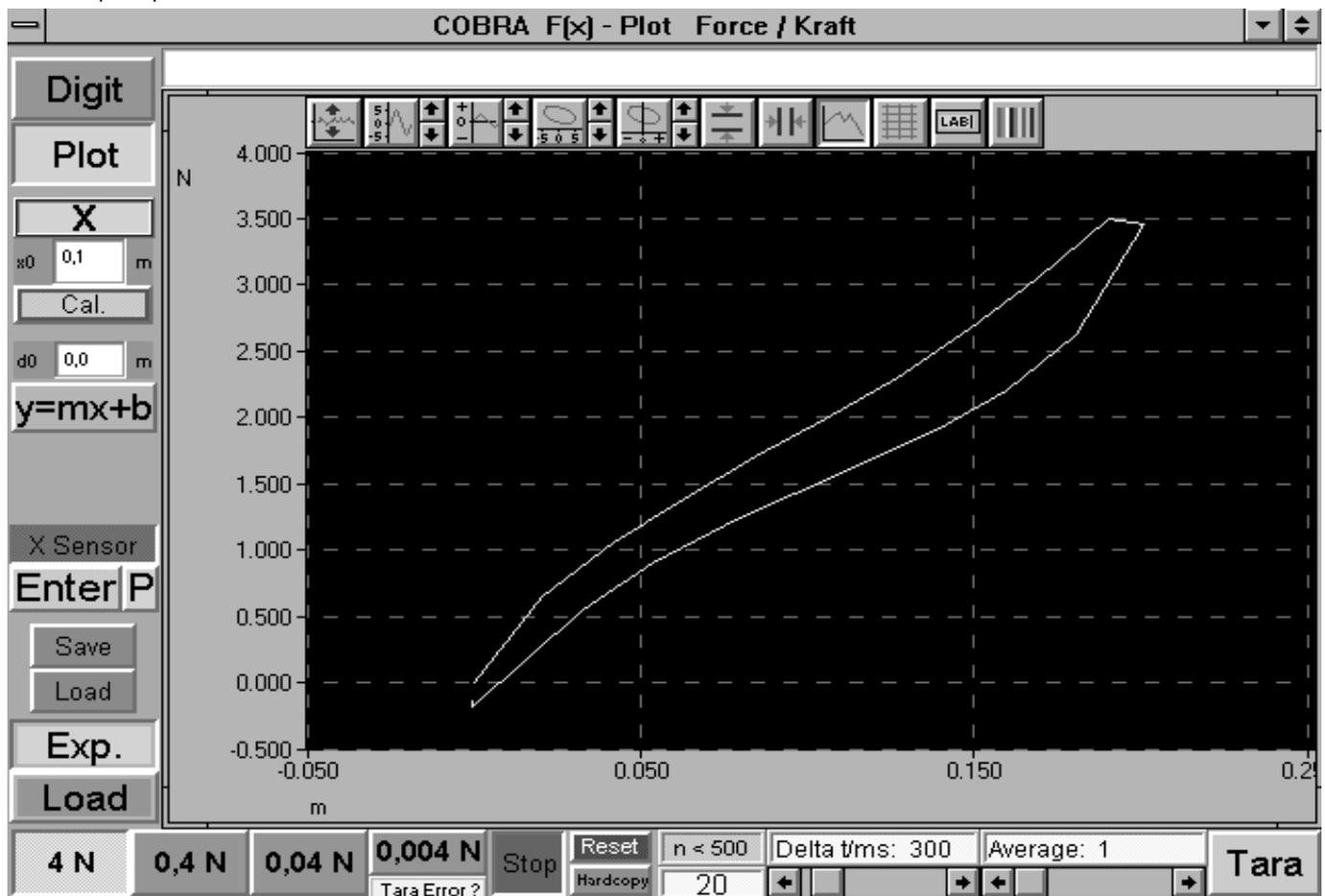
ly altered due to the rearrangement of its molecules. This is known as a plastic deformation. If the tension is increased further, then the material of the solid object disintegrates and the body breaks.

An example of material which does not follow Hooke's Law when subjected to even small forces is a rubber band. Figures 6 and 7 show the curve of a rubber band. The upper branch of the curve is the product of increased tension and the lower branch of the curve was produced by the subsequent relaxation of the rubber band. On the one hand, the relationship between the acting force and the change in length is no longer linear, and, on the other, the extent of stretching is dependent upon how much the rubber band has already been stretched. This behaviour is referred as elastic hysteresis. There are two reasons for the hysteresis of the curve. Firstly, only part of the deformation recedes immediately after stretching while the rest of the deformation only returns to its initial state gradually over a period of several hours. This reversible process is called elastic hysteresis, the material reacts in a elasticoviscous manner. Secondly, when the elasticity limit is exceeded, the material is rearranged internally, altering the shape of the body permanently. This process is irreversible since work is converted into heat. In this experiment, the elastic hysteresis effect is dominant.

#### Notes

- <Save> (green) saves the recorded measurement data as an ASCII file on the hard drive or on a floppy disk. This data can be read by and altered using common spread sheet or word processing programs. The files should have the ending \*.AFD (**A**SCII **F**ile **D**ata) so that the measurement data files can be easily identified.
- <Load> (green) loads saved measurement files into the computer's memory for evaluation and display. To display the loaded series of measurements graphically, click on the blue <Load> button beneath the <Exp.> button. Clicking on the <Exp.> button, however, always displays the graph of the current experiment. In other words, you can use the <Exp.> and <Load> buttons to switch back and forth between the loaded measurements and the current ones.
- There is a white line above the graph. You can enter a heading here after clicking on the line.
- The <Hardcopy> button is for printing out the entire screen display on the printer set under WINDOWS. Before using this function, you should change the colour combination of the graph to save on ink or toner. To do this, click on the fainbow-coloured button in the upper right above the

Fig. 6: Measurement example of the extension and compression of a rubber band. This series of measurements was taken with a path probe and manual confirmation of measurement values.



graph (to the right of the <LAB> button). The following settings are recommended for the printout:

Scope	white
Data	black

- The other buttons above the graph have the following functions from left to right:
  - Autoscale function, to display all measurement points.
  - Stretch and shrink y-axis, use the arrows to do this in set steps, click on the button with the curve to enter maximum and minimum values via the keyboard.
  - Shift the curve along the y-axis, without changing scale (arrows).
  - Stretch and shrink x-axis, use the arrows to do this in set steps, click on the button with the curve to enter maximum and minimum values.
  - Shift curve along x-axis, without changing scale (arrows).
- Show horizontal cursor lines which can be moved with the left mouse button. The absolute and relative values of each position are displayed beneath the graph.
- Show vertical cursor lines which can be moved with the left mouse button. The absolute and relative values of each position are displayed beneath the graph.
- Connect measurement points with a lines.
- Show grid, a coordinates of the origin system or a single-colour background.
- Change x- and y-axis labels.
- Change colour of the graph.
- The <->x> button, when it is visible and is clicked, enlarges the measurement points along the line of the measurement curve. Depending on the number of measurement points, this function may take a few seconds to execute.
- You can end the program by double-clicking on the symbol at the extreme upper left.
- The graphic system used has a resolution of 640 x 480 pixels. If you are using a monitor with a higher resolution, then the program only uses a specific segment in the upper left of the screen on the monitor.

Fig. 7: Measurement example of the extension and compression of a rubber band. This series of measurements was taken with a path probe and permanent, automatic recording of measurement values.

