Related topics

Hooke's law, spring constant, limit of elasticity, elastic hysteresis, elastic after-effect

Principle and task

The validity of Hooke's law is determined for two helical springs with different spring constants. The elongation of the helical spring, which depends on the deforming force, is studied by means of the weights of masses. For comparison, a rubber band, for which no proportionality exists between the exerted force and the resulting elongation, is submitted to the same forces.

Equipment

Tripod base -PASS-	02002.55	1
Barrel base -PASS-	02006.55	1
Support rod -PASS-, square, l = 1000 mm	02028.55	1
Right angle clamp -PASS-	02040.55	1
Cursors, 1 pair	02201.00	1
Weight holder f. slotted weights	02204.00	1
Slotted weight, 10 g, black	02205.01	2
Slotted weight, 10 g, silver bronze	02205.02	2
Slotted weight, 50 g, black	02206.01	1
Slotted weight, 50 g, silver bronze	02206.02	2
Helical spring, 3 N/m	02220.00	1
Helical spring, 20 N/m	02222.00	1
Silk thread, 200 m	02412.00	1
Meter scale, demo, l = 1000 mm	03001.00	1
Holding pin	03949.00	1
Square section rubber strip, l 10 m	03989.00	1

Problems

1. Determining the spring constants of helical springs
2. Study of the elongation of a rubber band

Set-up and procedure

The experimental set-up to measure the spring constants is shown in Fig.1. To start with, the helical spring is submitted to no stress; the sliding pointer is set to the lower end of the spring and its corresponding position x_0 on the measuring scale is recorded. The load on the helical spring is then increased in steps of 10 g, using the weight holder and the slotted weights, until a maximum load of 200 g is reached. Noting the equilibrium (stabilised) position of the lower end of the helical spring x_1, the corresponding increase $\Delta l = |x_1 - x_0|$ of the spring is assessed. Weight F_W, which causes elongation, is plotted as a function of elongation Δl. The elasticity of the helical spring is controlled by repeating the determination of Δl for some weights and by comparing the results to those obtained during the first measurement. This procedure is repeated for both helical springs (D = 3 N/m and 20 N/m).

To determine the characteristic curve of the rubber band, a piece of band of about 50 cm length is cut off. Both ends of the rubber band are tied to small loops with silk thread. One loop is slipped onto the holding bolt (cf. Fig. 2) and the weight holder is suspended from the other loop. In the same way as for the helical springs, forces (weights) are increased in steps of 10 g up to a maximum of 200 g. The momentary elongation of the rubber band must be maintained by hand during the exchange of weights, because the elongation depends on the previous history of the material (cf. theory). Subsequently, weight is decreased in 10 g steps by removing slotted

weights. The equilibrium position x_0 of the rubber band without slotted weights is calculated approximately through application of Hooke's law to equilibrium positions $x_1(10 \text{ g})$ and $x_1(20 \text{ g})$:

$$x_0 = x_1(10 \text{ g}) - (x_1(20 \text{ g}) - x_1(10 \text{ g})).$$

Weight F_W, which causes elongation, is plotted as a function of elongation $\Delta l = |x_1 - x_0|$.
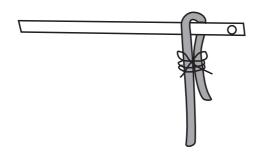
Fig.1: Experimental set-up: Hooke's law.

Fig. 2: Fixing the rubber band to the holding bolt.

Theory

When forces act on a solid body, the resulting deformation (translation and rotation movements are suppressed in the following) depends to a large extent on the material as well as on the size and on the direction along which the exterior forces act. When the solid body regains its original shape after the exterior force stops acting, that is, the interior restoring forces of the material can bring the solid body back to its original equilibrium position, the material is called elastic.

A helical spring is a very simple example of an elastic body (cf. Fig.3). In addition, if deviations Δl from the equilibrium position l_0 of the helical spring are not very large, the restoring force F_R of the spring is found to be proportional to its elongation (or to its compression) Δl:

$$\vec{F}_R = -D\,\vec{\Delta l} \qquad (1)$$

This is Hooke's law or the linear law of forces, where the proportionality constant D, which is a general magnitude of reference, is called the spring constant in the case of a helical spring. If an exterior force acts on the spring, such as the weight $F_W = m \cdot g$ of a mass m (g = 9.81 m/s^2: acceleration of terrestrial gravity) in this experiment, a new stable equilibrium is reached for the length of the spring l_1, for which the weight mass m is equal to the restoring force of the spring:

$$F_R = D\,\Delta l = mg = F_W \qquad (2)$$

The elongation of the helical spring is therefore proportional to the forces F_W exerted by the weights:

$$\Delta l = \frac{1}{D}\,F_W \qquad (3)$$

as is also shown by the characteristic curves of the two helical springs (Figs. 4 and 5). The slope of the characteristic curves is the respective spring constant D of the helical springs. Measurement values from Fig. 4 yield a spring constant of:

$$D = 3.03 \text{ N/m},$$

measurement values from Fig.5 yield a spring constant of:

$$D = 19.2 \text{ N/m}.$$

Thus, forces required to cause a given elongation of the spring increase proportionally with the spring constant. Using equation (3), the new equilibrium length l_1 is found to be:

$$l_1 = l_0 + \frac{mg}{D} \qquad (4)$$

Proportionality between the restoring forces, as long as they are small, and the elongation of the solid body are ascertained not only for the helical spring, but also for all other materials which are in a state of stable equilibrium: the potential energy of forces between molecules is approximately parabolic around a stable point of equilibrium. Restoring forces obtained by differentiating the potential are thus proportional to the deviation from the rest position.

Taking for example a rod or wire of a given material of length l and cross-section A, to which a traction force F is applied, Hooke's law is expressed through:

$$\frac{\Delta l}{l} = \alpha\,\frac{F}{A} \qquad (6)$$

or

$$\varepsilon = \alpha \cdot \sigma$$

where $\varepsilon = \Delta l / l$ is the relative elongation of the rod, the proportionality factor α is the coefficient of elasticity of the rod material and $\sigma = F/A$ is the tension of the rod.

Proportionality only holds up to a characteristic limit stress. A schematic stress-elongation diagram for a metal wire is shown in Fig. 6. The limit of proportionality (σ_P) generally lies below the elastic limit (σ_E), above which the form of the solid body changes permanently, due to interior molecular re-arrangements. In this range of stresses, the material is said to be plastic. If the deforming forces exceed the limit of solidity (σ_B), the solid material begins to flow and the body breaks.

An example of a material which does not follow Hooke's law, even when submitted to small forces, is a rubber band. Fig. 7 shows the characteristic curve of a rubber band, with continuously increasing stress between point O and point A and with gradual relief between point A and point B. On the one hand, the relation between acting weight F_w and resulting
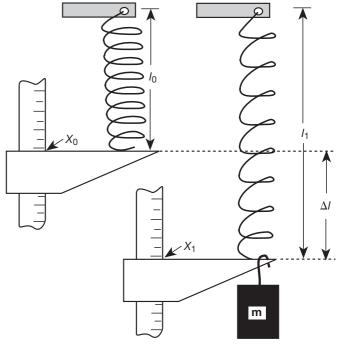

Fig. 3: Measurement of the elongation of the helical spring.

Fig. 4: Weight F_w of a mass m which acts on the helical spring, plotted as a function of elongation Δl for a helical spring with constant D = 3 N/m.
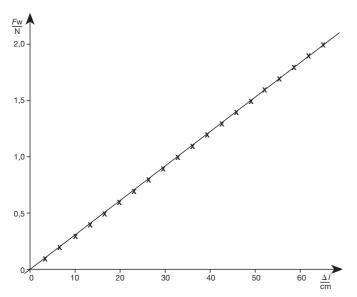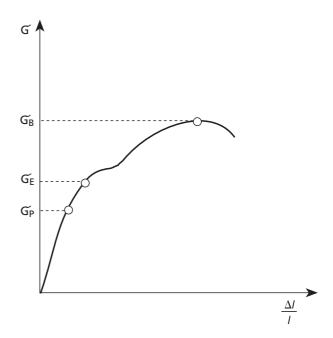

Fig. 6: Stress-elongation diagram (schematic).

elongation Δl is no longer linear: elongation is larger than expected according to Hooke's law, considering the measurement values for small stresses (dotted line).

On the other hand, the degree of elongation depends on the previous history of the rubber band. In the characteristic curve of the rubber band, part OA (gradual increase of stress) does not coincide with part AB (gradual relief of stress), which is contrary to what is observed for the helical spring, as long as it remains within the limit of elasticity. This phenomenon is called elastic hysteresis. If the same rubber band is stressed again, elongation Δl will now be significantly larger than had been the case for the new rubber band.

The hysteresis of the characteristic curve has two causes: on one hand, only part of the deformation reverts back to the original form momentarily, whereas the rest of the deformation reverts back over a period of several hours. This reversible process is called elastic after-effect, the material reacts visco-elastically. On the other hand, once the elastic limit is exceeded, interior re-arrangements take place within the material, which results in permanent changes of shape. This process is irreversible, because work is converted to heat. In this experiment, the effect of the elastic after-effect dominates.
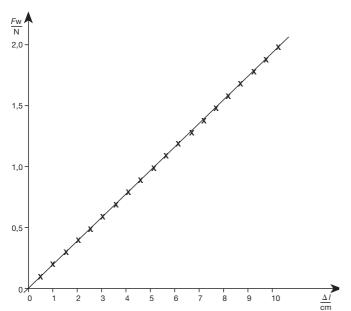

Fig. 5: Weight F_w of a mass m which acts on the helical spring, plotted as a function of elongation Δl for a helical spring with constant D = 20 N/m.
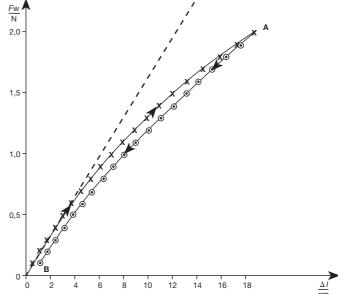

Fig. 7: Acting weight F_w as a function of the extension Δl for a rubber band (elastic hysteresis).