

### Related topics

Young's modulus, modulus of elasticity, stress, deformation, Poisson's ratio, Hooke's Law.

### Principle and task

A flat bar is supported at two points. It is bent by the action of a force acting at its centre. The modulus of elasticity is determined from the bending and the geometric data of the bar.

### Equipment

|                                      |          |    |
|--------------------------------------|----------|----|
| Dial gauge 10/0.01 mm                | 03013.00 | 1  |
| Holder for dial gauge                | 03013.01 | 1  |
| Flat bars, set                       | 17570.00 | 1  |
| Knife-edge with stirrup              | 03015.00 | 1  |
| Bolt with knife-edge                 | 02049.00 | 2  |
| Weight holder f. slotted weights     | 02204.00 | 1  |
| Spring Balance 1 N                   | 03060.01 | 1  |
| Tripod base -PASS-                   | 02002.55 | 2  |
| Support rod -PASS-, square, l 250 mm | 02025.55 | 2  |
| Support rod -PASS-, square, l 630 mm | 02027.55 | 1  |
| Right angle clamp -PASS-             | 02040.55 | 5  |
| Slotted weight, 10 g, black          | 02205.01 | 10 |
| Slotted weight, 50 g, black          | 02206.01 | 6  |
| Measuring tape, l = 2 m              | 09936.00 | 1  |

### Problems

- Determination of the characteristic curve of the dial gauge
- Determination the bending of flat bars as a function
  - of the force
  - of the thickness, at constant force
  - of the width, at constant force
  - of the distance between the support points at constant force
- Determination the modulus of elasticity of steel, aluminium and brass.

### Set-up and procedure

The set up is as shown in Fig. 1. The dial gauge is mounted on the knife edge with stirrup. The flat bars must be positioned accurately **on** the two supporting knife edges having the possibility to move in x and y direction. The geometric data of the set up and the bars must be recorded several times or at different positions.

As the dial gauge possesses a restoring force which obeys Hooke's Law  $F \sim s$ , its characteristic curve must first of all be determined.

Fig.1: Experimental set-up for determining the modulus of elasticity.

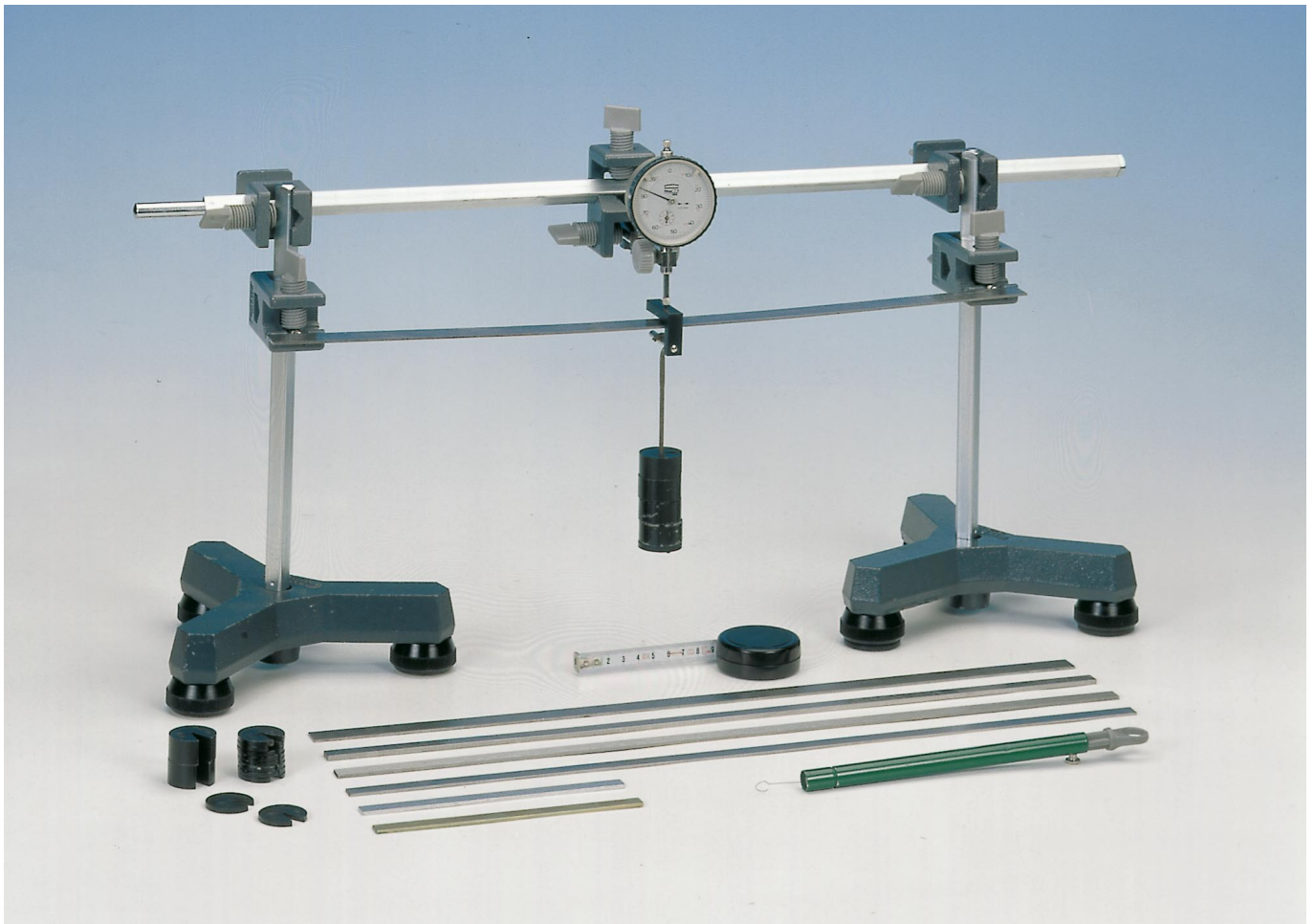
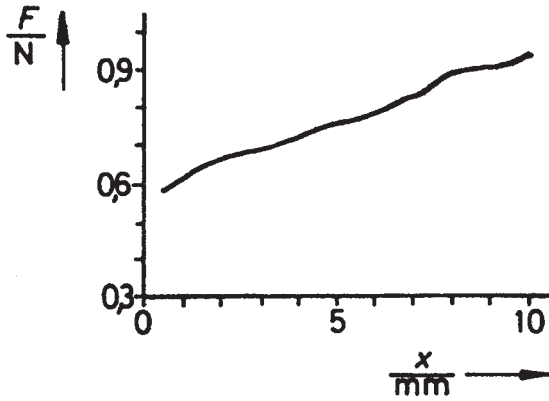


Fig. 2: Characteristic curve of a dial gauge.



The resulting force  $F_r$  of the dial gauge is the sum of

$$F_r = F_h + F_f$$

where

$$F_h = \text{Static friction force (constant)}$$

$$F_f = \text{restoring force } (F_f \sim s)$$

Since the static friction force always acts opposed to the direction of motion, care should be taken of constant direction of force while recording of the characteristic curve of the dial gauge as well as during performance of the experiment. Additionally, the plunger is manually raised (the probe is unloaded) and then gently lowered. Due to this procedure the resulting force  $F_r$  is:

$$F_r = F_f - F_h.$$

a) Characteristic curve of the dial gauge.

Dynamometer and plunger of the dial gauge are so mounted, that the dial gauge shows full deflection.

Through a reduction of the tension of force to the dynamometer a force results according to the above conditions.

With the help of the spring balance the characteristic curve of the dial gauge has to be recorded.

During experimentation the forces must be corrected accordingly. This means the resulting force  $F_r$  must be calculated.

Hence, the effective force is the sum of the weights of the additional masses and the resulting force of the dial gauge.

### Theory and evaluation

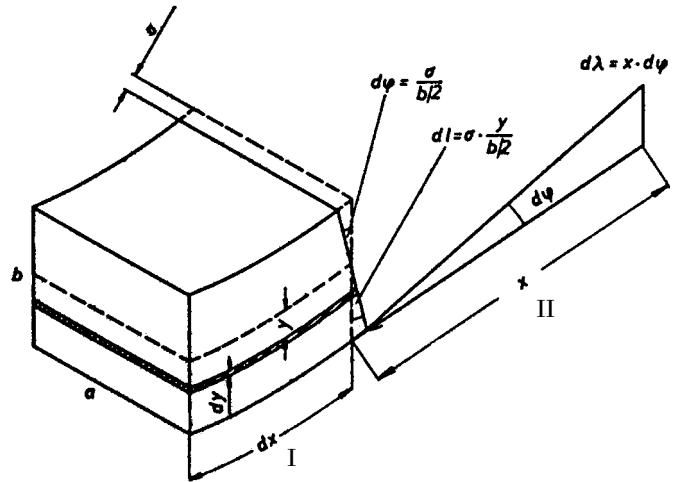
If a body is regarded as a continuum, and  $\vec{r}_0$  or  $\vec{r}$  defines the local vector of a point  $P$  in its undeformed and deformed state respectively, then, for small displacement vectors

$$\vec{u} = \vec{r} - \vec{r}_0 \equiv (u_1, u_2, u_3)$$

the deformation tensor  $\hat{d}$  is:

$$d_{ik} = \frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}$$

Fig. 3: Deformation of a bar.



The forces  $d\vec{F}$  which act upon an element of volume of the body whose edges are intersected parallel to the coordinate surfaces will be described by the stress tensor  $\hat{\tau}$ .

This allocates a stress  $\vec{p}$  to each area element  $dA$  defined by the unit vector  $\vec{e}$  in the direction of the normal.

$$\vec{p} = \frac{d\vec{F}}{dA}$$

$$\vec{p} = \vec{e} \odot \hat{\tau}$$

From Hooke's Law we obtain the relationship between  $\hat{d}$  and  $\hat{\tau}$ :

$$\tau_{ik} = \sum_{l,m} c_{ik}^{lm} d_{lm}$$

The tensor  $\hat{c}$  is symmetrical for an elastic body so that of the 81 components only 21 remain. This number is reduced to 2 for the isotropic elastic body, i.e. the modulus of elasticity  $E$  and either the shear modulus  $G$  or the Poisson ratio  $\mu$ :

$$\tau_{11} = \frac{E}{1+\mu} (d_{11} + \frac{\mu}{1-2\mu} (d_{11} + d_{22} + d_{33}))$$

$$\tau_{12} = G d_{12} = \frac{1}{2} \frac{E}{1+\mu} d_{12} \quad (1)$$

and analogously for  $\tau_{22}$ ,  $\tau_{33}$ ,  $\tau_{13}$ ,  $\tau_{23}$ .

If a force acts in only one direction then

$$\tau_{22} = \tau_{33} = 0,$$

so we obtain

$$\tau_{11} = E \cdot d_{11} \quad (2)$$

If a bar of height  $b$  and width  $a$ , supported at both ends by supports (separated by a distance  $L$ ), is subjected to a force  $F_y$  acting at its centre, it behaves like a bar supported in the middle, its two ends being subjected to a force  $F_y/2$  in the opposing direction. In order to express the bending  $\lambda$  as a

function of the modulus of elasticity  $E$ , let us first consider an element of volume

$$dV = dx \cdot a \cdot b$$

the upper layer of which is shortened on bending, and the bottom layer lengthened. The length of the central layer remains unchanged (neutral fibre).

In fig. 3, I and II denote the sides before and after deformation.

Using the symbols given in fig. 3, we obtain:

$$d\lambda = x \cdot d\phi = \frac{2\sigma x}{b}$$

The elastic force  $dF_x$  which produces the extension  $dI$ , according to (1), is

$$\frac{dF_x}{ds} = E \frac{dI}{dx}$$

where  $ds = a \cdot dy$  is the area of the rotated layer.

The force produces a torque

$$dT_z = y dF_x = \frac{2Ea\sigma}{b \cdot dx} y^2 dy$$

The sum of these torques produced by the elastic forces must be equal to the torque produced by the external force  $Fy/2$ .

$$\frac{Ea\sigma b^2}{6dx} = \frac{Fy}{2} \cdot x$$

from which we obtain

$$d\lambda = \frac{6Fyx^2}{Eab^3} dx$$

and, after integration, the total bending

$$\lambda = \frac{1}{4} \left(\frac{L}{b}\right)^3 \cdot \frac{1}{a} \cdot \frac{Fy}{E}$$

Table 1: The modulus of elasticity for different materials

| Material  | Dimensions [mm] | $E$ [ $N \cdot m^{-2}$ ] |
|-----------|-----------------|--------------------------|
| Steel     | $10 \times 1.5$ | $2.059 \cdot 10^{11}$    |
| Steel     | $10 \times 2$   | $2.063 \cdot 10^{11}$    |
| Steel     | $10 \times 3$   | $2.171 \cdot 10^{11}$    |
| Steel     | $15 \times 1.5$ | $2.204 \cdot 10^{11}$    |
| Steel     | $20 \times 1.5$ | $2.111 \cdot 10^{11}$    |
| Aluminium | $10 \times 2$   | $6.702 \cdot 10^{10}$    |
| Brass     | $10 \times 2$   | $9.222 \cdot 10^{10}$    |

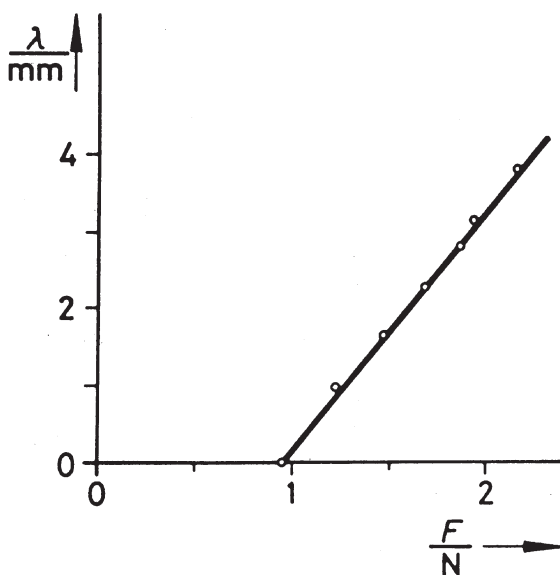


Fig. 4: Bending of a bar as a function of the force (steel,  $L = 0.48$  m,  $a = 10$  mm,  $b = 1.5$  mm).

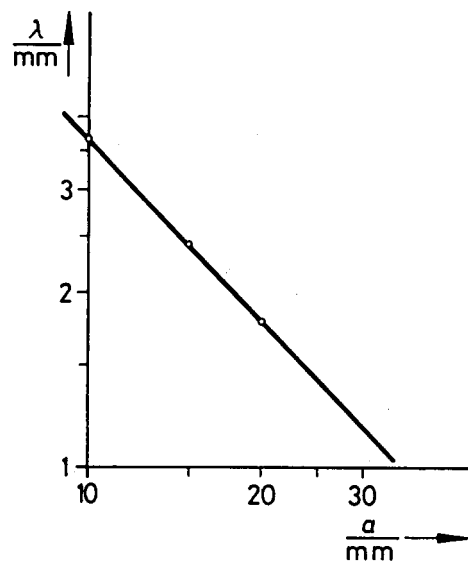


Fig. 5: Bending of a bar as a function of its width, at constant force (steel, thickness = 1.5 mm).

Fig. 6: Bending of a bar as a function of its length, at constant force.

Fig. 7: Bending of a bar as a function of its thickness, at constant force (steel, width = 10 mm).

