chapter 3

5) Neutral mesons are special in the sense that they can oscillate from particle to anti-particle, $P^0 \leftrightarrow \bar{P}^0$. Is this also true for the π^0 ? Explain.

6) We consider the system of oscillating neutral mesons $|P^0\rangle$ and $|\bar{P}^0\rangle$, and describe them quantum mechanically as a coupled system, $i\frac{\partial\psi}{\partial t} = H\psi$ with $\psi(t) = a(t)|P^0\rangle + b(t)|\bar{P}^0\rangle$ written in the subspace of P^0 and $\bar{P}^0$ as: $\psi(t) = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$.

   (a) What do we get for $a(t)$ and $b(t)$ if

$$H = \begin{pmatrix} m_{P^0} & 0 \\ 0 & m_{\bar{P}^0} \end{pmatrix}$$

   (b) Show that we can add an imaginary part $\Gamma/2$ to the Hamiltonian to obtain the decay. What is the lifetime of the P^0 in terms of Γ?

   (c) If off-diagonal elements are introduced in the Hamiltonian, $P^0 \to \bar{P}^0$ transitions become possible, but the $|P^0\rangle$ and $|\bar{P}^0\rangle$ states are no longer mass eigenstates. The mass eigenstates are obtained by finding the eigenvalues and eigenvectors of the Hamiltonian (Section 3.3). Finally, the meson will decay.
   What is the probability for the P^0 to decay as P^0, $|\langle P^0(t)|P^0\rangle|^2$, at time t?

   (d) Does the P^0 have a well-defined lifetime? If yes, what is that lifetime?

   (e) The value of $\Delta\Gamma$ differs for the B^0 ($\Delta\Gamma/\Gamma \approx 0$) and B_s^0 system ($\Delta\Gamma/\Gamma \approx 0.1$). What does that imply for the relative lifetimes of the mass eigenstates?