chapter 2

4) The unitarity triangle is not unique to the B^0 meson, there are in fact six of them. Construct the unitarity triangle for the K^0 system, i.e. the triangle that follows from the orthogonality condition on the 2^{nd} row and 1^{st} column, from $V^\dagger V = \mathbb{1}$, see Eq. (2.1).

   (a) Write out the orthogonality condition of the 2^{nd} row and 1^{st} column, from $V^\dagger V = \mathbb{1}$ (ignore $O(\lambda^7)$ terms).

   (b) Represent the condition as a triangle in the complex plane.

   (c) Rescale and rotate the triangle such that the basis of the triangle is formed by the points (0,0) and (1,0).

   (d) What are the coordinates of the apex of the triangle in terms of the Wolfenstein parameters?

   (e) Using the values of A, λ, ρ and η ($A = 0.82, \lambda = 0.22, \rho = 0.25$ and $\eta = 0.35$), make a drawing of the K^0 triangle that accurately reflects the shape of the triangle (i.e. the height-to-width ratio).

   (f) Calculate the surface area of the K^0 triangle in terms of the Wolfenstein parameters and compare it to the surface area of the B^0 triangle.

   (g) If we now include $O(\lambda^6)$-terms in the expression of the CKM-matrix with phases, on which element is an extra phase missing? (See Eq. 2.16):

$$V_{CKM,\text{Wolfenstein}} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{i\beta_s} & |V_{tb}| \end{pmatrix} + \mathcal{O}(\lambda^5)$$

5) The uncertainties on the measurements of moduli of the CKM-elements, $|V_{ij}|$ is larger than in expression Eq. (2.6):

$$V_{CKM} = \begin{pmatrix} 0.97419 & 0.2257 & 0.00359 \\ 0.2256 & 0.97334 & 0.0415 \\ 0.00874 & 0.0407 & 0.999133 \end{pmatrix} \pm \begin{pmatrix} 0.00022 & 0.0010 & 0.00016 \\ 0.0010 & 0.00023 & 0.0011 \\ 0.00037 & 0.0010 & 0.000044 \end{pmatrix} \quad (6.2)$$

   (a) Explain why a 4^{th} generation can affect the result.

   (b) What would be your guess of a CKM-matrix with 4 generations (in terms of λ only) ?

   (c) Check if your guess is compatible with observations, by calculating one unitarity relation numerically, taking into account the uncertainty on the measured moduli $|V_{ij}|$.