Exercises

chapter 1

1) Consider the hydrogen wave function

$$\psi(r,\theta,\phi) = \chi(r) Y_l^m(\theta,\phi) = \chi(r)\sqrt{\frac{(2l+1)(l-m)!}{4\pi(l+m)!}} P_l^m(\cos\theta) e^{im\phi}.$$

with $P_m^l(x) = \frac{(-1)^m}{2^l l!}(1-x^2)^{m/2}\frac{d^{l+m}}{dx^{l+m}}(x^2-1)^l$.
What is the dependence on m and l of the eigenvalue of the parity operation P?

2) Consider the spinor that describes the electron with positive helicity:

$$\psi^{(1)} = u^{(1)}(\vec{p})e^{-ip\cdot x} = \begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ \frac{p_x+ip_y}{E+m} \end{pmatrix} e^{-ip\cdot x}.$$

(a) Check that the C operation transforms an electron into a positron. (Remember that the anti-particle is described by the particle with negative E and $\vec{p}$.)

(b) What happened to the momentum and spin? And what happened to the helicity? Suppose the exercise involved a neutrino (instead of an electron) what can you say about the C-conjugated version?

(c) Check that subsequently the P operation changes the anti-particle to the state with opposite momentum and helicity.

(d) Make a sketch of the situation after C and combined CP operation, indicating the direction and spin with arrows. What happens to the spin after the CP operation?

3) Show that CPT invariance implies that the mass of the particle is equal to the mass of the anti-particle. Start from Eq. (1.2), $H\psi(\vec{p},\vec{\sigma}) = m\psi(\vec{p},\vec{\sigma})$, and use that CPT invariance implies $[H, CPT] = 0$.