

# Erratum for the time-like evolution in QCDNUM

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## Abstract

A recent comparison of the evolution programs QCDNUM and APFEL showed a discrepancy in the time-like evolution of the singlet fragmentation function at NLO. It was found that the splitting functions of this evolution were wrongly assigned in QCDNUM, and also that the fragmentation functions were not correctly matched at the flavour thresholds. These errors are corrected in a new release of the program.

## 1 Introduction

QCDNUM [1] is a fast QCD evolution program that can evolve parton densities (space-like evolution) and fragmentation functions (time-like evolution). Up to NLO, the evolution kernels are taken from publications by Furmanski and Petronzio for the flavour non-singlet [2] and singlet evolutions [3].<sup>1</sup>

A recent comparison [4] of QCDNUM and the evolution program APFEL [5] has shown very good agreement between the codes, except for the singlet evolution of fragmentation functions at NLO. This is because QCDNUM used a NLO time-like splitting function matrix in the index notation of [3], instead of properly taking its transpose. It also appeared that the fragmentation functions were not correctly matched at the flavour thresholds when running the evolution in the variable flavour number scheme at NLO.

The transposed matrix is implemented in the new release 17-00/07 of QCDNUM, together with the NLO threshold matching of the fragmentation functions as described in [6].<sup>2</sup>

In Figure 1 we show the time-like evolution at NLO in the variable flavour number scheme of the gluon, singlet and valence distributions up to a scale of  $\mu = 100$  GeV with old (dashed curves) and new versions of QCDNUM (full curves). There are sizeable differences except in the valence evolution which is not affected by the error in the splitting function matrix since it is a non-singlet. In the lower panel of the plot is shown

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<sup>1</sup>Well known misprints in [3] can be found in a footnote of [1] and are corrected for.

<sup>2</sup>The errors are also fixed in the beta releases version 17-01/12 and higher. All current QCDNUM releases can be downloaded from <http://www.nikhef.nl/user/h24/qcdnum>.

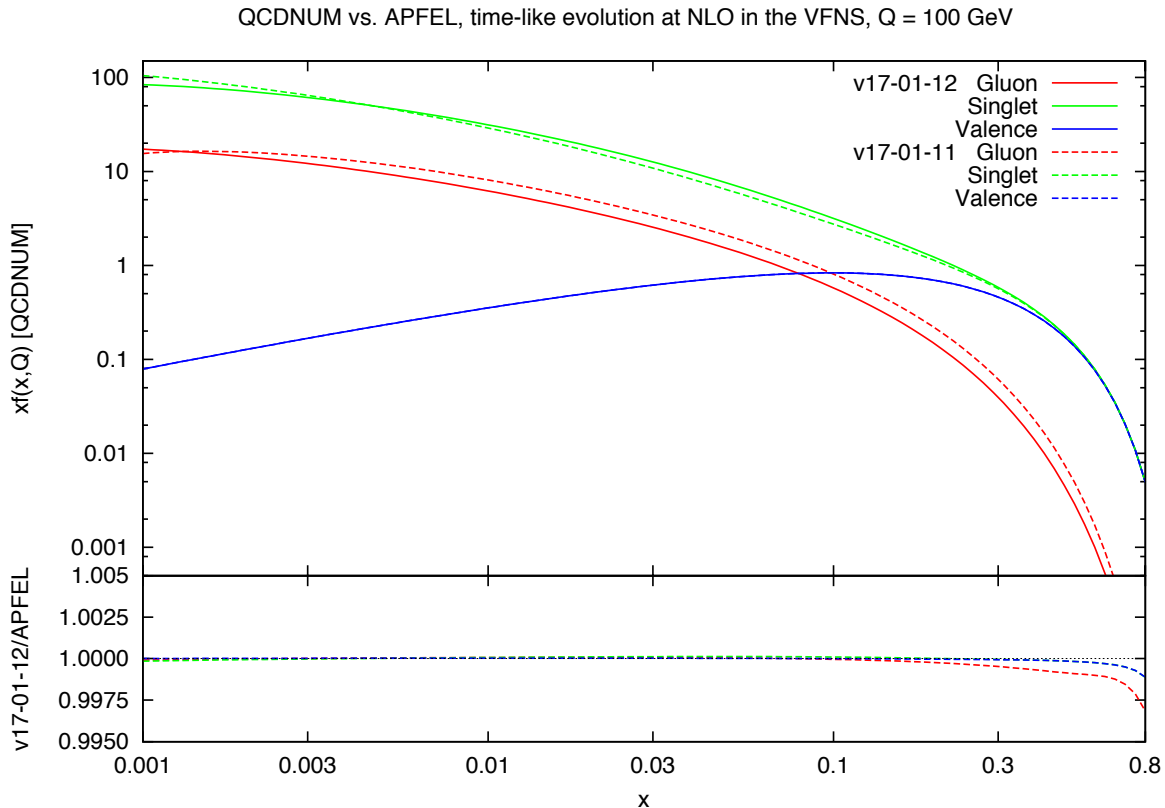


Figure 1: Time-like evolution at NLO in the variable flavour number scheme of gluon, singlet and valence distributions to a scale of  $\mu = 100$  GeV using an uncorrected (dashed curves) and corrected version of QCDNUM (full curves). The lower panel shows the ratio of distributions evolved with the corrected QCDNUM version and APFEL.

the comparison of the new QCDNUM version with APFEL. It is seen that, after the correction in QCDNUM, the agreement between the two evolution programs is excellent.

To clarify the index notation, we present in the next section the splitting function matrices that are currently implemented in QCDNUM.

## 2 Singlet evolution

We write the singlet evolution (coupled to the gluon) in matrix notation as

$$\frac{\partial \mathbf{V}}{\partial \ln \mu^2} = \mathbf{M} \otimes \mathbf{V} \quad \text{with} \quad \mathbf{V} = \begin{pmatrix} F \\ G \end{pmatrix} \quad \text{and} \quad \mathbf{M} = \begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix}.$$

Here the symbol  $\otimes$  denotes the Mellin convolution

$$[f \otimes g](x) = \int_x^1 \frac{dz}{z} f\left(\frac{x}{z}\right) g(z).$$

For space-like evolution  $G$  is the gluon density and  $F$  is the quark singlet density  $\sum_{i=1}^{n_f} (q_i + \bar{q}_i)$  where  $q_i$  ( $\bar{q}_i$ ) is the (anti)quark number density of flavour  $i$  in the proton

and  $n_f$  is the number of active flavours. For time-like evolution  $G$  and  $F$  stand for the corresponding fragmentation functions.

Below we will be only concerned with a splitting function expansion up to NLO,

$$\mathbf{M} = a_s \mathbf{M}^{(0)} + a_s^2 \mathbf{M}^{(1)} \quad \text{with} \quad a_s \equiv \frac{\alpha_s}{2\pi}.$$

The following four functions are defined in [3]

$$\begin{aligned} p_{\text{FF}} &= (1+x^2)/(1-x) & p_{\text{GF}} &= x^2 + (1-x)^2 \\ p_{\text{FG}} &= [1+(1-x)^2]/x & p_{\text{GG}} &= 1/(1-x) + 1/x - 2 + x - x^2. \end{aligned}$$

The four LO splitting functions are then written as

$$\begin{aligned} P_{\text{FF}}^{(0)} &= C_F [p_{\text{FF}}]_+ & P_{\text{GF}}^{(0)} &= 2T_R n_f p_{\text{GF}} \\ P_{\text{FG}}^{(0)} &= C_F p_{\text{FG}} & P_{\text{GG}}^{(0)} &= 2C_G x^{-1} [xp_{\text{GG}}]_+ - \frac{2}{3} T_R n_f \delta(1-x) \end{aligned}$$

with the colour factors and the regularisation prescription given by

$$C_F = \frac{4}{3}, \quad C_G = 3, \quad T_R = \frac{1}{2} \quad \text{and} \quad [f(x)]_+ \equiv f(x) - \delta(1-x) \int_0^1 f(y) dy.$$

The NLO splitting functions for space-like (S) and time-like (T) processes are

$$\begin{aligned} P_{\text{FF}}^{(1,U)} &= \hat{P}_{\text{FF}}^{(1,U)} - \delta(1-x) \int_0^1 dx x \left[ \hat{P}_{\text{FF}}^{(1,T)} + \hat{P}_{\text{FG}}^{(1,T)} \right] \\ P_{\text{GF}}^{(1,U)} &= \hat{P}_{\text{GF}}^{(1,U)} \\ P_{\text{FG}}^{(1,U)} &= \hat{P}_{\text{FG}}^{(1,U)} \\ P_{\text{GG}}^{(1,U)} &= \hat{P}_{\text{GG}}^{(1,U)} - \delta(1-x) \int_0^1 dx x \left[ \hat{P}_{\text{GG}}^{(1,T)} + \hat{P}_{\text{GF}}^{(1,T)} \right], \end{aligned}$$

where  $U = \{S, T\}$ . The functions  $\hat{P}_{\text{AB}}^{(1,U)}$  are given in Eqs. (11) and (12) of [3].<sup>3</sup>

Because the authors of [3] do not clearly define their index notation (hence the confusion), we identify the splitting functions not by their indices but, instead, by their overall colour factors which should be the same at LO and NLO.

For space-like evolution the colour factors of  $P_{\text{qg}}$  and  $P_{\text{gq}}$  are  $2T_R n_f$  and  $C_F$  while those for time-like evolution are  $2C_F n_f$  and  $T_R$ , respectively.

Identifying the splitting functions by these factors we obtain the LO and NLO space-like evolution matrices (note that these were always correctly implemented in QCDNUM):

$$\mathbf{M}^{(0,S)} = \begin{pmatrix} P_{\text{FF}}^{(0)} & P_{\text{GF}}^{(0)} \\ P_{\text{FG}}^{(0)} & P_{\text{GG}}^{(0)} \end{pmatrix}, \quad \mathbf{M}^{(1,S)} = \begin{pmatrix} P_{\text{FF}}^{(1,S)} & P_{\text{GF}}^{(1,S)} \\ P_{\text{FG}}^{(1,S)} & P_{\text{GG}}^{(1,S)} \end{pmatrix}. \quad (1)$$

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<sup>3</sup> Modulo the misprint in  $\hat{P}_{\text{FF}}^{(1,T)}$  which does not affect the colour factors.

It is well known that the LO time-like matrix is the transpose of the space-like matrix [7]. To get the same colour factors at NLO it can be seen from inspection of Eq. (12) in [3] that also the NLO matrix must be transposed. Accounting for factors  $2n_f$ , we thus have

$$\mathbf{M}^{(0,T)} = \begin{pmatrix} P_{\text{FF}}^{(0)} & 2n_f P_{\text{FG}}^{(0)} \\ \frac{1}{2n_f} P_{\text{GF}}^{(0)} & P_{\text{GG}}^{(0)} \end{pmatrix}, \quad \mathbf{M}^{(1,T)} = \begin{pmatrix} P_{\text{FF}}^{(1,T)} & 2n_f P_{\text{FG}}^{(1,T)} \\ \frac{1}{2n_f} P_{\text{GF}}^{(1,T)} & P_{\text{GG}}^{(1,T)} \end{pmatrix}. \quad (2)$$

The mistake made in the previous QCDNUM releases is that the NLO time-like matrix  $\mathbf{M}^{(1,T)}$  was *not* transposed, contrary to what is done in APFEL [8].

### 3 Acknowledgements

I am grateful to V. Bertone for spotting the problem, for running several QCDNUM versus APFEL comparisons to locate the error, and for providing the plot. I thank A. Vogt for useful discussions. Finally, my apologies to those who have used previous versions of QCDNUM to evolve their singlet fragmentation functions at NLO.

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