Lecture notes Particle Physics II

Quantum Chromo Dynamics

8. The Structure of the Proton

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December 3, 2013

QCD as a predictive theory

- We have seen that perturbative QCD suffers from collinear and soft singularities but that so-called infrared-safe observables are insensitive to the number of partons in the final state which means that they are insensitive to the collinear merging of two daughter partons or the disappearance of one daughter parton in the soft limit. It can be shown that for infrared-safe observables there is a precise cancellation of the soft and collinear divergences in the contributing Feynman diagrams at all orders.[47]

- However, it is clear that if QCD predictions would be restricted to infrared-safe observables only, it would not be a very useful theory. Fortunately, there a large class of cross-sections that **factorise** into a perturbatively calculable infrared-safe short-distance (hard) part, and a long-distance (soft) part which is infrared-singular but has the virtue of being *universal*, that is, process-independent.

- An example is the **deep-inelastic scattering** (DIS) cross-section

$$\sigma(\ell\,\mathrm{p} \to \ell' X)$$

  where a lepton ℓ (electron, muon or neutrino) scatters on a proton which breaks-up into the (uspecified) system X. The DIS cross-section can be factorised in a hard lepton-quark cross-section and a so-called **parton distribution** which is process independent, but non-perturbative and infrared-singular. However, it can be replaced by a measurement, like we did for the running coupling where *ultraviolet divergences* were replaced by a measured value of the coupling constant at some renormalisation scale (Section 6).

- In this section we will study DIS in more detail and see how it leads to the famous **quark-parton model** of the proton.

[47] So-called Bloch-Nordsieck and Kinoshita-Lee-Lauenberg theorems.

Inward bound

- One way to probe the internal structure of matter is to bombard it with high energy particles, and then see what happens. For instance, in the Rutherford experiment (1911), alpha particles (helium nuclei) were deflected on a thin gold foil. Rutherford found that the deflections followed his famous inverse $\sin^4(\theta/2)$ law (see page 0–13), and concluded that the alpha particles were scattered from electrically charged point-like nuclei inside the gold atoms.

- Experiments using probes of higher energy later revealed that the point-like scattering distributions were damped by **form factors** which are essentially the Fourier transform of a charge *distribution.* This clearly showed that nuclei are not point-like and indeed, after the discovery of the neutron by Chadwick (1932), it became clear that nuclei are bound states of protons and neutrons.

- Also the protons and neutrons were found not to be point-like and a real breakthrough came with a series of *deep inelastic scattering* experiments in the 1960's at SLAC, where electron beams were scattered on proton targets at energies of about 20 GeV, large enough to reveal the proton's internal structure.

- The SLAC experiments showed that the electrons were scattering off quasi-free point-like constituents inside the proton which were soon identified with quarks. This was the first time that quarks were shown to be dynamical entities, instead of bookkeeping devices to classify the hadrons (Gell-Mann's eightfold way). The Nobel prize was awarded in 1990 for this spectacular discovery, and the lectures of Friedman, Kendall and Taylor are a fascinating record of the struggle to understand what these data did tell us.

Deep inelastic scattering (DIS)

- The pioneering SLAC experiments were followed by a series of other **fixed-target** experiments[48] with larger energies at CERN (Geneva) and at Fermilab (Chicago), using electrons, muons, neutrino's and anti-neutrinos as probes.

- The largest centre-of-mass energies were reached at the HERA collider in Hamburg (1992–2007) with counter-rotating beams of 27 GeV electrons and 800 GeV protons.

  Exercise 8.1: [1.0] Calculate the centre-of-mass energies at SLAC (20 GeV electrons on stationary protons) and at HERA (27 GeV electrons on 800 GeV protons). You can neglect the electron mass and, at HERA, also the proton mass.

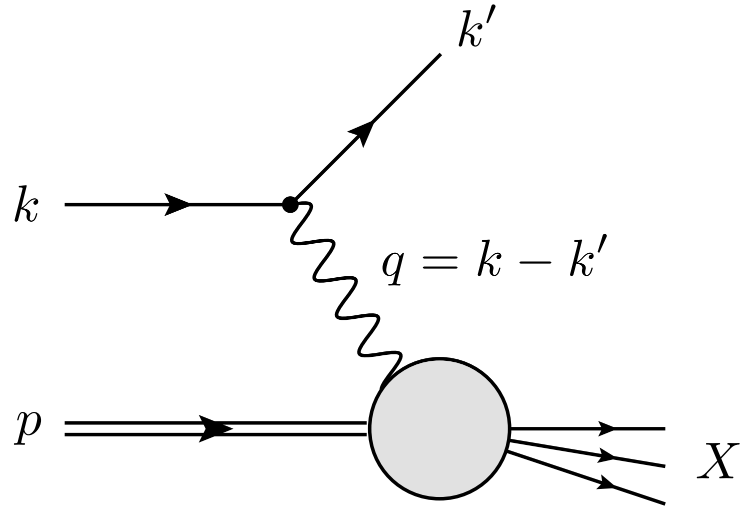
- Deep inelastic scattering (DIS) data are very important since they provide detailed information on the momentum distributions of the **partons** (quarks and gluons) inside the proton.

- Parton distributions are crucial ingredients in theoretical predictions of scattering cross-sections at hadron colliders like the Tevatron (Fermilab, proton-antiproton at 2 TeV) or the LHC (CERN, proton-proton at 8–14 TeV). The reason for this is simple: the colliding (anti)protons have a fixed centre-of-mass energy but not the colliding partons, since their momenta are distributed inside the (anti)proton. Clearly one has to fold-in this momentum spread to compare theoretical predictions with the data.

- Apart from providing parton distributions, DIS is also an important testing ground for perturbative QCD, as we will see.

[48] In a fixed-target experiment, beam particles interact with a stationary target in the laboratory, and the debris is recorded in a downstream detector. In a collider experiment, on the other hand, counter-rotating beams collide in the centre of a detector, which is built around the interaction region.

DIS kinematics I



- The graph above shows the kinematics of deep inelastic scattering:

  k = incoming lepton k' = outgoing lepton
  p = incoming proton X = hadronic final state
  $q = k - k'$ momentum transfer

- The interaction between the exchanged photon (or W in case of $\nu\mathrm{p} \to \ell X$ neutrino scattering) and the proton depends on p and q, from which we can build the two Lorentz scalars:

$$\boxed{Q^2 = -q^2 \quad \text{and} \quad x = \frac{Q^2}{2p \cdot q}}$$

- Other scalars that are often used to characterise the event are

$M^2 = p^2$	Proton mass squared
$s = (p + k)^2$	Centre of mass energy squared
$W^2 = (p + q)^2$	Invariant mass of X squared
$y = (p \cdot q)/(p \cdot k)$	Fractional energy transfer in the lab
$\nu = (p \cdot q)/M$	Energy transfer in the lab system

DIS kinematics II

- In case of **elastic scattering**, the proton leaves the collision without breaking-up or being in an excited state. Thus we have, for elastic scattering, ep → e′p′ with[49]

$$p^2 = p'^{\,2} = (p+q)^2 = W^2 = M^2.$$

- Here are a few useful relations, which we will prove below

1. $q^2 < 0$ (hence the minus sign in the definition of Q^2)
2. $0 \leq x \leq 1$
3. $0 \leq y < 1$
4. $W^2 = M^2 + Q^2(1-x)/x \geq M^2$

- Proof

Since all kinematic variables are Lorentz invariants, it is often useful to calculate them in frames which are convenient.

1. In the rest frame of the incoming electron with the outgoing electron along the z axis we have $k = (m, 0, 0, 0)$ and $k' = (E', 0, 0, k')$ so that

$$q^2 = (m-E', 0, 0, -k')^2 = m^2 - 2mE' + E'^2 - k'^2 = m^2 - 2mE' + m^2 = 2m(m-E') < 0.$$

2. Obviously $x = 0$ when $Q^2 = 0$. For the other limit set $W^2 = M^2$. This gives

$$W^2 = (p+q)^2 = p^2 + 2p\cdot q + q^2 = M^2 + 2p\cdot q - Q^2 = M^2 \quad \rightarrow \quad Q^2 = 2p\cdot q \quad \rightarrow \quad x = 1.$$

3. For the limits on y it is easiest to work in the lab frame where the proton is at rest and the electron comes in at the z direction. We then have $k = (E, 0, 0, k)$, $p = (M, 0, 0, 0)$ and $q = (E - E', q_x, q_y, q_z)$ so that $p \cdot k = ME$ and $p \cdot q = M(E - E')$. Therefore

$$y = \frac{p \cdot q}{p \cdot k} = \frac{E - E'}{E} \quad \text{with} \quad m_{\rm e} \leq E' \leq E.$$

From this it immediately follows that $0 \leq y < 1$.

4. With $x = Q^2/(2p \cdot q)$ we find

$$W^2 = (p+q)^2 = p^2 + 2p \cdot q + q^2 = M^2 + Q^2/x - Q^2 = M^2 + Q^2(1-x)/x.$$

[49] From relation (4) below it follows that the elastic scattering limit is also given by $x = 1$.

Exercise 8.2:

(a) [0.5] Show that $Q^2 \approx xys$ for large $s \gg M^2$ (so that we can neglect the proton mass). What is, in this approximation, the largest Q^2 that can be reached at the SLAC experiments ($\sqrt{s} = 6.4$ GeV) and at HERA ($\sqrt{s} = 294$ GeV).[50]

(b) [1.5] All DIS kinematic variables can be determined from a measurement of the scattered electron energy E' and angle θ with respect to the incident beam. In particular, show that for fixed-target experiments (proton at rest and the electron coming in from the z direction) we have the relations

$$\begin{aligned} Q^2 &= 4EE'\sin^2(\theta/2) \\ \nu &= E - E' \\ x &= Q^2/(2M\nu) \\ y &= \nu/E \\ W^2 &= M^2 - Q^2 + 2M\nu \\ s &= M(M+2E) \approx 2ME \end{aligned}$$

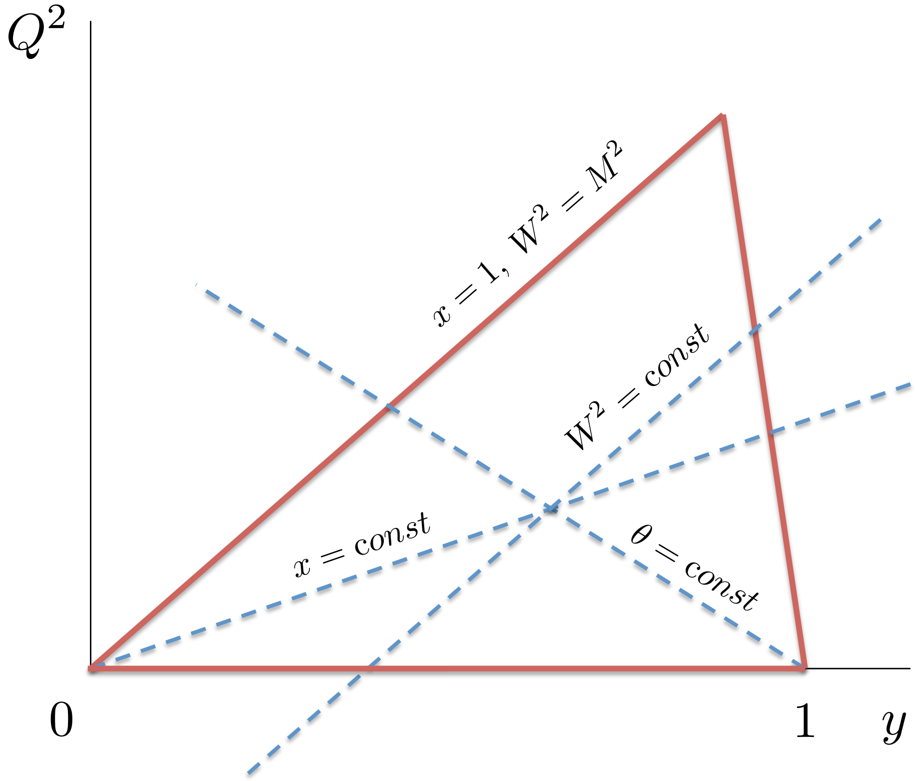
Hint: For the expression of Q^2 use the half-angle formula

$$\cos\theta = 1 - 2\sin^2(\theta/2).$$

- It is convenient to plot the allowed kinematical region in the y-Q^2 plane ($\rightarrow$ **Fig**.)

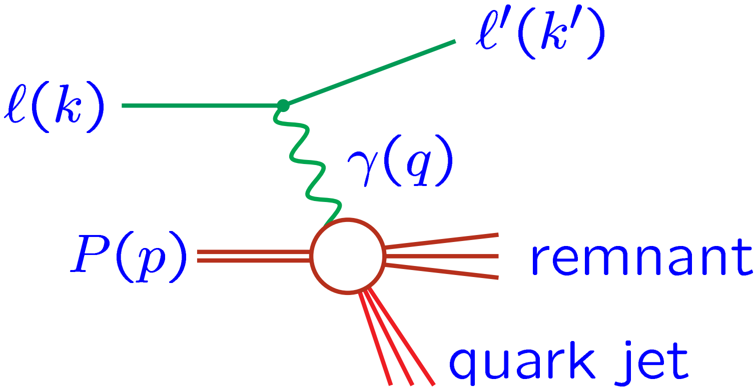
[50] We will see later that the cross-section drops like Q^{-4} so that DIS events at very large Q^2 are rare. It is thus difficult to collect data in this kinematic region.

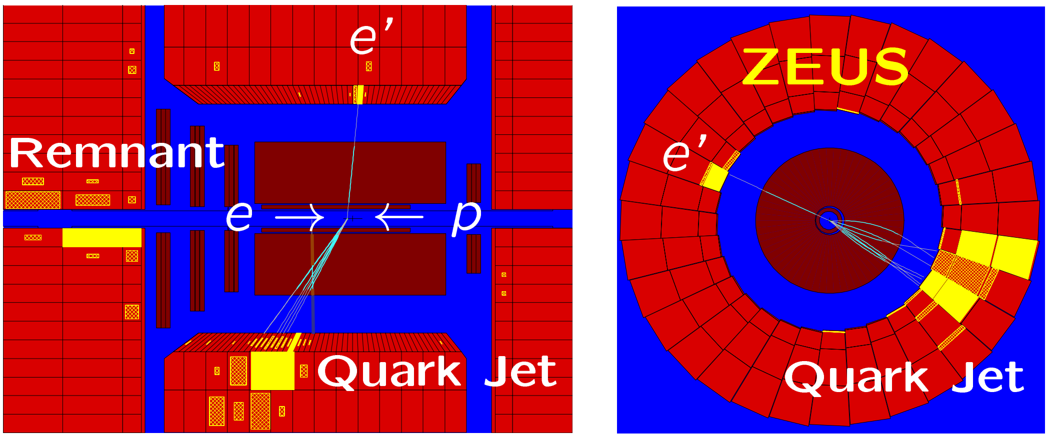
DIS kinematic plane



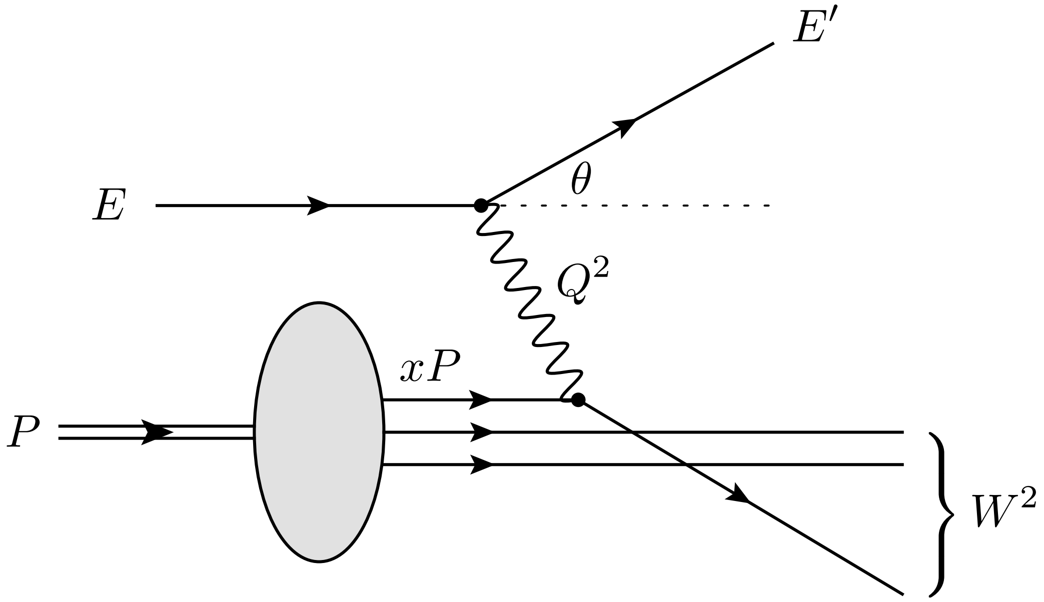
$$
\begin{aligned}
Q^2 &= 2MEy + M^2 - W^2 \\
&= 2MExy \\
&= 4(1-y)E^2 \sin^2 \theta/2
\end{aligned}
$$

DIS kinematics in real life





The quark-parton model



- To explain the DIS measurements at SLAC, Feynman, Bjorken, and others (1969) proposed the so-called **parton model** which states that

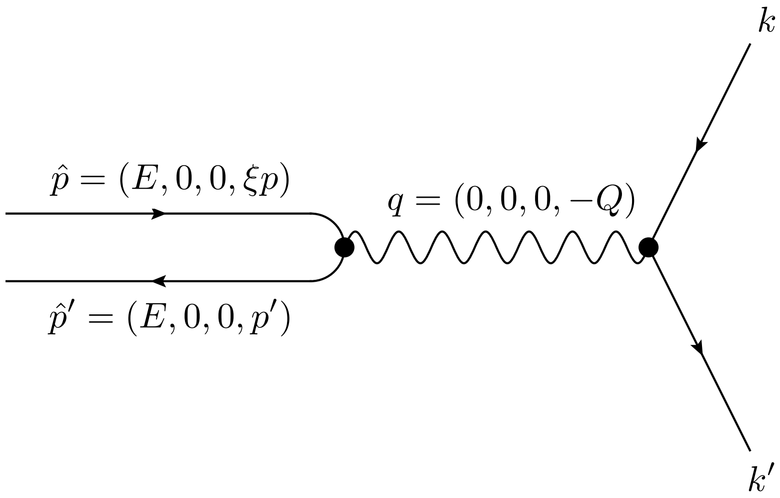
  Assumption I: A fast moving hadron appears as a jet of partons (quarks and gluons) moving in more or less the same direction as the parent hadron and sharing its 3-momentum.

  Assumption II: The reaction cross-section is the incoherent sum of partonic cross-sections, as calculated with free partons.[51]

- We will now use the quark-parton model and results from the PP-I course to derive the DIS cross-section. The kinematics is best understood in the so-called Breit-, or infinite-momentum frame.

[51] By 'incoherent sum' we mean that cross-sections are added, instead of amplitudes.

The Breit frame I



- Because the virtual photon is space-like ($q^2 < 0$) it follows that we can boost the photon along its direction of propagation (which points to the proton) such that q^0 vanishes. This frame is called the **Breit frame** or **infinite momentum frame** since the proton then moves with very large momentum towards the virtual photon.
- In this frame the incoming quark moves with a 3-momentum ξp_z along the z axis, where ξ is the fraction of the proton 3-momentum p_z. The virtual photon moves with a 3-momentum Q along $-z$.
- We take the incoming quark to be point-like, so that the scattering is necessarily elastic:[52]

$$\hat{p}^2 = (\hat{p} + q)^2 \;\to\; \hat{p}^2 = \hat{p}^2 + 2\hat{p} \cdot q - Q^2 \;\to\; Q^2 = 2\hat{p} \cdot q$$

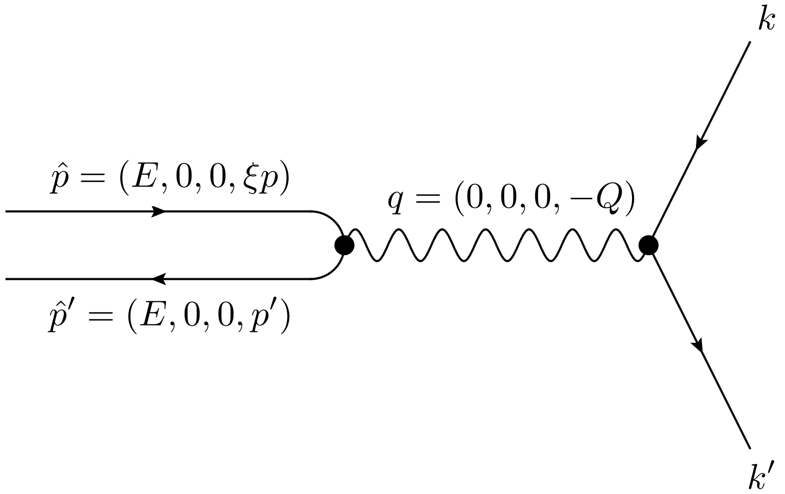
- If we denote the proton 4-momentum by p then, in the Breit frame,

$$\begin{aligned} \hat{p} \cdot q &= (E, 0, 0, \xi p_z) \cdot (0, 0, 0, -Q) = \xi p_z Q \\ \xi p \cdot q &= \xi(E_p, 0, 0, p_z) \cdot (0, 0, 0, -Q) = \xi p_z Q \end{aligned}$$

  Thus $\hat{p} \cdot q = \xi\, p \cdot q$ but remember that this is *only* true in the Breit frame where the virtual photon does not transfer energy.

[52]We indicate the unobservable partonic kinematic variables by a hat, like $\hat{p}$ for a partonic 4-momentum.

The Breit frame II



- The elastic scattering condition now becomes

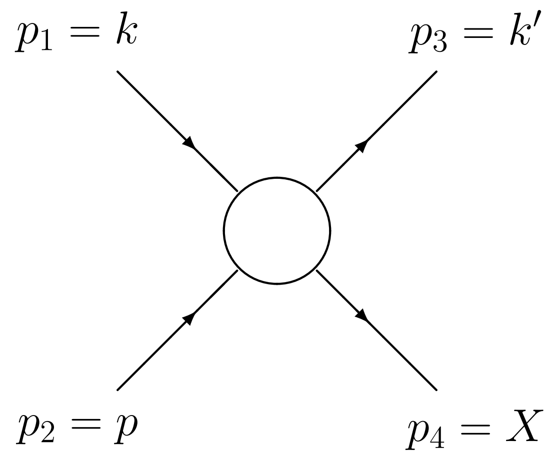
$$Q^2 = 2\,\hat{p}\cdot q = 2\xi\, p\cdot q \quad \rightarrow \quad \xi = \frac{Q^2}{2\,p\cdot q} = x$$

- So we can identify the Bjorken-x variable as the 3-momentum fraction of the struck quark <u>in the Breit frame</u>.

- Let us at this point introduce the notion of a **quark distribution** $f_i(x)\mathrm{d}x$, which gives the number of quarks of flavour i which carry a 3-momentum fraction (in the Breit frame) between x and $x+\mathrm{d}x$.

- <u>Remark</u>: note that in the Breit frame the proton moves very fast towards the photon, and is therefore Lorentz contracted to a kind of pancake. The interaction then takes place on the very short time scale when the photon passes that pancake. On the other hand, in the rest frame of the proton, the inter-quark interactions take place on time scales of the order of r_p/c but because of time dilatation these interactions are like 'frozen' the Breit frame. During the short interaction time, the struck quark thus does not interact with the spectator quarks and can be regarded as a free parton.

Intermezzo: Mandelstam variables



$$
\begin{aligned}
s &= (p_1 + p_2)^2 \approx 2p_1 \cdot p_2 \\
t &= (p_1 - p_3)^2 = (p_4 - p_2)^2 \approx -2\,p_1 \cdot p_3 \approx -2\,p_2 \cdot p_4 \\
u &= (p_1 - p_4)^2 = (p_3 - p_2)^2 \approx -2\,p_1 \cdot p_4 \approx -2\,p_3 \cdot p_2
\end{aligned}
$$

- **Exercise 8.3**: [0.5] Show that $s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$
- **Exercise 8.4**: [0.5] Show that, if we neglect the electron and proton mass,

$$
\begin{aligned}
Q^2 &= -t \\
x &= -t/(s+u) \\
y &= (s+u)/s \\
W^2 &= s + t + u
\end{aligned}
$$

Note that we immediately get $Q^2 = xys$ (if we ignore the masses).

Parton density

- On page 8–13 we have introduced the **parton density function** $f_i(x)\,\mathrm{d}x$ which gives the number of quarks of flavour $i = \mathrm{d}, \mathrm{u}, \mathrm{s}, \ldots, \bar{\mathrm{d}}, \bar{\mathrm{u}}, \bar{\mathrm{s}}, \ldots$ between momentum fraction x and $x + \mathrm{d}x$.
- Huh? But does not the proton consist of *three* quarks? No, not in a dynamical picture: inside the proton there are also gluons from the QCD splitting $q \to q\mathrm{g}$ and quark-antiquark pairs from the splitting $\mathrm{g} \to q\bar{q}$ (→**fig**). What is true is that there is a *net excess* of three quarks that carry the quantum numbers of the proton.
- Now we use the second assumption of the parton model and write the cross section as an incoherent sum of partonic cross sections

$$\mathrm{d}\sigma = \sum_i \mathrm{d}\hat{\sigma}(\hat{s}, \hat{t}, \hat{u})\ f_i(x)\mathrm{d}x$$

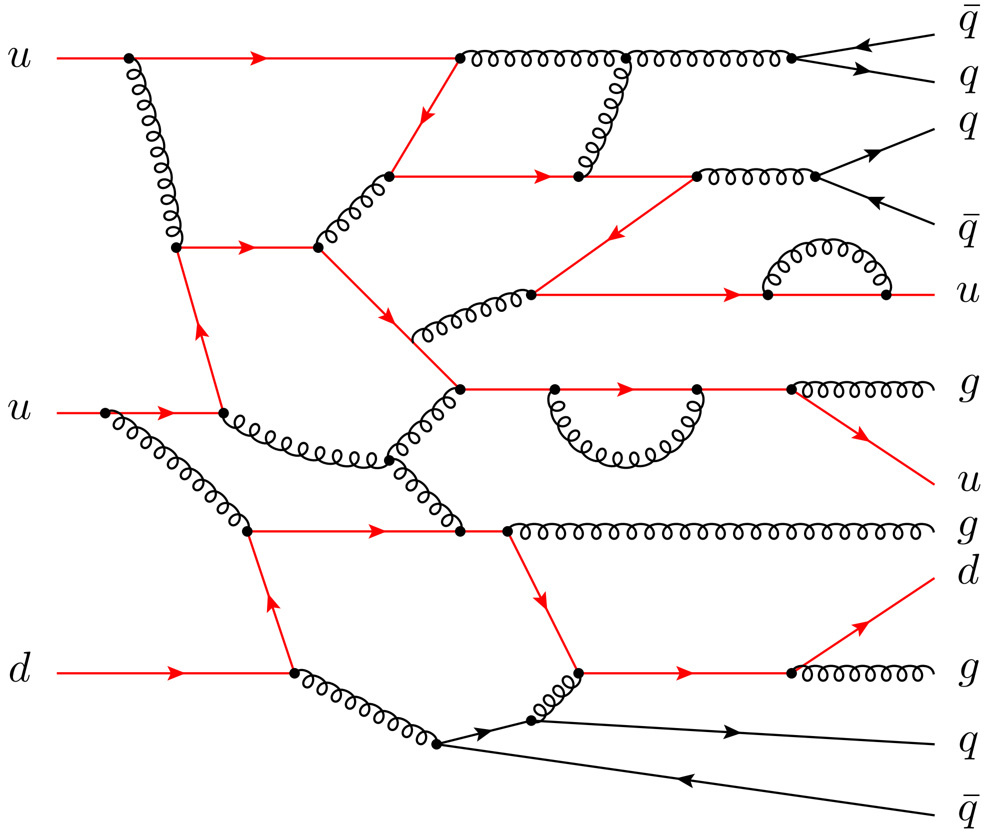
  Here we have introduced the parton kinematic variables

$$\hat{s} \approx 2\,xp\cdot k = xs, \quad \hat{t} = (k - k')^2 = t, \quad \hat{u} \approx -2\,xp\cdot k' = xu$$

- For the partonic cross section we just take $\sigma(\mathrm{e}\mu \to \mathrm{e}\mu)$ as calculated in PP-I (lecture 8), with the muon charge replaced by the quark charge.

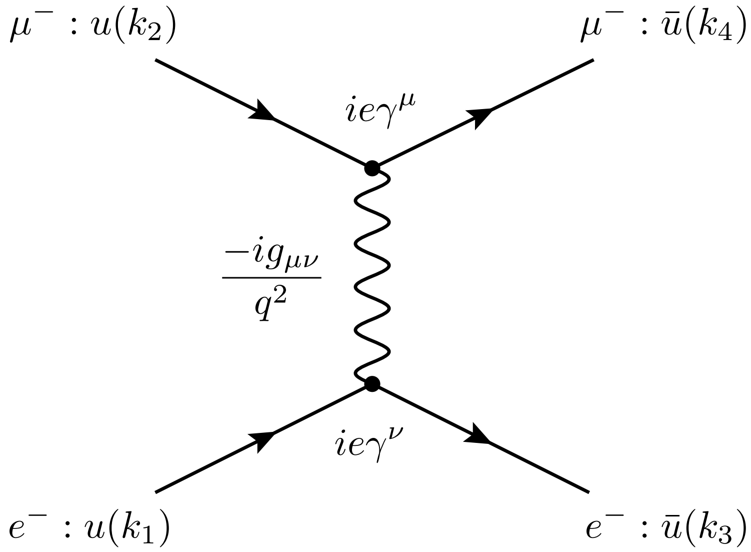
  Exercise 8.5: [0.5] Why do we take the cross section for $\mathrm{e}\mu \to \mathrm{e}\mu$ as our reference, and not that of $\mathrm{ee} \to \mathrm{ee}$?

Dynamical picture of the proton



Schematic picture of the QCD proton structure. The uud **valence quarks** that carry the quantum numbers of the proton enter the diagram on the left. This corresponds to a low-resolution 3-quark picture of the proton that only accounts for its quantum numbers. At the right of the diagram we see a high-resolution picture (at large Q^2) of the proton where the valence quarks are dressed with gluons and a **sea** of $q\bar{q}$ pairs. Note that the valence quarks can zig-zag through the diagram but will never disappear so that the proton quantum numbers are the same in both the low- and high-resolution pictures.

Cross section for e-μ scattering



- In PP-I Section 8.2 the e-μ cross section is calculated as

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{com}} = \frac{\alpha^2}{2s}\left(\frac{s^2+u^2}{t^2}\right)$$

- COM frame with momentum k_1 along x and scattering angle θ

$$\begin{aligned} k_1 &= (k, \quad k, 0, 0) & \qquad k_3 &= (k, \quad k\cos\theta, \quad k\sin\theta, 0) \\ k_2 &= (k, -k, 0, 0) & \qquad k_4 &= (k, -k\cos\theta, -k\sin\theta, 0) \end{aligned}$$

$$s = 4k^2, \quad t = -2k^2(1-\cos\theta), \quad u = -2k^2(1+\cos\theta)$$

- Cross section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\mathrm{d}\sigma}{\sin\theta\mathrm{d}\theta\mathrm{d}\phi} = \frac{\mathrm{d}\sigma}{\sin\theta\mathrm{d}\phi\mathrm{d}t}\left|\frac{\mathrm{d}t}{\mathrm{d}\theta}\right| = 2k^2\frac{\mathrm{d}\sigma}{\mathrm{d}\phi\mathrm{d}t} = \frac{s}{2}\frac{\mathrm{d}\sigma}{\mathrm{d}\phi\mathrm{d}t}$$

- Integrating over ϕ then gives

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = \frac{4\pi}{s}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{2\pi\alpha^2}{s^2}\left(\frac{s^2+u^2}{t^2}\right)$$

DIS cross section

- Using the partonic variables, and multiplying the charge at the muon vertex by the fractional quark charge e_i, we get the partonic cross section
$$\frac{\mathrm{d}\hat{\sigma}_i}{\mathrm{d}\hat{t}} = \frac{2\pi\alpha^2 e_i^2}{\hat{s}^2}\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right)$$
It is a simple matter to re-write this in terms of the DIS kinematic variables (see page 8–14).

- **Exercise 8.6**: [1.0] Show that
$$\frac{\mathrm{d}\hat{\sigma}_i}{\mathrm{d}\hat{t}} \to \frac{\mathrm{d}\hat{\sigma}_i}{\mathrm{d}Q^2} = \frac{2\pi\alpha^2 e_i^2}{Q^4}\left[1+(1-y)^2\right]$$

- Now we can put this result in our master formula on page 8–15
$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2} = \sum_i \frac{2\pi\alpha^2 e_i^2}{Q^4}\left[1+(1-y)^2\right] f_i(x)\mathrm{d}x$$
or
$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}Q^2} = \frac{2\pi\alpha^2}{Q^4}\left[1+(1-y)^2\right] \sum_i e_i^2 f_i(x)$$

- The F_2 **structure function** is defined as the charge weighted sum of the parton **momentum densities** $xf_i(x)$
$$\boxed{F_2(x) = \sum_i e_i^2 x f_i(x)}$$
so that the DIS cross section can be written as[53]
$$\boxed{\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\frac{\left[1+(1-y)^2\right]}{2x}F_2(x)}$$

[53] One can think of the y dependence as being an angular dependence through the relation $1-y=\frac{1}{2}(1+\cos\theta^*)$.

The Gallan-Gross relation

- We have given a rather simple derivation of the parton model cross section and established the relation between parton density functions and the F_2 structure function.
- However, a more formal derivation (see H&M Section 8), which does not use the parton model, leads to the result

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{xQ^4}\,\left[(1-y)F_2(x,Q^2) + xy^2F_1(x,Q^2)\right]$$

  Here another structure function shows up, which turns out to be proportional to the absorption cross-section of *transversely polarised* photons: $2xF_1 \propto \sigma_\mathrm{T}$. Because the exchanged photon is virtual, is also has a *longitudinally polarised* component. The F_2 structure function is proportional to the sum of the transverse and longitudinal absorption cross sections: $F_2 \propto \sigma_\mathrm{T} + \sigma_\mathrm{L}$.
- In the Breit frame, where the quarks are highly relativistic without transverse momenta, the quark spins will be aligned parallel or antiparallel to the direction of motion (z axis) so that it can absorb a head-on photon with helicity ±1, just by flipping the spin.
- However the quarks cannot absorb a longitudinally polarised photon because for this the quark spin must have a non-vanishing transverse component. Thus, in the parton model, $\sigma_\mathrm{L} = 0$ and $F_2 = 2xF_1$. This is called the **Gallan-Gross relation**. Setting $F_2 = 2xF_1$ above, we recover the formula on page 8–18.[54]
- In the QCD improved parton model, gluon radiation imparts small transverse momenta to the quarks so that now $\sigma_\mathrm{L} \neq 0$ and another (small) structure function shows up, $F_\mathrm{L} \equiv F_2 - 2xF_1$, with its own characteristic y-dependence.

[54]Historically, experimental verification of the Gallan-Gross relation was a proof that quarks carry spin $\frac{1}{2}$.

Bjorken scaling

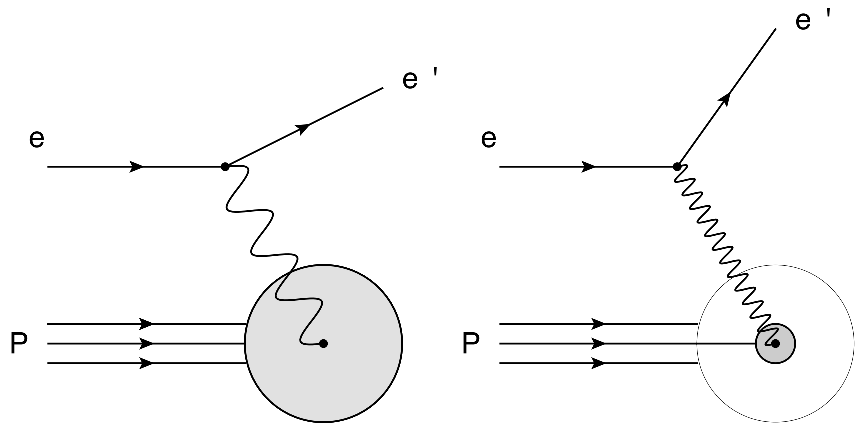
- In principle, the structure functions depend on two variables, x and Q^2 say, but in the parton model derivation on page 8–18 we have defined

$$F_2(x) = \sum_i e_i^2 x f_i(x)$$

which depends on x only. This Q^2 independence is called **Bjorken scaling** and is formally stated as follows (in terms of lab variables):

$$\text{If} \quad \left.\begin{array}{r} Q^2 \to \infty \\ \nu \to \infty \end{array}\right\} \text{ with } x = \frac{Q^2}{2M\nu} \text{ finite, then } \quad F_2(x, Q^2) \to F_2(x)$$

- This scaling behaviour is easy to understand by noticing that the wavelength of the virtual photon $\lambda \sim 1/Q$. But the resolving power is irrelevant when we scatter on *point-like* objects, hence the independence on Q^2. In short, scaling $\to$ point-like scattering.



(a) (b)

- Indeed, measurements of F_2 at different Q^2 values seem to fall on a universal curve ($\to$ **fig**) but close inspection reveals a characteristic scale-breaking pattern. This Q^2 dependence is caused by the QCD processes of gluon radiation and $q\bar{q}$ formation, as we will see later.

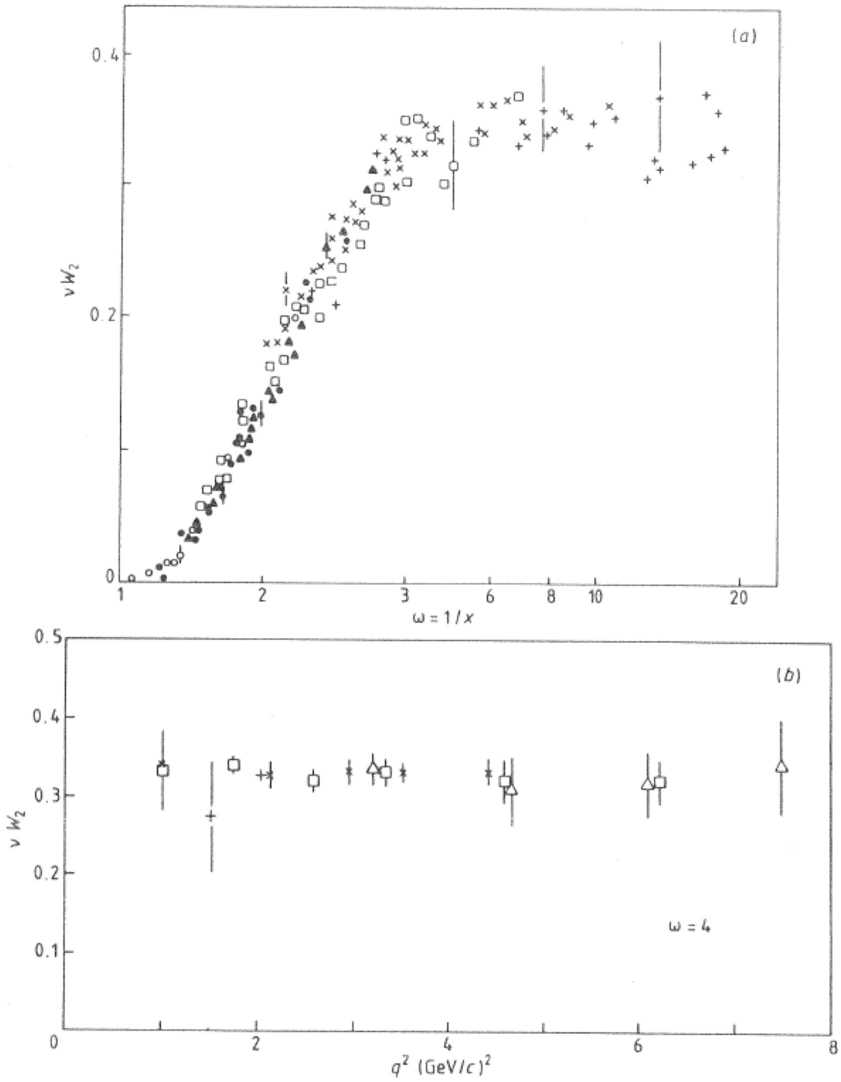
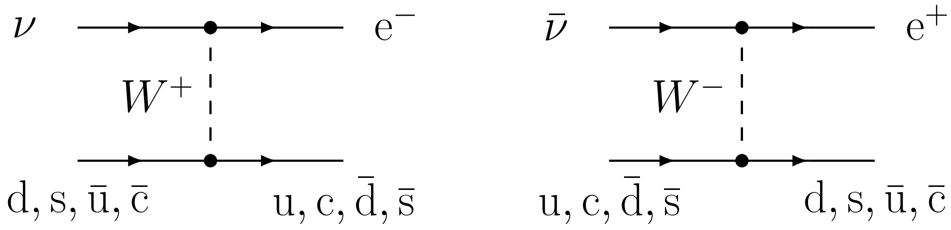


Figure 4.2 Bjorken scaling: the structure function νW_2 (a) plotted against $\omega = 1/x$ for different q^2 values (Miller *et al* 1972) (b) plotted against q^2 for a single value of $x = 0.25$ ($\omega = 4$) (Friedman and Kendall 1972).

Neutrino scattering



- Much information on the proton structure comes from (anti-)neutrino deep inelastic scattering. We will not derive here the expressions for the cross-sections (see H&M) but simply list the result for $\nu\mathrm{p} \to \mathrm{e}^-\mathrm{X}$ scattering and for $\bar{\nu}\mathrm{p} \to \mathrm{e}^+\mathrm{X}$ scattering[55]

$$\frac{\mathrm{d}^2\sigma^{\nu\mathrm{p}}}{\mathrm{d}x\mathrm{d}Q^2} = \sigma_0 \left\{ \left[1 + (1-y)^2\right] F_2^{\nu\mathrm{p}} + \left[1 - (1-y)^2\right] xF_3^{\nu\mathrm{p}} \right\}$$

$$\frac{\mathrm{d}^2\sigma^{\bar{\nu}\mathrm{p}}}{\mathrm{d}x\mathrm{d}Q^2} = \sigma_0 \left\{ \left[1 + (1-y)^2\right] F_2^{\bar{\nu}\mathrm{p}} - \left[1 - (1-y)^2\right] xF_3^{\bar{\nu}\mathrm{p}} \right\}$$

- Here we encounter a new structure function, xF_3, which is sensitive to the *difference* of quark and anti-quark distributions.

- In the parton model, the neutrino structure functions are

$$\begin{aligned} F_2^{\nu\mathrm{p}} &= 2x(d + s + \bar{u} + \bar{c}) \\ xF_3^{\nu\mathrm{p}} &= 2x(d + s - \bar{u} - \bar{c}) \end{aligned}$$

$$\begin{aligned} F_2^{\bar{\nu}\mathrm{p}} &= 2x(u + c + \bar{d} + \bar{s}) \\ xF_3^{\bar{\nu}\mathrm{p}} &= 2x(u + c - \bar{d} - \bar{s}) \end{aligned}$$

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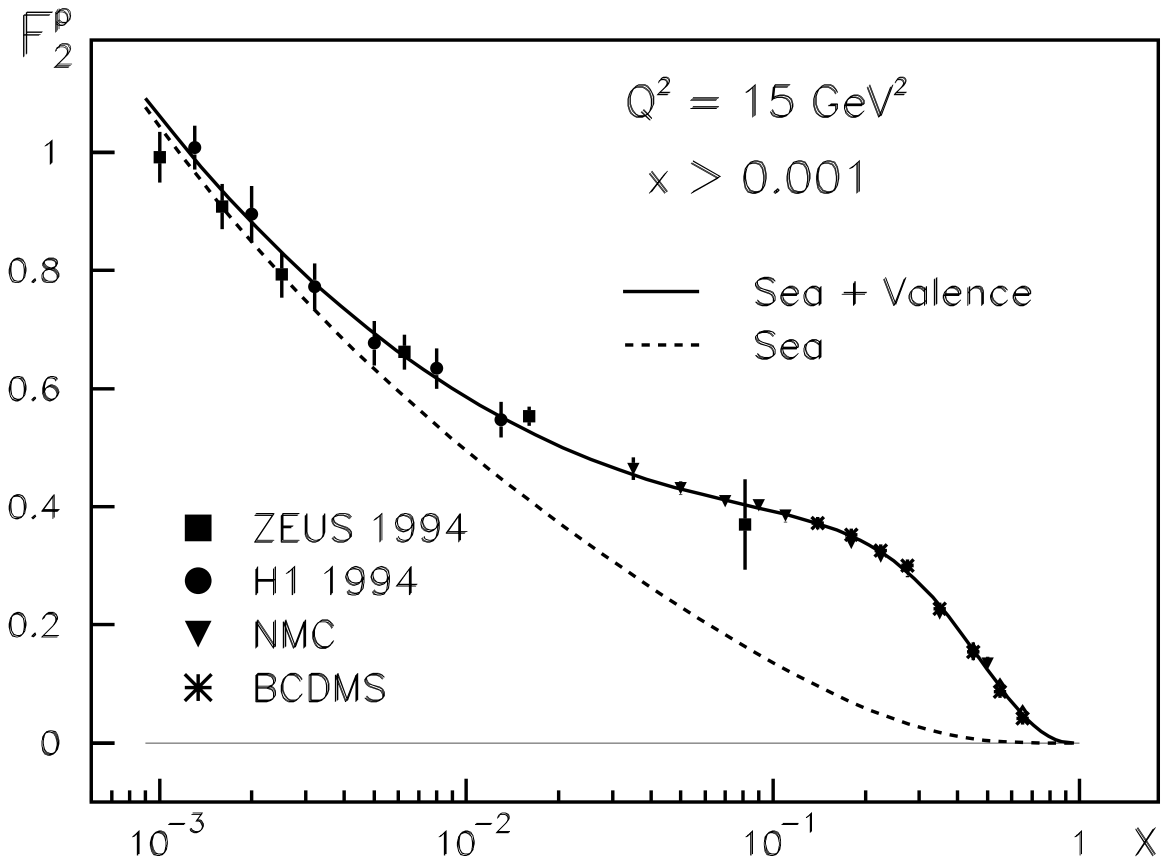
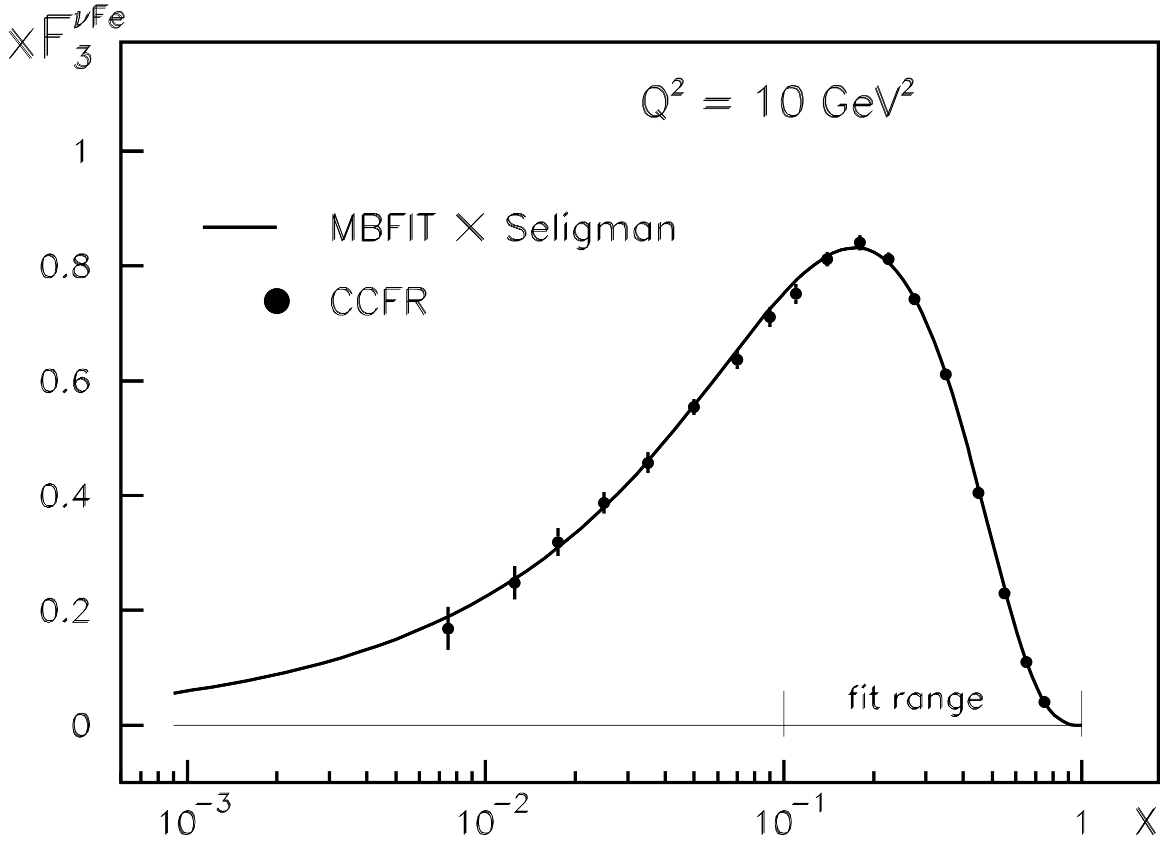
55 The factor in front is $\sigma_0 = \frac{G_\mathrm{F}^2}{4\pi x}\left(\frac{M_\mathrm{W}^2}{Q^2 + M_\mathrm{W}^2}\right)^2$


F_2^p
$Q^2 = 15\ GeV^2$
$x > 0.001$
Sea + Valence
Sea
ZEUS 1994
H1 1994
NMC
BCDMS
0
0.2
0.4
0.6
0.8
1
10^{-3}
10^{-2}
10^{-1}
1
X


$xF_3^{\nu Fe}$
$Q^2 = 10\ GeV^2$
MBFIT × Seligman
CCFR
fit range
0
0.2
0.4
0.6
0.8
1
10^{-3}
10^{-2}
10^{-1}
1
X

Valence and momentum sum rules

- We have introduced the quark number distributions $f_i(x)$ which we will write as $u(x), \bar{u}(x), d(x), \bar{d}(x)$, *etc.* It is convenient to define the **valence** and **sea** distributions by

$$\begin{array}{llll} u_{\rm v} = u - \bar{u}, & d_{\rm v} = d - \bar{d}, & s_{\rm v} = s - \bar{s} = 0, & \cdots \\ u_{\rm s} = 2\,\bar{u}, & d_{\rm s} = 2\,\bar{d}, & s_{\rm s} = 2\,\bar{s}, & \cdots \end{array}$$

  so that $u + \bar{u} = u_{\rm v} + u_{\rm s}$, *etc.* See also the diagram on page 8–16.

- Because the quantum numbers of the proton must be carried by the surplus of quarks over antiquarks, we get the counting rules

$$\int_0^1 u_{\rm v}(x)\,{\rm d}x = 2 \quad \text{and} \quad \int_0^1 d_{\rm v}(x)\,{\rm d}x = 1$$

- The momentum distributions $xf_i(x){\rm d}x$ give the **probability** that a quark carries a momentum fraction between x and $x+{\rm d}x$.[56] Thus if all quarks carry the momentum of the proton we should have

$$\sum_i \int_0^1 xf_i(x){\rm d}x = 1$$

- But integration of the quark distributions obtained from deep inelastic charged lepton and neutrino scattering gives

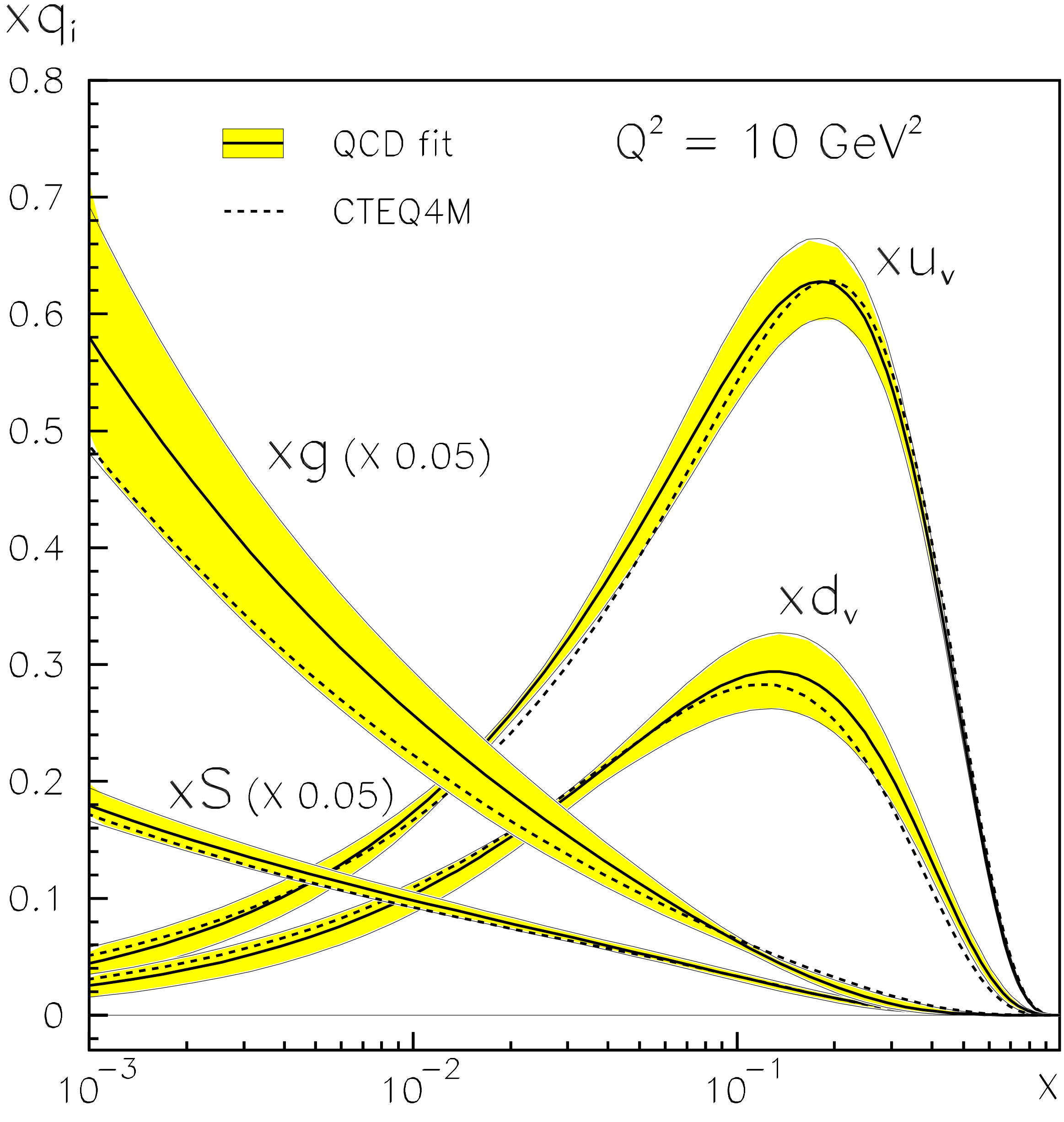
$$\sum_i \int_0^1 xf_i(x){\rm d}x \approx 0.5$$

- Where is the missing momentum? The answer is that it is carried by gluons. Introducing a gluon momentum distribution $xg(x)$, the correct momentum sum rule is

$$\sum_i \int_0^1 xf_i(x){\rm d}x + \int_0^1 xg(x){\rm d}x = 1$$

[56] If $f(x){\rm d}x$ is the *number* of quarks carrying a fraction x of the proton momentum P, then the total momentum carried by these partons is $p = xPf(x){\rm d}x$. The *probability* to carry a fraction x is thus $p/P = xf(x){\rm d}x$.

Example of a pdf set



Remark: The widely used abbreviation 'pdf' stands for 'parton density function'. Usually, but not always, it is clear from the context or notation (xf or f) if a *momentum* or a *number* density is meant.

Exercise 8.7: Universality of pdfs

- The **isospin symmetry** assumption says that the u (anti)quark distribution in the proton is equal to the d (anti)quark distribution in the neutron, and *vice versa*. Thus we have

$$\begin{aligned} F_2^{\mathrm{ep}} &= \tfrac{1}{9}x(d+\bar{d}) + \tfrac{4}{9}x(u+\bar{u}) + \tfrac{1}{9}x(s+\bar{s}) + \cdots \\ F_2^{\mathrm{en}} &= \tfrac{1}{9}x(u+\bar{u}) + \tfrac{4}{9}x(d+\bar{d}) + \tfrac{1}{9}x(s+\bar{s}) + \cdots \end{aligned}$$

The same applies to (anti)neutrino scattering: $F_2^{\nu\mathrm{n}} = F_2^{\nu\mathrm{p}}(u \leftrightarrow d)$. Note that the parton distributions, by convention, always refer to those of the proton.

  (a) [0.5] Use isospin symmetry to write down the parton model expressions for $F_2^{\nu\mathrm{n}}$ and $F_2^{\bar{\nu}\mathrm{n}}$ (F_2^{ep} and F_2^{en} are already given above). Define the *nucleon* structure function $F_2^{\mathrm{e}N}$ by averaging the proton and neutron F_2. Likewise define $F_2^{\nu N}$ by averaging over proton and neutron and *also* over ν and $\bar{\nu}$.

  (b) [0.5] Neglect charm and assume 3 flavours $(\mathrm{d}, \mathrm{u}, \mathrm{s})$. Show that

$$\frac{F_2^{\mathrm{e}N}}{F_2^{\nu N}} = \frac{5}{18}\left[\,1 - \frac{3}{5}\,\frac{(s+\bar{s})}{\sum(q+\bar{q})}\,\right] \approx \frac{5}{18}$$

  Because the (anti)strangeness content of the nucleon turns out to be small, it follows that the strangeness correction term above is also small; correction for charm would be even smaller.

- The plot on the next page shows an early experimental verification of $F_2^{\nu N} \approx \frac{18}{5}F_2^{\mathrm{e}N}$. This not only tests the (fractional) quark charges, but also that electron and neutrino DIS probe the *same* parton distribution: parton distributions are a process independent property of the nucleon (universality of pdfs).

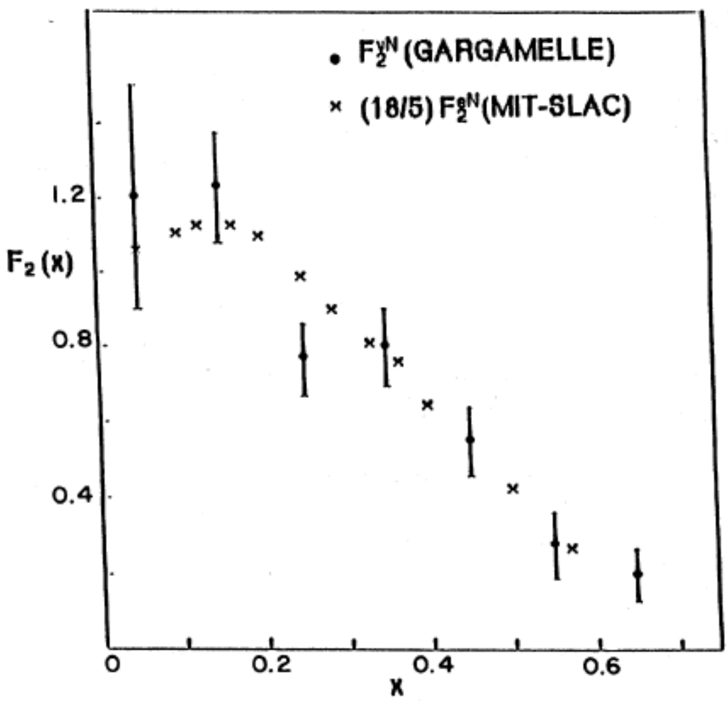


FIG. 10. Early Gargamelle measurements of $F_2^{\nu N}$ compared with $(18/5)F_2^{eN}$ calculated from the MIT-SLAC results.

Early verification that $F_2^{\nu N} \approx \frac{18}{5} F_2^{\mathrm{e}N}$. Figure taken from Jerome Friedman Nobel lecture, Rev. Mod. Phys. 63(1991)615.