Lecture notes Particle Physics II

Quantum Chromo Dynamics

1. Introduction

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The quest for elementary particles and forces

- Particle physics is the search for the fundamental constituents of matter and their interactions.
- The idea that matter is built from indivisible constituents dates back to the Greek philosopher Demokritos (400 BC) but it took a long time to prove him right: the proof that chemical elements are indeed made of atoms came only at the beginning of the last century, together with the development of statistical mechanics and quantum theory.
- The discovery of radioactivity indicated, however, that atoms could not be the fundamental constituents of matter and, indeed, after the experiments of Rutherford (1909) it was realised that atoms are complex objects with electrons orbiting a small heavy nucleus.
- The discovery of the neutron by Chadwick (1932) showed that atomic nuclei are made up of protons and neutrons. It was also clear that, in addition to gravitation and the electromagnetic force, there should exist two short-range forces in nature: a strong force which binds the nucleons together and a weak force which is responsible for radioactive β-decay. These forces had to be short-range because they were not felt at atomic scales.
- So nowadays four types of interaction are known: the **strong interaction** (~ 1), the **electromagnetic interaction** ($\sim 10^{-2}$), the **weak interaction** ($\sim 10^{-6}$) and **gravity** ($\sim 10^{-38}$), where we have indicated the relative strength in brackets.

Interactions

- Today we know that nucleons are made up of quarks and that the strong force between the nucleons in an atomic nucleus is a van-der-Waals type residual force of a more fundamental strong interaction between quarks. The field theory of this interaction between quarks is called **Quantum Chromodynamics** (QCD).
- QCD is a so-called **gauge theory**, like quantum electrodynamics (QED) and the theory of the weak interactions. In such a theory, the constituent fields are described by representations of a symmetry group while the interaction between the fields is described by the exchange of so-called **gauge bosons**. These interactions follow from the requirement that the Lagrangian is invariant under arbitrary local symmetry transformations of the constituent fields.
- The underlying U(1) symmetry of QED gives rise to the photon γ as the gauge boson. The weak interaction is governed by an SU(2) symmetry and is mediated by the three vector bosons $W^{\pm}$ and Z^0. The SU(3) symmetry of QCD generates generates eight types g_a of gluon as the quanta of the gauge field. Here are the properties of these so-called **intermediate vector bosons** (spin 1).

Interaction	Boson	Spin	Q	A	L	Mass
Electromagnetic	γ	1	0	0	0	0
Weak (CC)	$W^{\pm}$	1	± 1	0	0	80.4 GeV
Weak (NC)	Z^0	1	0	0	0	91.2 GeV
Strong	$g_1, \ldots, g_8$	1	0	0	0	0

- Note that all particles participate in the weak interaction, all charged particles in the electromagnetic interaction and that only the quarks participate in the strong interaction. Gravity is so weak that it can be neglected at subatomic scales.

Elementary particles

- The elementary particles can be classified into **leptons** (without strong interaction) and **quarks** (with strong interaction):

Lepton	Spin	Q	L_e	L_μ	L_τ	Mass	Lifetime
e	$\frac{1}{2}$	–1	1	0	0	0.5 MeV	
ν_e	$\frac{1}{2}$	0	1	0	0	~ 0	
μ	$\frac{1}{2}$	–1	0	1	0	106 MeV	2×10^{-6}
ν_μ	$\frac{1}{2}$	0	0	1	0	~ 0	
τ	$\frac{1}{2}$	–1	0	0	1	1.8 GeV	3×10^{-13}
ν_τ	$\frac{1}{2}$	0	0	0	1	~ 0	

Quark	Spin	Q	A	I_3	S	C	B	T	Mass
d	$\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{2}$	0	0	0	0	~ 7 MeV
u	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	0	0	0	0	~ 3 MeV
s	$\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{3}$	0	–1	0	0	0	~ 120 MeV
c	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{3}$	0	0	1	0	0	~ 1.2 GeV
b	$\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{3}$	0	0	0	–1	0	~ 4.3 GeV
t	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{3}$	0	0	0	0	1	~ 172 GeV

- The additive quantum numbers Q, L, A, I_3, S, C, B, T all change sign under charge conjugation (particle $\rightarrow$ antiparticle).[10]
- The charge (Q), lepton number ($L_{e,\nu,\tau}$) and baryon number (A) are aways conserved in every type of interaction (electromagnetic, weak, strong).

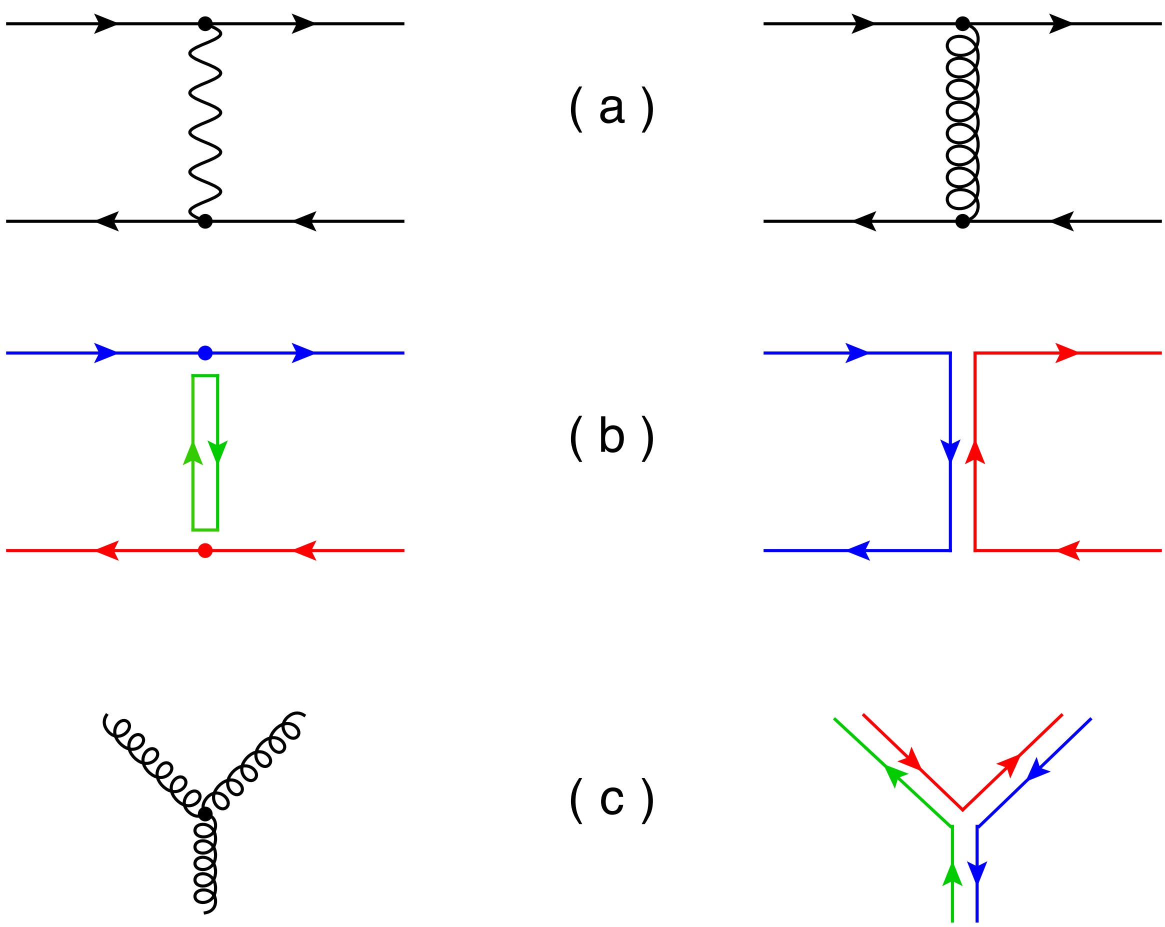
[10]Note that the flavour quantum numbers I_3, S, C, B, T have, by convention, the same sign as the charge.

Colour charge

- Free quarks have never been observed because their coupling is so strong that with increasing separation it becomes easier to produce a quark-antiquark pair than to isolate the quark.
- Quarks therefore bind permanently into **hadrons** which can be classified as **mesons** ($q\bar{q}$) and **baryons** (qqq).
- A problem with this is that there exist baryons such as the spin $\frac{3}{2}$ resonance $\Delta^{++} = u\uparrow u\uparrow u\uparrow$ with a ground state wave function that is fully symmetric under the exchange of two quarks. But for fermions (half-integer spin) the wave function should be antisymmetric.
- A way-out is provided by the **colour hypothesis** which states that each quark comes in one of three colours red (r) , green (g) or blue (b). Antiquarks are anticoloured: $\bar{\mathrm{r}}$, $\bar{\mathrm{g}}$ and $\bar{\mathrm{b}}$. The hypothesis furthermore states that hadrons are **colour singlets** ('white'), that is, they are invariant under rotations in colour space. The colour hypothesis thus naturally explains the existence of $q\bar{q}$ and qqq hadronic states[11] and also that of particles like the Δ^{++} since its colour wave function can always be made fully antisymmetric.
- In QCD, colour plays the role of charge, and gluons are the quanta of the colour gauge field that binds the quarks into hadrons. Unlike the photons in QED, the gluons themselves carry a colour charge, so that self-coupling 3- and 4-gluon vertices do exist (→ **fig**).
- As we will see later, this self-coupling of gluons has dramatic implications for the effect of *charge screening* in QCD, which turns out to be completely different from that in QED.

[11] It also allows for $q\bar{q}[q\bar{q}]^n$ and $qqq[q\bar{q}]^n$ exotic states. It is controversial if such states actually exist.

Basic diagrams of QED and QCD



(a) Electromagnetic interaction of a quark and an antiquark through photon exchange (left). Strong interaction of a quark and an antiquark through gluon exchange (right).

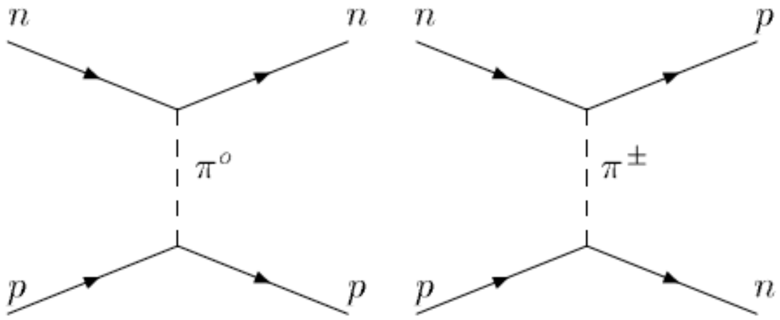
(b) Two possible colour flows in the $q\bar{q}$ strong interaction.

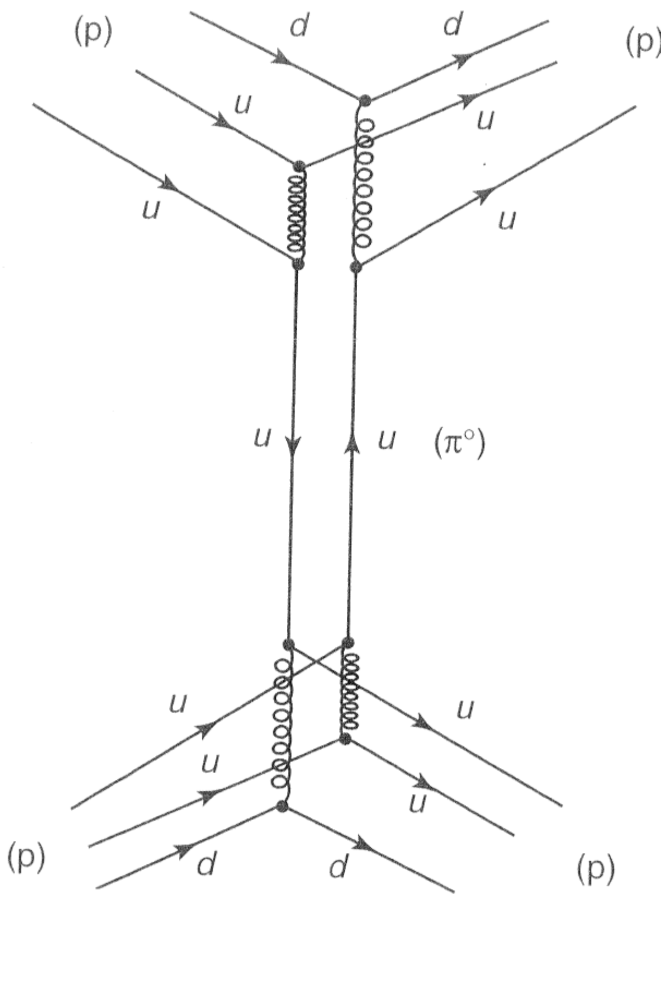
(c) Gluon interaction (left) and a possible colour flow through the 3-gluon vertex (right). Note that gluons always carry one unit of colour and one unit of anticolour.

Two paradoxes

- To explain the short range of the nuclear force, Yukawa (1934) proposed that this force is mediated by the exchange of *massive* field quanta which he called mesons (→ **fig**). In his theory, the range of the force is inversely proportional to the mass of the intermediate vector boson. He estimated a mass of about 140 MeV and indeed a candidate (the π meson) was later found in cosmic rays (1937).
- But, as we will see later, massive gauge field quanta break the gauge symmetry so that the exchanged boson must necessarily be massless. For instance, the U(1) symmetry of the QED Lagrangian forces the photon to be massless, which indeed it is. As a consequence the electromagnetic interaction has an infinite range.
- It follows that the SU(3) gauge symmetry of the QCD Lagrangian forces the gluons to be also massless, like the photon. But if these gluons are massless, how can the strong force then be short-range?
- Another puzzle came with a series of high-energy electron-proton scattering experiments at SLAC ($\sim$ 1970) which proved the existence of quarks but also showed that they seemed to behave like free particles, in spite of the fact that they are strongly bound inside the proton.
- The solution to both these paradoxes was found by Gross, Politzer and Wilczek by their discovery of **asymptotic freedom**. They could explain why, as Wilczek put it in his Nobel lecture, 'Quarks are Born Free, but Everywhere They are in Chains'.
- As we will see in these lectures, the phenomena of asymptotic freedom and confinement are caused by the self-interaction of gluons which, in turn, is a consequence of the non-abelian nature of SU(3).

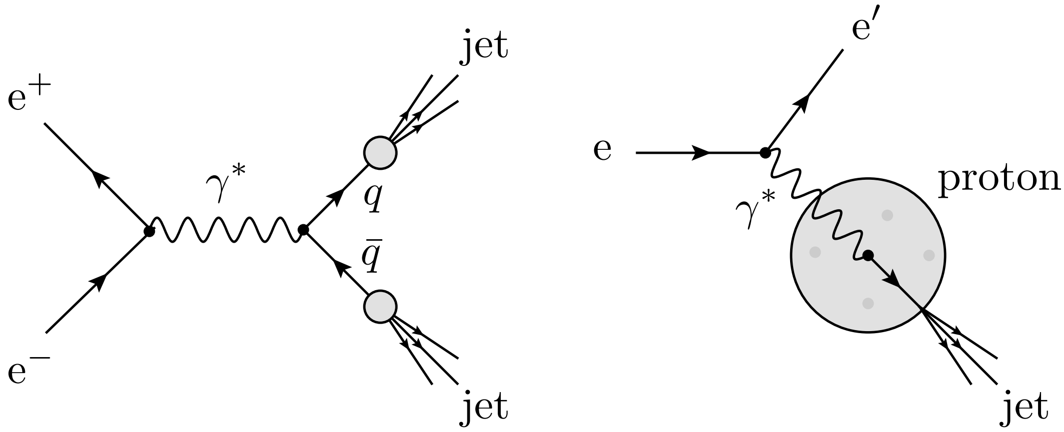
Old and modern views of pion exchange





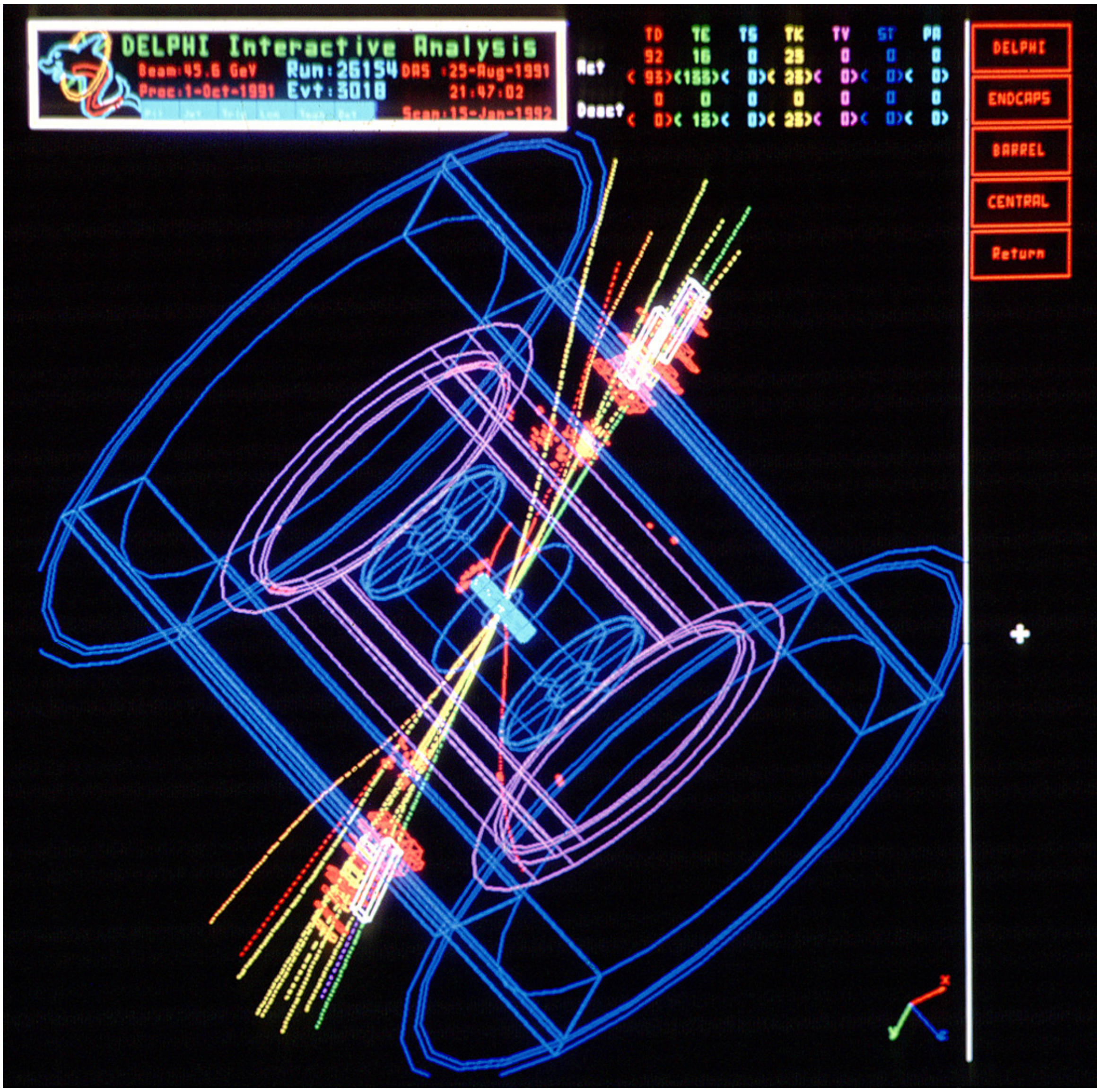
In the lower diagram π^0 exchange in a proton-proton interaction is described in terms of constituent quarks by the exchange of an $u\bar{u}$ pair.

Are quarks real?



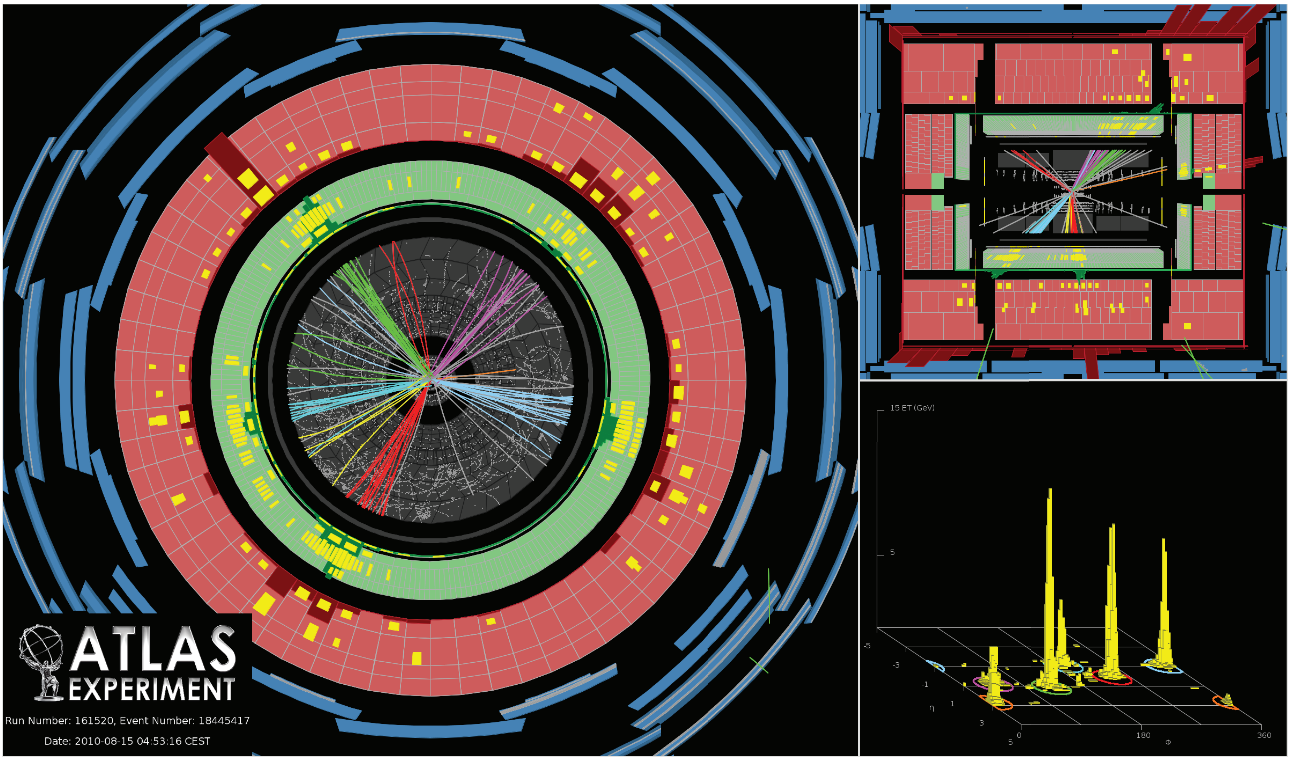
- If quarks cannot be observed in isolation, how do we know that they actually exist and are not mere theoretical constructs?
- One way is to resolve quarks by illuminating protons with photons of large momentum Q and therefore small Compton wavelength $1/Q$. These very short wavelength photons are radiated off highly energetic electrons when they scatter on protons (right-hand diagram above). This process is called **deep inelastic scattering** which indeed acts as a microscope to reveal the internal quark structure of the proton. How this actually works, will be the subject of the last two lectures in this course.
- Furthermore it turns out that highly energetic quarks produced in hard e^+e^-, $\bar{p}p$ and pp scattering hadronise into collimated sprays of particles, known as jets (left-hand diagram above). Thus we can more or less directly probe the dynamics of quarks by measuring jets in experiments at high energy colliders ($\rightarrow$ **fig**). Jet production is clearly a very important tool to confront QCD with experiment and can certainly produce spectacular events in particle colliders ($\rightarrow$ **fig**) but, unfortunately, we cannot cover the large field of jet physics in these eight lectures.

Two-jet event in a e^+e^- collision at LEP



Two back-to-back jets observed by the DELPI experiment at LEP in an e^+e^- collision at 90 GeV centre-of-mass energy.

Six-jet event in a proton-proton collision at the LHC



Candidate six-jet event recorded by the Atlas experiment in a 7 TeV proton-proton collision at the LHC.

About this course

- QCD, and strong interaction physics in general, is a huge subject, so no two courses on QCD are the same since they necessarily reflect the choices made by the lecturer.
- My first choice is to devote ample time (three lectures) to build the QCD Lagragian, much based on Chapter 10 of Griffiths. Here a good understanding is important because QCD lectures which you may attend later (*e.g.* at the CERN summer school) often start from the Lagrangian, without much further ado ($\rightarrow$ **fig**). Of course there will be some overlap with PP-I but I prefer to tell the full story instead of relying on what you presumably know already.
- QCD calculations quickly become technically complicated so we have to limit ourselves to some simple colour factor calculations which, however, nicely explain why the colour force is attractive for meson and baryon quark states and repulsive for others.
- Of course, asymptotic freedom is a crucial property of QCD and we will devote a full lecture to the running coupling constant and its implications. Here we will encounter ultraviolet singularities which will be dealt with by a simple cut-off regularisation.
- Infrared singularities are the subject of the another lecture where we will explain how they are related to long-distance physics, as opposed to short-distance perturbative QCD.
- The last two lectures are devoted to the structure of the proton, the quark-parton model, and the so-called QCD improved quark-parton model. This subject is not covered in Griffiths but extensively treated in Halzen and Martin.

First page of a QCD course at CERN

Feynman rules of QCD

- Feynman rules follow from QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4} F^A_{\alpha\beta} F_A^{\alpha\beta} + \sum_{\text{flavours}} \bar{q}_a (i\not{D} - m_q)_{ab} q_b + \mathcal{L}_{\text{gauge-fixing}}$$

  $F^A_{\alpha\beta}$ is field strength tensor for spin-1 gluon field $\mathcal{A}^A_\alpha$,

$$F^A_{\alpha\beta} = \partial_\alpha \mathcal{A}^A_\beta - \partial_\beta \mathcal{A}^A_\alpha - g f^{ABC} \mathcal{A}^B_\alpha \mathcal{A}^C_\beta$$

  Capital indices A, B, C run over 8 colour degrees of freedom of the gluon field. Third 'non-Abelian' term distinguishes QCD from QED, giving rise to triplet and quartic gluon self-interactions and ultimately to asymptotic freedom.

- QCD coupling strength is $\alpha_S \equiv g^2/4\pi$. Numbers f^{ABC} $(A, B, C = 1, ..., 8)$ are structure constants of the SU(3) colour group. Quark fields q_a $(a = 1, 2, 3)$ are in triplet colour representation, while gluon fields $\mathcal{A}^A_\alpha$ are in adjoint representation.

- D is covariant derivative:

$$\begin{aligned} (D_\alpha)_{ab} &= \partial_\alpha \delta_{ab} + ig \left(t^C \mathcal{A}^C_\alpha\right)_{ab} \\ (D_\alpha)_{AB} &= \partial_\alpha \delta_{AB} + ig (T^C \mathcal{A}^C_\alpha)_{AB} \end{aligned}$$

Flying start on the first page of the course 'Introduction to QCD' in the CERN postdoc lecture series, given by Bryan Webber in October 2003.