Comment on Einstein's original (1905) derivation of the Lorentz transformations

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Consider a light source, equiped with a clock. Consider a mirror equiped with an identical clock. Source and mirror are placed at the ends of a rigid rod. Light is emitted by the source when the clock reads t_0 and reflected when the second clock reads t_1 and it arrives at the source again when the clock there reads t_2. The clocks are said to be synchronous (run at the same rate) when:

$$(t_2 - t_0)/2 = t_1 \tag{1}$$

The readings are done by observers at the place of the clocks.

Now the setup just described is put in uniform motion, velocity v. For convenience we consider motion in the x-direction, the rigid rod being aligned along the x-direction. The reference frame S is the frame in which the setup was originally at rest. The reference frame S' is the frame in which the setup is at rest after having been put in motion (Fig. 1). Seen from S the moving clocks are no longer synchronous, due to the constancy of the speed of light. (Easily verified).

However, due to the principle of relativity, the setup cannot 'know' it is in motion (absolute motion is an invalid concept) and in S' the clocks should still be synchronous, leading to the condition

$$(t'_2 - t'_0)/2 = t'_1 \tag{2}$$

Together with the postulate of the constancy of the speed of light these equations should lead to the establishment of the relationship between the 'primed' (reference system S') and the 'unprimed' (reference system S) variables:

$$\left[t'(x_2, t_2) - t'(x_0, t_0)\right]/2 = t'(x_1, t_1) \tag{3}$$

where the indices refer to the three events referred to above: emission (0), reflection (1) and reception (2) at the point of emission of the light ray. We chose $x_0 = 0$ and $t_0 = t$ where t is the time variable; $x_0 = 0$. It is straightforward to show that (introducing the space variable

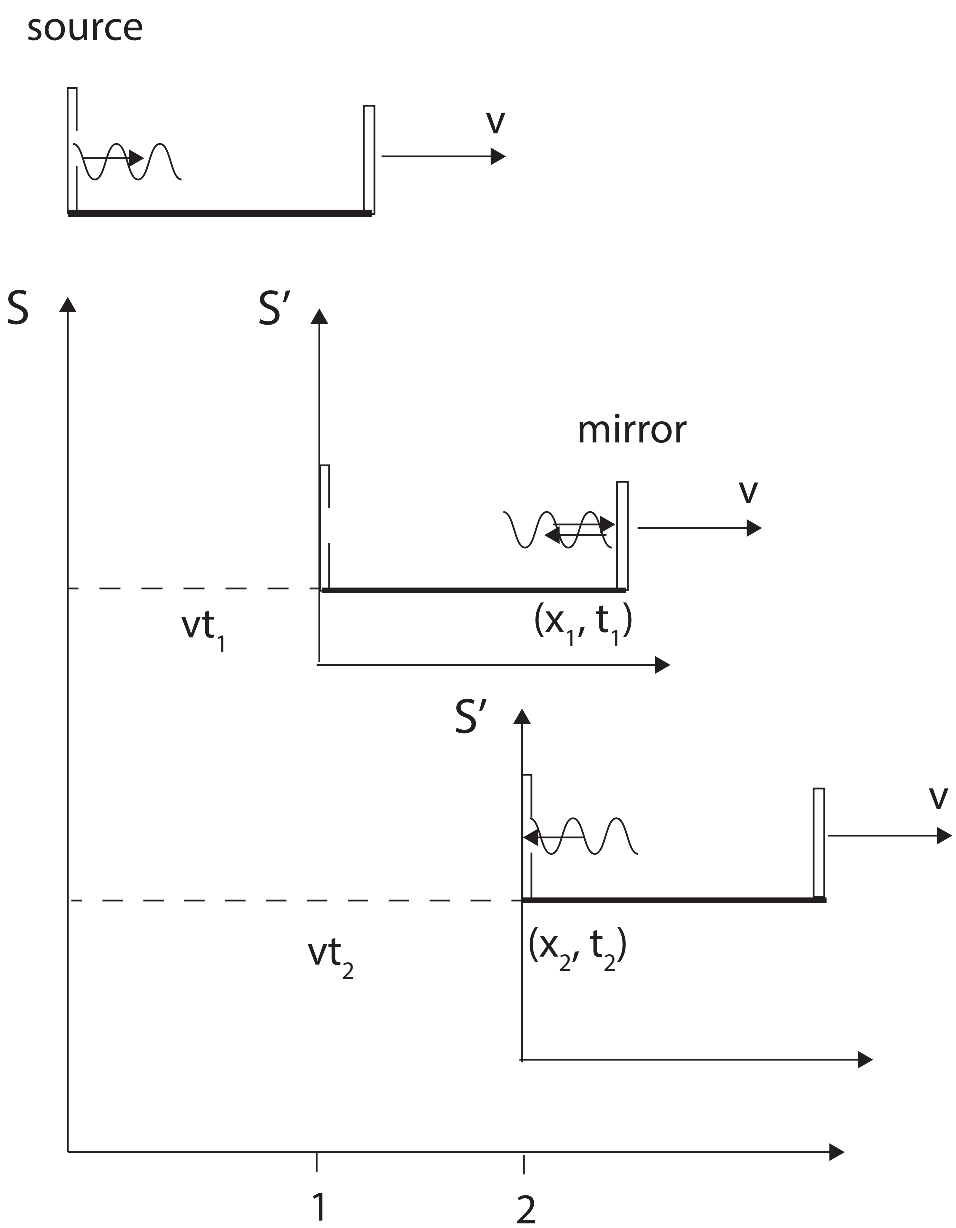

Figure 1: A light pulse from left to right and back

$x = x_1 + vt_1$, where $t_1 = (x_1 + vt_1)/c$, etc.):

$$\begin{aligned} x_1 &= x + \frac{vx}{c-v} && (4) \\ t_1 &= t + \frac{x}{c-v} && (5) \\ x_2 &= \frac{vx}{c-v} + \frac{vx}{c+v} && (6) \\ t_2 &= t + \frac{x}{c-v} + \frac{x}{c+v} && (7) \end{aligned}$$

so we find:

$$[t'(0,t) + t'(\frac{vx}{c-v} + \frac{vx}{c+v}, t + \frac{x}{c-v} + \frac{x}{c+v})]/2 = t'(x + \frac{vx}{c-v}, t + \frac{x}{c-v}) \qquad (8)$$

So this equation should be solved to find the transformation $t' = t'(x,t)$. We will not do that here, but it is easy to verify that this equation indeed is satisfied if we apply the Lorentz transformations (for the time coördinate t) to it, by taking

$$t'(x,t) = \gamma(t - vx/c^2) \qquad (9)$$

with $\gamma = 1/\sqrt{(1 - v^2/c^2)}$

The issue is that Einstein, in his 1905 paper, follows the approach we have copied above, but does not arrive at the equation 8. Quite abruptly he states that the transformations he is looking for should satisfy:

$$[t'(0,t) + t'(0, t + \frac{x'}{c-v} + \frac{x'}{c+v})]/2 = t'(x', t + \frac{x'}{c-v}) \qquad (10)$$

where a new variable $x' = x - vt$ has been introduced. We have not been able to derive this equation and it certainly is not satisfying the Lorentz transformation from S to S' that eq.8 does satisfy. Yet, and that is hard to understand, Einstein derived the Lorentz transformations from it.

For completenes I mention two relevant references, the first one by Martinez [1] goes through great length to explain what Einstein's considerations must have been when he formulated eq. 10. Martinez also derives the Lorentz transformations in a way similar to Einstein, but starting from eq. 8. I still have to scrutinize this reference in order to be able to fully appreciate it.

The article by Freeman [2] concludes that Einstein's original derivation is wrong. I also still have to scrutinize this reference to fully appreciate it.

References

[1] A. A. Martinez, Kinematic subtleties in Einstein s first derivation of the Lorentz transformations, Am. J. Phys., vol. 72, no. 5, pp. 1-9, May, 2004. Click here: `www.nikhef.nl/~h02/deriv_lt_martinez.pdf`

[2] Jon C. Freeman, in The General Science Journal, https://www.gsjournal.net/Science-Journals/Research%20Papers-Relativity%20Theory/Download/6206 Click here: www.nikhef.nl/~h02/deriv_lt_freeman.pdf