

# ParFORM: Parallel Version of the Symbolic Manipulation Program FORM

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**Abstract.** The symbolic manipulation program FORM is specialized to handle very large algebraic expressions. Some specific features of its internal structure make FORM very well suited for parallelization.

After an introduction to the sequential version of FORM and the mechanisms behind, we report on the status of our project of parallelization. We have now a parallel version of FORM running on Cluster- and SMP-architectures. This version can be used to run arbitrary FORM programs in parallel.

## 1 Introduction

FORM [1] is a program for symbolic manipulation of algebraic expressions specialized to handle very large expressions of millions of terms in an efficient and reliable way. That is why it is widely used in Quantum Field Theory, where the calculation of the order of several hundred (sometimes thousands) of Feynman diagrams is required. Currently the actual version of FORM is called FORM3.

In context with this goal an improvement of efficiency is very important. Parallelization is one of the most efficient ways to increase performance. So the idea to parallelize FORM is quite natural.

This paper reports the present status of our project of FORM parallelization and the result is called ParFORM.

The main goal of parallel processing is to reduce wall-clock time<sup>1</sup> i.e. the user's waiting time. Parallelism does not come for free; it always has some overhead with respect to serial execution, but it can significantly reduce the wall-clock time.

Not every problem can be divided into parallel tasks. An example of a parallelizable problem is the multiplication of two matrices. An example of a non-parallelizable problem is the calculation of the Fibonacci series (1,1,2,3,5,...) by means of the recurrence formula  $F(k+2) = F(k+1) + F(k)$ .

The last example has to be taken with caution. It does not imply that every recursion is necessarily non-parallelizable. For example, in the field of perturbative calculation, where FORM has become a standard tool, we often exploit recurrence relations. The latter, however, are special in a sense, that they are applied to every separate term in one expression and they are examples of local operations.

Of course, ParFORM cannot handle non-parallelizable problems, but ParFORM is quite natural to apply it to any kind of parallelizable problem. So, if we have to solve a parallelizable problem which essentially involves local operations, the interior structure of FORM permits us to parallelize each step of a process.

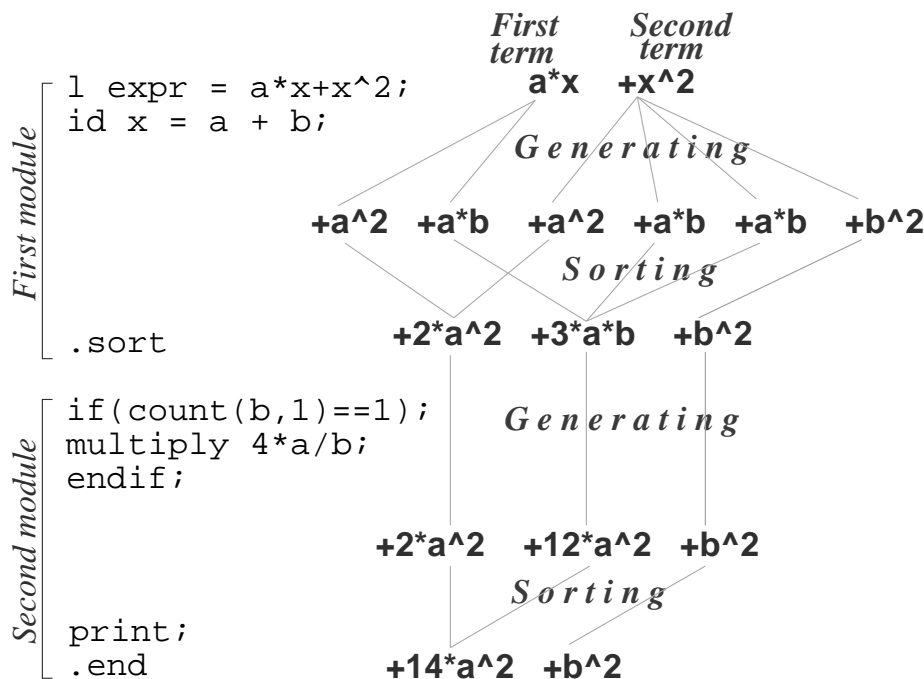
There are some internal mechanisms of FORM that become important in its parallel version and this will be described in the next section.

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<sup>1</sup> The elapsed time from start to finish of a process.

## 2 The Sequential Version of FORM

FORM is used non-interactively by executing a program that contains several parts called modules. Modules are terminated with “dot”-instructions that cause the execution of the module, see the example on the left of Fig. 1.



**Fig. 1.** The fragment of a typical FORM program. In the first module, the expression  $expr = ax + x^2$  is introduced, and then the substitution  $x \rightarrow a + b$  is performed. In the second module, only terms in which the degree of  $b$  is exactly 1 are multiplied by  $4a/b$  (there is only one such term in the expression).

This example consists of only two modules. There are two “dot”-instructions: a `.sort` and a `.end`. In both cases the result is sorted. `.end` additionally terminates the program.

The execution of each module is divided into three steps:

- **Compilation:** The input is translated into an internal representation.
- **Generating:** For each term of the input expressions the statements of the module are executed. This in general generates a lot of terms for each input term.
- **Sorting:** All the output terms that have been generated are sorted and equivalent terms are summed up.

FORM only allows local operations on single terms, like replacing parts of a term or multiplying something to it. Together with a sophisticated pattern matcher, this at first strong limitation allows the formulation of general and efficient algorithms. The limitation to local operations makes it possible to handle expressions as “streams” of terms, that can be read sequentially from a file and processed independently.

The restriction to local operations allows to deal with expressions that are larger than the available main memory and thus in addition allows parallelism.

## 3 The Parallelization of FORM

The limitation of performing only local operations makes FORM very well suited for parallelization. The concept of parallelization is straightforward and indicated in Fig. 2: Distribute the input terms

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l expr = a*x+x^2+b*x+...
id x = a + b;

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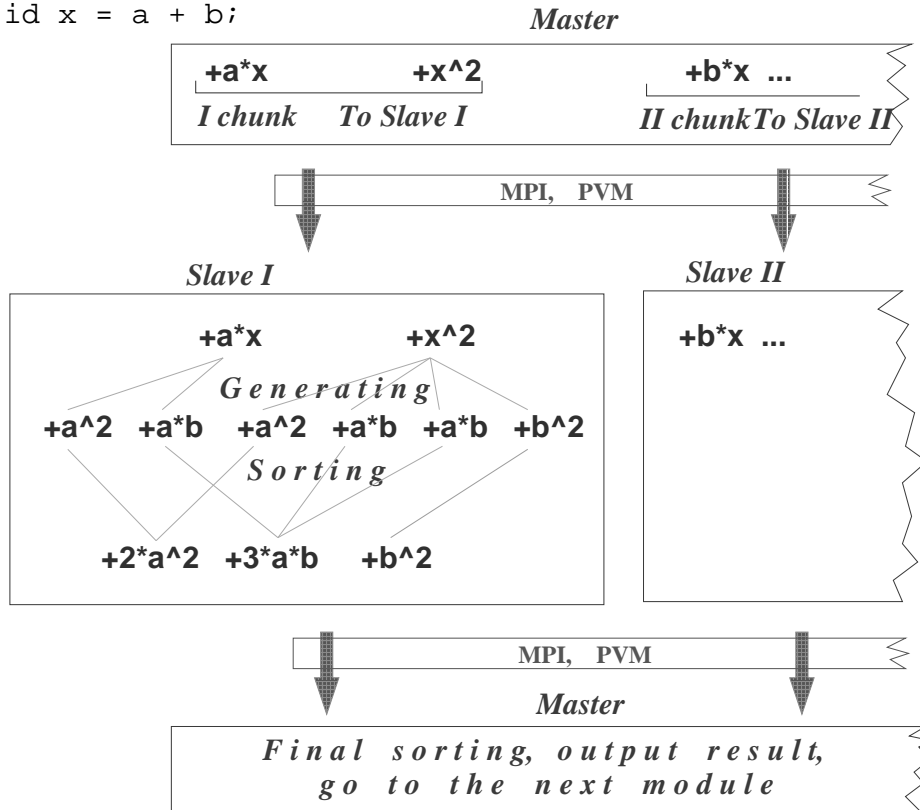


Fig. 2. General conception of ParFORM.

among available processors, let each of them perform local operations on its input terms, generate and sort the arising output terms. At the end of a module the sorted streams of terms from all processors have to be merged to one final output stream again.

This concept indicates to use a master-slave structure for parallelization. The master would store expressions, distribute and recollect all terms of each expression.

The master communicates with slaves by means of some message passing library. Message passing permits to parallelize FORM on computer architectures with shared or distributed memories, but on the other hand, this leads to some overhead due to huge data transfers. Formerly two libraries were used, MPI and PVM, but we decided to skip PVM support. The reason was because many vendors announced to discontinue further development of PVM. On the other hand, with the announced development of MPI almost all useful PVM features should appear in MPI.

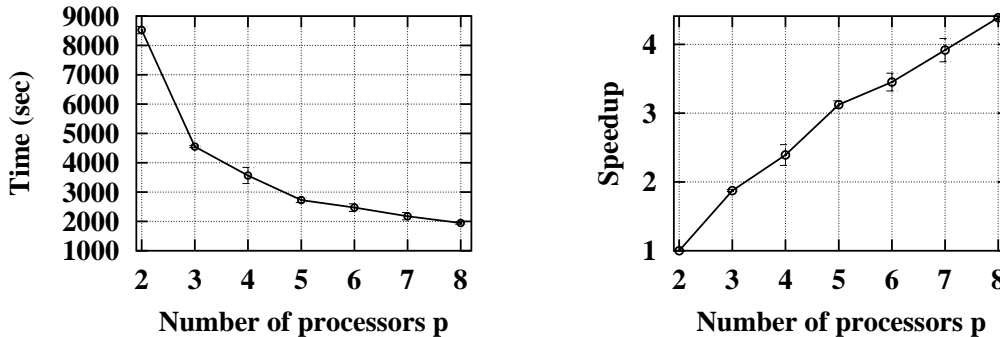
The master simply distributes and collects data. With a lower number of processors, the master becomes almost idle. For that case one can try to force the master to participate in real calculations, too. On the other hand, with increasing numbers of slaves, the master spends more and more time to control slaves, which may lead to early speedup saturation. Our estimations show that for more than four processors our Master-Slave model is adequate.

A working parallel FORM prototype ParFORM[2] was completed in 2000, this was a preliminary version with the syntax of FORM 3, but without complete FORM 3 features [3]. During the last years, the real FORM 3 version 3.1 was parallelized. At present, a number of real physical applications exist which would not have been possible without ParFORM [4].

As a typical example for physics applications we consider a packet called “BAICER” written by P. Baikov. This is a FORM-packet developed for reduction of 4-loop propagator massless integrals to some small set of so-called “master integrals”. The algorithm is based on recently developed

techniques of solving recurrence relations using an integral representation for their solutions [5]. It requires the calculation of large  $D$  expansion coefficients (here “ $D$ ” is the dimension of the integration space). As a result, the mathematical complexity of the original problem can be transformed to the necessity to make simple manipulations but with very large polynomial expressions (billions of terms and more).

Both working prototypes of ParFORM and BAICER were developed using the Karlsruhe Compaq-AlphaServer GS60e, 8 Alpha (EV67) processors (700 MHz), manufactured in 1999. Results for a “typical” test of ParFORM using the BAICER packet are shown in Fig. 3.



**Fig. 3.** Computing time and speedup for the test program BAICER on Compaq-AlphaServer with 8x Alpha (EV67) processors 700 MHz.

The ParFORM structure assumes that almost all real calculations are performed by slaves while the master only distributes and collects data. This is the reason why we calculate speedups normalized to the time spent by programs running on two processors.

Our test program demonstrates an almost linear speedup up to 8 processors available.

From a practical point of view this means that the wall-clock time for some real tasks can be reduced from months to weeks, which sometimes is a really essential feature.

Since February 2004 we had a SGI Altix 3000 server available, and some technical details will be given:

SGI Altix 3700 Server 32x 1.3 GHz/3MB-SC Itanium2 CPUs

64 GB DDR/116 MHz mem

2.4 TB SCSI disks

Red Hat Linux Advanced Server release 2.1AS (Derry).

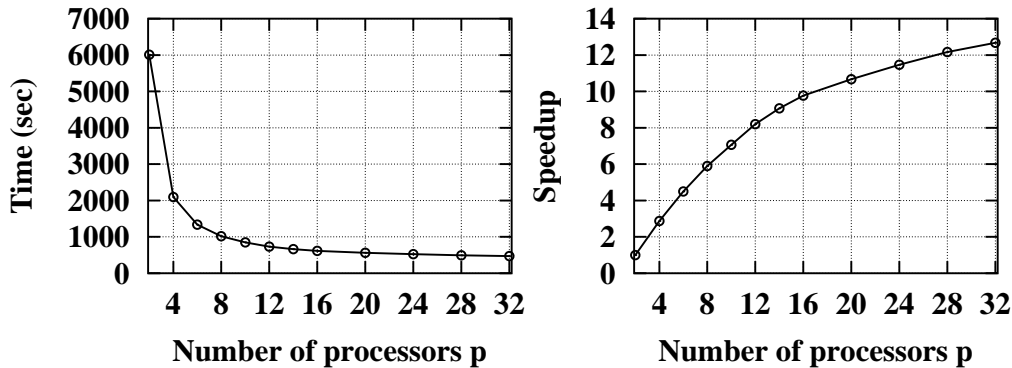
The results for our test program BAICER are shown in Fig. 4. The speedup is almost linear up to 12 processors. Of course hereafter the speedup is not linear, but it is still considerable.

An achieved speedup of 12 means that a FORM job that would need one year of computing time can be run as ParFORM job in less than one month. This leads physics to a qualitatively new level, because it would practically be impossible to run jobs for years whereas months are feasible nowadays.

Fig. 4 shows that with 16 processors we have a speedup of 10. This means that we can run on our 32-processor computer two jobs simultaneously, having the speedup of 10 for each of them.

## 4 Conclusion

We have shown that the internal structure of FORM permits a “natural” parallelization of parallelizable problems. It is worth mentioning that these results have been achieved with FORM programs that were written for the sequential version and have not been modified. Generally the FORM user does not have to know anything about the mechanism behind the parallel version to



**Fig. 4.** Computing time and speedup for the test program BAICER on the SGI Altix 3700 server with 32x Itanium2 processors (1.3 GHz).

run existing programs in parallel. Still, some knowledge can help to tune them and achieve a higher speedup.

Of course the speedup that can be achieved strongly depends on the problem under consideration.

In some cases, i.e. ideal FORM input code and adequate problem size, the achieved speedup is almost linear in the number of slave processors. For realistic complex applications, the speedup is still considerable even with a larger number of processors.

Colleagues who are interested to use ParFORM should contact M. Tentyukov by e-mail [tentyukov@particle.uni-karlsruhe.de](mailto:tentyukov@particle.uni-karlsruhe.de)

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