

Tuning FORM with large calculations

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Some recent additions to FORM are discussed. In particular large file support and the tablebases are presented.

1. introduction

Traditionally FORM [1] is called a program for particle theory. This is however a misconception that follows from a desire of putting labels on things. FORM is a program for many fields of science in which large formulae occur, like in deep perturbative expansions. Its dealing with non-commutative objects makes it also very suitable for mathematics calculations [2,3]. And it has also been used successfully in the field of Euler-Zagier sums [4] in which the results of certain categories of sums can only be obtained by solving large sets of equations. Of course FORM has been mostly tested in perturbative quantum field theory. However its speed and the potential size of its expressions should make FORM very attractive for many scientists.

New features are always looked at from a more generic viewpoint. This makes them useful for as many people as possible. Some of these new features are:

- \$-variables which allow a high level of control over the organisation of a program. Version 1 and 2 of FORM never had this flexibility.
- Write to file facilities. This allows even dynamical addition to running programs.
- Large file support. Now also 32-bits processors can deal with intermediate expressions and files of more than 2 Gbytes, provided the operating system supports this (as in the ext-3 file-system versions of Linux).
- Better support for large tables.
- The tablebase. This is a database-like facility for extremely large tables. It was in-

spired by a calculation [5] in which there were more than 20000 table elements, each of which occupied on average more than 20 Kbytes. To compile all these table elements in each program that might need some of the elements would be wasteful and slow, even in FORM. Now there are facilities by which the program can determine what is needed and when, and only those elements will be compiled at the proper moment.

2. Some examples

The first example concerns a run which is much like a benchmark used originally by D. Fliegner [6] to test the parallel version of FORM. Later this test was taken over by R. Kreckel [7] to compare the GiNaC system with other symbolic systems. Here we have modified it somewhat to allow the intermediate expression to surpass 4 Gbytes¹.

FORM by J.Vermaseren, version 3.1(Jul 22 2002)
Run at: Mon Jul 22 15:06:39 2002

```
#: SmallSize 10000000
#: LargeSize 100000000
#: TermsInSmall 1000000
#define MAX "700"
S a0,...,a'MAX';
L F = (a0+...+a'MAX')^3;
id a1 = -a2-...-a'MAX';
Print +f;
.end
```

Time =	1.73 sec	Generated terms =	417057
	F	1 Terms left =	133668
		Bytes used =	1882524
		.	
		.	

¹A direct extension of the original test in which the power is 2 would run into the limit of 6000 variables (on 32-bit systems) before the 4 Gbyte limit would be reached

```

Time = 2611.92 sec      Generated terms = 457333100
      F              1 Terms left      = 390169111
                        Bytes used      = 5494312698

Time = 2613.52 sec      Terms active   = 390169111
      F              Bytes used      = 5496905398

Time = 2959.48 sec      Generated terms = 457333100
      F              Terms in output = 1
                        Bytes used    = 18

F =
  a0^3;

```

The run was on a notebook computer with a 850 MHz pentium, 500 Mbytes memory and Red-Hat 7.3 Linux.

The next example shows the dynamic extension of tables during a run. It uses the $\$$ -variables and the resulting table elements are also appended to a file. This way each new run can start by reading all results of the previous jobs. This mechanism was used to run and tabulate more than 20000 integrals in the computation of basic building blocks for the three loop structure functions in deep inelastic scattering. Sometimes more than 1000 integrals were done in a single (rather lengthy) run.

```

#include BE88fill.h
#do NUM = 1,500

  L FFK =
*   get an integral from a list
  #call intlist(BE88,'NUM')
  ;
*   for example this one:
*   +BE(0,1,1,1,1,2,1,3+N,0,1,0,0,0,0,N,0)
*
*   Here we compute the integral. Next we do
*
  L FFL =
*   get the integral again
  #call intlist(BE88,'NUM')
  ;
  id BE(n1?,...,n7?,n8?!number_,k1?,k2?
      ,0,0,0,0,0,N?!number_,k9?) =
      BE88fil(n1,...,n7,n8-N,k1,k2,k9,N)*f(be88);
* load arguments into $args and type into $ltype
  id fx?{...,BE88fil,...}(?a$args,N)*f(x?$ltype)
      = 0;

  .sort
*   put the result in $expr
  #$expr = FFK;
*   'construct' a fill statement to add to table
  Fill '$ltype'fill('$args') = '$expr';

```

```

.global
*   make sure file is ready for appending
#append <'$ltype'fill.h>
*   and append to file
#write <'$ltype'fill.h> \
      "Fill '$ltype'fill('$args') = %E;";FFK
.store
#enddo
.end

```

3. The tablebase

Faced with hundreds of megabytes of table elements of which we may typically need only a few in each job (but we cannot say in advance which) we need a special database structure. We want a database for FORM with the features:

- FORM reads at first only an index of the database.
- At a specified time FORM can determine which elements are actually needed.
- At a specified time FORM will load and compile these elements.
- When the user specifies it, the elements will be used.
- The elements can be stored in gzipped [8] form (saves a factor 4).

Of course such 'tablebases' need a number of control commands among which should be commands for

- Creating a new tablebase
- Adding tables and table elements to the tablebase.
- Investigating what is in the tablebase
- Removing elements from the tablebase
- Cleaning up a tablebase
- Loading the index and compiling 'stubbs'
- Loading and compiling individual elements.
- Loading and compiling complete tables.
- Loading and compiling indicated elements.

- ... and probably more

The stubbs are intermediate expressions. They replace an object by an indicator that this table element exists in the tablebase. The advantage of this is that the object does not need to be manipulated by other routines that would deal with cases that are not in the tablebase, but yet we do not replace it by potentially lots of terms until we are ready for manipulating those terms. Let us see how this works out.

```
#-
#define EXPANDEP "6"
#include ensum.h
#if 'EXPANDEP' > 0
S ep(:'EXPANDEP');
#endif
.global
L F = x1+x2;
.sort:start;
#include be11fill.h
#include be22fill.h
#include be55fill.h
#include be66fill.h
#include be88fill1.h
#include be88fil2.h
#include be88fil3.h
#include be88fil4.h
#include la11fill.h
#include la22fill.h
#include la77fill.h
#include no11fill.h
#include no22fill.h
.sort:after 4;
.sort:complete reading;
TableBase "three.tbl" create;
.sort:create;
TableBase "three.tbl" addto be11fill,be22fill
,be55fill,be66fill,be88fill
,la11fill,la22fill,la77fill
,no11fill,no22fill;
.sort:addto;
.end
```

This program gives the output

```
Time = 0.04 sec    Generated terms = 2
          F        Terms in output = 2
          start Bytes used = 32

Time = 148.75 sec    Generated terms = 2
          F        Terms in output = 2
          after 4 Bytes used = 32

Time = 148.76 sec    Generated terms = 2
          F        Terms in output = 2
          complete reading Bytes used = 32
```

```
Time = 148.76 sec    Generated terms = 2
          F        Terms in output = 2
          create Bytes used = 32
We add the name be11fill
We add the name be22fill
We add the name be55fill
We add the name be66fill
We add the name be88fill
We add the name la11fill
We add the name la22fill
We add the name la77fill
We add the name no11fill
We add the name no22fill
```

```
Time = 241.65 sec    Generated terms = 2
          F        Terms in output = 2
          addto Bytes used = 32
```

```
Time = 241.65 sec    Generated terms = 2
          F        Terms in output = 2
          Bytes used = 32
```

The running times refer to a pentium 850. The first part shows the reading and compilation of the entire tables. The second part is the compression and the writing into the tablebase. How big are these files?

lines	bytes	
21527	1665848	be11fill.h
13123	1030971	be22fill.h
12420	968211	be55fill.h
19035	1486649	be66fill.h
679908	53221903	be88fill1.h
490372	38477216	be88fil2.h
410549	32158987	be88fil3.h
165495	12920526	be88fil4.h
798355	61843593	la11fill.h
37895	2896918	la22fill.h
120615	9421657	la77fill.h
48035	3647629	no11fill.h
14177	1090916	no22fill.h
2831506	220831024	total

---> 51875476 three.tbl

254795	20008483	ta0fill.h
66790	5270950	ta1fill.h
317318	24834908	ta2fill.h
553843	43037497	ta3fill.h
252002	19610471	ta5fill.h
568589	44627190	tb0fill.h
277903	21738353	tb1fill.h
21553	1689338	tb5fill.h
2312793	180817190	total

---> 38599445 two.tbl

```

35338 2348216 gtab00.prc
49784 3313386 gtab01.prc
48620 3065428 gtab10.prc
68647 4383913 gtab11.prc
51825 3283619 gtab20.prc
51077 3401328 gtab02.prc
305291 19795890 total

```

```

---> 5005177 one.tbl

```

These are the three loop, two loop and one loop tabulated integrals respectively.

There is already one pleasant spinoff. When we just load this one file three.tbl and enter and compile all elements we have

```

Time = 0.07 sec  Generated terms = 2
      F          Terms in output = 2
            start Bytes used      = 32

Time = 0.20 sec  Generated terms = 2
      F          Terms in output = 2
            open  Bytes used      = 32

Time = 0.59 sec  Generated terms = 2
      F          Terms in output = 2
            load  Bytes used      = 32

Time = 124.80 sec Generated terms = 2
      F          Terms in output = 2
            enter Bytes used      = 32

Time = 124.80 sec Generated terms = 2
      F          Terms in output = 2
            Bytes used            = 32

```

and we see that the fact that the file is compressed saves much time on the reading. Loading alone, the process of reading the index and compiling a complete list of 'stubbs' takes about 0.4 sec. which indicates that we have eliminated the whole problem of slow startup.

Let us now try to use this.

```

#-
.global
L F = LA(1,N+1,1,1,1,1,1,1,0,N,0,0,0,0,0,0,3)
      +LA(1,N+1,1,8,1,1,1,1,0,N,0,0,0,0,0,0,3);
Print +f +s;
.sort

Time = 0.03 sec  Generated terms = 2
      F          Terms in output = 2
            Bytes used            = 172

F=
+LA(1,1+N,1,1,1,1,1,1,0,N,0,0,0,0,0,0,3)

```

```

      +LA(1,1+N,1,8,1,1,1,1,0,N,0,0,0,0,0,0,3)
;
TableBase "three.tbl" open;
TableBase "three.tbl" load;
.sort

Time = 0.40 sec  Generated terms = 2
      F          Terms in output = 2
            Bytes used            = 172

id LA(n1?pos_,n2?!number_,<n3?pos_>,...,
      <n8?pos_>,k1?,k2?!number_,k3?,0,0,0,0,
      0,k9?) = LA22(n1,...,n8,k1,k2,k3,k9);
Print +f +s;
.sort

F=
+LA22(1,1+N,1,1,1,1,1,1,0,N,0,3)
+LA22(1,1+N,1,8,1,1,1,1,0,N,0,3)
;
*
* Shift to table notation and back
* Whatever is in the table will be intercepted
*
id LA22(n1?,...,n8?,k1?,k2?,k3?,k9?) =
      la22fill(n1,n2-k2,n3,...,n8,k1,k3,k9,k2);
id la22fill(n1?,...,n8?,k1?,k3?,k9?,k2?) =
      LA22(n1,n2+k2,n3,...,n8,k1,k2,k3,k9);
Print +f +s;
.sort

F=
+tbl_(la22fill,1,1,1,1,1,1,1,0,0,3,N)
+LA22(1,1+N,1,8,1,1,1,1,0,N,0,3)
;
TestUse la22fill;
Print +f +s;
.sort

F=
+tbl_(la22fill,1,1,1,1,1,1,1,0,0,3,N)
+LA22(1,1+N,1,8,1,1,1,1,0,N,0,3)
;
TableBase "three.tbl" use;
Print +f +s;
.sort

Time = 0.45 sec  Generated terms = 2
      F          Terms in output = 2
            Bytes used            = 152

F=
+tbl_(la22fill,1,1,1,1,1,1,1,0,0,3,N)
+LA22(1,1+N,1,8,1,1,1,1,0,N,0,3)
;
PolyFun acc;
Apply;
id Nval(N?)*R(n?,x?) = den(x+N)^n;
id Nval(N?)*R(n?,x?,?a) =
      den(x+N)^n*S(R(?a),x+N);
id S(R,x?) = 1;

```

```

id Nval(N?) = 1;
id z3?{z3,z4,z5,z6} = acc(z3);
id ep^n? = acc(ep^n);
Print +f +s;
B theta,delta;
.end

```

Time =	0.45 sec	Generated terms =	50
	F	Terms in output =	32
		Bytes used =	2550

```

F=
+theta(-2+N)*(
+den(-1+N)*acc(23/8+ep^-2+1/3*ep^-1+4/3*z3)
+den(-1+N)^2*acc(-7/2-ep^-1)
+den(-1+N)^3*acc(3)
+den(-1+N)^3*S(R(1),-1+N)*acc(2/3)
+den(-1+N)^2*S(R(1),-1+N)*acc(-31/18)
+den(-1+N)*S(R(1),-1+N)*acc(37/36+ep^-1)
+den(-1+N)*S(R(1,1),-1+N)*acc(1)
+den(-1+N)*S(R(2),-1+N)*acc(-29/18)
+den(-1+N)*S(R(3),-1+N)*acc(-2/3) )
+theta(-1+N)*(
+den(N)*acc(-17/18-ep^-2-1/3*ep^-1+16/3*z3)
+den(N)^2*acc(-73/18+ep^-1)
+den(N)^3*acc(11/3)
+den(N)^3*S(R(1),N)*acc(8/3)
+den(N)^2*S(R(1),N)*acc(-29/9)
+den(N)*S(R(1),N)*acc(23/9-ep^-1)
+den(N)*S(R(1,1),N)*acc(-1)
+den(N)*S(R(2),N)*acc(5/9)
+den(N)*S(R(3),N)*acc(-8/3) )
+theta(N)*(
+den(1+N)*acc(4*z3)
+den(1+N)^2*acc(8*z3)
+den(1+N)^4*S(R(1),1+N)*acc(4)
+den(1+N)^3*S(R(1),1+N)*acc(2/3)
+den(1+N)^2*S(R(2),1+N)*acc(-2/3)
+den(1+N)^2*S(R(3),1+N)*acc(-4)
+den(1+N)*S(R(1),1+N)*acc(-8*z3)
+den(1+N)*S(R(1,2),1+N)*acc(2/3)
+den(1+N)*S(R(1,3),1+N)*acc(4)
+den(1+N)*S(R(2,1),1+N)*acc(-2/3)
+den(1+N)*S(R(3,1),1+N)*acc(-4) )
+delta(-1+N)*(
+acc(1001/648+4/3*ep^-3-2/9*ep^-2
-13/108*ep^-1-1/3*z3)
)
+delta(N)*(
+acc(-1087/81-1/3*ep^-3-1/9*ep^-2
-97/54*ep^-1+20/3*z3)
)
+LA22(1,1+N,1,8,1,1,1,1,0,N,0,3)*acc(1);

```

4. Some extra remarks

The above features are released in version 3.1 of FORM at its regular address <http://www.nikhef.nl/~form>.

Of course there are still features that FORM does not have and would be much appreciated. One would be proper GCD and factorization algorithms. This would make it much easier to solve sets of equations. These are anticipated, but lack of manpower is the main problem.

It seems that the inherent speed of FORM comes from its internal data representation. The locality of its operations seems to be less important in this matter than was previously believed. This plays mainly a role when expressions are so big that they reside on disk. But even in that case a good use of the .sort instructions helps.

Currently a study is under way to see whether FORM can be made into an open source project. This would need a considerable amount of manpower, because the sources may have to be reprogrammed, several levels of documentation will have to be made and a number of additions will have to be made.

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