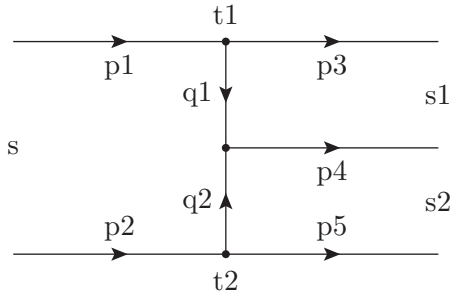
1 Multiperipheral diagrams

Here we are going to look at the single most complicated kinematics. This does not mean that with the routines we study here we can calculate all reactions trivially, because, when there are many Feynman diagrams in a reaction, there can be many different types of peaks and the number of potential propagators that can cause peaking grows like $\mathcal{O}(2^n)$, while the number of non-trivial integration variables is at most $3n-4$. In the case that there are many non-trivial peaks one needs to split the phase space into pieces that each have a limited number of peaks/poles and use for each a dedicated phase space configuration.

What we are going to look at here is the phase space configuration that by itself is the most difficult and lies at the core of the peaks that can mess up the Monte Carlo integration most. The basic diagram for which we are going to construct the kinematics is given by:



The history of the routines that are presented here is (to my knowledge):

- Solving phase space this way was suggested by Byers and Yang (1964).
- The book of Byckling and Kajantie gives a good description.
- A first version of the pickin/orient system was made in 1976.
- The eventual routines were made in 1982/1983 and the method is described in the paper (J.A.M.Vermaseren, Nucl. Phys. B229 (1983) 347-371). We will refer to the paper frequently.

As you can see, these are old routines but still very good.

What is really the problem?

Let us study the reaction $e^-e^+ \to e^-e^+\pi^0$. The π^0-particle is a pseudoscalar state and we will assume here for a moment that we do not have to worry about formfactors. We could also try a scalar state, which at the moment is more topical because of the Higgs particle, but the tricky point that we want to study shows itselve more directly with a pseudo scalar state. The coupling of such a pseudoscalar state is given in formula 3.1 of the paper. The essential part is the Levi-Civita tensor $\epsilon^{\mu q_1 \nu q_2}$. Working out the fermion traces we obtain formula 3.2 and the Levi-Civita tensor terms are effectively Gram determinants of the system. The most important part is that the first term combines with $1/(t_1t_2)^2$ which by itself would cause problems with physics because it would give a crosssection that grows with powers of s. Only a behaviour like $1/(t_1t_2)$ could give a decent crosssection. This means that when t_1 and t_2 become small,

the numerator must become very small as well. But the numerator, if we would contract the Levi-Civita tensors and write them in terms of dotproducts, has terms that are very big ($\mathcal{O}(s^2 M_\pi^4)$). This indicates that there must be very bad cancellations between the various terms. At the same time, the terms in the propagators prefer to be small and put us in this danger zone. This means that the kinematics need great care.

Let us start with looking at the formulas A.1 and A.2. A.1 is the basic formula and in principle one may derive the boundaries of the variables by the condition that the Gram determinant (amazingly equal to the numerator in our bad term above) be negative to allow the square root to be taken. This is not always the easiest way. Appendix A describes how to get around all practical problems here. The routine pickin follows this method closely.

The routine orient determines the laboratory variables once pickin has done its work in terms of invariants. These can be used for cuts and histograms. The derivation of the formulas for these variables is given in appendix B. Here we need the Levi-Civita tensors already. Take for instance $e^{p_1 p_2 p_3 p_4}$. Because there are only 5 external momenta there are only 4 independent momenta and hence only one nontrivial Levi-Civita tensor that is contracted with four momenta. When the lab frame is a colliding frame p_1 and p_2 have only energy and z components. If we let p_4 determine the xz plane we find that $e^{p_1 p_2 p_3 p_4} = 2\ E\ p\ p_4^x\ p_3^y$. This indicates that the transverse momenta p_4^x and p_3^y must be very small in the region where the crosssection is big. Let us check this with a computer program.

The program runpi, made with makepi, will run the total crosssection and produce a few histograms. Because the x-axes are logarithms of a variable, these variables run over a large rangle. For the first two histograms the $^{10}\log$ runs from -20 to $+10$ while for the three transverse momenta the logarithms run from -10 to $+5$.

The production of a muon pair in the center requires more care. One can in principle insert a two body decay for particle 4. The main problem is that there will be peaking in the CM frame, due to the incoming virtual photons. This kind of memory is not present in the mgoto2 routine. This means that in the gamgam routine we have to rotate the m4 system first to align the photons along the z-axis, then do the two body decay and then rotate and boost back. All along we have to worry about numerical accidents.

Section 3 describes how to deal with the matrix element for the muon production. In the case of the pseudoscalar, the Levi-Civita tensor writes the matrix element in a manifestly gauge invariant way and no further cancellations can occur. For a scalar one obtains similar Levi-Civita tensors but there are also other terms. Those terms all have at least two powers of combinations of t_1, t_2 and m_e^2, and do not contribute to the major numerical disasters.

The essence of the derivation of the matrix element for the muons is to try to obtain it also in a form in which the physical fields $F^{\mu\nu}$ are present to have a manifestly gauge invariant form. This can be done but it is not entirely trivial. For a fermion line with two photons (hence only two diagrams) the formula is obtained in the paper. I do not know of a similar result for three or more photons, although it has been possible to express the one loop amplitude for

the reaction $\gamma\gamma \to \gamma\gamma$ in terms of four $F^{\mu\nu}$ tensors.

Finally we can produce the program and run some distributions.

Homework (optional): We have now a program for the reaction $e^-e^+ \to \tau^-\tau^+ \to e^-\mu^+ + ...$ and a program for $e^-e^+ \to e^-e^+\mu^-\mu^+$. In the last reaction most particles cannot be seen because they stay in the beam pipe.

1. Design some more or less realistic detector acceptance and see what the total crosssection for each reaction is within this acceptance, assuming that of the second reaction you will only observe an e^- and a μ^+ (hence the other two particles are outside the acceptance). Can you tell the two reactions apart? Is the second a serious background for the first?

2. Scale up the energies to those of the ILC or the FCC. Assume a new fermionic particle that behaves like a lepton. Ignore the Z (unless you manage to make a more complete matrix element of course). Will there be any problems now?

Hint: it helps a lot when you make histograms of variables that might be important.

About matrix elements. If you have a large number of diagrams, it could be far more efficient to evaluate the amplitude (which is a complex number) and square it after evaluation. This has disadvantages (complex arithmetic, need to evaluate all different spin (or color) configurations) and advantages (linear in the number of diagrams, numerical problems are somewhat less). For a small number of diagrams the method we used is usually better. For large numbers and/or massless fermions the amplitude method is usually to be preferred. For the reaction $e^-e^+ \to e^-e^+\mu^-\mu^+$ in the low energy region (only QED) there are in total 12 diagrams. Already then the amplitude method is much better, provided the code is designed properly.