RasClic: A Long Base, Low Frequency, High Precision Seismograph

M.Sc. Thesis

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Abstract

Particle accelerators have been pushing the limits of modern technology over the past 80 years. The quest for higher and higher energies has created big challenges for accelerator technologies. One of these is accelerator alignment. RasClic, a three point, long base alignment system has proven it can meet the stringent alignment tolerances of the proposed next generation accelerator, the Compact Linear Collider. It consists of a light source, a diffraction plate and an image sensor. By monitoring the position of the a diffraction pattern on the image sensor a measure for the relative positions of the three components is obtained.

Development beyond these alignment tolerances could lead to applications in other fields of science. By following the movement of the Earth’s crust on a nanometer scale, RasClic can be used as a low frequency, high precision seismograph. The development of the system and review of it’s potential as a seismograph are the main objectives of this report.

Developments of a RasClic test setup have led to a measurement resolution of 20 nm at a light source - sensor distance of 140 m. Simulated images have shown that improvements to the shift estimator have increased the accuracy by 0.3 μm. The construction of a duplicate system has shown that measurements from two RasClic systems comply to within 1 μm of each other.

The static measurement technique implemented by RasClic makes it particularly suited to measuring low frequency seismic movements. The earth-hum and earth-tide are very low frequency seismic phenomena that are investigated closely in this report. Calculations on the RasClic response to these oscillations provide feasible design constraints for the construction of a RasClic seismograph. A RasClic device 7 km long will be sufficiently sensitive to both the earth-tide and the earth-hum. Finally the future prospects of a RasClic capable of detecting gravitational waves are also investigated. The Earth’s and the Moon’s response to these gravitational fluctuations form the basis of this analysis.
Acknowledgements

This project certainly wouldn’t have reached completion without the help and support of a number of people. Firstly I would like to thank my colleagues at NIKHEF, Rogier, Geert, Hidde and in particular Elmar for their endless stream of discussions and good ideas. The NIKHEF workshop staff also deserve thanks for their brilliant skills and patience in delivering the required goods, sometime under considerable time pressure. I would like to thank Tom Touzé for his technical expertise on location at CERN and the numerous visits made to the tunnel to help perform various experiments.

A big thanks goes to my supervisors for their supervision, but in particular to Harry van der Graaf for sharing his office with me, always being able to find time in his busy schedule and an undying enthusiasm for the project.

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Chapter 1

Introduction

Curiosity, the impulse to uncover the fundamental laws that make our universe what it is and come to understand the origin of our existence has been the driving force behind modern physics since the time of man. Each step closer to a final answer has unveiled a new set of complex problems, theorems and hypotheses which in turn need to be addressed.

Next year it is exactly a century ago that Ernest Rutherford carried out the famous gold foil experiment that would later be the basis for the development of Rutherford’s orbital theory for atoms. This theory was a founding step that would eventually bring nuclear physicists to a model containing all the elementary building blocks of our universe, the Standard Model. The Standard Model has over the past few decades been put to the test by experimental research, yet some major questions still remain unsolved.

To keep pushing the boundaries of experimental research more and more complex apparatus are needed. To delve into the world of the elementary particles continuously bigger instruments are needed that can reach higher and higher energies. The main instrument of choice for particle physicists is the particle accelerator. Over the years they have been accelerating and colliding particles with ever increasing energies. The pinnacle of particle physics to date is the completion of the Large Hadron Collider (LHC) at CERN (European Organization for Nuclear Research), this is a proton - proton accelerator that is expected to reach collision energies of 14 TeV. Looking beyond the LHC, to the next generation of particle accelerators physicists consider the use of linear accelerators, the reasons for which will be explained in the following section. These linear accelerators will still be massive instruments with the planned Compact Linear Collider being 44 km long. The specifications required to ensure optimal use of these colliders will form huge technological challenges that engineers will need to overcome.

One such challenge is the alignment of the accelerators. The size of the particle beams is proposed to be much smaller for linear colliders. This means that the magnets that accelerate and confine the particles have a much smaller margin of error, with the accurate alignment of one magnet to the next being crucial in minimizing transverse spread of the particle bundles. While the ultra-fine alignment of the magnetic field geometry during operation is done by the beam itself, a very precise pre-alignment is
needed for the initial operation and after shutdowns. For the CLIC the pre-alignment tolerances are 10 micron over 200 meters. This report describes developments of a new technique suitable to obtain this precision and go beyond the scope of a precision alignment system.

A system that can monitor movement at a nanometer scale, could provide opportunities in other fields of science. Instead of monitoring positions of the particle accelerators, a system closely following the movement of the Earth’s crust could also provide an innovative new apparatus for studying the subtle movements of the Earth. Breathing, twisting, rising and falling, the Earth is in constant movement. Undetectable to humans, with the exception of earthquakes these movements can tell us a lot about the subsurface of the Earth and its dynamics. Due to its static measurement principle, this technique could be excellent in detecting very slow oscillations, < 10 mHz. For this reason four examples of seismic movement have been chosen for further investigation; Earth-hum, the continuous background oscillations of the Earth; The earthquake excited versions of the same earth-hum vibrations; Earth-tide, the movement of the Earth due to the gravitational attraction of the Moon, Sun and other planets; and finally the Earth’s response to gravitation waves will be discussed as a future application for a nanoscale seismic device.

1.1 Physics needs a Linear Collider

1.1.1 State of particle physics

With the Large Hadron Collider (LHC) recently becoming operational the world waits in suspense for the first discoveries in the realm of TeV particle physics. Obviously it can be expected that much new physics will be unraveled in this frontier of physics. Much anticipation lies with finding the Higgs boson. This particle holds the answer to the fundamental question; what is the origin of a particle’s mass? The Higgs mechanism was first postulated in 1964 by Peter Higgs and is the last remaining piece of the Standard Model jigsaw puzzle yet to be detected. The Higgs field is thought to be a field filling the universe that interacts with particles in different ways, thus making certain particles seem heavy and others, like the photon, massless. The Higgs particle, or Higgs boson, is a disturbance in the field, analogous to the photon being a disturbance of an electro-magnetic field. The discovery of the Higgs particle will be the most exciting achievement in physics this decade and could lead to completely new discoveries in particle physics. Physicists also believe it could shed more light on theories beyond the Standard Model, such as Supersymmetry and Quantum gravity.

The LHC however has its limitations. A proton-proton ($p^+$) collision like that in the LHC are much more complex compared to lepton-lepton collisions proposed by linear collider collaborations. This is due to the fact that a proton consists of two up (u) quarks and a down (d) quark that are held together by gluons. A $p^+ - p^+$ collision should be regarded as a quark-quark collision, the total energy being distributed over the 6 quarks and resulting in a large spread of energies between collisions. Leptons (such as electrons and positrons) on the other hand are elementary particles. During
1.1. PHYSICS NEEDS A LINEAR COLLIDER

An electron($e^-$) - positron($e^+$) collision all the center-of-mass energy is available for the formation of new particles and consistent between collisions thus improving the energy resolution of the experiments. A lepton-lepton collider is therefore an essential tool in verifying and testing discoveries done by proton-proton collision experiments. An example is the detection of the W and Z bosons by the proton - antiproton experiments of the SPS at CERN and the subsequent precision measurements and testing of the properties of the electro-weak force in the Large Electron Positron Collider (LEP) $e^- - e^+$ experiments[1]. The hunt is now on for a particle accelerator that will best complement the LHC.

![Figure 1.1](image1.png)

**Figure 1.1:** Proton - proton collisions like those done in the LHC, each proton is made up of two up quarks and a down quark

![Figure 1.2](image2.png)

**Figure 1.2:** Positron - electron collisions like those done in the LEP and proposed for the CLIC

To be totally complementary to the LHC experiments a collider with center-of-mass energies of up to 3 TeV will be required. Laws of physics require the accelerator to be linear (or circular with an exceptionally large diameter). This is due to the fact that a constant energy supply is required to keep a particle traveling through a circular trajectory. This loss of energy is known as synchrotron radiation and is inversely proportional to the mass of the particle, according to,

$$\Delta E = \frac{4\pi e^2 p^4}{3m^4\beta \rho}$$  \hspace{1cm} (1.1)$$

where, $\Delta E$ is the energy lost per revolution, $e$ the charge, $p$ the momentum and $m$ the mass of the particle. $\beta$ is the ratio between the particles speed and the speed of light ($\approx 1$ for high energy particles) and $\rho$ is the radius of the circular trajectory. Because the mass of a proton is roughly 2000 times larger than that of an electron, an electron would loose 2000^4 times more energy than a proton on the same circular path, at the same momentum. Using a linear accelerator circumvents this problem but puts restraints
on the accelerator technology. Being able to accelerate an electron or positron to the required energy depends on the accelerating gradient and the length of the accelerator. One such proposed linear accelerators is the Compact Linear Collider (CLIC) being developed at CERN.

1.1.2 CLIC Alignment

The hunt for new particles such as the Higgs boson is governed by statistical properties that require many events to be of any physical relevance (the LHC experiment ATLAS is expecting to see just 1 Higgs event per day out of the $10^7$ daily events). The number of events per second is determined by the luminosity, $L$, and cross-section, $\sigma$. So that the collision rate is given by,

$$ N = L\sigma $$

Luminosity is therefore an important factor for colliders. Increasing the luminosity involves squeezing the particle bundle into an as small a beam area as possible. This however places enormous pressure on the accelerator alignment. It is particularly important to limit transverse beam offsets in the accelerating structures because these offsets induce disruptive wakefields which adversely affect the transverse emittance of the beam. Before moving on to explain CLIC alignment a quick introduction of the CLIC setup is required.

The Compact Linear Collider is one of the proposed next generation linear colliders, another is the International Linear Collider (ILC) [2]. CLIC is unique in that it makes use of an innovative parallel drive beam system. A drive beam with a low energy, high intensity electron stream is used to obtain the high frequency (RF) power required to accelerate the high energy, low intensity particles in the main beam. The devices used to generate the high frequency power by decelerating the electrons in the drive beam are known as Power Extraction Structures (PETS). A basic overall set-up of CLIC is shown in figure 1.3. To simplify the construction and alignment of CLIC it is divided into modules that are pre-aligned during construction onto girders. Each module consists of 4 accelerating structures (CAS) and a beam position monitor (BPM). In the same

![Figure 1.3: Proposed CLIC setup showing drive beam with PETS and main beam](image)

Figure 1.3: Proposed CLIC setup showing drive beam with PETS and main beam
way the drive beam structures will be mounted on a separate girder. A CLIC module showing the configuration of the different components is shown in 1.4.

![Figure 1.4: A proposed CLIC module](image)

Alignment of CLIC involves two separate alignment systems; the pre-alignment and the beam based alignment systems. The pre-alignment system has no monitoring purpose, its readings are used to determine the positions of the accelerator modules relative to a straight line. Then according to these readings the active pre-positioning can be done using a network of actuators. Once the pre-alignment has been completed a pilot beam is sent through the accelerator allowing the beam based alignment system to take over. The pre-alignment tolerance for CLIC is a running tolerance over a length of 200 m. Within this distance the deviation from a straight line must not exceed an rms value of 10 μm. The pre-alignment network will consist of short range and long range aligning components. The short range alignment will be responsible for the alignment of adjacent girders over a distance of approximately 5 m. The long range alignment will observe the overall straightness of the accelerators, with a range of over 100 m [3]. Various techniques are being investigated for use in the long-range system, these include the wire positioning system (WPS) and the hydrostatic leveling system (HLS) [3] [4]. The short range alignment will be carried out by the Red Alignment System NIKHEF (RasNIK). The remainder of this chapter will discuss the working of this short-range alignment system, and how it lead to the development of a long range variant which is the subject of this report.

### 1.2 Optical Alignment

#### 1.2.1 From RasNIK to RasClic

RasNIK is a three point alignment system [5]. It consists of a light source (typically a light emitting diode (LED)) that illuminates a coded mask. The resulting image is projected through a lens onto a CCD image sensor. In figure 1.5 the general setup is
CHAPTER 1. INTRODUCTION

shown schematically and part of the coded mask as seen by the image sensor is shown. If any of the three components moves relative the the others the image will become displaced, scaled or rotated. The images are sent to a computer that registers these movements. These values are fed back into the analysis programs of the experiments. RasNIK was originally designed for the alignment of muon drift chambers that NKHEF contributed to the L3 experiments, operational at CERN from 1989 to 2000. More recently the system has been developed with higher precision for use in the LHC ATLAS detector. Here 5317 RasNIK systems are in place as proof of the large applications of this cheap and effective system.

The RasNIK system typically has a resolution of around 50 nm and a maximum alignment distance of about 15 m. It is the most favored alignment system for the short range alignment of CLIC. Scaling this system up to the length required for long range alignment has practical drawbacks. The main issues are image distortions due to turbulent air movement and the diffraction-limited resolution of the image of the mask. To achieve an image-lens distance of 50 m, the Rayleigh criterion requires a lens diameter of at least 0.4 m. This makes RasNIK an expensive and impractical system for long range alignment.

A solution is to take the unavoidable diffraction at the lens aperture as a source of position information in its own right. The LED and mask are replaced by a coherent point light source, realised by HeNe-laser light. Without a mask pattern to be imaged, the lens can be left out as well. The circular aperture is replaced by a diffraction plate with a more elaborate pattern of open and closed surfaces, optimized for a diffraction pattern with high position information over the full sensor surface. This is the idea that lead to RasClic(Red alignment system for CLIC). RasClic has been under development for 2 years now and proven itself a viable candidate for the long range CLIC alignment [4]. Going beyond alignment tolerances RasClic has reached resolutions of 20 nm and has potential to improve even further. With such accuracy, new applications for RasClic can be considered. Developing RasClic for use as a seismograph will be the next challenge and the subject of the following chapter.
Chapter 2

RasClic as a seismograph

2.1 Introduction

Seismology is a relatively young science. Although the Earth’s heaving and shuddering has been a subject of many ancient myths and legends, modern seismology dates back only about 100 years to the development of the first instruments capable of recording seismic movements [6]. Investigations into the cause, effect and prediction of earthquakes dominated initial research. This led to important improvements in quake-proof building design. More recently seismology has found applications in the search for oil and gas as well as the monitoring of underground nuclear explosions to verify compliance with test ban treaties.

Much of what is known about the Earth’s interior composition and mineralogy was learnt from analysis of the Earth’s slow movements, and has led to a better understanding of its dynamics. Low frequency data is the only seismic data sensitive to the density of the Earth, and the attenuation is only weakly frequency dependent [7].

Conventional seismographs measure the inertial movement of the Earth’s surface, i.e. the acceleration. A simple model of a seismic wave, equation 2.1, shows that the amplitude of the acceleration scales quadratically with the frequency. For low frequencies ($\omega \ll 1$) the acceleration becomes very small.

\[
\begin{align*}
  s &= A \sin(\omega t) \\
  \frac{d^2s}{dt^2} &= -\omega^2 A \sin(\omega t)
\end{align*}
\]

RasClic avoids this issue by measuring the actual displacement of the seismic waves instead of the acceleration.

The goal of this chapter is to explore the various seismic phenomena for which a RasClic system could provide new insights. These are dealt with one by one in the following sections.
2.2 Seismic Phenomena

2.2.1 Earth-hum

The earth-hum is a connotation for the fundamental resonant oscillations of the Earth. The source of these oscillations has been the cause of much dispute in the geophysical world since their discovery in 1960 [8]. Certain caution should be taken when referring to earth-hum. This report proposes to discuss the earth-hum in two separate cases. One is that of the earth modes being excited by earthquakes, and the other is what most people consider as the Earth’s background oscillations, referring to the continuous oscillation of the Earth’s vibrational modes. The former of the two is much easier to verify experimentally because the signal to noise ratio is more favorable. The excited modes after an earthquake do however damp out relatively quickly and although the lower frequency modes may persist for a number of weeks the higher frequency modes quickly fade away into the background oscillations. What many geophysicists consider the true earth "hum" are the incessant oscillations of the Earth’s vibrational modes and that, as discussed shortly, cannot be attributed to continuous excitation by earthquakes. This form of earth-hum is much more difficult to locate experimentally due to the much lower signal to noise ratio. In 1998 Suda, Nawa [9], for the first time produced experimental evidence of incessant excitation of the Earth’s free oscillations. Originally the source of the earth-hum was solely attributed to earthquakes. However, although humans cannot feel these low frequency movements, the summed amplitude is equivalent to a continuous earthquake of magnitude 6. An earthquake of this magnitude occurs on average only once every 3 days so it is impossible that earthquakes are the sole cause. Other candidate theories involve changing atmospheric pressures and oceanic waves colliding with continental shelves. The basis of these theories is the 12 month periodicity of the earth-hum that is suggested to be related to the respective northern and southern hemisphere winter [10] [11].

Because the Earth is roughly spherical, the geographical patterns of its vibrational modes can be expressed in terms of the spherical harmonics, $Y_{lm}(\sigma, \phi)$, where $l$ is the angular degree, $m$ is the azimuthal order, $\theta$ and $\phi$ give the position on the Earth’s surface in latitude and longitude respectively. The Earth has been modeled in the Preliminary Reference Model (PREM) that provides theoretical values for the expected frequencies of the modes [12]. In these models the oscillations follow the spheroidal ($S$) or toroidal ($T$) vibrational patterns and have the notation $nS_{lm}$ and $nT_{lm}$, where $m = -l, ..., l$, and $n$ is the radial overtone number. $l$ is directly related to the number of wavelengths of the oscillation around the circumference of the globe and is therefore quantized. $m$ represents the degeneracy of each mode and is often left out of the notation. The $1S_2$ and $0S_3$ vibrational modes are illustrated in figure 2.2a and b.

The spheroidal patterns are the most predominate, these involve movement of the Earth’s crust perpendicular to its surface. $0S_0$ represents the expanding a contracting of the Earth’s radius and is therefore referred to as the breathing mode. $0S_2$ has two wavelengths around that globe and has the connotation, “football” mode. The more recently introduced toroidal modes involve different parts of the Earth’s surface twisting
2.2. SEISMIC PHENOMENA

Figure 2.1: (a) The $1S_2$ earth-hum mode and (b) the $0S_3$ mode. In both cases the radial over tone, $n$, represents the axial displacement of the Earth’s movement. (c) the toroidal mode $0T_2$ and (d) $0T_3$.

around in the horizontal plane [13]. This is shown schematically in figure 2.2c and d.

In a model of the Earth all $2l+1$ vibrations of $S$ and $T$ modes have the same frequency, however on the real Earth departures from the reference model cause the modes to couple and hybridize. At very low frequencies ($< 1$ mHz) the modes even suffer from fine-scale splitting caused mainly by the Earth’s rotation, similar to Zeeman splitting of electrons in an external magnetic field [14].

Earthquake excitation

A significant earthquake excites the free oscillations far above the usual background level, at frequencies $< 1$ mHz, they remain observable for many weeks on global seismic networks. A recent example is that of the monumental 24 December 2004 Sumatra-Andaman earthquake [15]. During this earthquake peak ground motion exceeded 1 cm at all locations on Earth’s solid surface. Observing the free oscillations after an earthquake can offer information on the position and magnitude of the shock. For example the $m = \pm 1$ vibrations of the $0S_2$ mode would be zero for an earthquake on the equator. A comparison of the relative amplitudes of the $m = \pm 1$ and $m = -2, 0, 2$
vibrations of $0S_2$ indicates how far north the Sumatra earthquake took place. Data from the Canberra station of the Geoscope network after the Sumatra-Andaman earthquake is shown in figure 2.2.

Figure 2.2: Schematic of the motion of the $0S_2$, $0T_3$, $0S_3$ and $0S_0$ modes, superimposed on a spectrum computed from 240 hours of vertical seismic motion recorded at the Canberra station of the Geoscope Network.

Incessant excitation

But even if the Earth hasn’t been excited by a recent earthquake there still exist continuous seemingly coherent oscillations of the Earth’s vibrational modes. As mentioned above these incessant excitations are much harder to detect due to the lower signal to noise ratio compared to their earthquake excited cousins. To combat this measurements concerning these oscillations are done over a much longer period of time and make use of stacking. In general the following steps are taken.

- Most modern seismographs use inertial based techniques that output the ground motion in terms of an acceleration. In seismology it is common place to use the units, nanogalileo (nGal, $1 \text{nGal} = 10^{-11}\text{ms}^{-2}$).
- To isolate the true earth-hum from activity due to earthquakes only data is used from seismically “quite” days. Days in which the seismic noise or instrument noise exceed a certain level are discarded.
- The data is then divided up into segments and fourier transformed to obtain an amplitude spectrum. The power spectral density (PSD) can then be calculated.
2.2. SEISMIC PHENOMENA

- By summing over a large number of segments the signal-to-noise ratio can be improved. If \( N \) segments of data are stacked then the signal-to-noise ratio improves by a factor \( \sqrt{N} \), this has the same result as measuring for a long period of time however this makes it more difficult to extract noisy data segments.

In an example of this technique being used by Nishida, Kobayashi and Fukao [10] 1 day segments of data are stacked over 3 months from 25 different seismometers. Nine years of data was analyzed in this way to produce a single spectrogram, a contour plot of the PSD on the frequency-time domain. This is shown in figure 2.3.

![Spectrogram produced by Nishida, Kobayashi and Fukao (2000) from seismically quite days from 1989 to 1998, the \( 0S_{23} \) and \( 0S_{37} \) spheroidal modes are indicated by tick marks on the upper edge. \( 0S_{29} \) and \( 0S_{37} \) seem more predominate than the other modes, possibly due to atmospheric coupling. There is also an apparent annual variation in the amplitude of the hum.](image)

The apparent vertical lines indicate that particular frequencies are always and more strongly present than others. These lines coincide remarkably well with the theoretical spherical harmonic modes mentioned earlier and indicated by tick marks along the upper edge of the spectrogram. These lines are also remarkably equidistant, each one caused by the a higher harmonic of the \( 0S \) mode, i.e. another wavelength around the circumference of the Earth. There is also an apparent annual variation in the amplitude of the earth-hum.
From this and similar experiments the Earth’s background free oscillations have been firmly established. The amplitude of the oscillations has been determined to be in the order of 0.5 nGal for periods less than 240 min and monotonically decreasing for higher periods, down to around 0.02 nGal for the $0S_2$ mode. This corresponds well with theoretical predictions used by Ben-Menahem [16]. Relating this back to an actual ground displacement gives values between 4 and 50 nm depending on the frequency of the oscillations.

In chapter 5 investigations will continue to determine the feasibility of developing a RasClic system sensitive enough to hear the Earth hum.

2.2.2 Earth-tide

The universal $1/r$ shape of gravitational potentials results in a gradient of gravitational forces across extended objects that are freely falling in the potentials of other objects. The gradients of the Moon’s and the Sun’s potentials are the origin of tides on Earth’s oceans, which is why the forces in inhomogeneous gravitational fields are called ‘tidal forces’ in general.

The tidal forces of the Moon and the Sun do not only affect the oceans, but also the body of the Earth itself, resulting in periodic movements of the Earth’s crust. This movement can have amplitudes of up to 30 cm and can therefore have dramatic effects on particle accelerators that need to be aligned to micrometer precision. At CERN [17] and JAERI [18] corrections are applied to account for the earth-tide. A detailed account of the basics of theory and observations of earth-tides can be found in [19]. The long period of the oscillation gives a static measurement of displacement, like that in RasClic, an advantage over measuring the related acceleration. Other static measurement techniques include space borne interferometric synthetic aperture radar (InSAR), or measurements of the variation of the local $g$ and of the local vertical, rather measuring the surface deformation itself.

In lowest order, tidal forces acting on a freely falling body can be approximated as quadratic distortions of its own gravitational field. Therefore changes of the equipotential surfaces are given by second order spherical harmonics; this leads to constituents of the earth-tide with periods around 12 hours. These are known as the semi-diurnal tides and are due to the rotation of the Earth.

Further, the inclination of the Earth’s rotation axis with respect to the ecliptic plane leads to constituents with roughly 24 hour periods. These are known as the diurnal tides, most pronounced at high latitudes. The change of inclination of Sun and Moon over the year and the month, respectively, lead to modulations of both diurnal and semi-diurnal tides; these can be expressed as sidebands in the tidal frequency spectrum. Another important source of extra frequencies is the ellipticity of the lunar orbit.

The spatial distribution of tidal amplitudes can be expressed in spherical harmonics oriented along the Earth’s rotation axis (see figure 2.4): The sectorial function, with two nodal planes along meridians, applying to semi-diurnal tidal components; the tessal function, with one meridian nodal plane, applying to diurnal components; and the zonal function without meridian dependence describing some slow tidal contributions.
2.2. SEISMIC PHENOMENA

Figure 2.4: Geographical distribution of tidal potential. (A) Sectorial function, (B) Tesseral function, (C) Zonal function [19].

Semidiurnal and diurnal modes with corresponding theoretically derived periods and amplitudes are given in table 2.1 and 2.2, adapted from [19].

<table>
<thead>
<tr>
<th>Tidal constituent</th>
<th>Period(hr)</th>
<th>Vertical amp.(mm)</th>
<th>Horizontal amp.(mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_2$</td>
<td>12.421</td>
<td>384.83</td>
<td>53.84</td>
</tr>
<tr>
<td>$S_2$</td>
<td>12.000</td>
<td>179.05</td>
<td>25.05</td>
</tr>
<tr>
<td>$N_2$</td>
<td>12.658</td>
<td>73.69</td>
<td>10.31</td>
</tr>
<tr>
<td>$K_2$</td>
<td>11.967</td>
<td>48.72</td>
<td>6.82</td>
</tr>
</tbody>
</table>

Table 2.1: Theoretical amplitudes of the semi-diurnal earth-tide modes.
CHAPTER 2. RASCLIC AS A SEISMOGRAPH

<table>
<thead>
<tr>
<th>Tidal constituent</th>
<th>Period(hr)</th>
<th>Vertical amp.(mm)</th>
<th>Horizontal amp.(mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_1$</td>
<td>23.934</td>
<td>191.78</td>
<td>32.01</td>
</tr>
<tr>
<td>$O_1$</td>
<td>25.819</td>
<td>158.11</td>
<td>22.05</td>
</tr>
<tr>
<td>$P_1$</td>
<td>24.066</td>
<td>70.88</td>
<td>10.36</td>
</tr>
<tr>
<td>$\Phi_1$</td>
<td>23.804</td>
<td>3.44</td>
<td>0.43</td>
</tr>
<tr>
<td>$\Psi_1$</td>
<td>23.869</td>
<td>2.72</td>
<td>0.21</td>
</tr>
<tr>
<td>$S_1$</td>
<td>24.000</td>
<td>1.65</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Table 2.2: Theoretical amplitudes of the diurnal earth-tide modes.

Amplitudes are given for the latitude where they have their maximum. Figure 2.5 shows the dependence on latitude of the vertical ground motion for the most important tidal contributions; horizontal motions have different dependencies, so a grid of measurement points at low, mid, and high latitudes, measuring displacement in three directions can help reveal the relative contributions of the tidal modes.

![Figure 2.5](image.png)

Figure 2.5: Amplitude variation of the principal waves as a function of latitude for the vertical component of tidal force; $K_1$, $O_1$, $P_1$: tesseral diurnal waves; $M_2$, $S_2$, $N_2$: sectorial semi-diurnal waves. Unit = $10^{-8}\text{ms}^{-2}$ [19]

Love numbers are a measure of how much a planet’s surface and interior move in response to a gravitational pull. Measurements of these numbers on Earth support deviation of real earth-tides from the deformation of equipotential surfaces by the tidal forces. The real amplitudes are only about 60% of the equilibrium value, and differ by mode as well as by location. Further, the Earth rotation causes a phase lag between the theoretical and the real tides of a few degrees. These effects make it possible for the oceans to exhibit tides with respect to the Earth surface at all.

The presence of ocean tides acts back on the deformation of the Earth’s crust, especially
in coastal areas; this is known as tidal loading. At high ocean tide there is an excess (or at low tide a deficit) of water and the adjacent ground falls or rises in response to the resulting differences in weight. As coastlines inhibit the free propagation of ocean tides, they are quite out of step with the earth-tide; resonances of local oceanic basins with diurnal or semi-diurnal oscillation periods can enhance the respective tidal components by more than an order of magnitude. Tidal loading modifies the earth-tides accordingly and can become a significant contribution locally. In an analogous way, atmospheric loading occurs as a result of tidal motion of the air, with a much smaller amplitude due to the smaller masses involved.

Being driven by celestial motions, earth-tides have an essentially infinite coherence time; in contrast to seismic motions of all kinds, which are subject to attenuation by internal friction. Therefore, by increasing recording time any small signal will ultimately come out of an instrumental background noise. Long measurement times are needed anyway for spectral separation of tidal components within the group of diurnal and semi-diurnal modes; e.g., the time required to separate the important diurnal modes $K_1$ and $P_1$ is $T >> \left(\frac{1}{24.066h} - \frac{1}{23.934h}\right)^{-1} = 0.5 \text{ yr}$. Therefore, a measurement time of at least one year can be assumed for the requirement of detecting all the amplitudes as given in tables 2.2. For spectral separation of the main semi-diurnal modes, $M_2$, $S_2$ and $N_2$, 2 months of measuring would suffice.

2.2.3 Gravitational waves

Einstein’s general theory of relativity defines gravity in terms of a space and time geometry. Gravitational waves are transient fluctuations that are supposed to stretch and compress that geometry. Sources of gravitational waves are large masses that are subjected to large accelerations, for example, two black holes orbiting around each other or exploding or colliding stars. Gravitational waves have alluded mankind despite several attempts to detect them. Laser interferometers like that used in the Virgo project [20] attempt to detect gravitational waves but are yet to provide conclusive evidence. Evidence for generation of gravitational waves supporting theoretical hypothesis was obtained by Hulse and Taylor [21]. They studied 2 neutron stars orbiting each other, known as a pulsar. Over many years the spinning speeded up, evidence that the system was losing energy. The rate at which energy was lost matched very precisely the power with which relativity predicted would be emitted by gravitational waves.

In 1983 Ben-Menahem [16] published an article in which the Earth’s response to gravitational waves is derived. It is shown that gravitational waves from astrophysical sources such as pulses from supernova explosions can excite the $0S_2$ mode to an acceleration level of $10^{-3} \text{ nGal}$. Monochromatic wave sources from a binary orbiting system excite the same mode to an acceleration level of $10^{-5} \text{ nGal}$. All other modes are smaller in amplitude and not likely to be observed. Building a RasClic capable of measuring these tiny changes in the vibrational modes will be a huge technical challenge. Analysis of the signal response will need to distinguish between excitation by gravitational waves and earthquakes, for example a network of RasClic’s over the globe will simultaneously
be excited by a gravitational wave, whereas excitation by an earthquake will notably originate from a certain point on the Earth’s surface, showing up in the higher order modes as well.

Since the discovery of the free oscillations of the Earth in 1960, the possibility of similar effects on the Moon have been considered. Bolt [22] used computational models of the Moon to determine the periods of the Moons free oscillations. For the $0S_2$ mode values between 12.5 and 15.1 mins (1.1 - 1.3 mHz) were predicted. These vibrations will most likely also be effected by gravitational waves in a similar way to their terrestrial counterparts. As it will be shown later the sensitivity of a RasClic system is inversely proportional to the square of the Earth radius. The smaller radius of the Moon will therefore greatly improve the sensitivity. Using the Moon as the worlds biggest gravitational antenna would also have other benefits. There may not be a need for expensive and space consuming vacuum tubes as the atmosphere on the Moon is ail enough. The Moon is seismically a lot more quiet then the Earth, reducing the noise signal. Noise due to atmospheric and man made disturbances could also be neglected. A network of lunar RasClics could also provide conclusive evidence as to the composition, mineralogy and origin of the Moon.
Chapter 3

The RasClic System

Now that the origin of and the motivation behind the RasClic system have been introduced, this chapter will describe, in more detail, the RasClic system itself and will explore various aspects of RasClic data analysis.

3.1 Overview

RasClic is a long base 3 point alignment system. It consists of a laser, a diffraction plate and an image sensor. The basic principle of the system is as follows; the laser illuminates the diffraction plate. The resulting diffraction pattern is projected onto the sensor where it is analyzed to determine its position relative to a reference image. The position is measured in terms of the horizontal and vertical displacement perpendicular to the beam, in this case \( x \) indicates horizontal, and \( y \) vertical displacement. This position gives an indication of the central point with respect to the two outer points, this is characterized by the sagitta, \( s \), which is the distance between the middle point and a straight line between the two outer points (figure 3.1). If \( d_1 \) and \( d_2 \) are the displacements of the two outer points relative to a reference line, then the displacement \( d \), measured by the image sensor is given by,

\[
d = -d_1 + 2s - d_2
\]

RasClic can be extended to a N-point alignment system by creating a network if overlapping RasClics. This was shown extensively by Kea(2007)[4].

An experimental test setup of RasClic has been built in an old proton transport tunnel at CERN. This device is 140 meters long. To eliminate distortions of the light path due to thermal and density gradients and turbulence the entire trajectory of the light is confined within a vacuum tube. This vacuum is kept in the order of \( 10^{-4} \) mbar by three vacuum pumps. The diffraction plate is situated half way between the laser and sensor systems, and has the form of a circular cut-out. A schematic of the setup is shown in figure 3.2.
CHAPTER 3. THE RASCLIC SYSTEM

3.1.1 Laser system

The coherent light is produced by a He-Ne 20mW laser. In the first version of RasClic, the light was directed straight from the laser into the vacuum tube. From analysis of the resulting data, it was found that the direction of the light was effected by the laser’s own thermal stabilization feedback system, which adversely affected the accuracy of the data. To overcome this, the light is now first coupled into a single mode optical fiber with a length of approximately 2 meters. The light is then coupled into the vacuum tube via a fiber out-coupler that is mounted on a meteorological plate (figure 3.3). The meteorological plate is situated on top of a cast iron pillar. The laser light propagates into the main vacuum tube via flexible bellows to ensure that the out-coupler is independent of movements by the vacuum tube itself. The laser and optical fiber coupler are shown in figure 3.4.

3.1.2 Diffraction plate

The diffraction plate is responsible for the formation of a suitable diffraction pattern on the image sensor. The basis of all RasClic diffraction patterns is a circular slit,
3.1. OVERVIEW

Figure 3.3: Technical drawing of the optical-fiber out-coupler. A small flexible vacuum bellows connect the out-coupler to the main vacuum tube.

Figure 3.4: Laser and optical-fiber coupler. The optical fiber on its way to the vacuum tube can be seen in the background.
the inner circle being held in place by support structures (figure 3.6). The design of the diffraction plate is done to optimize the information available for image processing. Practical design elements need to be kept in mind as well. A large slit diameter will produce more diffraction maxima and minima, i.e. more radial information, but a too large a diameter will increase the chance of unwanted reflections off the inside of the vacuum tube. Because 2D Fourier transformations are used to determine the image displacement (see section 3.1.3), as much information gradient along the \( x \) and \( y \) axis as possible is desirable; a chess board pattern being a good example of such a pattern. Work done by van der Geer(2008) [23] showed that the position and width of the inner circle support structures could be designed in such a way to increase the angular information density of the diffraction pattern. This was done by using software developed for the simulation of RasClic diffraction patterns. One such diffraction plate with corresponding simulated and actual diffraction pattern is shown in figure 3.5, as a demonstration of the effectiveness of the software. The diffraction plates are fabricated out of 1.5 mm thick aluminum sheets. They are then anodized to produce a mat black surface that reduces the risk of reflected light corrupting the images. An earlier version of a diffraction plate and resulting diffraction pattern is shown in figure 3.6.

![Figure 3.5](image)

**Figure 3.5:** (a) An example of a diffraction plate with (b) the simulated diffraction pattern and (c) the actual diffraction image.

### 3.1.3 Image system

The sensor consists of a black and white CCD pixel sensor with 1000 × 1000 pixels (see appendix A for further sensor specifications). The sensor is read out with a 2 × 2 binning process that produces a 500 × 498 pixel image. A quartz-stabilized function generator is used to trigger the image capture. The images are then analyzed by specially developed software run on a linux machine. The algorithms in this software make use of a FFT shift estimator (described below) to determine the image position. An overview of the image processing software can be found in appendix B. The resulting \( x \) and \( y \) displacement values are then stored in data files.
FFT shift estimator

A Fast Fourier Transform (FFT) shift estimator is used to determine the position of the diffraction pattern relative to a reference image. The reference image is, in this case, usually the first image in a series of measurements but it can also be any previously made image. The FFT shift estimator makes use of the fact that a shift in the space domain results in a phase shift in the fourier domain.

\[ f(x - s_x, y - s_y) \Rightarrow F(k_x, k_y)e^{i(k_x s_x + k_y s_y)} \] (3.2)

Here \( s_x \) and \( s_y \) are the displacements that need to be determined and \( k_x \) and \( k_y \) are the spatial frequencies of the image. The Fourier transform of a shifted image, \( \tilde{D} \), can therefore be expressed as a phase shifted version of the Fourier transform of the reference image, \( \tilde{R} \),

\[ \tilde{D}(k_x, k_y) = \tilde{R}(k_x, k_y)e^{i(k_x s_x + k_y s_y)} \] (3.3)

By dividing the FFT of the shifted image by that of the reference image and taking the argument of the result one finds a linear relationship between the displacements and the spatial frequencies.

\[ \arg\left(\frac{\tilde{D}}{\tilde{R}}\right) = k_x s_x + k_y s_y \] (3.4)

this is equivalent to a 2D plane in the Fourier domain where \( s_x \) and \( s_y \) are the respective slopes in the \( k_x \) and \( k_y \) directions. By calculating the slopes of the phase plane fit, one arrives at the displacement of the shifted image. It should be noted that the above derivation is only precise for infinitely big images. When dealing with finite images the periodicity of the Fourier transform needs to be taken into account as well. The reason for this and how it is done for RasClic is explained further in section 4.1.2.
To explain how RasClic makes use of 2D image FFT techniques an example is used in the form of a concentric ring diffraction pattern. The image is $X \times X$ large and sampled by $N \times N$ pixels. For this example, $X = 15$ mm and $N = 500$, see figure 3.7a. Taking the 2D Discrete Fourier Transform (DFT) of this image results in a $N \times N$ DFT image. Each point is a function of the spatial frequency components, $k_x \in [0, 1/X, ..., (N - 1)/X]$ and $k_y \in [0, 1/X, ..., (N - 1)/X]$ (the conventional $2\pi$ factor is left out for simplicity) and defined by a complex number with a magnitude and phase. In fact the highest frequency component is $N/2X$, the rest of the components represent negative frequencies that are the complex conjugates of their positive counterparts. For the image shown in figure 3.7a the 2D DFT is illustrated in figure 3.7b. The oscillations in the example image have a wavelength of $\sim 0.4$ mm so the most predominate frequency component will be at $k_x, k_y = 1/0.4 = 2.5$ mm$^{-1}$. The highest frequency component is $\frac{500}{2\pi} = 16.67$ mm$^{-1}$. Each quadrant of this DFT image can be arranged so that the null frequency component is centered in the image. Because there is very little high spatial frequency information in the image only a small segment of each quadrant from 3.7b is used to construct the DFT-bin, figure 3.7c. This essentially implements a low pass filter and is used to filter high frequency signals in RasClic images. This also greatly reduces the amount of information stored per image and improves the computational efficiency.

Figure 3.7: (a) An example of a diffraction pattern with (b) the 2D DFT and (c) the 2D DFT-bin.

### 3.1.4 Vacuum tube

The vacuum pipe is a RVS tube with a internal diameter of 155 mm. It is made up of pipes some 6 m long joined by flexible bellows. The air inside the vacuum tube is evacuated by three vacuum pumps consisting each of a turbo-molecular pump backed by a rotary pump. These reach a pressure of down to $10^{-4}$ mbar.
3.2 Noise Analysis

The following noise analysis was done on data from a continuous measurement over 12 days. The data is plotted in figure 3.8.

![Figure 3.8: 12 continuous days of RasClic measurements.](image)

The data exhibits periodicities of roughly 24 hours. This is most likely a result of temperature effects due to day and night changes. Sudden glitches in data values are most likely due to human presence, a certain drift in the data has suspected origins that will be discussed later. To start the noise analysis a discrete fourier transform (DFT) of the data is done. The signal is a finite time discretely sampled signal, $x(k)$. The DFT is then given by,

$$X(\omega) = \sum_{k=0}^{N-1} u(n)e^{-i\frac{2\pi}{N}nk}$$

(3.5)

The Power Spectral Density (PSD), $\Phi_u$ is a commonly used way of expressing frequency dependence. It is calculated from the Fourier transform by,

$$\Phi_x(\omega) = \frac{1}{N}|X(\omega)|^2$$

(3.6)

The PSD of the above data is plotted in figure 3.9.

The following properties are evident from the noise spectrum,
• White noise at high frequencies, > 5 Hz, comparable in $x$ and $y$.

• $1/f$ noise at lower frequencies, < 0.5 Hz, also comparable in $x$ and $y$.

• A number of sharp peaks at higher frequencies

### 3.2.1 White noise

The white noise at higher frequencies corresponds to uncorrelated, normally distributed, individual position results with a variance, $\sigma^2$. The power spectral density of the white noise can be determined from the auto-correlation function, $R_n(\tau)$. The expectation value of the the auto-correlation will then lead to an expected white noise level in the PSD. Consider a white noise signal, $n(t)$, as described above then,

$$ R_n(k) = \frac{1}{N - k} \sum_{t=0}^{N-1-k} n(t) \cdot n(t - k) \quad (3.7) $$

$$ \mathbb{E}[R_x(k)] = \frac{1}{N - k} \sum_{t=0}^{N-1-k} \mathbb{E}[n(t) \cdot n(t - k)] \quad (3.8) $$

![Figure 3.9: Power spectral density of 12 days of RasClic data.](image-url)
because all \( n(t) \) are independent random variables,
\[
\mathbb{E}[R_n(k)] = \begin{cases} 
\frac{1}{N-k} \sum_{t=0}^{N-1-k} \mathbb{E}[n(t)] \cdot \mathbb{E}[n(t-\tau)] & \text{for } t \neq \tau \\
\frac{1}{N-k} \sum_{t=0}^{N-1-k} \mathbb{E}[n(t)^2] & \text{for } t = \tau 
\end{cases}
\] (3.9)
\( n(t) \) is normally distributed with a zero mean and a variance of \( \sigma^2 \), \( n(t) \in N(0, \sigma^2) \), so the expectation value of the auto-correlation function is simply,
\[
\mathbb{E}[R_n(k)] = \sigma^2 \delta(k) \tag{3.10}
\]
where \( \delta \) is the dirac delta function. The PSD is then given by the Fourier transform of the auto-correlation function,
\[
\Phi_n(\omega) = \sum_k R_x(k) e^{-i\omega k} = \sum_k \sigma^2 \delta(k) e^{-i\omega k} \tag{3.11}
\]
\[
= \sigma^2 \tag{3.12}
\]
This suggests that the expectation value of the white noise level is only dependent on the variance of the data points and independent of the measurement time or sampling frequency. Figure 3.10 shows the PSD of 7 days of RasClic data with 3 different sampling rates. Indeed the expected white noise level is not affected by the sampling frequency. The same is true for data from different measurement lengths. It can also be shown [24] that the variance of the PSD of white noise is given by,
\[
\text{var}[\Phi_n(\omega)] = \sigma^4 \tag{3.13}
\]
This suggests that the variance of the PSD of white noise is not affected by the measurement time either. In fact, a longer measurement time will only increase the number of Fourier components spread evenly in \( f \in [0, \frac{1}{2T_s}] \), \( T_s \) is the sampling period) but each component will still be governed by the same variance.

### 3.2.2 \( 1/f \) noise

The static measurement principle of RasClic, i.e. measuring the displacement instead of the acceleration means the measurement technique has very little intrinsic \( 1/f \) noise. Compare this to a conventional inertial seismograph that measures acceleration, to arrive at an actual displacement the acceleration needs to be integrated twice (see equation 2.1). Integrating over any signal spectrum with (white) noise results in \( 1/f \) characteristics in the new signal. By directly measuring the displacement a RasClic system avoids this intrinsic property of seismic measurements. However, in reality, \( 1/f \) noise can still be present as a result of drift over longer periods of time. The origin of these drifts for RasClic are thought to be a result of the external environment. Influences such as temperature, rainfall and atmospheric pressure changes, or human presence have been seen to have an effect on the long term stability of RasClic. Westra [25] determined a correlation between atmospheric pressure and RasClic measurements. To check that this noise is not an intrinsic property of the measurement system, a comparison needs to be made between two closely identical RasClics. In this way identical drift patterns can rule out intrinsic errors and rather be attributed to external factors. See chapter 4 for a more detail discussion on this subject.
CHAPTER 3. THE RASCLIC SYSTEM

3.3 Sensitivity Analysis

To get an idea of the sensitivity with which RasClic can measure seismic phenomena, a sensitivity analysis is done. It would be expected that the longer the wavelength of the ground motion the smaller the measured RasClic signal, the sagitta in such a case is obviously much smaller. At higher frequencies with wavelengths smaller than half of RasClic length ($\lambda \leq L/2$) the measured amplitude could be as much as 4 times the ground amplitude. This is due to the fact that movement at the diffraction plate is amplified twice at the sensor location, this on top of the shift in the laser and sensor positions. To do a more quantitative analysis around the earth-hum modes a slice of the Earth was modeled as a disk, each earth-hum modes represented a whole number of wavelengths that could fit around the circumference of the disk. The $0S_l$ modes are investigated, so that the angular degree, $l$, corresponds directly to the number of circulating wavelengths. For this simple earth model the local radius can then be described by the equation,

$$r(\phi) = R + A \cos l\phi$$  \hspace{1cm} (3.14)

Where $R$ is the Earth radius, $A$ the amplitude of the mode and $\phi$ indicates the position of the RasClic on the Earth. From this the Earth’s radius of curvature at any given
3.3. SENSITIVITY ANALYSIS

point can be calculated using the spherical curvature equation,

\[
\frac{1}{\rho} = \frac{r^2 + 2r'^2 - rr''}{(r^2 + r'^2)^{3/2}}
\]  (3.15)

Where \(\rho\) is the radius of curvature and \(r'\) is the derivative of \(r\) with respect to \(\phi\). So that \(r' = -A l \sin l \phi\) and \(r'' = -A l^2 \cos l \phi\). We assume \(A \ll R\), so neglect all terms leading with \(A^2/R^2\) or \(A^3/R^3\). It then follows that the local curvature is given by,

\[
\frac{1}{\rho} \approx \frac{1}{R} (1 + (l^2 - 1) \frac{A}{R} \cos l \phi)
\]  (3.16)

The distance, \(d\), between a tangent line to a circle of radius \(R\), and the circle itself, at a distance \(z\) from the tangent point, is given by,

\[
d = R - R \cos \phi
\]

For small angles, i.e, \(z \ll R\), the approximation, \(z \approx R \phi\) is used, so that the second order derivative of \(d\) becomes,

\[
\frac{d^2d}{dz^2} \approx \frac{1}{R}
\]  (3.17)

The displacement measured by RasClic is the difference between the displacement as a result of the Earth’s radius, or any spherical body for that matter, and that of the local radius of curvature due to the mode. Therefore equation 3.17 becomes,

\[
\frac{d^2d}{dz^2} \approx \frac{1}{R} - \frac{1}{\rho}
\]  (3.18)

Furthermore, \(\phi_0\) is introduced to allow for the location of the RasClic on the Earth. Now \(\phi\) can be substituted by, \(\phi = z/R + \phi_0\) (we take \(z = 0\) and \(\phi = \phi_0\) at the diffraction plate). Combining equations 3.16 and 3.18 we get,

\[
\frac{d^2d}{dz^2} \approx \frac{1}{R} - \frac{1}{\rho(z/R + \phi_0)} = -(l^2 - 1) \frac{A}{R^2} \cos l(z/R + \phi_0)\]  (3.19)

Double integration of the above equation then leads to a final result for the measurable displacement.

\[
\frac{d}{dz} \approx \int_0^z \frac{1}{R} - \frac{1}{\rho(\zeta/R + \phi_0)} d\zeta
\]

\[
= (l^2 - 1) \frac{A}{lR} \sin l(z/R + \phi_0)
\]  (3.20)

Because the first integral results in an odd function, the final integral from \(-L/2\) to \(L/2\), will always be zero. The displacements measured by RasClic are however sensitive to both positive and negative displacements of the Earth so the absolute values from each side of \(z = 0\) are taken.

\[
d = \int_0^{L/2} (l^2 - 1) \frac{A}{lR} \sin l(z/R + \phi_0)dz + \int_0^{-L/2} (l^2 - 1) \frac{A}{lR} \sin l(z/R + \phi_0)dz
\]  (3.22)

\[
d = 2A(1 - \frac{1}{l^2})(1 - \cos (\frac{lL}{2R})) \cos (l\phi_0)
\]  (3.23)

This result has the following properties:
CHAPTER 3. THE RASCLIC SYSTEM

- The factor \( \cos l \phi_0 \) accounts for the location on the globe with respect to the nodes and anti-nodes of a standing wave.

- For low frequencies, i.e. low-order modes with \( l \ll \frac{2R}{L} \), the sensitivity reduces to,

\[
d = 2A(l^2 - 1)\frac{L^2}{8R^2}
\]  

(3.24)

showing a quadratic increase in sensitivity with length.

- For high frequencies, i.e. high-order modes with \( l \gg 1 \) the sensitivity reduces to,

\[
d = 2A(1 - \cos \frac{L}{2R})
\]  

(3.25)

which oscillates between \( 4A \) where the three points are on, for example, consecutive peak, trough and peak, and 0 where all three points are at the same elevation.

The sensitivity is defined as \( \alpha(n, L) = d/A \). To relate the sensitivity to the frequency of the modes, the values of the observed periods of the \( \varnothing S_l \) earth-hum modes from A.M.Dziewonski et.al. [12] were used. This provided values up to \( l = 160 \), for higher modes the frequencies were extrapolated with assumption of a constant phase velocity. A sample of these periods are given in table 3.1. A plot of sensitivity as a function of frequency is plotted in figure 3.11 for a RasClic of length 140 and 500 meters.

<table>
<thead>
<tr>
<th>( \varnothing S_l ) mode</th>
<th>Period(s)</th>
<th>( \varnothing S_l ) mode</th>
<th>Period(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3233.25</td>
<td>12</td>
<td>502.36</td>
</tr>
<tr>
<td>3</td>
<td>2134.67</td>
<td>13</td>
<td>473.17</td>
</tr>
<tr>
<td>4</td>
<td>1545.60</td>
<td>14</td>
<td>448.21</td>
</tr>
<tr>
<td>5</td>
<td>1190.13</td>
<td>15</td>
<td>426.15</td>
</tr>
<tr>
<td>6</td>
<td>963.18</td>
<td>16</td>
<td>406.76</td>
</tr>
<tr>
<td>7</td>
<td>811.45</td>
<td>17</td>
<td>389.30</td>
</tr>
<tr>
<td>8</td>
<td>707.66</td>
<td>18</td>
<td>373.94</td>
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</tr>
<tr>
<td>11</td>
<td>536.89</td>
<td>20</td>
<td>347.50</td>
</tr>
</tbody>
</table>

Table 3.1: Earth-hum \( \varnothing S \) mode periods [12]

It is obvious from this figure that the sensitivity indeed increases with an increased length of the system. At higher frequencies the singularities are seen and have a period that is also related to the length of the system. In doing a simple calculation for the sensitivity of RasClic to earth-hum modes we use the average sensitivity over all the nodes, \( \alpha \approx 10^{-7} \) and assume an earth-hum mode amplitude of 1 cm. This results in a maximum measurable amplitude of 1 nm. Compared to the noise level at the same frequency of 300 nm it is apparent that much work still needs to be done on the sensitivity and reduction of noise before conclusive evidence of the earth-hum will be detected on the current RasClic system. Chapter 5 will explore the possibilities of finding the earth-hum and other seismic phenomena with an improved RasClic system.

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3.4 Cramer-Rao Analysis

Any estimator, like the FFT shift estimator used by RasClic, has a lower bound in the variance of the determined parameter. This is known as the Cramer-Rao lower bound (CRLB). The CRLB is not necessarily unsurpassable, however an estimator that achieves this lower bound is said to be very accurate. The CRLB is derived from the Fisher information matrix, it can be shown [26] that for a 2D shift estimator the CRLB is given by,

$$\text{var}(s_x) \geq \frac{\sigma_n^2 \sum_S I_y^2}{\det(T)} \quad \text{var}(s_y) \geq \frac{\sigma_n^2 \sum_S I_x^2}{\det(T)}$$ (3.26)

Where $I_x = \frac{\partial I}{\partial x}$ and $I_y = \frac{\partial I}{\partial y}$ are the spatial derivatives of the uncorrupted image $I$ over the region $S$, $\sigma_n^2$ is the variance in the pixel noise and $T$ is the gradient structure tensor derived from the Fisher information matrix, $F$ according to,

$$T = \sigma_n^2 F = \left[ \begin{array}{c} \sum_S I_x^2 \sum_S I_x I_y \\ \sum_S I_y^2 \sum_S I_x I_y \end{array} \right]$$ (3.27)

Put into a RasClic perspective, the lower bound in the variance for the estimated translations is proportional to the pixel noise and inversely proportional to the total gradient energy in the image. This would suggest that the estimator can be optimized by decreasing the pixel noise or by increasing the gradient energy. Decreasing the pixel
noise could be done by improving the properties of the CCD pixel sensor by for example cooling the device to reduce thermally excited electrons or using a larger CCD surface. The gradient energy can be increased by increasing the amount of light that falls on the image sensor or the amount of gradient information by increasing the number of maxima or minima. Increasing the amount of light can be done by placing a lens in front of the diffraction plate thus raising the amount of light propagating towards the sensor. Two diffraction patterns extensively used in the RasClic system were used as inputs for this Cramer-Rao analysis. Because the CRLB needs to be calculated from uncorrupted images, an image was simulated to resemble the particular diffraction pattern. The first to be discussed is the diffraction pattern shown in figure 3.6, which was first used in the 140 meter RasClic system and will be referred to as RasClic140. The second is the diffraction pattern that was later used in the double RasClic system. Both the real and the simulated images have been shown in figure 3.5. This diffraction pattern will be coined RasClic Double. In each of the two cases the CRLB was calculated and compared to the variance found in the real data measurements. The outcome of this analysis is given in table 3.2 and figure 3.12 shows a snapshot of data for comparison. It must be noted that the data is taken from two very different instances of time, but plotted on one graph for ease of comparison. A piece of the data has been zoomed in on to illustrate the difference in the variance of the two diffraction patterns. The 20 nm variance found in the RasClic140 data corresponds very well with the $\sigma^2 = 0.0004 \mu m$ white noise level indicated in the PSD (figure 3.9). For each of the two diffraction patterns we see that the CRLB gives an indication of the data variance, roughly 4 times that of the CRLB. Finding the cause and possibly a solution to the difference between the two variances will be investigated as part of the following 2 chapters.

<table>
<thead>
<tr>
<th>Pattern</th>
<th>$\sigma_{CRLB}$ (nm)</th>
<th>$\sigma_{data}$ (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RasClic140</td>
<td>5</td>
<td>20</td>
</tr>
<tr>
<td>RasClic$^2$</td>
<td>27</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 3.2: Results of the Cramer-Rao analysis for RasClic140 and RasClic$^2$ diffraction patterns compared with real data variance.
3.4. CRAMER-RAO ANALYSIS

Figure 3.12: Comparison of the two diffraction patterns used in RasClic. Data is taken at different times but plotted together for comparison. The zoomed in plot shows the big difference in variance of the data points.
Chapter 4

Development of the RasClic System

This chapter describes several improvements that have been implemented on the RasClic system since January 2008. The improvements have been categorized into three different groups: improvements on the image analysis algorithms, improvements to the image processing and finally developments that involve the physical setup of RasClic.

4.1 Image analysis

4.1.1 Weighted shift estimator

The phase plane fit in the shift estimator described in the previous chapter can be optimized by using a weighted least squares fitting algorithm. The full derivation of this is given in appendix C.1. Equation 3.4, of the phase relation can be written in matrix form, by,

\[ \arg \left( \frac{\tilde{D}}{\tilde{R}} \right) = \Phi = [k_x \quad k_y] \cdot \begin{bmatrix} s_x \\ s_y \end{bmatrix} \] (4.1)

where \( k = \{k_x, k_y\} \) is the wave vector and \( s = \{s_x, s_y\} \) the displacement vector which needs to be calculated. Using the normal non-weighted fit the displacements can be calculated using,

\[ s = (k^T k)^{-1} (k^T \Phi) \] (4.2)

When one wants to implement a weighted fit estimate the above equation can be written as,

\[ s = (k^T w k)^{-1} (k^T w \Phi) \] (4.3)

where \( w \) are the weighting factors given by,

\[ w = \frac{|\tilde{D}|^2 \cdot |\tilde{R}|^2}{|\tilde{D}|^2 + |\tilde{R}|^2} \] (4.4)
The implementation of the weighted fit improved the displacement resolution by a factor of 10 to a standard deviation of 20 nm as shown in section 3.4. For comparison a set of weighted and unweighted data from the RasClic 140 system are shown (figure 4.1). It must be noted that the two sets of data are from separate instances in time and are only plotted together for comparison.

![Figure 4.1: Comparison of two separate instances of RasClic 140 data with and without use of a weighted fit.](image)

### 4.1.2 Interpolation

It was seen that the errors within the analysis algorithm grow larger with the displacement of the image. This is due to the fact that the reference image and the image being compared become more and more dissimilar as the displacement increases. As the image shifts to one side more information becomes available on the opposite side that isn’t present in the reference image thus distorting the DFT. To clarify this further figure 4.2 illustrates the problem in terms of a simple 1D example. The Fourier transform of a finite time, sampled signal is in effect the Fourier transform of an infinite time periodic signal (4.2a). A DFT from a shifted image (4.2b) is therefore not the same
as a phase shifted version of the reference DFT. The larger the shift the more dissimilar the DFT’s become. To remedy this effect an interpolation process was implemented.

![Diagram](image)

**Figure 4.2:** Illustration of the motivation behind the interpolation process. a) the reference image with periodic DFT signal. b) a shifted image with periodic DFT signal. Notice that this is not the same as a shifted version of the reference image signal, a. c) the reference image with DFT signal from ROI. d) a shifted image with DFT of ROI. e) a reference image with DFT signal of a shifted ROI, notice that this signal much more closely resembles a shifted version of the signal in d.

A major difference to the old system is that this works within a region of interest (ROI) that is 480 by 480 pixels wide. Instead of taking the DFT of the whole image just the DFT of the region of interest is taken (4.2 c). Was well as this the region of interest is shifted by a whole number of pixels in all directions thus creating $20 \times 18 = 360$ other ”reference” images that correspond to an image shifted by a linear combination of pixels in both directions. Now the DFT of the target image within the region of interest is also taken, $\tilde{D}$ (4.2 d). A comparison with the first DFT of the reference image provides an initial estimate for the shift, this is in principle the same as the old algorithm except that now a region of interest has been used.

From here on the interpolation algorithm takes over. The initial estimate is used to pick an alternative reference DFT of a shifted reference image that better resembles the image (4.2 e), this shifted reference DFT will be referred to as $\tilde{R}_{shift}$. For example, if the initial shift estimator returns the values $(x, y) = (2.3, 4.6)$, then the reference DFT with the region of interest shifted by $(2, 5)$ is chosen. The $\tilde{R}_{shift}$ is then interpolated to resemble the DFT of the reference image with shift $(x, y) = (2.3, 4.6)$, this will be referred to as $\tilde{R}_{interp}$. For a detailed description of how this is done, see appendix C.2. $\tilde{R}_{interp}$ is then compared to $\tilde{D}$ in the usual way. The resulting shift estimate provides a correction for the initial estimate. This interpolation process is repeated 4 times resulting in an improved shift estimate. Obviously this method only works while the image has not shifted further than the region of interest can move within the image.
This means that the image cannot shift further than 9 pixels, or 133 microns. A schematic view of the interpolation algorithm is shown in figure 4.3.

![Schematic for the working of the interpolation algorithm](image)

**Figure 4.3**: Schematic for the working of the interpolation algorithm

The performance of the interpolation algorithm was tested on simulated images. The difference between the known displacement of the simulated image and that determined by the shift estimator was calculated and defines the error. The results of this test are shown in figure 4.4. Here it is obvious to see that the error in the error incurred by the shift estimator increases with the size of the displacement. Using the interpolation algorithm greatly reduces this effect.

## 4.2 Image processing

### 4.2.1 Dark-current and flat-field correction

A flat-field correction is a technique often used in astronomy applications for the accurate calibration of telescopes and cameras. A CCD camera consists of a matrix of silicon based transistors that convert photons into a measurable electronic signal. Even when there are no photons falling on the CCD electrons in the transistor can still be excited, via thermal excitation, into the conduction band of the semiconductor thus creating an electron flow. This current that is measured when there is no light shining on the CCD is known as the dark-current and is responsible for a biased noise in all images. It is assumed that this noise is unique but constant for each transistor, i.e. each pixel. This is by approximation true if the camera is operating at a constant temperature. The dark-current field can be accounted for by simply subtracting the bias value from the measured pixel response.

Besides the dark-current a CCD camera also needs to be calibrated for flat-field errors.
Each pixel is assumed to have a linear response to the number of incoming photons. This linear response is again expected to be unique but constant for each pixel. An ideal camera would have the same linear response for each pixel. In correcting for this flat-field error each measured pixel value is multiplied by a weighted factor normalized over the whole CCD. If $\tilde{I}_i$ is the value of a measured pixel then its flat-field and dark-current corrected value is given by,

$$I_i = \alpha_i \tilde{I}_i - \beta_i$$  \hspace{1cm} (4.5)

where $I_i$ is the corrected value, $\alpha$ is the flat-field correction factor and $\beta$ is the dark-current correction factor. $\alpha$ and $\beta$ are statistically obtained values determined using a large set of images. Two sets of images are required; dark-current images, $DC^j_i$, these have been taken when the camera is in darkness; and flat-field images, $FF^j_i$, are those taken when the camera is uniformly illuminated by a constant light source. For each case the indices $i$ and $j$ correspond to the pixel and image number respectively. To calculate $\alpha$ and $\beta$, the mean over all $N$ images is calculated for both dark-current and
4.2. IMAGE PROCESSING

dark-current corrected flat-field images,

\[ \overline{DC}_i = \frac{1}{N} \sum_{j=0}^{N} DC^j_i \]  

(4.6)

\[ FF_i = \frac{1}{N} \sum_{j=0}^{N} (FF^j_i - DC_i) \]  

(4.7)

The mean flat-field image is normalized by the mean pixel intensity \( \mu \),

\[ \mu = \frac{1}{w \cdot h} \sum_{i=1}^{w \cdot h} FF_i \]  

(4.8)

where \( w \) and \( h \) are the width and height, respectively, of the image in pixels, such that the normalized final flat-field image becomes,

\[ FF_i = \frac{FF_i}{\mu} \]  

(4.9)

Subsequent calculation of the correction factors \( \alpha \) and \( \beta \) is done by,

\[ \alpha = \frac{1}{FF_i} ; \beta = \frac{DC_i}{FF_i} \]  

(4.10)

The alpha and beta values as found for the camera used in the RasClic test setup are depicted graphically in figure 4.5. In both alpha and beta there appears to be a obvious separation exactly halfway across the pixel sensor. This is most likely due to a fabrication artifact. There are also discernible gradients visible in both the dark-current and flat-field images. As mentioned before both of these factors are susceptible to thermal variance. Heat dissipated by electronics in the camera will result in a temperature gradient across the CCD, causing the gradients seen in the figures. Care must therefore be taken to ensure the camera has reached a thermal equilibrium before the dark-current and flat-field images are recorded. Another artifact to note are the visible steps in the graphics. This is caused by the fact that the \( \alpha \) and \( \beta \) values are in the regime of the lowest few bits. It is expected that this will have a non-negligible effect on the flat-field correction. This can be negated by increasing the number of bits per pixel, this is discussed in the following section. In testing the effect of the dark-current field on the image analysis algorithm the following test was devised. RasClic was turned on as normal and allowed to thermalize. Then the light into the laser coupler was blocked off, creating a dark image. The analysis programs were left running, thus calculating the shift estimation on information produced by the dark-current images. The values given had a very large offset(\( \approx 400 \mu m \) in both \( x \) and \( y \)) suggesting there is a lot of position dependent information in the background images. These images were then used to construct a dark-current correction image that was then implemented in the analysis program. Once the analysis program was applying the dark-current correction the laser light was again blocked off. This time the analysis program returned values
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(a)

(b)

Figure 4.5: The \(a) \alpha \) and \(b) \beta \) values as calculated for the sensor. The brightness and contrast of the images have been modified for clarity

with a large variance but centered around zero, suggesting the corrected images have very little position dependent information (figure 4.6). Exactly how big the effect of dark-current noise is on the image analysis during normal operation is not clear, further investigations will be made using simulated images in chapter 5. It is also not clear how the dark-current correction will evolve in time. A dark-current correction image made today may not necessarily provide an effective correction next week under different external conditions. A work around could be devised where dark-current correction images are recalculated at regular intervals during normal operation.

4.2.2 8 to 16 bits per pixel

The image sensor provides 2 options for the encoding of the image pixel; in 8 or 16 bits, giving values between 0 and 255 or 65535 respectively. As mentioned earlier the choice between these two options could have a measurable effect on the effectiveness of the image analysis algorithms. Take for example an area of the images where the signal is in the same order of magnitude as the background signal, i.e. in the lowest few bits. Because the values are discretized in relatively large steps the effect of a signal on top of, for example a sloping dark-current background noise, will have a small contribution to the measured images. Shifting the signal while the background stays the same will only have a minimal effect on the overall outcome. By increasing the number of discretization steps (by a factor 256 for 8 to 16 bits) means the signal is followed more closely and the effect of a shifted signal will be more noticeable to a Fourier transformation.
4.3 System Improvements

4.3.1 Duplicate system

The assumption that the $1/f$ noise is a result of external factors is tested by constructing a duplicate system. If the noise is indeed a result of external factors then the effect should be the same for both systems, as apposed to factors due to intrinsic properties of the system. A double RasClic system was implemented by passing two parallel laser beams through the same vacuum tube. Figure 4.7 is a schematic showing the setup of the duplicate parallel system. The diffraction plate had to be modified to allow for the two beams. Because of the spatial restrictions in the vacuum tube a much smaller diffraction plate had to be used. The diffraction plate and resulting diffraction pattern used in the parallel system were shown in figure 3.5, while the whole diffraction plate mounted in the vacuum tube is depicted in figure 4.8. The two laser de-couplers are now mounted together on a solid steel plate(figure 4.9). Similarly a camera mounting system was constructed to hold two image sensors (figure 4.11).
CHAPTER 4. DEVELOPMENT OF THE RASCLIC SYSTEM

Figure 4.7: Setup of the double RasClic system

Figure 4.8: Double RasClic diffraction plate
4.3. SYSTEM IMPROVEMENTS

**Figure 4.9**: Double RasClic laser de-couplers mounted to the decoupler holder. Each decoupler is connected to the vacuum tube with small flexible vacuum tubes.

**Figure 4.10**: Double RasClic field stop. Field stops are used to confine the light to a trajectory that doesn’t allow for reflections off the inside of the vacuum tube.
CHAPTER 4. DEVELOPMENT OF THE RASCLIC SYSTEM

Figure 4.11: Double RasClic camera holders. The image sensors are mounted in sideways. Aluminum foil is used to minimize light contamination from the direct surroundings.

20 hours of data recorded by the parallel system is shown in figure 4.12. The difference between the two systems has also been calculated and plotted in the same figure. The two systems agree with each other to within $1\, \mu m$. This is proof that to this degree of accuracy movements recorded by the RasClic are primarily caused by external factors, whether they be of seismic, man-made or meteorological origin. Finding a relationship between the difference measurements and, for example, the temperature or the pressure will be the next step in determining the correlation between these influences and the RasClic measurements.
4.3. SYSTEM IMPROVEMENTS

Figure 4.12: 20 hours of double RasClic data
Chapter 5

Simulations of the RasClic System

This chapter describes the simulations done on the RasClic system. To get a better understanding of the effect of certain image characteristics image analysis simulations were done by testing the accuracy of the analysis algorithms. Calculations were also made to quantify the effectiveness of RasClic as a seismograph.

5.1 Image analysis simulations

To determine the error associated with the analysis algorithm, the calculated displacement needs to be checked against the true displacement. In reality it is very difficult to control, to the required accuracy, the positions of the laser, diffraction plate and sensor. To overcome this problem simulated images of the expected diffraction pattern were used. These can be made to simulate any desired displacement, which is then known exactly. For the simulation experiments three different diffraction patterns were used, they are named; simple, non-symmetric and chess and are shown in figure 5.1.

- **Simple** - A basic circular symmetric damped cosine function that corresponds to a perfectly circular diffraction plate with a circular cut out (without holders to keep the inner disk from falling out).

- **Non-symmetric** - A diffraction pattern that was created using the simulation algorithms developed by van der Geer [23]. The corresponding diffraction plate is a narrow circular slit, with inner and outer diameters of 142 and 145 mm respectively. In this case the inner disk is attached to the outer ring by two stretches of aluminum top and bottom, roughly 30 mm wide. It is the shadow of the inner disk support structures that cause the diffraction pattern not to be circular symmetric, but rather have a higher frequency signal in the y direction.

- **Chess** - is again a pattern simulated by an analytical function and was designed to mimic the more square form of the diffraction pattern that resulted from the smaller diffraction pattern implemented on the double RasClic set-up.
5.1. IMAGE ANALYSIS SIMULATIONS

In each of the following experiments the diffraction pattern was created 400 times with randomly generated displacements between -1 and 1 pixel in both the $x$ and the $y$ direction. The error of one of these images is defined as being the the analyzed displacement minus the actual displacement. The root mean squared (rms) error can be calculated over all 400 images. Mathematically this is given by,

$$dx_{rms} = \left( \frac{1}{M} \sum_{i}^{M} (x_a - x_r)^2 \right)^{\frac{1}{2}}$$  

(5.1)

where $M$ is the number of images, $x_a$ is the analyzed displacement and $x_r$ the real displacement. The resulting errors of the 400 images for each of the three diffraction patterns are shown in figure 5.2. There are a few things to note from this experiment. The most obvious is that the simple and chess diffraction patterns have got a much smaller error but a larger variance in the analysis. The rms error of simple and chess are respectively 19 and 35 pm (comparable in $x$ and $y$) compared to the 160 pm in $x$ and 150 pm in $y$ for non-symmetric. The non-symmetric diffraction pattern has a much larger rms error but the resolution with which the displacement of neighboring images is analyzed is much better. This may suggest that the higher amount of information in this diffraction pattern helps in producing consistent results but also contributes to the systematic errors in the shift estimation algorithms. This effect seems to be lost...
at larger displacements. It is clear that the errors are periodic with a spatial period of exactly 1 pixel (14.8 μm). Bearing the interpolation process described in the previous chapter in mind it can be expected that the errors incurred during interpolation will be smallest around whole pixel displacements. For symmetry reasons, the error reduces to zero around half pixel displacements as well.

Most importantly this experiment shows that the largest absolute error incurred during the analysis procedure is just 0.3 nm. This is well within the Cramer-Rao lower bound which will be discussed later in this chapter.

The above diffraction patterns and analysis methods were used to determine the effect of certain image characteristics, that do not shift along with the pattern, on the accuracy of the shift estimator. Three major characteristics were identified. These were; gradient, background step and dust particles. Additionally the noise was studied to verify its relationship in the CRLB and effect on the shift estimator.

- **Gradient** - A gradient in the light intensity across the pixel sensor could be due to dark-current or thermal gradient effects.

- **Background step** - As identified in the previous chapter the dark-current noise imposes a sharp step from the left to the right hand half of the image.

- **Dust particles** - One of the anomalies detected on the images are small secondary
diffraction patterns assumed to be caused by dust particles on the windows of the vacuum tube. Cleaning the windows thoroughly reduces the number these blemishes.

- **Noise** - A property of the silicon transistors of the sensor is that there is a variance in the values that a chip gives at the same light intensity. This effect causes the images to appear "noisy". It is assumed that the variance is the same for each pixel.

Over the next few paragraphs each of these effects will be tested by the simulated images to arrive at a conclusion to their effect on the image analysis algorithm. The chess diffraction pattern will form the basis of these investigations because it most resembles the real diffraction pattern obtained by the most recent RasClic set-up. First the Cramer-Rao lower bound will be analyzed for each of the diffraction patterns.

### 5.1.1 Cramer-rao

The CRLB for the three diffraction patterns were calculated using equation 3.26. The results are given in table 5.1. It is obvious that the non-symmetric diffraction pattern has stronger gradients than the other two, the majority of which is in the y-direction. This is likewise reflected in a lower CRLB for $s_y$. It may seem strange that it is the smallest CRLB that produces the highest rms error in the simulated measurements. The CRLB is partially governed by the pixel noise, whereas the simulated images from the measurements above don’t have any pixel noise. Because the rms error is an order of magnitude smaller than the CRLB it is assumed this is governed mainly by the noise.

<table>
<thead>
<tr>
<th>Diffraction pattern</th>
<th>$\sigma_{s_x}(nm)$</th>
<th>$\sigma_{s_y}(nm)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>Non-symmetric</td>
<td>4.0</td>
<td>3.6</td>
</tr>
<tr>
<td>Chess</td>
<td>16</td>
<td>16</td>
</tr>
</tbody>
</table>

### 5.1.2 Gradient

The first effect to be studied is the gradient. This involved superimposing a gradient of varying slope over the simulated images. Figure 5.3 shows the gradient used in this experiment. The slope has been exaggerated for clarity, the resulting simulated image and its DFT-bin are also shown.
CHAPTER 5. SIMULATIONS OF THE RASCLIC SYSTEM

Figure 5.3: (a) The superimposed gradient (scaled for better visualization) and (b) the resulting diffraction pattern for a slope of 8, (c) the DFT bin of diffraction pattern without gradient, (d) the DFT bin of the diffraction pattern with gradient, notice the increased intensity of the components at the center of the DFT bin.

An interesting thing about the consequence of the gradient is that it only effects the lowest spatial frequency components of the DFT, and in this particular case also only the components on the wave number axis. This would lead one to assume that a filter removing the low frequency signals would reduce the effects of a gradient. The initial analysis algorithm made use of a low pass filter to remove unwanted high-frequency signals. By extending this filter to cut out low frequency signals as well, essentially becoming a band-pass filter, the above hypothesis can be tested. The results of this test are shown in figure 5.4. The curves initially undergo an almost identical linear increase, followed by a polynomial increase in which the bandpass filter out-performs the lowpass filter. This behavior can be understood as a result of weighing factors in the shift estimator. The effects of the gradient can be added in the frequency domain and effect both the reference and target image. A frequency factor due to gradient \( g \) will be added to the reference DFT, \( R \), and the target image DFT, \( D \). Recalling
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5.1. IMAGE ANALYSIS SIMULATIONS

In the low slope regime, $D^2 \gg g^2$, all the high-order effects of the gradient can be neglected ($g^2 \approx g^3 \approx g^4 \approx 0$), the additional effects of the gradient are small enough not to have a large effect on the analysis. The only impact the gradient has is in the mid-range frequencies that don’t get filtered by either filters. These effects have only a small effect on the weighting factors and in that way degrade the effectiveness of the analysis in a linear fashion.

In the high slope regime, $D^2 \approx g^2$ the effect of the gradient on the weighting factors is no longer negligible, the errors therefore scale to the power of 4. The low frequency gradient components now have a much stronger influence on the weighting factors, removing these components with the bandpass filter improves the analysis to a certain degree.

**Figure 5.4:** Analysis algorithm performance for randomly displaced simulated images with varying gradient with low pass and band pass filters

In equation 4.4 the weighing factors can then be given by,

$$w = \frac{|D + g|^2 \cdot |R + g|^2}{|D + g|^2 + |R + g|^2}$$

$$\approx \frac{(D^2 + 2Dg + g^2) \cdot (R^2 + 2Rg + g^2)}{D^2 + 2Dg + R^2 + 2Rg + 2g^2}$$

$$\approx \frac{D^2 R^2 + 2D^2 Rg + D^2 g^2 + 4D R g^2 + 2D g^3 + R^2 g^2 + 2R g^3 + g^4}{D^2 + 2Dg + R^2 + 2Rg + 2g^2}$$
To further test the influence of a gradient a signal with a broader frequency spectrum is preferable. The above experiment was therefore also performed on the 'non-symmetric' diffraction pattern.

![Figure 5.5: DFT of non-symmetric diffraction pattern a) with out gradient and b) with gradient](image)

This particular diffraction pattern has a higher frequency signal in the y than in the x direction, this is visible in the DFT of the image shown in figure 5.5. It is also clear that the gradient has a much larger effect on the information in the x direction, as the additional gradient components in the y direction don’t effect the higher frequency components. Bearing this in mind it can be expected that applying a bandpass filter would in this case remove the low frequency components in the x direction that are tainted by the gradient. This essentially involves chopping out a small circle from the center of the above DFTs. This will leave higher frequency components that are relatively less effected by the gradient to perform the shift estimation on. The results of this test are given in figure 5.6.

Indeed the errors in the x direction are greatly reduced whereas in the y direction they stay much the same. In fact there is even a slight decrease in the errors in the y direction, this is most likely due to the band pass filter removing information that was otherwise not tainted by the gradient.
5.1. **IMAGE ANALYSIS SIMULATIONS**

### 5.1.3 Background step

The background step described in chapter 4 is simulated in a similar way. In this case a fixed value was superimposed onto the right half of the image. The results are shown in figure 5.7.

Initially only the error in x increase with the step size. This is expected because the step occurs in the x-direction. As the step size increases the fourier components in the y direction start to become affected and an error in $s_y$ results. It is possible that this is due to the saturation of the pixel values. This creates images that are limited to the 16 bits pixel size causing discontinuities in the signal. This will adversely effect the DFT.

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**Figure 5.6**: Analysis algorithm performance for randomly displaced simulated images with varying gradient with just low pass and band pass
5.1.4 Dust particles

Another irregularity detected in the RasClic images are small secondary diffraction patterns assumed to be caused by dust particles on the windows of the vacuum tube. Cleaning the windows thoroughly reduces the number of blemishes. Once again simulations are done by superimposing dust particle effects into the simulated diffraction patterns. An example of a simulated dust particle is shown in figure 5.8. The inset shows a zoomed in image of one of the dust particles.

The position of these dust particles proved to have a large influence on the accuracy of the analysis. The number of dust particles had a much smaller effect. To understand this more fully, this experiment was done for an increasing numbers of dust particles, and for each number repeated 10 times with different random configurations of dust particles. Each time the rms error over 400 randomly shifted images was calculated. This resulted in, for each number of particles, a mean error, a maximum and minimum error from all 10 configurations. These results are plotted in figure 5.9.

From these results we see that the number of dust particles has little effect on the accuracy. The expected rms error is much the same for 4 particles as it is for 12. Simply the presence of dust particles however does have a large impact on the error, increasing to roughly 10 times that of the error for no dust particles at all. The dust particles have a high frequency signal. For this reason it was thought that by reducing
5.1. IMAGE ANALYSIS SIMULATIONS

**Figure 5.8:** Example of a simulated diffraction pattern with dust particle shown in inset

the bounds of the filter could remove more of the unwanted dust signal. This however had little to no effect on the performance of the analysis. The signal from the dust particles must therefore be within the range of the desired signal frequencies.

**Figure 5.9:** Analysis algorithm performance for randomly displaced images with randomly distributed dust particles. The mean, minimum and maximum from 10 different dust configurations are given.
5.1.5 Noise

Image noise as a result of a variance in the pixel values was simulated in a similar way as in the above examples. It was assumed that the pixel variance was the same in all pixels and a randomly generated gaussian distributed offset was added to each pixel. The variance was determined as a percentage of the maximum value of 65535 counts per pixel. The results are shown in figure 5.10.

![Figure 5.10: Analysis algorithm performance for randomly displaced simulated images with noise](image)

It is clear to see that the noise has a substantial effect on the accuracy of the shift estimator. The error scales very linearly which is in line with the linear dependence of the CRLB on pixel noise.

5.1.6 Conclusions

To get an idea of the relative effects of each of the image characteristics on a typical RasClic measurement each characteristic was assigned a typical value. This value was derived empirically from images taken by RasClic. The rms error at each characteristics typical values was then calculated and expressed as a ratio of rms error with anomaly to that of a perfect image. A ratio of 1 would suggest no effect at all and much greater than 1 a large effect. Finally a comparison is also made with the results of an image that has been affected by all characteristics. This can give an indication of the dependance
of the different effects on each other. The results are presented in table 5.2.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Typical value</th>
<th>Chess $dx_{\text{rms}}$(nm)</th>
<th>Effect ratio</th>
<th>Non symmetric $dx_{\text{rms}}$(nm)</th>
<th>Effect ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>x N/A</td>
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<td>1</td>
<td>0.16</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>y</td>
<td>0.16</td>
<td>1</td>
<td>0.16</td>
<td>1</td>
</tr>
<tr>
<td>Gradient</td>
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<td>1.5</td>
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<tr>
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<td>y</td>
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<td>600</td>
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Table 5.2: Summary of simulations results. If x and y are comparable only one value is given.

The simulations in this chapter only give a brief insight into the workings of the analysis algorithms and the effect of image characteristics on it. For a better understanding of the origin of the errors more extensive simulations will need to be done. It can be said that;

- A gradient has a small effect on the performance of the analysis algorithm in the low slope domain. In the high slope domain the error increases rapidly with higher order behavior thanks to the $4^{\text{th}}$ power scaling of the weighing factors. The use of a bandpass filter to minimize these effects only works if the gradient components in the DFT actually affect frequencies that play an important role in the shift estimation. In general it can be said that in the low slope regime where RasClic is expected to operate, the rms error increases linearly.

- The background step in x has a small effect on the effectiveness of the shift estimator. Logically the x position is more affected than the y position.

- The presents of dust particles, rather than the number of them seems to have a moderate effect on the algorithm performance.

- Compared to the other characteristics noise, by far, has the greatest effect. It is then not surprising that the CRLB is determined to a great extent by the pixel noise. Reducing this noise would be the best place to start in reducing the analysis errors. Once the noise is under control the other characteristics will start to play a more crucial role.

- As apposed to much better performance on characteristics without noise the non-symmetric diffraction pattern, with its lower CRLB, didn’t outperform the other.
diffraction pattern once noise was added.

- It seems apparent that noise has a relatively larger effect on the higher frequency $y$ direction of the non-symmetric diffraction pattern. This might suggest that noise will affect high frequency signals to a greater extent than low frequency signals. More simulations would need to be done to reach sound conclusions on this hypothesis.

The combined effects of all of these characteristics on analysis performance are still in the same order of magnitude as the CRLB, suggesting that the shift estimator is accurate.

### 5.2 RasClic seismograph simulations

Theoretically derived design constraints in order to construct a RasClic capable of measuring seismic phenomena will be established in this section. In order to do this a few assumptions are made:

- The drift effects can be neglected, i.e. white noise is the sole contributor to the data noise.

- The Cramer-Rao lower bound is assumed as being attainable. Two scenarios are used; that of a currently available RasClic diffraction pattern with a CRLB of 10 nm, and that of a future RasClic sensor with a $4 \times 4$ matrix of actively cooled CCD sensors which produce images with a CRLB of 0.1 nm.

- The signal-to-noise ratio needs to be greater than 2, the signal in this case being the power spectral density. This is the same as results found in the literature and ensures a 97.5% chance of correct classification.

Consider the RasClic response to a seismic signal such as the earth-hum as a sinusoidal plus a noise component,

$$ s(t) = D \cos(\omega_0 t) + n(t) $$

where $n(t)$ is described by normal distributed noise, $n(t) \in N(0, \sigma^2)$. Taking just the sinusoidal component of the signal, $u(t)$, the Fourier transform is then,

$$ U(\omega) = \sum_{t=0}^{N-1} D \cos(\omega_0 t) e^{-i \frac{2\pi}{N} \omega t} $$

$$ = \sum_{t=0}^{N-1} \frac{D}{2} \left( e^{-i(\omega-\omega_0)\frac{2\pi}{N}} + e^{-i(\omega+\omega_0)\frac{2\pi}{N}} \right) $$

$$ = \begin{cases} N \frac{D}{2} & \text{for } \omega = \pm \omega_0 \\ 0 & \text{elsewhere} \end{cases} $$
The power spectral density (PSD) is by definition given by,

\[
\Phi_u(\omega) = \frac{1}{N} |U(\omega)|^2
\]

\[
= \begin{cases} 
  N \frac{D^2}{4} & \text{for } \omega = \pm \omega_0 \\
  0 & \text{elsewhere}
\end{cases}
\]  

(5.7)

How well the signal can be distinguished from the noise is characterized by the signal-to-noise ratio (SNR) and in this case defined as the signal amplitudes divided by the standard deviation of the noise, or,

\[
SNR = \frac{\text{signal amplitude in PSD}}{\text{std. of noise in PSD}}
\]

(5.8)

The signal amplitude has just been calculated in equation 5.8 and the standard deviation of the noise is the square root of the noise variance already given in equation 3.13. So the SNR becomes,

\[
SNR = \frac{\Phi_u(\omega)}{\sqrt{\Phi_n(\omega)}} = \frac{N D^2}{4 \sigma^2}
\]

(5.9)

(5.10)

\(D\) is the amplitude of the signal measured by RasClic. This can be related back to the actual ground amplitude, \(A_x\) through the sensitivity factor, \(\alpha\), so that,

\[
SNR = \frac{A_x^2 \alpha^2 N}{4 \sigma^2}
\]

(5.11)

A signal to noise ratio larger than or equal to 2, gives a lower bound for the sensitivity,

\[
\alpha^2 \geq \frac{8 \sigma^2}{A_x^2 N}
\]

(5.12)

The sensitivity has been evaluated in chapter 3 from a spherical-mode perspective. Combining the above equation with equation 3.24 a relation for the length, \(L\), of RasClic is obtained.

\[
L^4 \geq \frac{128 \sigma^2 R^4}{(l^2 - 1)^2 A_x^2 N}
\]

(5.13)

where \(\sigma\) is the variance in the measured displacements, given in this case by the Cramer-Rao lower bound, \(R\) the radius of the Earth, \(l\) is the angular degree of the vibrational mode and \(N\) is the number of measurements. By filling values obtained in the previous chapters into equation 5.13 an indication for the required length of RasClic is given. This will be discussed for each seismic phenomena in the following sections. A summary of the results and an evaluation of the physical feasibility will be given in the concluding section.
5.2.1 Earth-hum

To be able to make valid comparisons with existing research on earth-hum, target modes are chosen within a frequency range consistent with comparable reports. Again the earth-hum was divided into the incessant and earthquake excited categories. In both cases modes of the fundamental \((n = 0)\) spheroidal oscillations were investigated. Angular orders of 20 to 50 were chosen for the incessant oscillations and 2 to 6 for the earthquake excited ones. The expected ground displacement, \(E[A_x]\) for the incessant modes range between 4 and 85 nm, this corresponds well with experimentally determined amplitudes [10]. Earthquake excited vibrations were derived for an earthquake of magnitude 8 on the richter scale. The expected ground amplitudes are then 6 to 84 \(\mu m\). These values are consistent with values determined experimentally by Park [15] and with theoretical values given by Ben-Menahem [16] via the equation,

\[
\log A_{x,max}(\mu m) = 1.6M - 10.9 + \log F
\]

where \(M\) is the earthquake magnitude and \(F\) is a number depending on the particular mode \((F \approx 1 \text{ for } \iota S_2)\). The measurement time was chosen as long as possible whilst still being sure of coherent oscillations. The incessant vibrations have a long coherence time, so measurement time is bounded by practicality. 90 days was chosen in line with other research.

The amplitudes of the earthquake excited modes diminish by a factor \(e^{-1}\) during the specific time, \(t_s\),

\[
t_s = \frac{QT}{\pi}
\]

\(T\) is the period and \(Q\) the quality factor of the oscillation. For the \(\iota S_2\) mode \((T = 3223\ \text{s}, Q = 581)\) the specific time is about 7 days. The earthquake-excited oscillations have been seen to be coherent within this time frame. The amplitudes by the end of this time will have reduced to \(2.2 - 30\ \mu m\). Filling all this into 5.13 produces minimum lengths for a RasClic of 7000 and 2800 m for incessant and excited oscillations respectively.

5.2.2 Earth-tide

In chapter 2 the properties of the earth-tide were introduced. Here it was shown that the main constituents could be divided into the categories; Semi-diurnal components with periods around 12 hours and diurnal component with periods around 24 hours. It was also concluded that the oscillations were always coherent. This means that the measurement time is bounded by the resolution required to distinguish the separate modes within each category. This results in a measurement time of 3 months for the semi-diurnal modes and 1 year for the diurnal modes. The amplitudes of the earth-tide oscillations were taken from the theoretical literature values and were seen to be as much as 23 cm for semi-diurnal and 12 cm for diurnal modes. Because the form of the tides is more or less consistent with the \(\iota S_2\) earth-hum mode, equation 5.13 with \(l = 2\), can be used to determine the required length of a RasClic system. The outcome for the semi-diurnal modes was 40 meters and for the diurnal modes 25 meters. Evidence
of the earth-tide should then be detectable from measurements from the current test setup. Unfortunately no long enough series of consistent data could be taken to achieve the required spectral resolution.

5.2.3 Gravitational waves

The idea of using the excitation of the Earth’s or the Moon’s vibrational modes to detect gravitational waves was also introduced in chapter 2. Following on this a proposal for the use of RasClic systems to detect these tiny fluctuations will be backed by suggested lengths for each scenario. The gravitational wave footprint can be detected in the oscillations of the $0S_2$ vibrational mode, so once again equation 5.13 with $l = 2$ can be used. The extremely small amplitudes associated with gravitational modes create a huge challenge for any seismic device. As well as this the oscillations caused by the larger amplitude pulses have a relatively small coherence time. After a specific time of 7 days the amplitudes will have reduced to 1 nm. Using the values as stated above produces lengths into the hundreds of kilometers. This obviously is far from practical. With further development of RasClic however suitable lengths may still be possible. If the image capture and analysis can be improved so as to reduce the attainable CRLB to 0.1 nm the a length of 45 km is obtained.

A few very crude assumptions are made to do a similar investigation for the Moon. The frequency of the Moon’s $0S_2$ mode has been shown to be in the order of 1 mHz, however little is known about the amplitude of these oscillations. Assuming the expected displacements scale with radius of the body then amplitudes of 0.3 nm can be expected. Thanks to the fact that the required length scales directly with the radius of the body, a smaller radius of the Moon this provides a lunar RasClic length of just 22 km.

5.2.4 Conclusions

All of the calculated RasClic lengths have been summarized in table 5.3. RasClic is a very new measurement technique and there is no seismograph like it with which to compare it. Investigations of the slow motions of the SLAC (Stanford Linear Accelerator Center) linac tunnel by A. Seryi [27] did use a similar technique over a distance of 3 km. A 4 quadrant diode was used instead of a CCD image sensor. Analysis of this data showed conclusive evidence of earth-tide movement. It was however not possible to identify the separate earth-tide components due to too short a measurement time and hence a restricted spectral resolution. Using values from this experiment in equation 5.13 gives a length of $\sim 2000$ m which is short of the actual 3000 m but in the right order of magnitude.

The RasClic earth-tide lengths of 25 and 40 meters seem rather short, this is mainly due to the large amplitude of the oscillations and length of measurement time. Lengths much larger than this have already been realized by the current RasClic. For practical reasons no consistent data measurements were able to be taken to allow a distinction between 24 hour temperature effects and tidal forces. To realize this with the current
CHAPTER 5. SIMULATIONS OF THE RASCLIC SYSTEM

Seismic Phenomena | Target modes | $E[A_x](\mu m)$ | $T_{meas}$ (days) | Length (m)
---|---|---|---|---
Earth-hum (background) | $0S_{20} - 0S_{50}$ | 0.004 - 0.085 | 90 | 7000
Earth-hum (quake) | $0S_{2} - 0S_{6}$ | 2.2 - 30 | 7 | 1700
Earth-tide (Semi-diurnal) | $M_2, S_2, N_2$ | 4.4 - 23 cm | 60 | 40
Earth-tide (diurnal) | $K_1, O_1, P_1$ | 4.3 - 12 cm | 365 | 25
Gravitational waves (Earth) $\sigma = 0.1$ nm | $0S_2$ | 0.001 | 7 | 45000
Gravitational waves (Moon) $\sigma = 0.1$ nm | $0S_2$ | $\sim 0.0003$ | 7 | 22000

Table 5.3: Summary of seismograph length results. $\sigma = 10$ nm unless otherwise stated.

RasClic setup, the challenge now lies in eliminating the $1/f$ noise, increasing consistent measurement times and ensuring sufficient coupling between the Earth’s crust and RasClic components.

Increasing the length of RasClic to the 7 km needed for earth-hum measurements will require a lot of engineering but is feasible. As mentioned above a similar experiment has already been conducted at a length of 3000 m. Gravitational wave experiments already use vacuum tubes 3 km long [20] and are proposed for lengths of 10 km [28]. The detection of gravitational waves on Earth seems a rather formidable challenge. Not only does is require a shift estimation resolution of 0.1 nm but a vacuum tube with a length of 45 km. On the Moon however such a vacuum tube is probably not required and the a distance of just 22 km is sufficient. Of course setting up a RasClic on the Moon brings with it a number of other technical challenges. Due to the limited amount of seismic information gathered on the Moon, these calculations rest on very crude estimates of lunar seismic activity. The actual expected displacements and the frequencies of the oscillations will need to be investigated in much more detail.
Chapter 6

Conclusions

Since January 2008 the RasClic test setup has been under continuous development. The displacement resolution has been considerably reduced and the software efficiency improved to allow sampling rates of up to 100 Hz. A double RasClic system has been setup and provides a viable means for comparison. Investigations into seismological phenomena and their relation to a RasClic system have allowed the feasibility of RasClic as a seismometer to be studied under more concrete assumptions.

6.1 Development and simulation of RasClic

Chapter 3 finished with a comparison of the CRLB and real data variance of RasClic systems, with the promise to determine the origin of the difference. A weighted fit had already been implemented for the data used here for comparison and had proven to be very effective in reducing the variance in the data. Improvement by a factor of 10 reduced the data variance from 200 to 20 nm. The implementation of the interpolation algorithm to amend periodicity problems of the DFT improved the accuracy of the shift estimator by up to 0.3 μm in rms error on simulated images. Although this improved accuracy it has no effect on the actual variance of the data. The same can be said for dark-current correction. Image simulations suggested that image properties such as a background step and gradient have little effect (~0.5% of total errors) on the shift estimation. Qualitative analysis of real data proved that dark-current images indeed contain position information and that use of dark-current correction can greatly reduce these influences. The exact quantitative effect this has on real data remains unknown but is assumed to be considerable.

Further simulations showed that pixel noise has got the biggest effect on analysis precision. Reducing this should be the next priority for the development of an ultra precision RasClic system. Reducing the pixel noise will of course also reduce the CRLB by a similar amount due to the linear response of both the CRLB and analysis error to pixel noise level. Therefore it seems the gap between CRLB and data variance can only marginally be improved upon by removing dust and background effects. Lowering the CRLB by reducing pixel noise will be the best way to likewise lower the data variance.
Both simulations and data analysis show that the data variance is of the same order of magnitude as the CRLB, differing by a factor 2 to 6 depending on the diffraction plate. This suggests that the shift estimator is efficient, but still leaves room for improvement. A double RasClic system has proven that two duplicate system are consistent to within 1 μm. The duplicate system provides a means of testing effects from external factors such as temperature and pressure.

### 6.2 RasClic as a Seismograph

RasClic has the potential to become a high precision seismograph. Because of the inherent properties of its static measurement technique RasClic could be developed beyond the limits of conventional seismographs, particularly in the very low frequency range. To accomplish this much work will still need to be done to identify and eliminate the source of the $1/f$ noise currently predominate at low frequencies. If this can be overcome, it has been shown that a RasClic with a length of 7 km will be able to study the earth-hum, both incessant and earthquake excited. With a much shorter length it will be possible to distinguish the earth-tides. No evidence of the earth-tides could be found in the current data because no consistent data measurements long enough to reach the required spectral resolution were available. The detection of gravitational waves will be much more difficult to realize. Even after the realization of a data variance of 0.1 nm a RasClic gravitational wave detector will need a length of 45 km, which is currently physically unfeasible. On the moon however a shorter length of 22 km will be sufficient. Of course this brings other technical problems and challenges with it.
Chapter 7

Recommendations

The most important conclusion of the RasClic simulations is that pixel noise plays a major role in the accuracy of the shift estimator. Reducing the pixel noise could be done by cooling the sensor to limit the thermal excitation of electrons. When designing a diffraction plate it should also be kept in mind that a maximum amount of gradient information in the diffraction pattern in desirable. Also maximizing the amount of light increases the signal to noise ratio of the image sensor. For the test setup the light intensity was sufficient to saturate the brightest pixel: if this is not the case for future designs the use of a lens attached in front of the diffraction plate could be a viable option to increase the light intensity in the diffraction pattern.

Tests of the effect of dark-current correction on the RasClic images showed a definite improvement though the extent of the effect remains vague. More detailed simulations and comparative tests with the duplicate system would provide more concrete evidence to the importance of dark-current correction. The same can be done for flat-field correction. For this a suitable method of acquiring flat-field images needs to be worked out. The time dependence of the dark-current correction will also need to be investigated to be sure the correction factors are in fact representative of the actual dark-current.

The double system that is now operational, provides an excellent way of testing the response of RasClic to external factors. The relationship between temperature or pressure and the difference between the two RasClic measurements could provide useful information to the origin of the $1/f$ noise. The temperature could be varied in the following ways; external temperature by heating the tunnel as a whole, the temperature of the separate components, laser, diffraction plate and sensors. The expansion and contraction of metal involved in the construction of laser, diffraction plate and sensor holders could have a considerable effect on the RasClic measurements. In a similar way the pressure in the vacuum tube as well as the atmospheric pressure in the tunnel should be investigated.

During the calculations to the feasibility of RasClic as a high precision seismograph a number of assumptions were made. Many of these could easily be defended as realistic, others may need further investigation. In order to detect gravitational waves, a displacement resolution of 0.1 nm was assumed, this is a factor 100 times better than
the current version and may require more engineering than reduction of pixel noise. The figures used to arrive at a lunar RasClic length were based on the very limited knowledge of the seismic dynamics of the moon. Further research into this will provide sounder assumptions on vibration mode amplitudes and periods.

The RasClic length derived for the detection of the earth-tide were smaller than the current RasClic test setup length. This would suggest that a thorough investigation of RasClic data should provide evidence of the earth-tide. An attempt should therefore be made to gather enough consistent RasClic data in order to perform analysis. Comparison can be made between the earth-tide level expected around the Geneva area and that found by RasClic.

Finally, thought should be taken to ensuring descent coupling of RasClic components to the Earth’s crust. The rigidity of the concrete tunnel in which the test setup is now situated could distort the response of RasClic to the real ground movement. Finding an alternative underground location where the components can be placed directly onto an earthen rock surface would minimize this effect.
Bibliography


Appendix A

Camera specifications

A.1 PIKE F100B Fiber from Allied Vision

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<td>Power consumption</td>
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Appendix B

RasClic program

This chapter will give a brief explanation of the computer programs that are involved in the shift estimation analysis of RasClic images. A schematic interpretation of the programs and their relation to each other is given in figure B.1.

Figure B.1: Illustration of the components of the RasClic computer program for shift estimation.

config.h

The configuration file. Most of the program configurations can be set in this file. The most important of these are the camera shutter time, gain and bits per pixel (BPP). The extent of the low pass filter is given in the variable RANGE. Controls such as flat-field correction and image output are also set here. Switching between simulation
and production mode is also done by setting the SIMULATION variable.

**interface**

The interface program produces a user friendly display of the status and progress of RasClis in a unix terminal. It can communicate with the other programs to turn them off or toggle certain parameters.

**image pipe**

A pipe is virtual memory storage buffer. It is used to passed images between programs. If there are no images in the pipe a program will wait idle until one is found. This has the benefit that images can be buffered without being lost if a program is temporary slowed down for whatever reason.

**acquire**

Acquire is responsible for the capture of images from the camera (or in simulation mode for the creation of images from formulas of files). Each image is passed on to the image pipe.

**imtee**

Imtee is used to produce human visible images, mainly for diagnostic purposes. How often a image will be outputted is set in the config file. This program is also useful for producing the images needed for flat-field correct. The output files are of tiff format and named according to: DYYYY-DDD-HH-MM.tif. DDD is the number of days since 1 January of the current year, 1 January being 000 (this can be found using the unix command `date +%j` and subtracting 1.

**analyze**

This is where most of the real work happens. Analyze performs the shift estimation and interpolation. All image DFT’s are made using the graphical processing unit (GPU) on the nvidia graphics accelerator card. Once a shift estimation has been done the image data is thrown away and memory space freed. The data calculated by analyze, which is the $x$ and $y$ displacement but also other properties of the image or shift estimator, such as the intensity of the image, are passed onto the data pipe.

**data pipe**

Like the image pipe, the data pipe is a virtual data buffer. This buffer is used to pass the data struct created by analyze onto the filewriter program.
filewriter

As the name suggests filewriter outputs the data from the data pipe into data files. To optimize performance a file is not written after each image has been analyzed. Rather filewriter stores a number of data instances (set in config.h) and prints these all at once into a file. The data files are named according to the format: YYYY-DDD-HH.dat, so that data from one hour is stored in one file.
Appendix C

Algorithms

C.1 Weighted phase fit algorithm

The idea behind the weighed phase fit algorithm is that a translation in the time
domain corresponds to a phase shift in the fourier domain. By plotting the phase of
each fourier component (relative to a reference image) against the wave numbers and
fitting a linear fit that passes through 0 the slope of the fit will provide an estimate of
the displacement of the image. The linear fit is essentially a linear regression with a
weighted least squares fit given by,

$$\chi^2 = \sum \frac{[y_i - f(x_i)]^2}{\sigma_i^2}$$  \hspace{1cm} (C.1)

where $f(x_i)$ is the function that is fitted to the data points $y_i$. The variance of the data
points $\sigma_i^2$ is used as a weighing factor. For the best fit $\chi^2$ should be minimized. In our
case the above equation can be written as,

$$\chi^2 = \sum_{j,k} w_{[j,k]} [\phi_{[j,k]} - (s_x j + s_y k)]^2$$  \hspace{1cm} (C.2)

where $j$ and $k$ are the spatial wave numbers in the $x$ and $y$ direction respectively, $s_x$
and $s_y$ the slope of the fitted regression and hence the displacement of the image, and
$\phi_{[j,k]}$ is the phase. The following paragraphs will discuss how to arrive at the weighing
factors, $w_{[j,k]}$. Firstly same definitions are clarified, to start with the phase;

$$\phi_{[j,k]} = arg \left( \frac{\hat{D}_{[j,k]}}{\hat{R}_{[j,k]}} \right)$$  \hspace{1cm} (C.3)

where $\hat{D}_{[j,k]}$ is the DFT-bin of the sampled image and $\hat{R}_{[j,k]}$ is the DFT-bin of the
reference image. Worked out further into real and imaginary parts of the DFT-bin
components one finds,

\[ \phi_{[j,k]} = \text{arg} \left( \begin{bmatrix} \Re D \cdot \Re R + \Im D \cdot \Im R \end{bmatrix} + \begin{bmatrix} \Im D \cdot \Re R - \Re D \cdot \Im R \end{bmatrix} \right) \]

\[ = \arctan \left( \begin{bmatrix} \Re D_{[j,k]} \cdot \Re R_{[j,k]} - \Re D_{[j,k]} \cdot \Im R_{[j,k]} \\ \Im D_{[j,k]} \cdot \Re R_{[j,k]} + \Im D_{[j,k]} \cdot \Im R_{[j,k]} \end{bmatrix} \right) \]  

(C.4)

Now the weighing factor will depend on the variation in the phase each DFT element, \( \sigma_{[j,k]}^2 \), assuming that the variance in the real and imaginary components of the phase is the same for each DFT element we use this as a normalization of the variance, and the following can be evaluated,

\[ \frac{\sigma_{[j,k]}^2}{\sigma^2} = \left( \frac{\partial \phi_{[j,k]}}{\partial \Re D} \right)^2 + \left( \frac{\partial \phi_{[j,k]}}{\partial \Im D} \right)^2 + \left( \frac{\partial \phi_{[j,k]}}{\partial \Re R} \right)^2 + \left( \frac{\partial \phi_{[j,k]}}{\partial \Im R} \right)^2 \]

(C.5)

To continue these calculations a few substitutions are made;

\[ \phi = \arctan(x) = \arctan \left( \frac{E}{F} \right) \]

\[ E = \Im D \cdot \Re R - \Re D \cdot \Im R \]

\[ F = \Re D \cdot \Re R + \Im D \cdot \Im R \]

We know that,

\[ \frac{\partial \arctan x}{\partial x} = \frac{1}{1 + x^2} = \frac{F^2}{E^2 + F^2} \]

(C.7)

\[ \frac{\partial x}{\partial \Re D} = \frac{-E\Re R - F\Im R}{F^2} \]

\[ \frac{\partial x}{\partial \Im D} = \frac{-E\Im R + F\Re R}{F^2} \]

\[ \frac{\partial x}{\partial \Re R} = \frac{-E\Re D + F\Im D}{F^2} \]

\[ \frac{\partial x}{\partial \Im R} = \frac{-E\Im D - F\Re D}{F^2} \]

(C.8)

Additionally,

\[ E^2 + F^2 = (\Im D \cdot \Re R - \Re D \cdot \Im R)^2 + (\Re D \cdot \Re R + \Im D \cdot \Im R)^2 \]

\[ = [(\Re R)^2 + (\Im R)^2] \cdot [(\Re R)^2 + (\Im R)^2] \]

\[ = |D|^2 \cdot |R|^2 \]
Filling this all back into equation C.5, one gets,

\[
\left( \frac{\partial \phi_{[j,k]}}{\partial R} \right)^2 + \left( \frac{\partial \phi_{[j,k]}}{\partial D} \right)^2 = \left( \frac{\partial \phi_{[j,k]}}{\partial x} \cdot \frac{\partial x}{\partial R} \right)^2 + \left( \frac{\partial \phi_{[j,k]}}{\partial x} \cdot \frac{\partial x}{\partial D} \right)^2 \tag{C.10}
\]

\[
= \frac{E^2(\Re R)^2 + F^2(\Im R)^2 + EF\Re R\Im R}{(E^2 + F^2)^2} \]

\[
+ \frac{E^2(\Re R)^2 + F^2(\Im R)^2 - EF\Re R\Im R}{(E^2 + F^2)^2} \]

\[
= 2 \frac{(\Re R)^2 + (\Im R)^2}{E^2 + F^2} \]

\[
= 2 \frac{|R|^2}{|D|^2 \cdot |R|^2}
\]

and

\[
\left( \frac{\partial \phi_{[j,k]}}{\partial \Re R} \right)^2 + \left( \frac{\partial \phi_{[j,k]}}{\partial \Im R} \right)^2 = \frac{E^2(\Re D)^2 + F^2(\Im D)^2 + EF\Re D\Im D}{(E^2 + F^2)^2} \]

\[
+ \frac{E^2(\Re D)^2 + F^2(\Im D)^2 - EF\Re D\Im D}{(E^2 + F^2)^2} \]

\[
= 2 \frac{(\Re D)^2 + (\Im D)^2}{E^2 + F^2} \]

\[
= 2 \frac{|D|^2}{|D|^2 \cdot |R|^2}
\]

so finally,

\[
\frac{\sigma_{[j,k]}^2}{\sigma^2} = 2 \frac{|D_{[j,k]}|^2 + |R_{[j,k]}|^2}{|D_{[j,k]}|^2 \cdot |R_{[j,k]}|^2} \tag{C.12}
\]

Putting this back into equation C.2, the weighing factor becomes,

\[
w_{[j,k]} = \frac{|D_{[j,k]}|^2 \cdot |R_{[j,k]}|^2}{|D_{[j,k]}|^2 + |R_{[j,k]}|^2} \tag{C.13}
\]

Evaluating \(\chi^2\) further we find,

\[
\chi^2 = \begin{bmatrix}
\sum_{j,k} w_{[j,k]} \phi_{[j,k]}^2 \\
\sum_{j,k} w_{[j,k]} |j|^2 \\
-2 \sum_{j,k} w_{[j,k]} \phi_{[j,k]} j \\
2 \sum_{j,k} w_{[j,k]} |j|^2
\end{bmatrix} + s_x \begin{bmatrix}
\sum_{j,k} w_{[j,k]} |j|^2 \\
-2 \sum_{j,k} w_{[j,k]} \phi_{[j,k]} j \\
2 \sum_{j,k} w_{[j,k]} |j|^2
\end{bmatrix} + s_y \begin{bmatrix}
\sum_{j,k} w_{[j,k]} |k|^2 \\
-2 \sum_{j,k} w_{[j,k]} \phi_{[j,k]} k \\
2 \sum_{j,k} w_{[j,k]} |k|^2
\end{bmatrix} \tag{C.14}
\]

And finally to arrive at the values of the weighted least squares fitted slope and hence the shifted image positions we must calculate \(s_x\) and \(s_y\) for which,

\[
\frac{\partial (\chi^2)}{\partial s_x} = 0, \quad \frac{\partial (\chi^2)}{\partial s_y} = 0 \tag{C.15}
\]
and thus,

\[
\begin{align*}
    s_x &= \frac{\sum_{j,k} w[j,k] \phi[j,k] \cdot \sum_{j,k} w[j,k] \phi[j,k]}{\sum_{j,k} w[j,k] j^2} - \frac{\sum_{j,k} w[j,k] \phi[j,k] k \cdot \sum_{j,k} w[j,k] j k}{\sum_{j,k} w[j,k] j^2} \\
    s_y &= \frac{\sum_{j,k} w[j,k] \phi[j,k] k \cdot \sum_{j,k} w[j,k] \phi[j,k] j \cdot \sum_{j,k} w[j,k] j k}{\sum_{j,k} w[j,k] j^2} - \frac{\sum_{j,k} w[j,k] \phi[j,k] j \cdot \sum_{j,k} w[j,k] j k}{\sum_{j,k} w[j,k] j^2} 
\end{align*}
\]

Lots of homework, minimal code;

```c
void fit_phase_plane(phase_fit_t data, dft_t ref, dft_t dft)
{
    int j, k;
    int ix;
    double rn, fn, w, phase;
    double wpj, wpk, wp2;
    double wj2, wk2, sns;
    double wjk, det;
    const double vf = PIX_VAR * DFTSIZE;
    const double vx = vf * DFTWIDTH * DFTWIDTH;
    const double vy = vf * DFTHEIGHT * DFTHEIGHT;
    wj2 = 0;
    wk2 = 0;
    wpj = 0;
    wpk = 0;
    wp2 = 0;
    sns = 0;
    wjk = 0;
    for (k = -RANGE, ix = 0; k <= RANGE; k++)
    {
        for (j = -RANGE; j <= RANGE; j++, ix++)
        {
            rn = ref[ix][REAL] * ref[ix][REAL] +
                ref[ix][IMAG] * ref[ix][IMAG];
            fn = dft[ix][REAL] * dft[ix][REAL] +
                dft[ix][IMAG] * dft[ix][IMAG];
            if ((rn + fn) < vf) continue;
            w = rn * fn / (rn + fn);
            phase = atan2(dft[ix][IMAG] * ref[ix][REAL] -
                           dft[ix][REAL] * ref[ix][IMAG],
                           dft[ix][REAL] * ref[ix][REAL] +
                           dft[ix][IMAG] * ref[ix][IMAG]);
            wj2 += w * j * j;
            wk2 += w * k * k;
            wpj += w * j * phase;
        }
    }
}
```
C.2 Interpolation Algorithm

A description of the workings of the interpolation algorithm is given in this section. For a full description of all variables please see the documentation made by Hanraads [29]. To start out with the following function is used to build all DFT's from a single reference image. As well as this the factors required for the interpolation are also calculated. This is part of the initialization phase of the image analysis.

```c
void build_dfts ( dftcol_t weights, frame_t in_frame )
{
    int j, k;
    for (k=0; k<DFTREFHEIGHT; k++)
        for (j=0; j<DFTREFWIDTH; j++)
        {
            build_dft ( weights[k][j], in_frame, j-NSHIFTX, k-NSHIFTY );
        }
    const double da_h = M_PI * 2.0 / DFTWIDTH ;
    const double da_v = M_PI * 2.0 / DFTHEIGHT ;
```
for (j=0; j<ASIDE; j++)
{
    a_v[j] = da_v * (j−RANGE);
    a_h[j] = da_h * (j−RANGE);
    sa_v[j] = sin(a_v[j]);
    sa_h[j] = sin(a_h[j]);
    omca_v[j] = 1.0 − cos(a_v[j]);
    omca_h[j] = 1.0 − cos(a_h[j]);
}

Because the fourier transform is a linear operation it is possible to interpolate between two FFT’s. The new interpolated Fourier component will become a linear combination of the original and neighbouring components, in both the real and imaginary parts. The scale to determine to which point the interpolation needs to take place is given by , w, and is dependent on the phase angle between the two components, α, and the new desired angle uα. u is given by the original shift estimate or the previous interpolation. The following figure shows how the interpolation is done. From the figure it is easy to

![Figure C.1: Illustration for derivation of interpolation parameters.](image)

show that,

\[
\tan(uα) = \frac{w(u, α)\sin(α)}{[1 − w(u, α)] + w(u, α)\cos(α)} \tag{C.17}
\]

from which \(w(uα)\) is calculated,

\[
w(u, α) = \frac{1}{[1 − \cos(α)] + \sin(α)/\tan(uα)} \tag{C.18}
\]

This is the formula used in the algorithm. It should be noted that the modules of the complex number are not conserved, This can be compensated for but is not implemented because of the very small effect it has on the phase plane fit used for the shift estimation.

```c
int comp_new_ref(dftcol_t weights ,
                 dft_t ref ,
                 double in_x , double in_y )
```
C.2. INTERPOLATION ALGORITHM

```c
{
  int j, k;
  double a, b;
  int u, v, uv;

  j = (int)(in_x);
  k = (int)(in_y);
  a = in_x - j;
  b = in_y - k;
  if (a < 0) { a++, j--; }
  if (b < 0) { b++, k--; }

  if ((j < -NSHIFTX) || (j >= NSHIFTX - 1)) return 1;
  if ((k < -NSHIFTY) || (k >= NSHIFTY - 1)) return 1;
  j += NSHIFTX;
  k += NSHIFTY;

  for (v=0; v<ASIDE; v++)
    if (((b==0.0)||(v==RANGE)) w_b[v] = 0.0;
    else w_b[v] = 1.0/(omca_v[v]+sa_v[v]/tan(b*a_v[v]));

  for (u=0; u<ASIDE; u++)
    if (((a==0.0)||(u==RANGE)) w_a[u] = 0.0;
    else w_a[u] = 1.0/(omca_h[u]+sa_h[u]/tan(a*a_h[u]));

  for (v=0, uv=0; v<ASIDE; v++)
    {
      double wb = w_b[v];
      double omwb = 1.0 - wb;
      for (u=0; u<ASIDE; u++, uv++)
        {
          double wa = w_a[u];
          double omwa = 1.0 - wa;
          ref[uv][REAL] =
            omwa*omwb*weights[k][j][uv][REAL] +
            wa*omwb*weights[k][j+1][uv][REAL] +
            omwa*wb*weights[k+1][j][uv][REAL] +
            wa*wb*weights[k+1][j+1][uv][REAL];

          ref[uv][IMAG] =
            omwa*omwb*weights[k][j][uv][IMAG] +
            wa*omwb*weights[k][j+1][uv][IMAG] +
            omwa*wb*weights[k+1][j][uv][IMAG] +
            wa*wb*weights[k+1][j+1][uv][IMAG];
        }
    }
}
```
} return 0;
}
#endif
Appendix D

Seismograph simulation calculations

D.1 Earth-tide

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## D.2 Earth-hum

### Background free oscillations

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Radius Earth (m): 6.36E+06

Sample rate (Hz): 100

Measurement time (days): 90.0
### D.3 Gravitational waves

#### Gravitational Wave excited oscillations - Earth

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<th>Amplitude (nGal)</th>
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#### Gravitational Wave excited oscillations - Moon

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<th>Angular degree</th>
<th>Frequency (Hz)</th>
<th>Amplitude (nGal)</th>
<th>Ground amp. (m)</th>
<th>Length^4</th>
<th>Length (m)</th>
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**Earthquake excited oscillations**

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<th>Ground amp. (m)</th>
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