

Announcements

Welcome to Gravitational Waves Msc course at Nikhef!

Instructor: Dr. Sarah Caudill (physarah@gmail.com)

Teaching Assistant: Dr. Khun Sang Phukon (k.s.phukon@nikhef.nl)

Let me know your preferred email address.

Homework #1, syllabus, final project documents, and lecture slides are uploaded to <https://www.nikhef.nl/~caudills/teaching.html>

Homeworks will be due 1 week after assignment. Please email to Khun Sang (k.s.phukon@nikhef.nl).

Some Lecture Notes

- Sean Carroll's GR notes and other resources: <https://www.preposterousuniverse.com/grnotes/>
- B.S. Sathyaprakash & Bernard Schutz, Physics, Astrophysics and Cosmology with Gravitational Waves, *Living Reviews in Relativity* **12**: 2 (2009): <https://link.springer.com/article/10.12942/lrr-2009-2>
- Alessandra Buonanno's GW lecture notes: <https://arxiv.org/pdf/0709.4682.pdf>

Summary of the field

Lecture 1: Gravitational Waves MSc Course

- What are gravitational waves?
 - Detectors
 - Sources
- A brief history of the field
- Some terminology
- Current state of the field
 - Notable detections
 - Science results
- Next steps
 - Detectors
- Homework

What are gravitational waves?

Einstein field equations:

$$G_{\mu\nu} = 8\pi \frac{G}{c^4} T_{\mu\nu}$$

description of
geometry of
spacetime

description of how
matter/energy is
distributed

Spacetime tells matter how to move; matter tells spacetime how to curve. - John Wheeler

What are gravitational waves?

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$$G_{\mu\nu} = 8\pi \frac{G}{c^4} T_{\mu\nu} = 0 \text{ (in vacuum)}$$

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What are gravitational waves?

Einstein field equations:

$$G_{\mu\nu} = 8\pi \frac{G}{c^4} T_{\mu\nu} = 0 \text{ (in vacuum)}$$

Far from the source: metric is flat with small perturbation

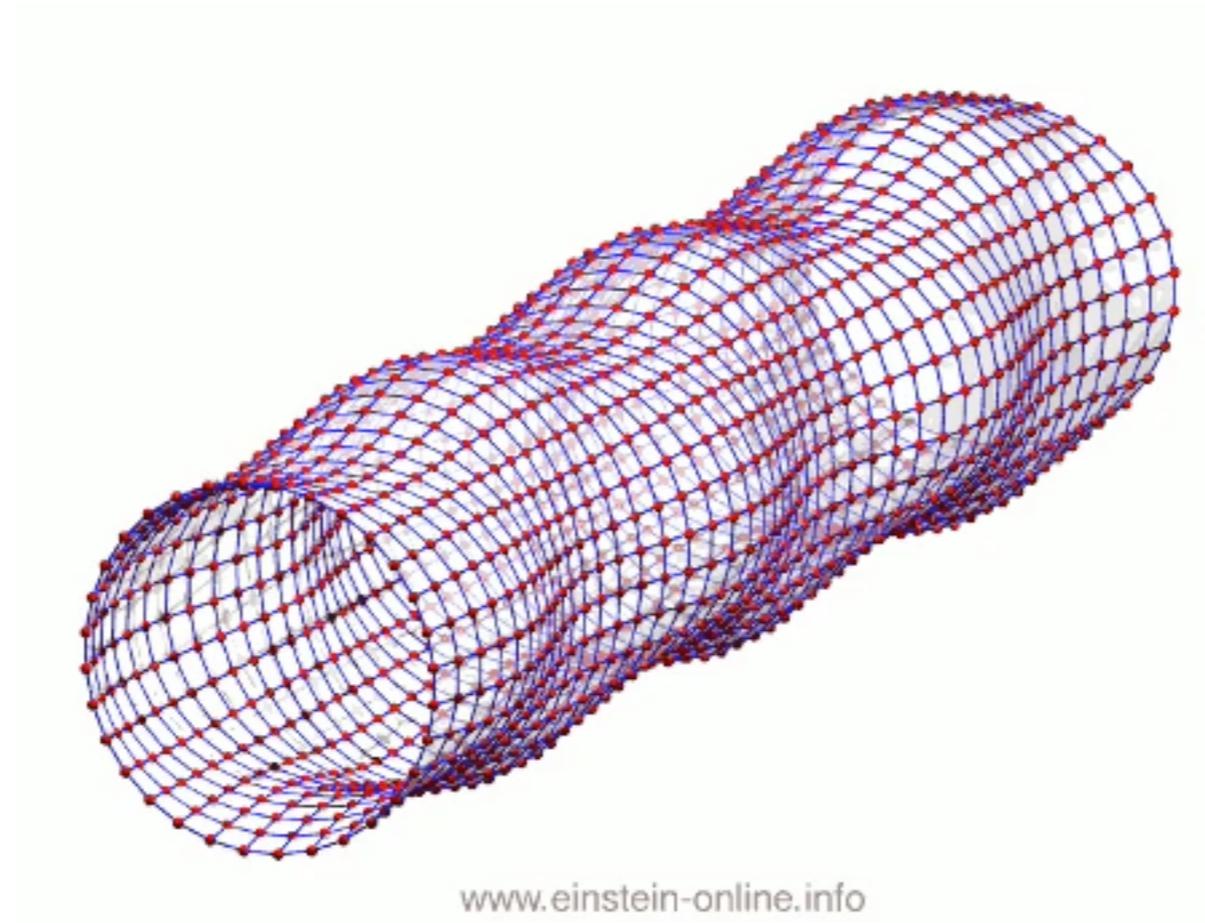
$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Far from the source: Einstein equations reduce to wave equation for the perturbation

$$\left(-\frac{\partial^2}{c^2 \partial t^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) h_{\mu\nu}^{\text{TT}} = 0$$

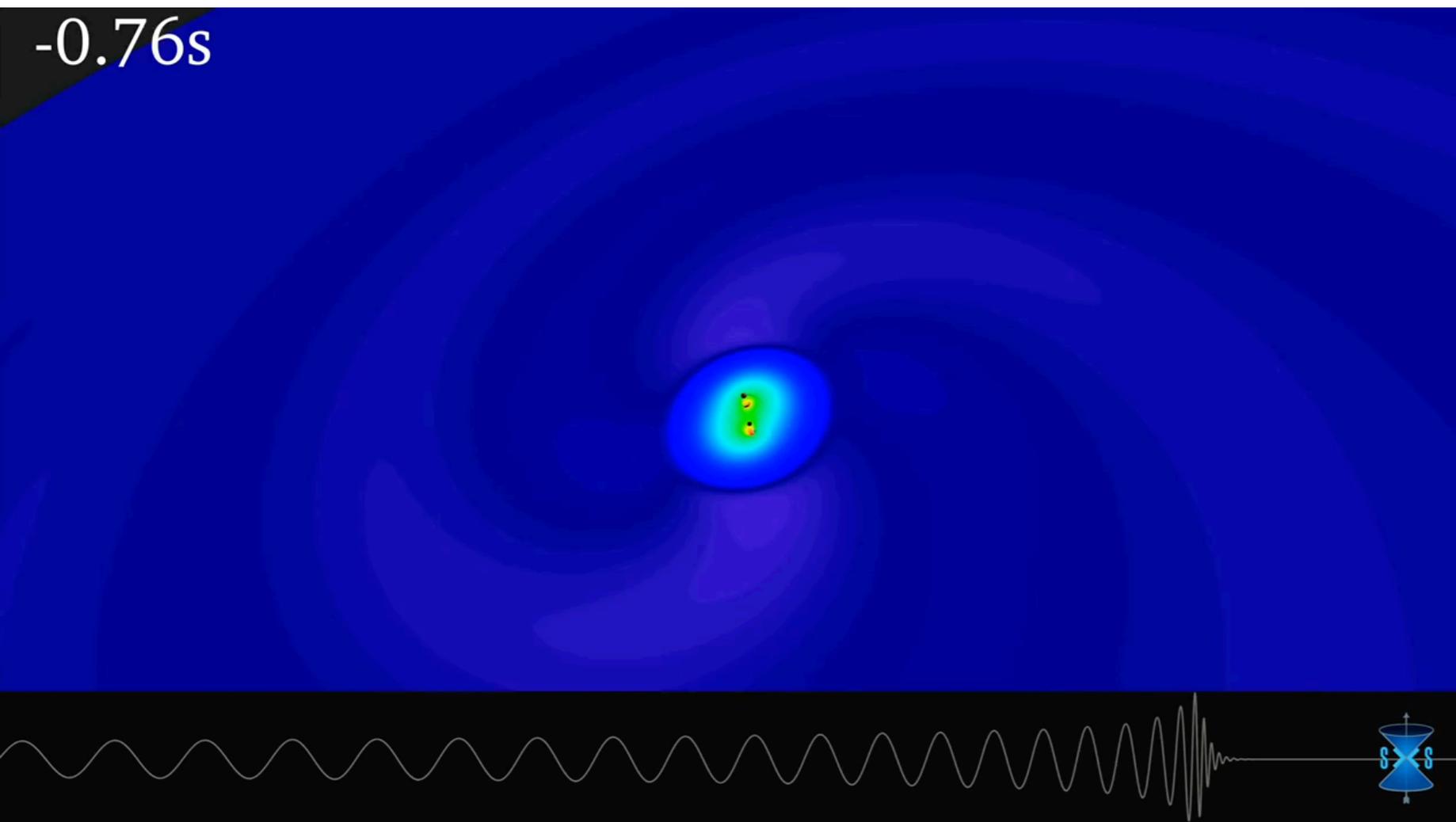
What are the properties of these waves?

Propagation: light speed, have effect of traveling tidal waves



What are the properties of these waves?

Generation: Accelerating masses (changing quadrupole and higher multipole moments)



$$h_{\mu\nu} \sim \frac{1}{R} \frac{d^2 Q_{\mu\nu}}{dt^2}$$

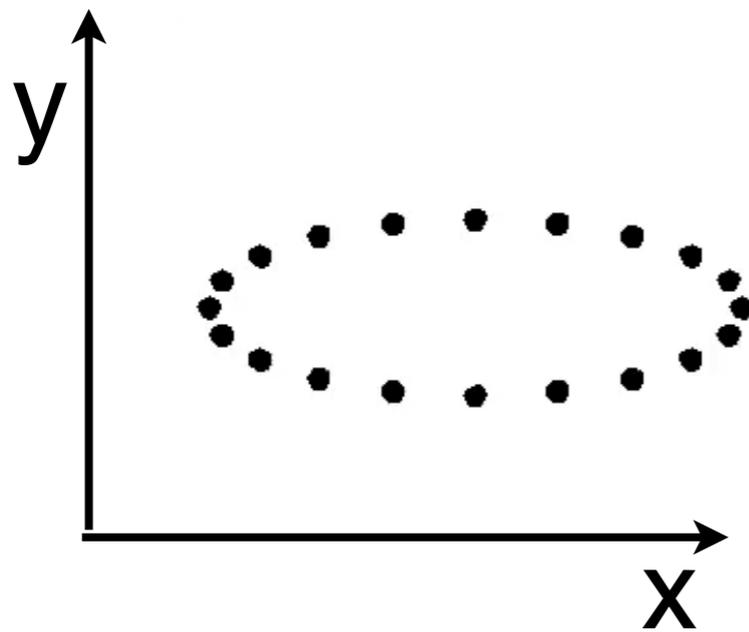
$$\sim \frac{G}{c^4} \frac{T}{R}$$

$$\sim 10^{-22}$$

T = Kinetic Energy

What are the properties of these waves?

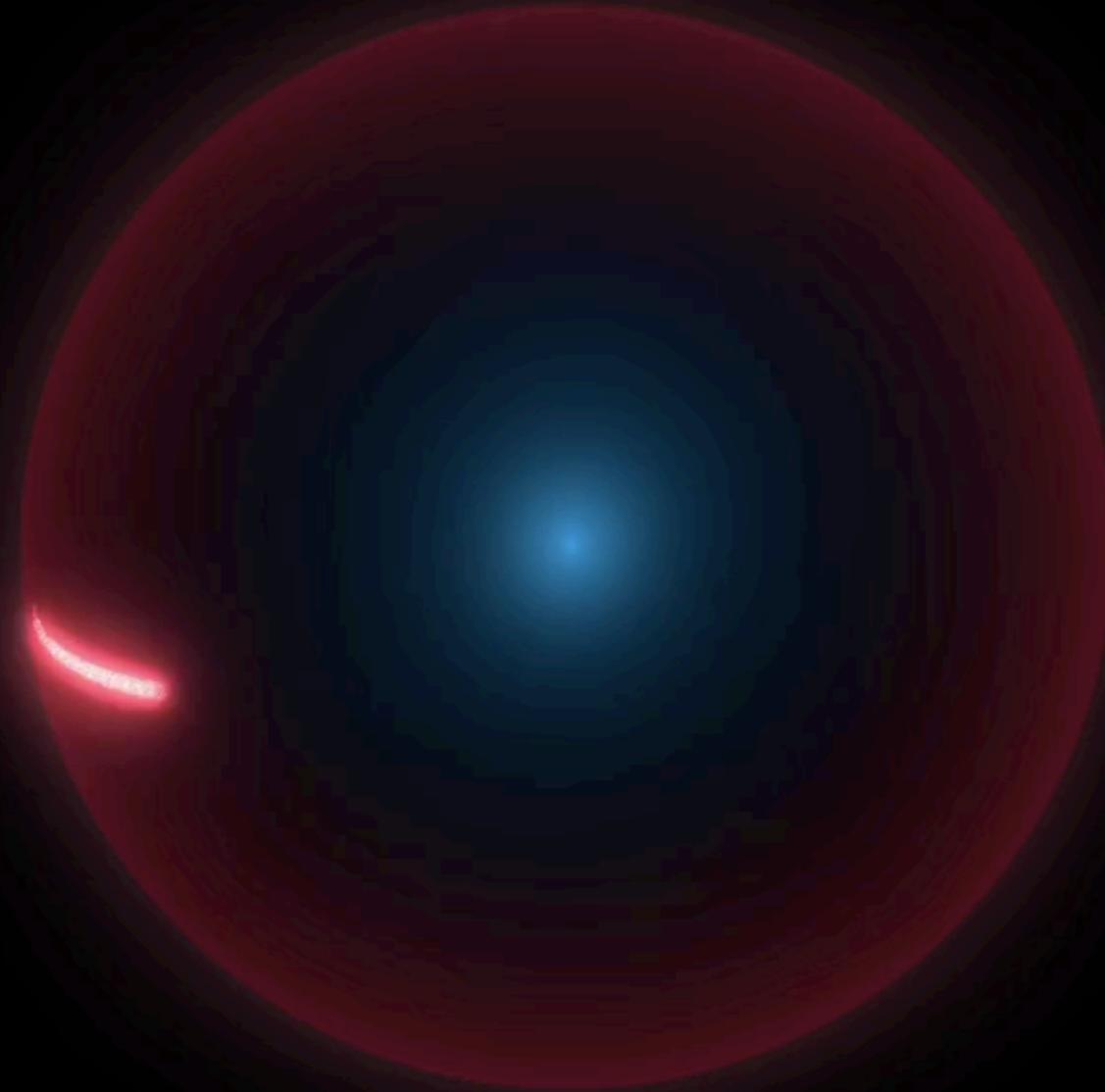
Measurability: Induce a strain on a ring of test masses; two independent polarizations



$$h = \frac{\delta L}{L}$$

$$\Delta L = L_x - L_y = hL$$

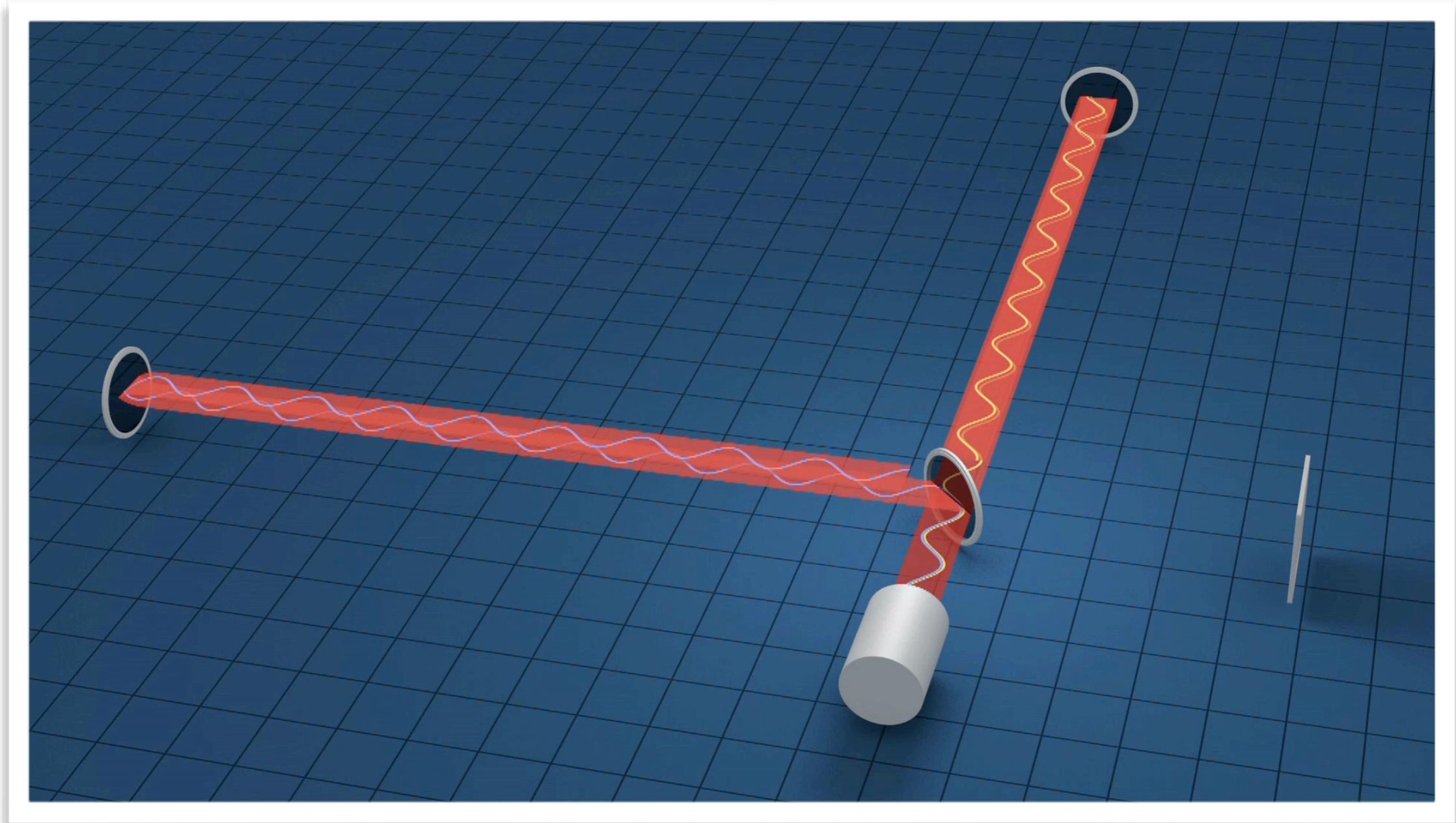
Example: Binary star system in Virgo cluster (16.5 Mpc away) would produce $h \sim 10^{-21}$. Over a distance of $L = 1$ AU, ΔL would be ~ 1 atomic diameter.



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Ground-based Interferometric Detector

$$h = \frac{\delta L}{L}$$



The Advanced LIGO Detectors



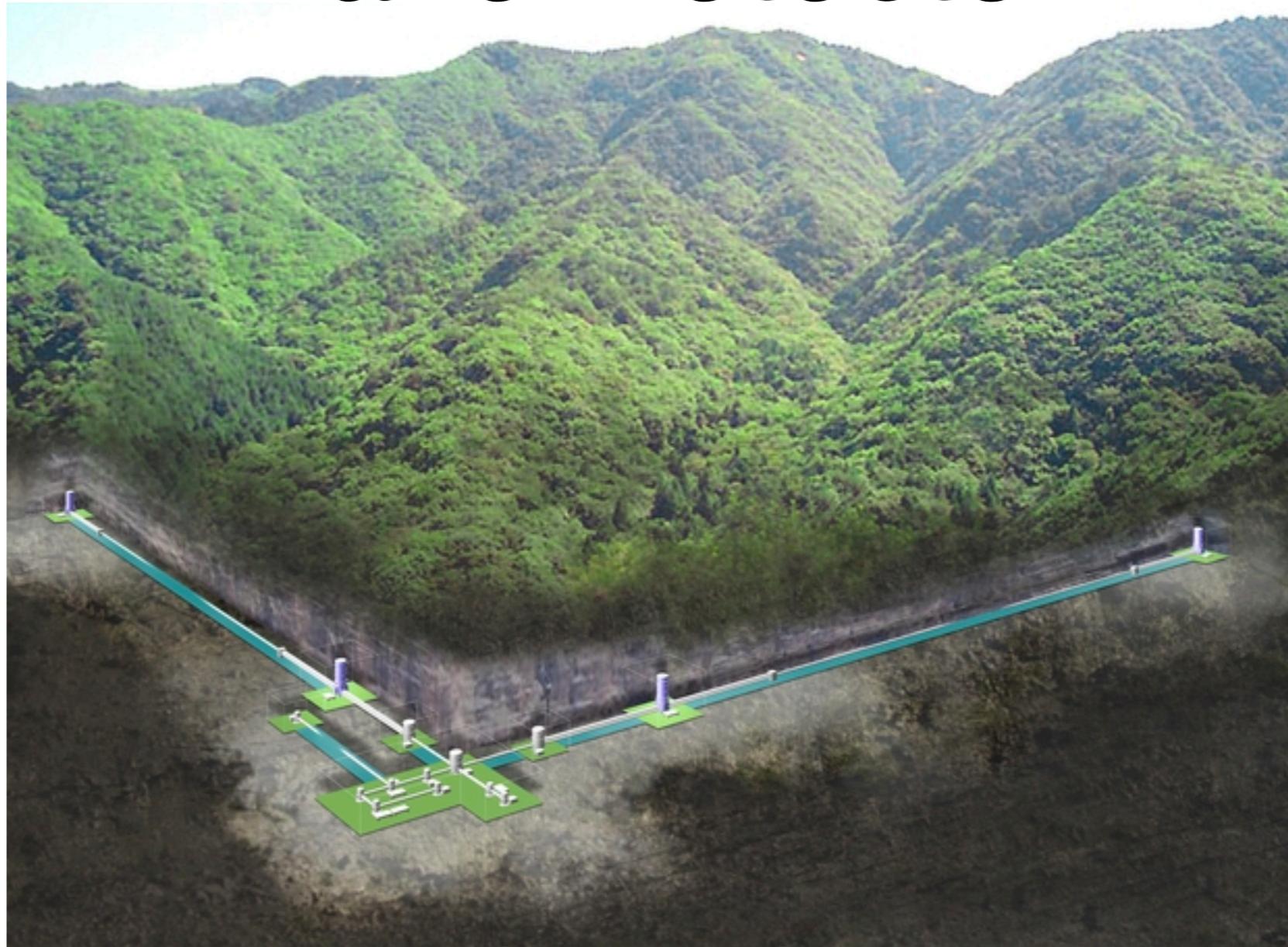
- LIGO - Laser Interferometer Gravitational-wave Observatory
- Operated out of Hanford, WA and Livingston, LA USA
- 4 km-long arms

Advanced Virgo Detector



- Operated out of Cascina, Italy
- 3 km-long arms
- France, Italy, Netherlands, Poland, Hungary

KAGRA - Kamioka Gravitational-wave Detector



- Built in Kamioka Observatory in Japan
- 3 km-long arms
- Ready for first observation run

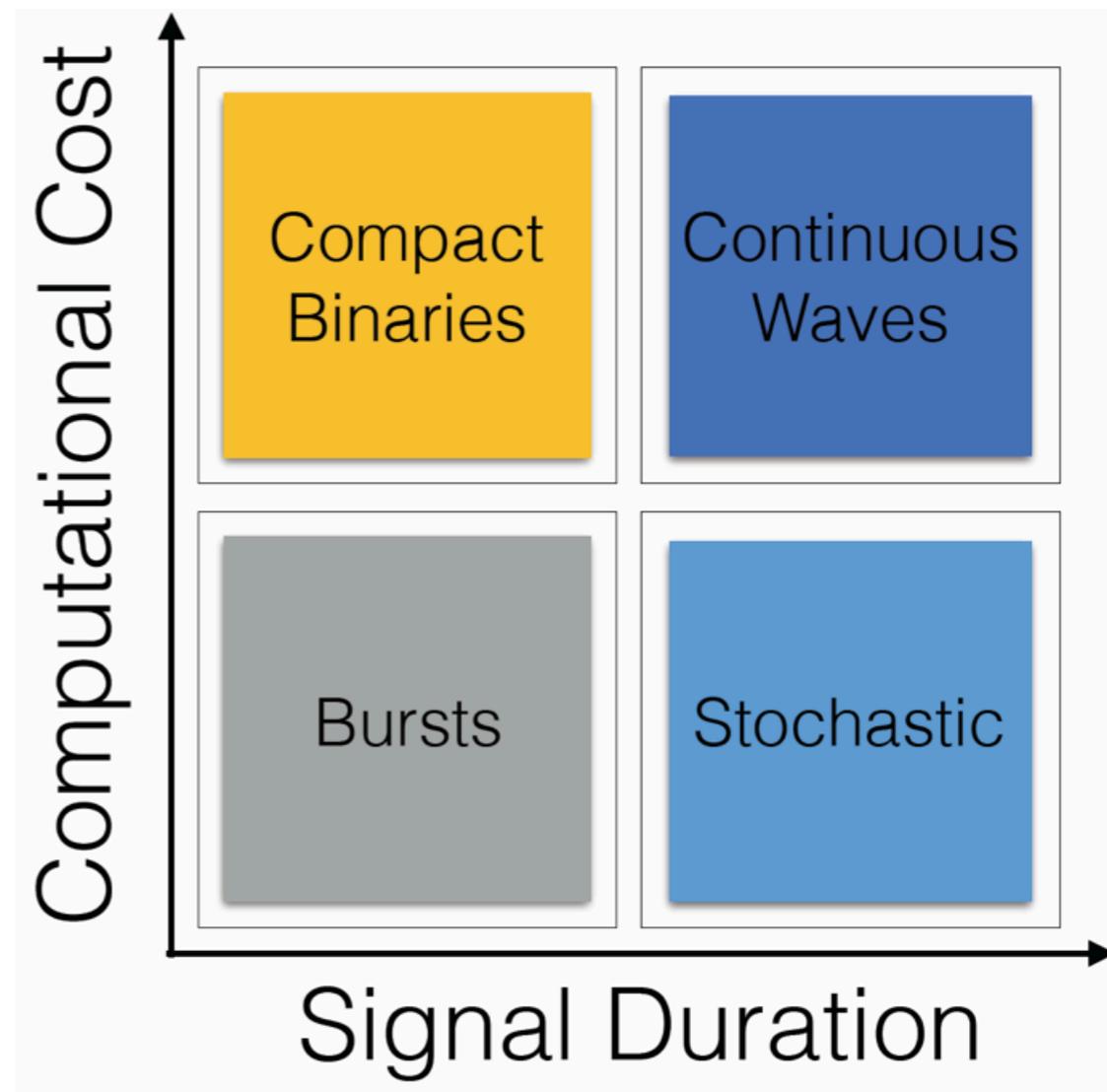
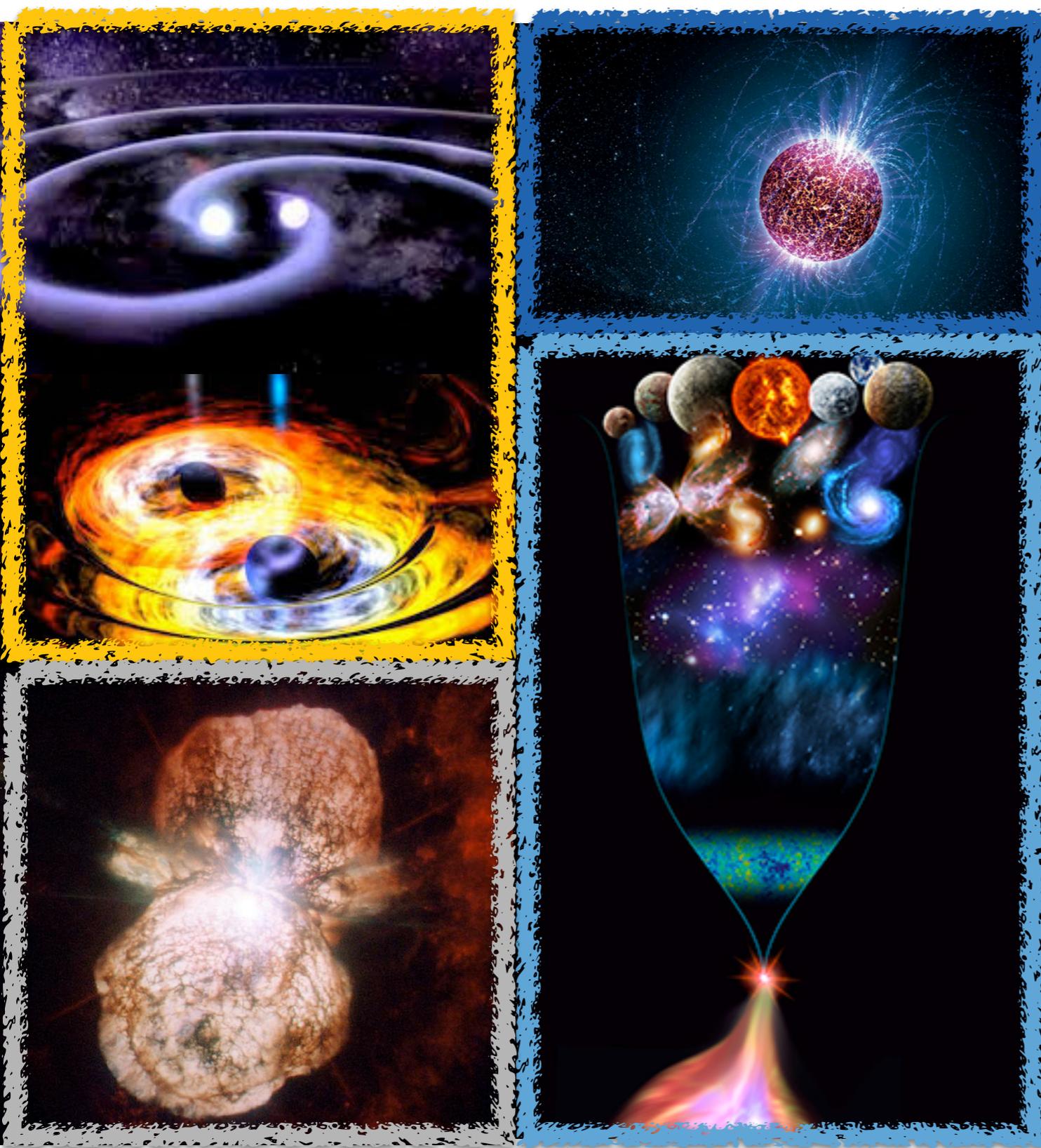
GEO600



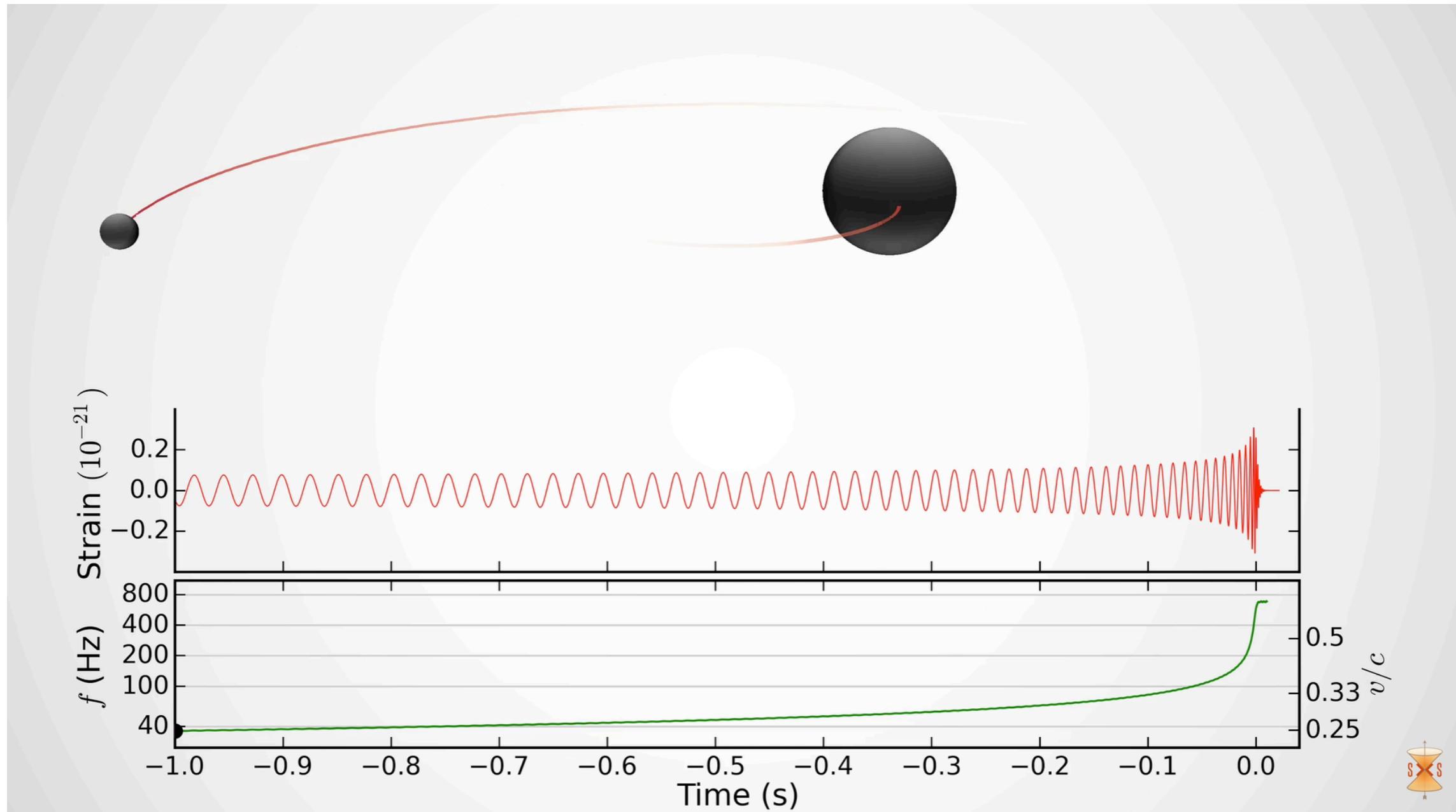
- GEO600, test-bed for new technologies: laser stabilization, absorption-free optics, control engineering, vibration damping, etc.
- “Squeezed light” technology - mitigating shot-noise

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Gravitational-wave Sources for Ground-based Detectors



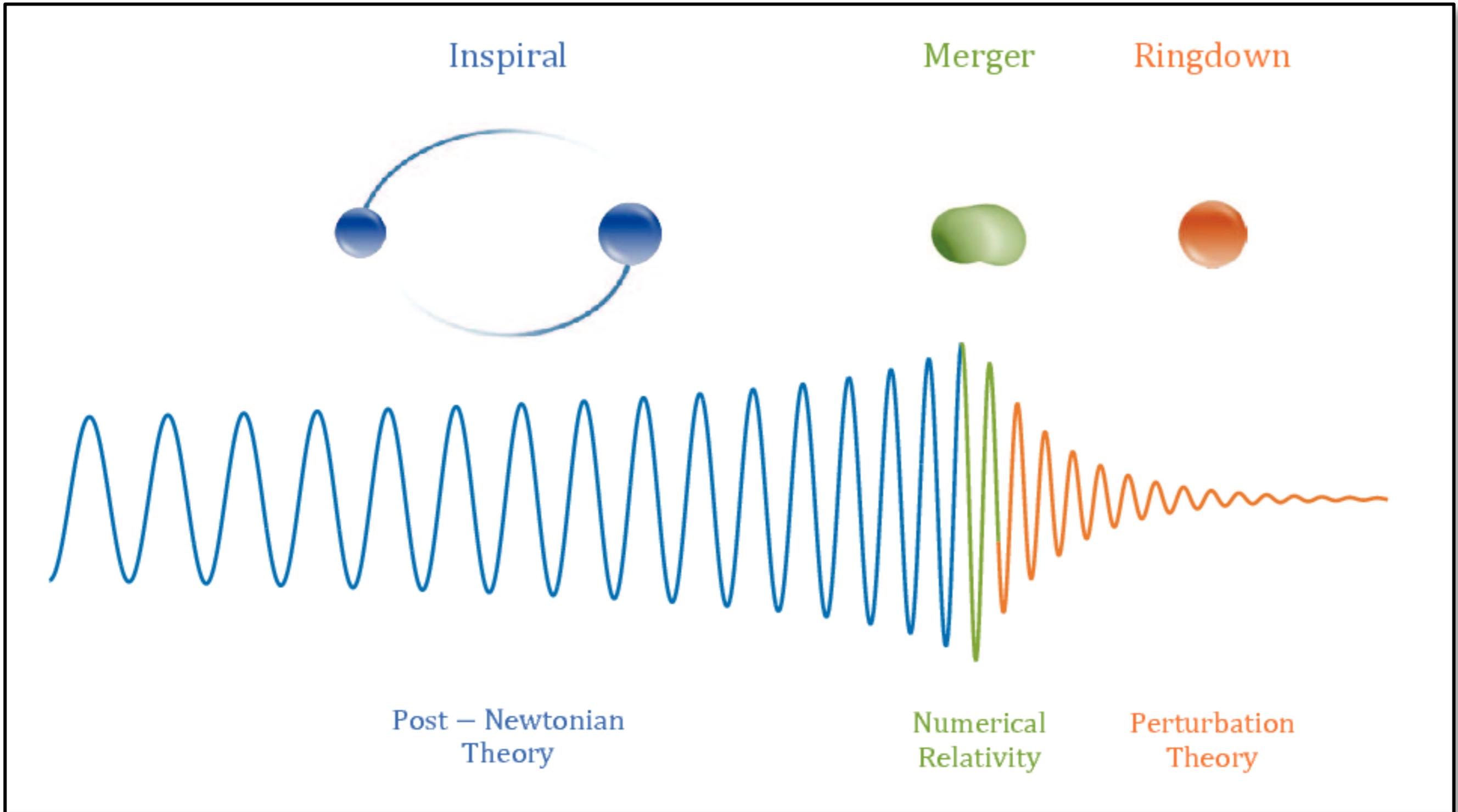
Compact Binary Coalescence Sources



Binaries in the last few seconds before merger:

- neutron stars
- stellar mass black holes
- intermediate mass black holes

Compact Binary Coalescence Waveforms

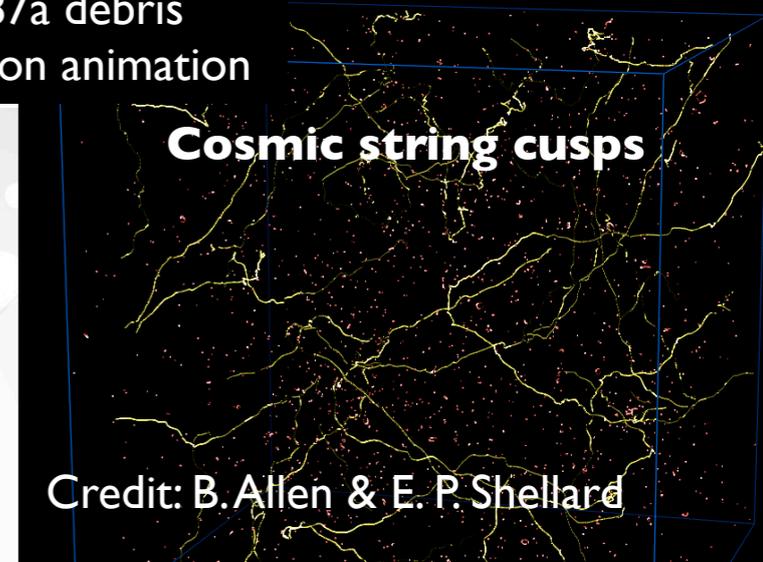


Burst Sources of Gravitational Waves

Transient GW astronomy using minimal assumptions.

Sources could include:

- core collapse SN
- merger phase of heavy binary compact objects
- NS instabilities
- accretion disk instabilities
- fall back accretion, cosmic string cusps/kinks
- unexpected...



Continuous Gravitational Wave Sources

Non-axisymmetric rotating neutron stars; asymmetry could arise from:

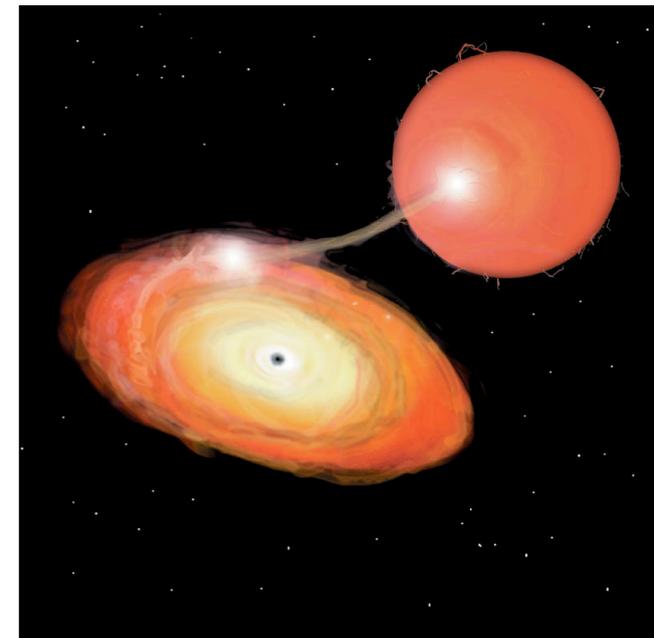
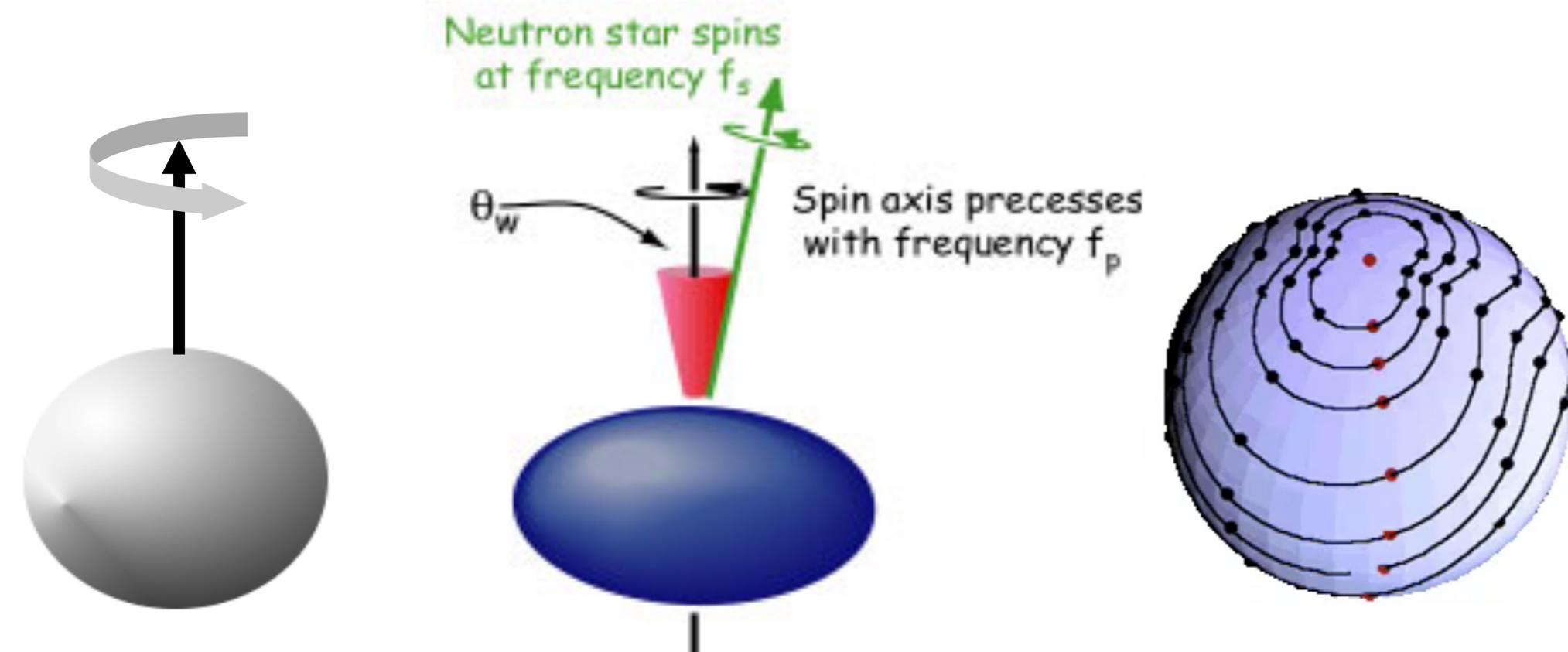
- equatorial ellipticity (mm-high mountain)
- free precession around rotation axis
- excitation of long-lasting oscillations
- deformation due to matter accretion

$$f_{\text{GW}} = 2f_{\text{rot}}$$

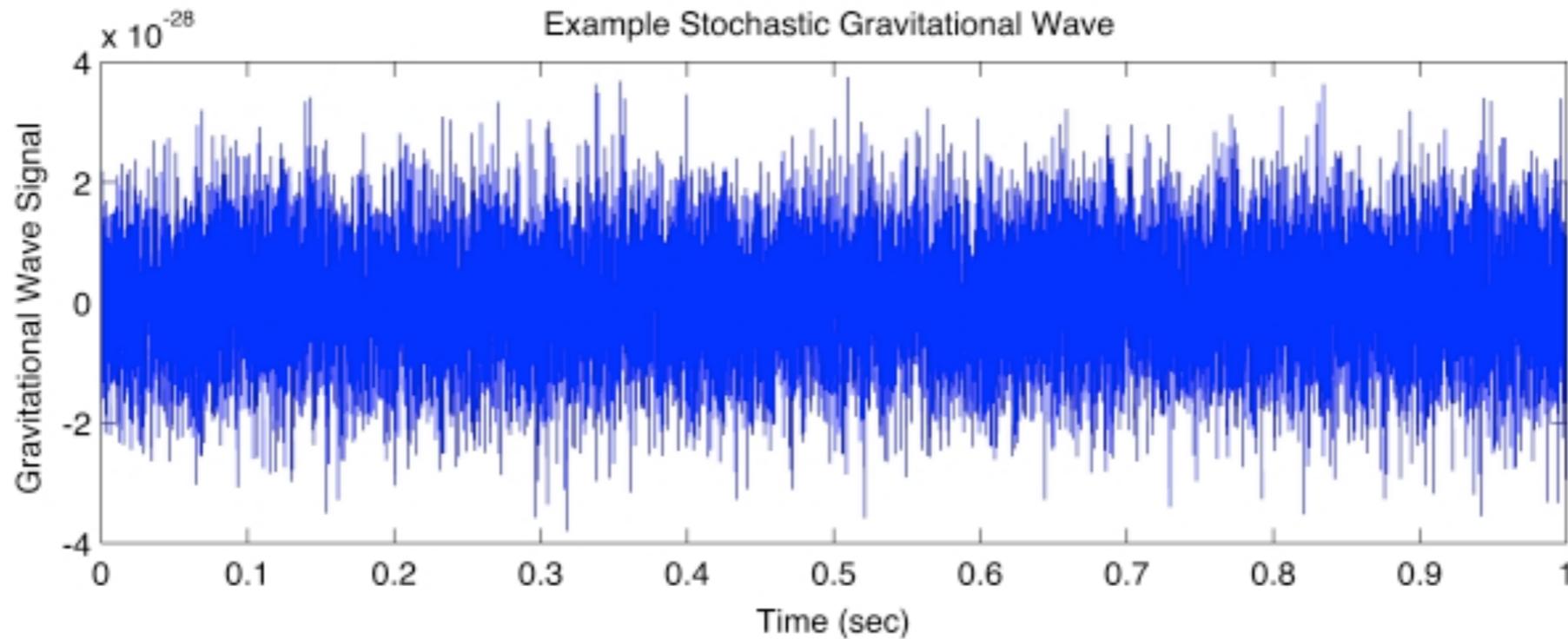
$$f_{\text{GW}} \sim f_{\text{rot}} + f_{\text{prec}}$$

$$f_{\text{GW}} \sim 4/3 f_{\text{rot}}$$

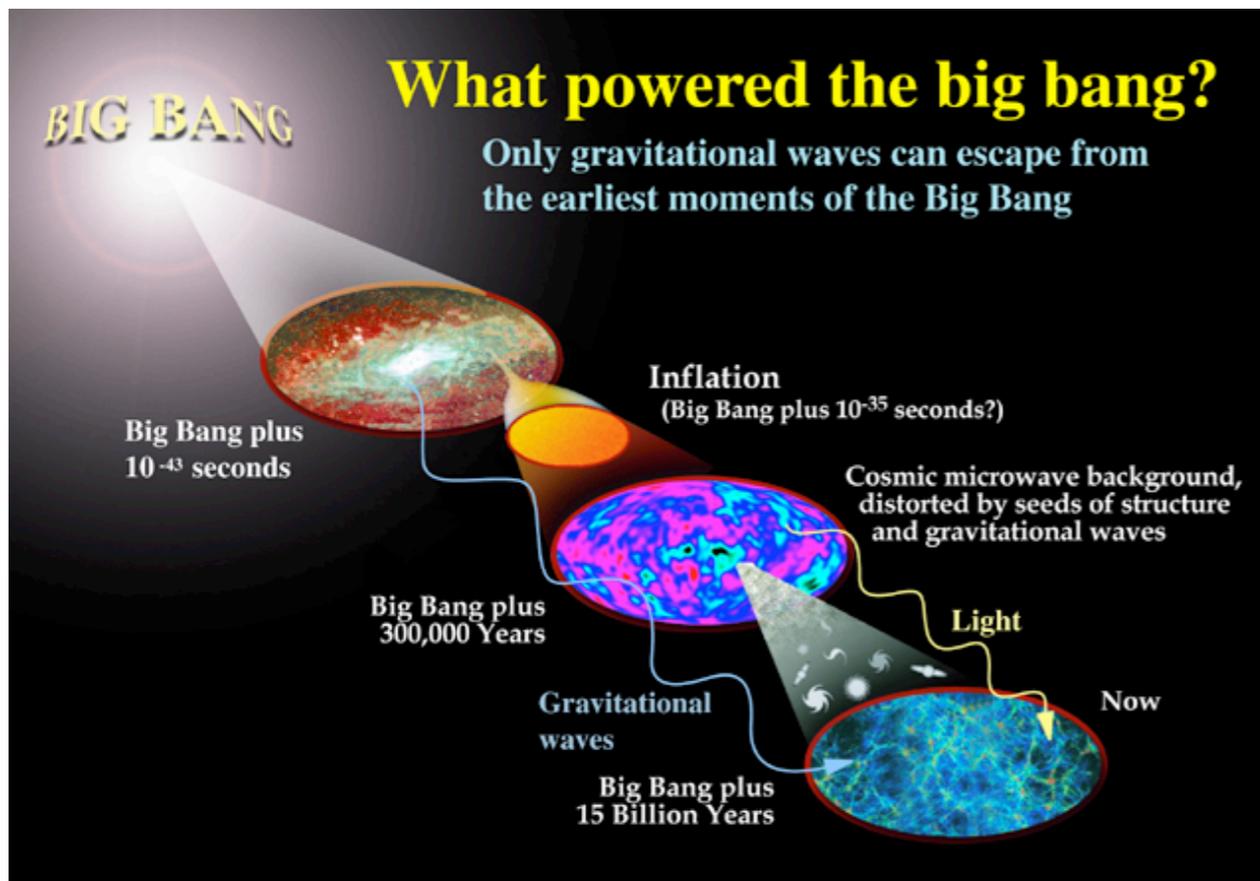
$$f_{\text{GW}} = 2f_{\text{rot}}$$



Stochastic Gravitational Waves



I. Cosmological origin



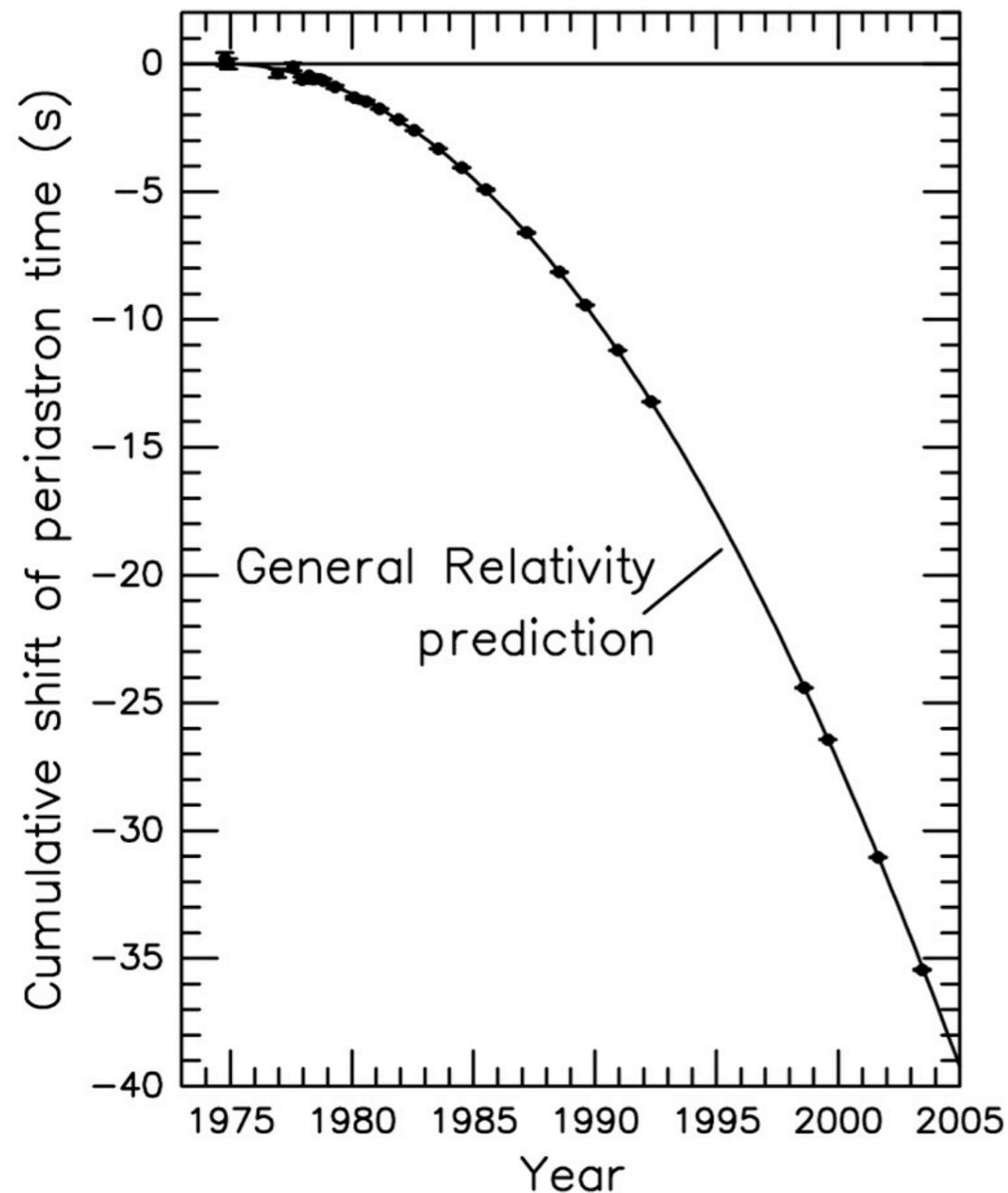
II. Astrophysical origin



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First Observational Evidence

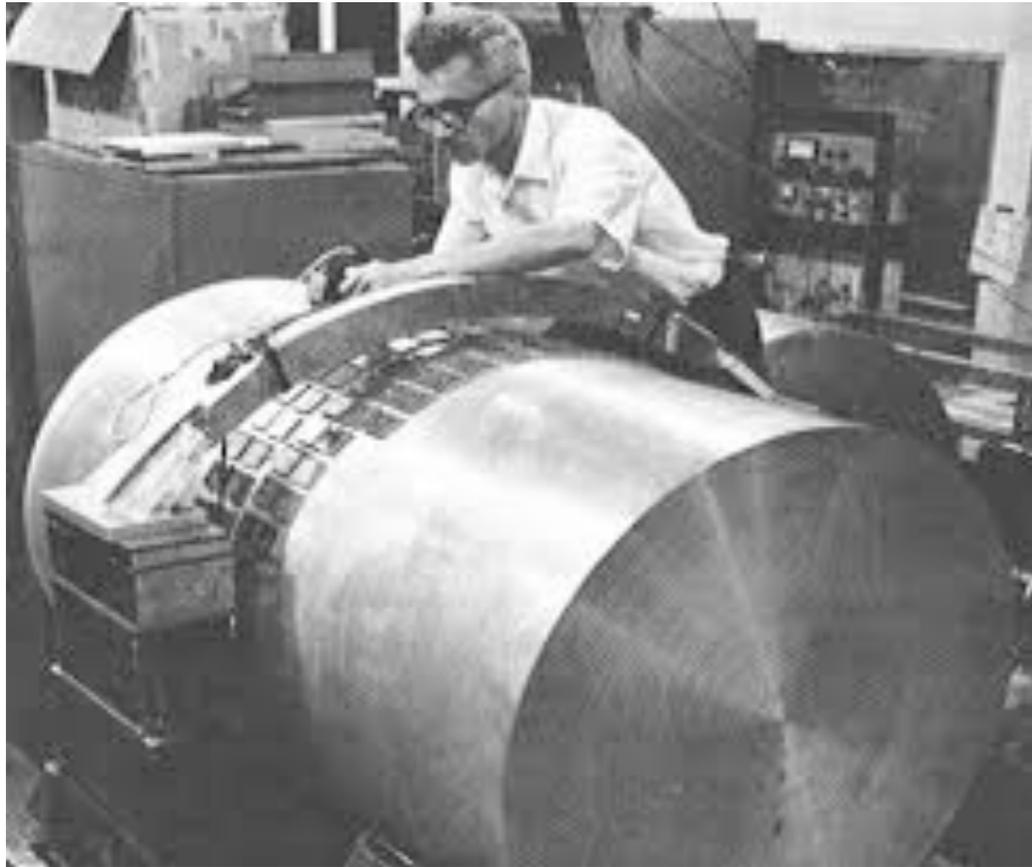
Hulse-Taylor Binary Pulsar



Pulsar-Neutron Star System

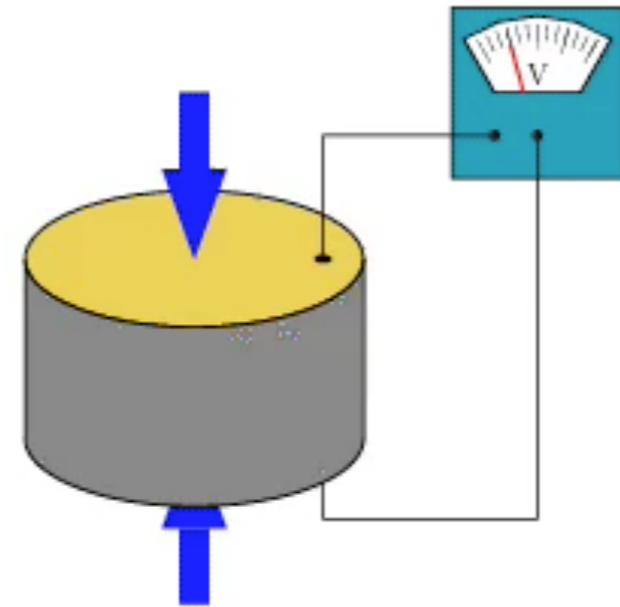
- Period: 7.75 hours
- Discovered by R. Hulse and J. Taylor
- Awarded 1993 Nobel prize

Bar Detectors



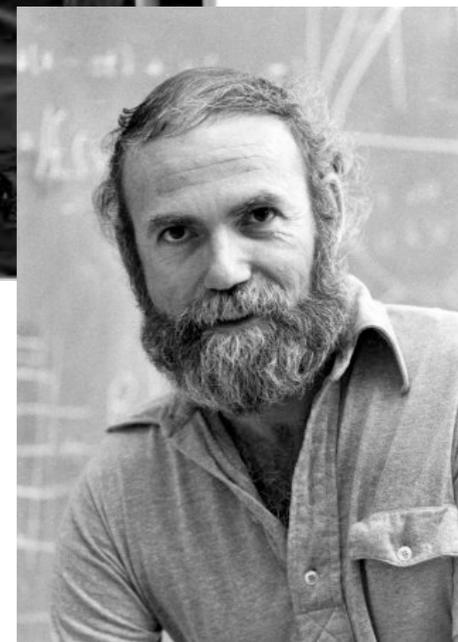
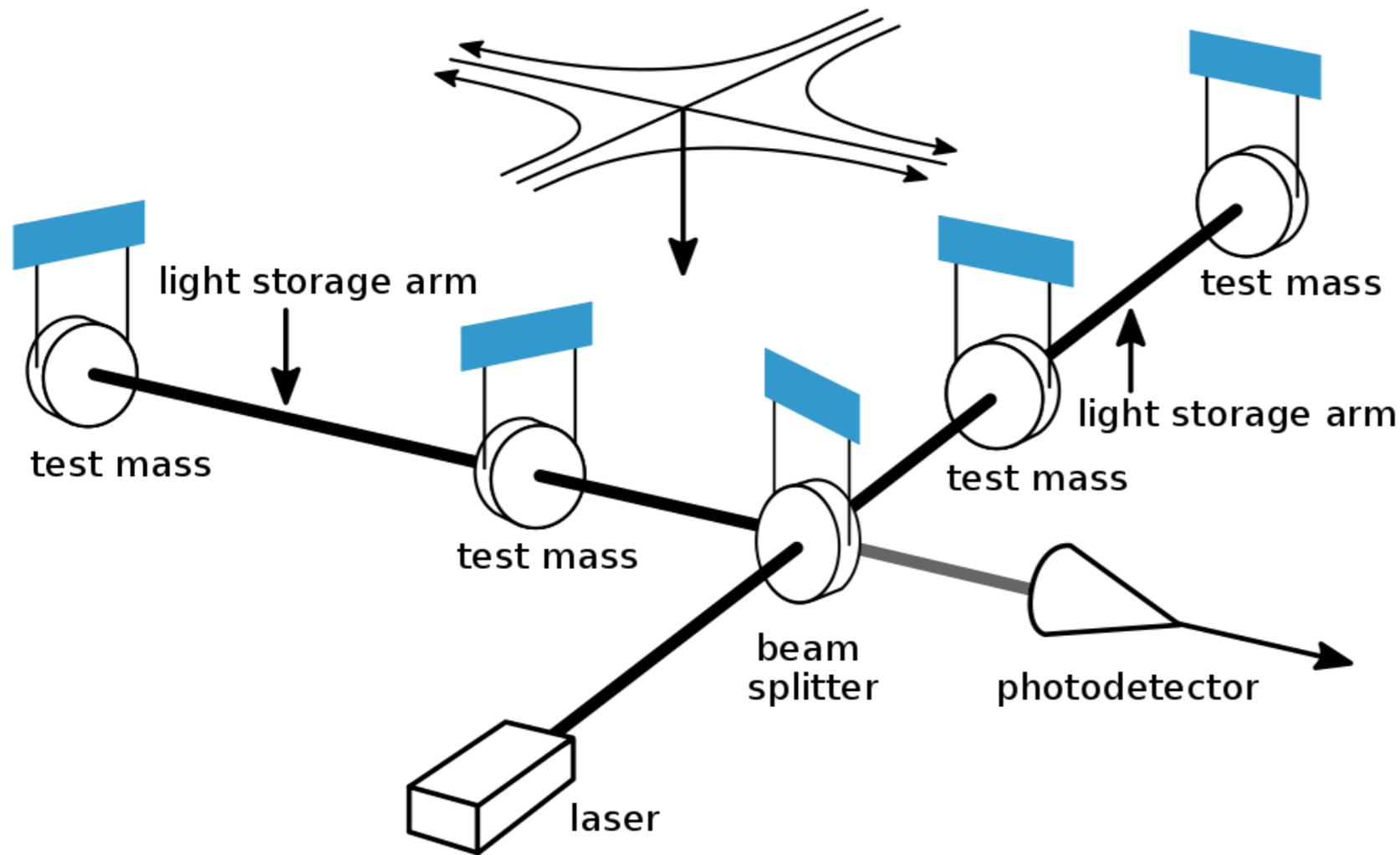
Joseph Webber -
University of Maryland
1961: proposed to use
resonant bar detectors
to detect GWs

Piezoelectric sensors



Only sensitive over
very narrow range of
frequencies

Interferometric Detectors



- 1970s: Development of precision measurement laser technology
- Early 1980s: Prototypes built in Glasgow, Garching, MIT
- 1994: LIGO construction begins...
- 1996: Virgo construction begins

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Power Spectral Density

$$s(t) = n(t) + h(t)$$

$$\tilde{x}(f) = \mathcal{F}\{x(t)\}(f) = \int_{-\infty}^{\infty} dt x(t) \exp(-2\pi i f t)$$

Energy in time series is $E = \int_{-\infty}^{\infty} |n(t)|^2 dt = \int_{-\infty}^{\infty} |\tilde{n}(f)|^2 df$

Parseval's theorem

f - frequency in Hz, i.e. cycles per second

$|\tilde{n}(f)|^2$ - density function describing energy per unit frequency in time series at frequency f

Energy spectral density of signal is defined as $S(f) = |\tilde{n}(f)|^2$

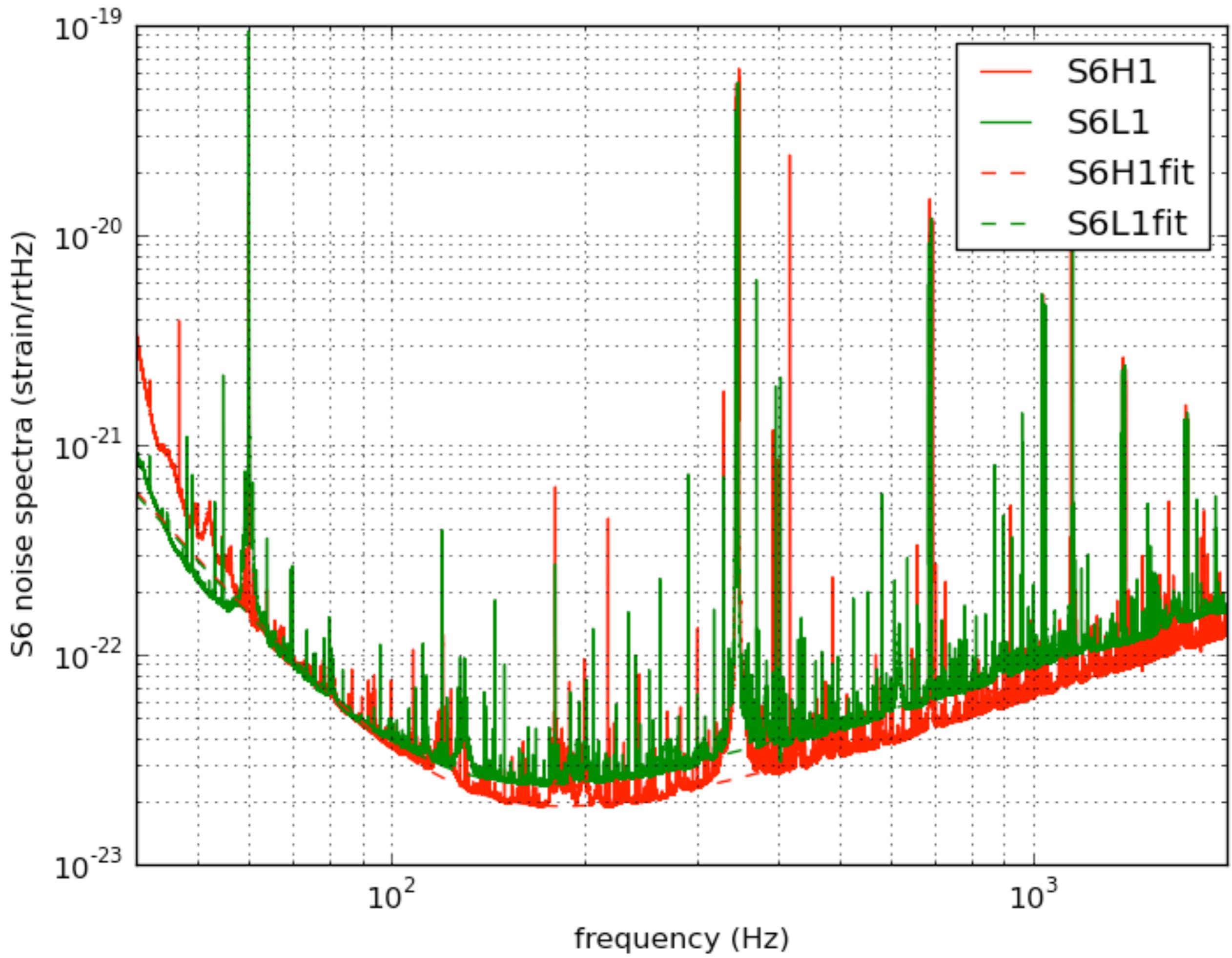
Power Spectral Density

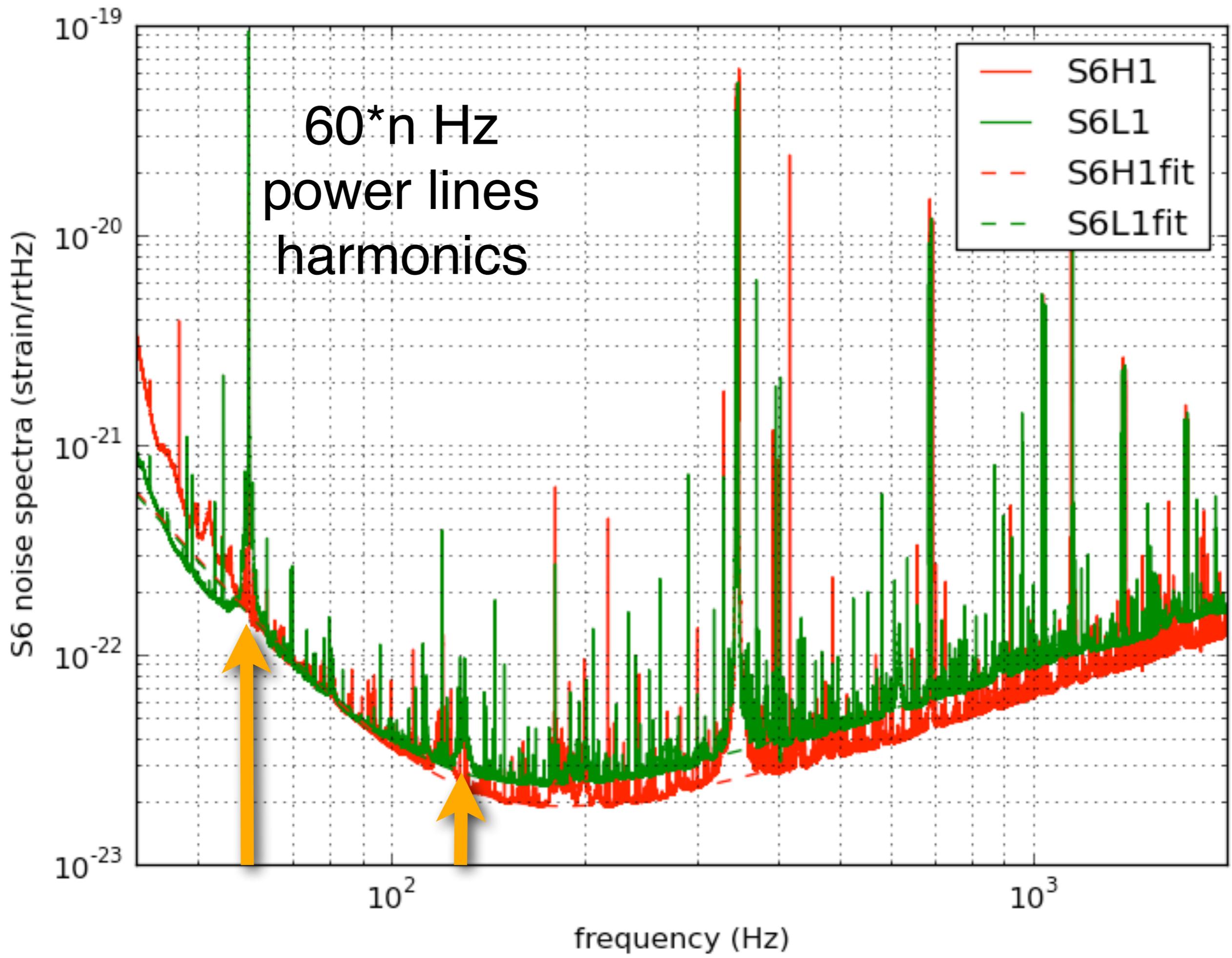
If the energy is spread over long-ish time interval, we instead define the power spectral density (PSD):

$$S(f) = \lim_{T \rightarrow \infty} \mathbb{E} \left[|\tilde{n}(f)|^2 \right]$$

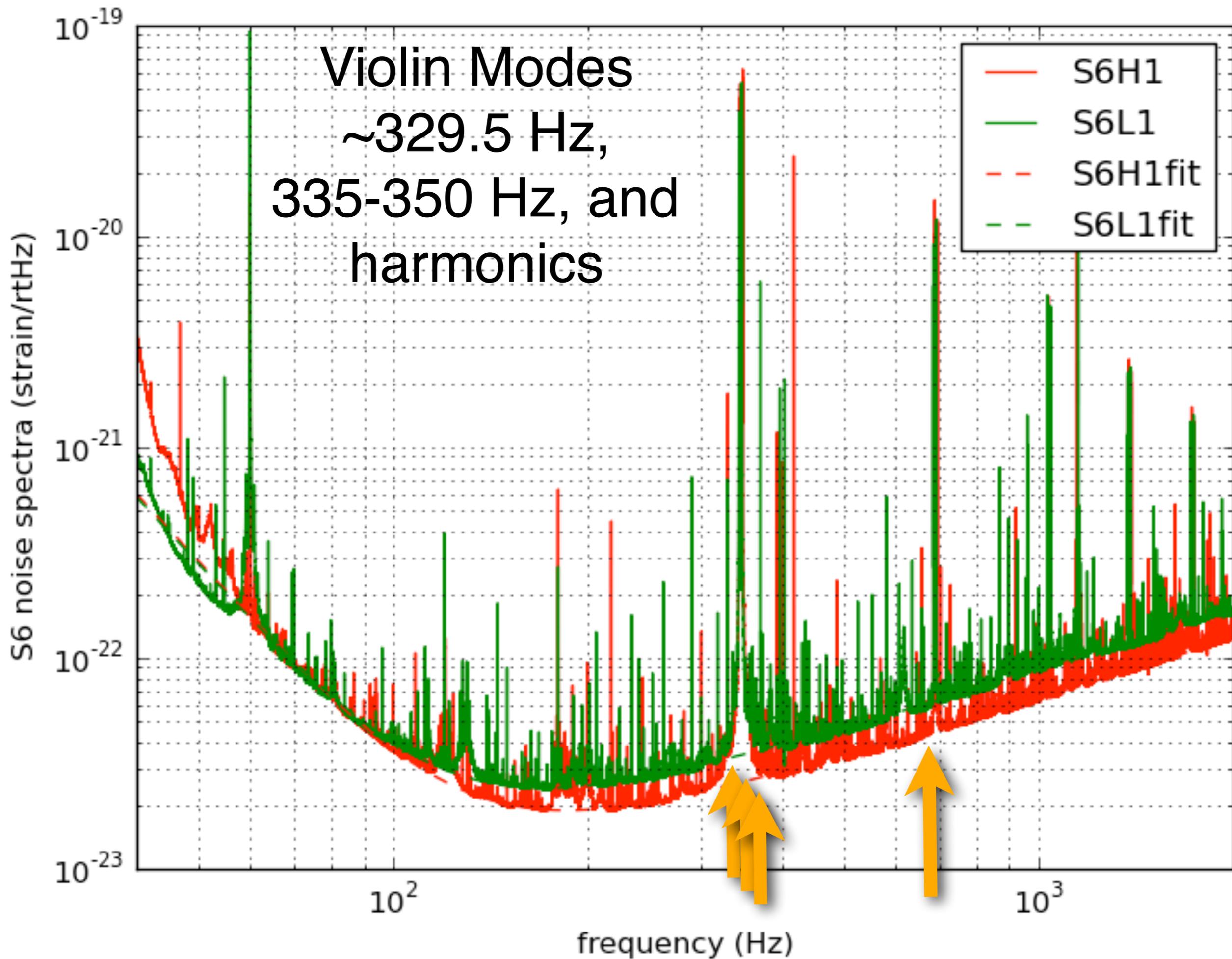
Power spectral density (PSD): $S(f) \sim \frac{\text{strain}^2}{\text{Hz}}$

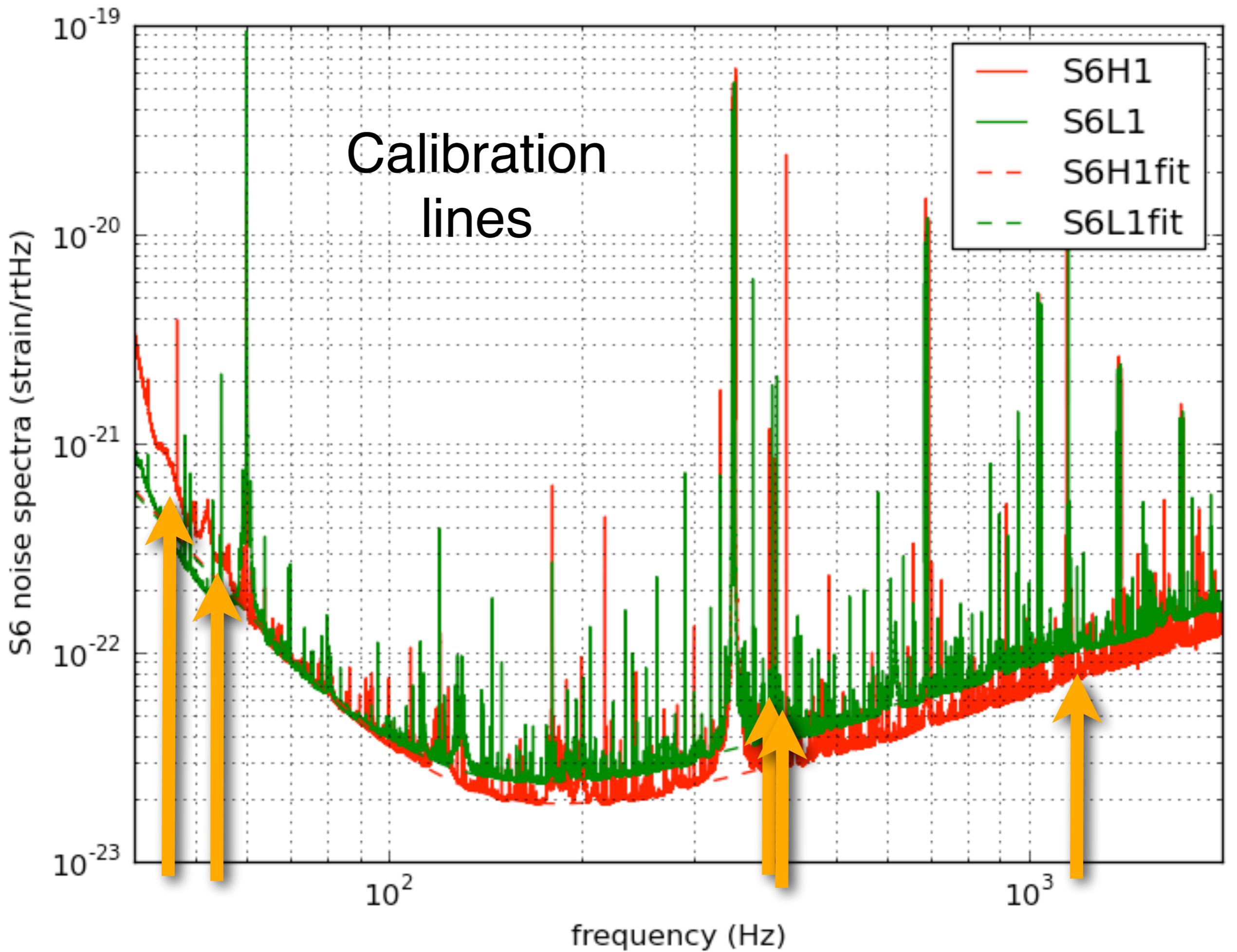
Amplitude spectral density (ASD): $\sqrt{S(f)} \sim \frac{\text{strain}}{\sqrt{\text{Hz}}}$



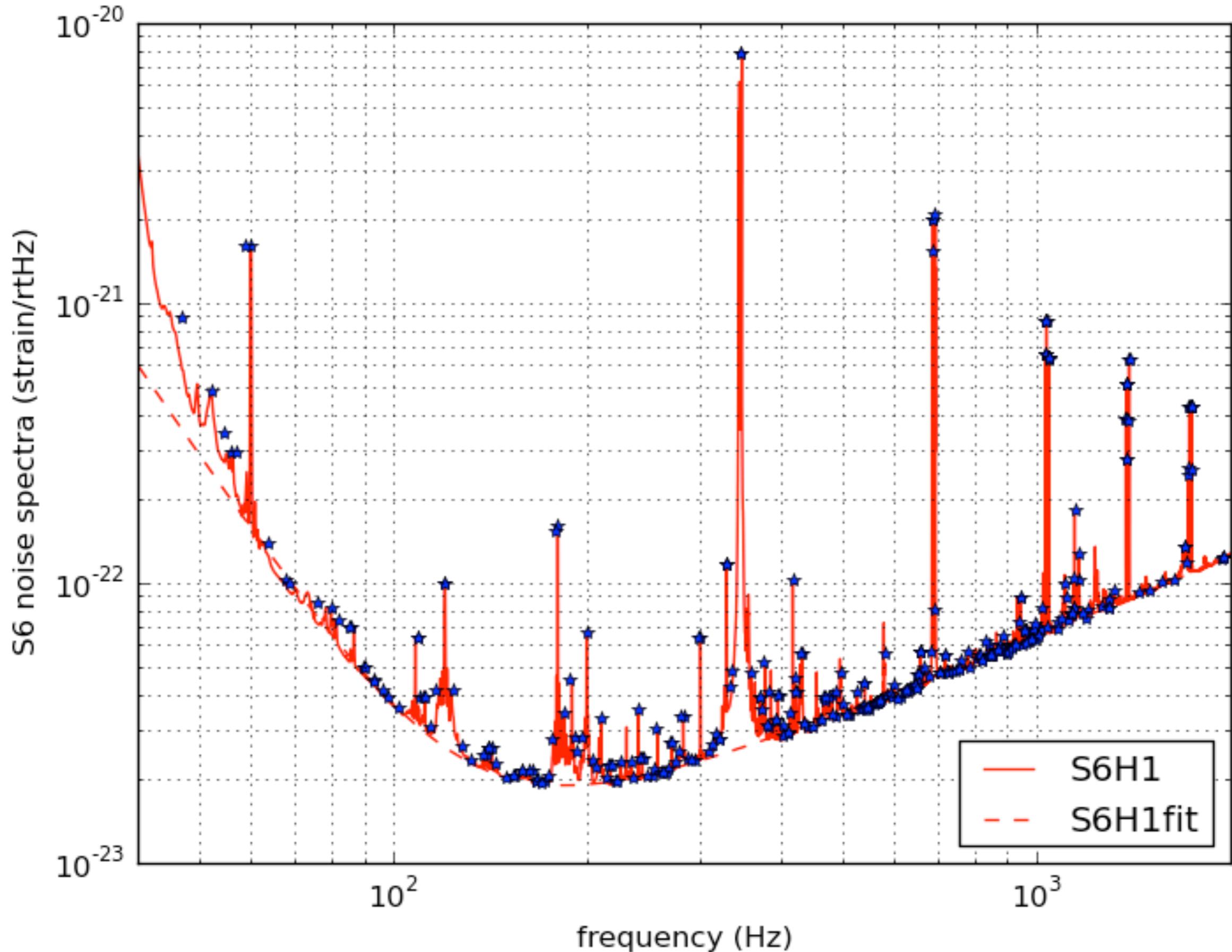


Violin Modes
~329.5 Hz,
335-350 Hz, and
harmonics





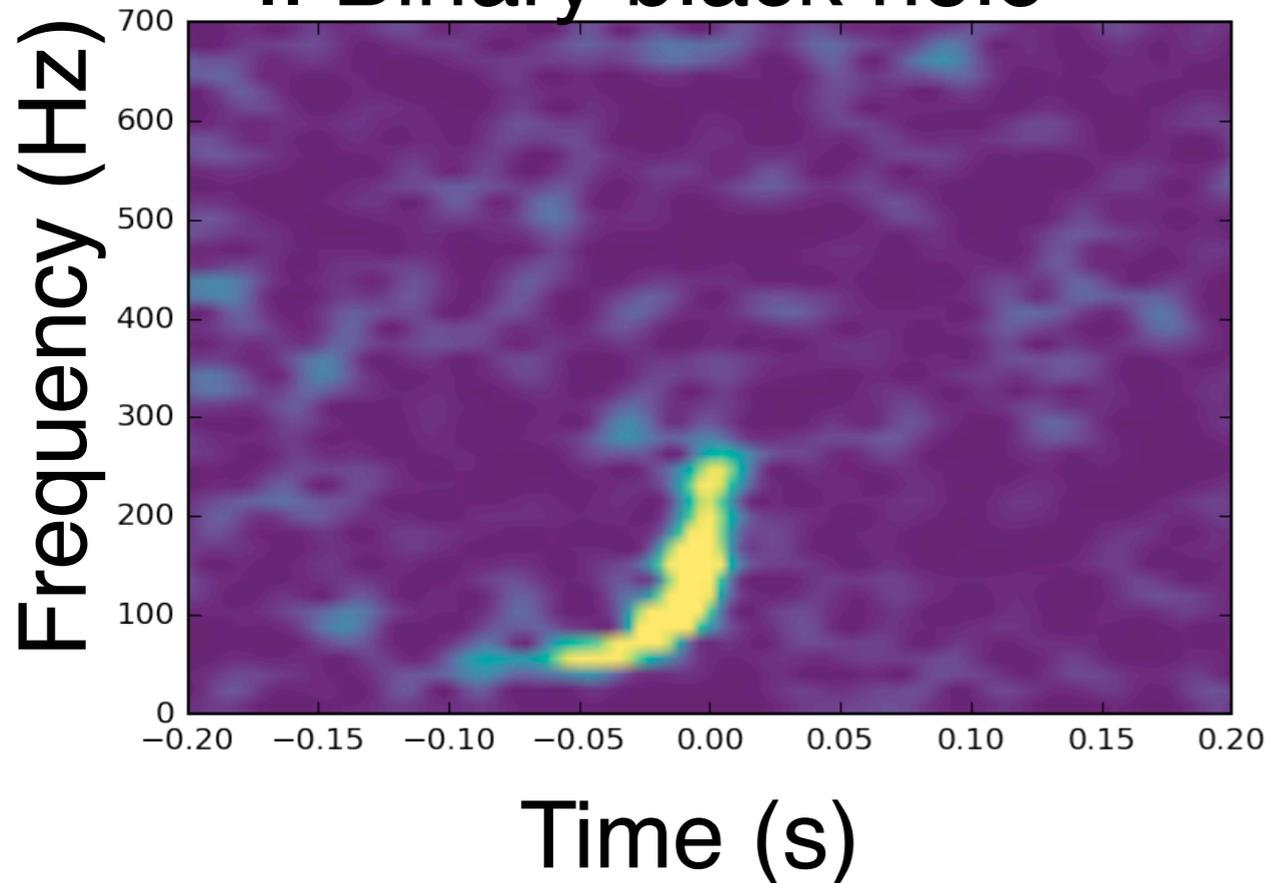
An example of identified spectral lines



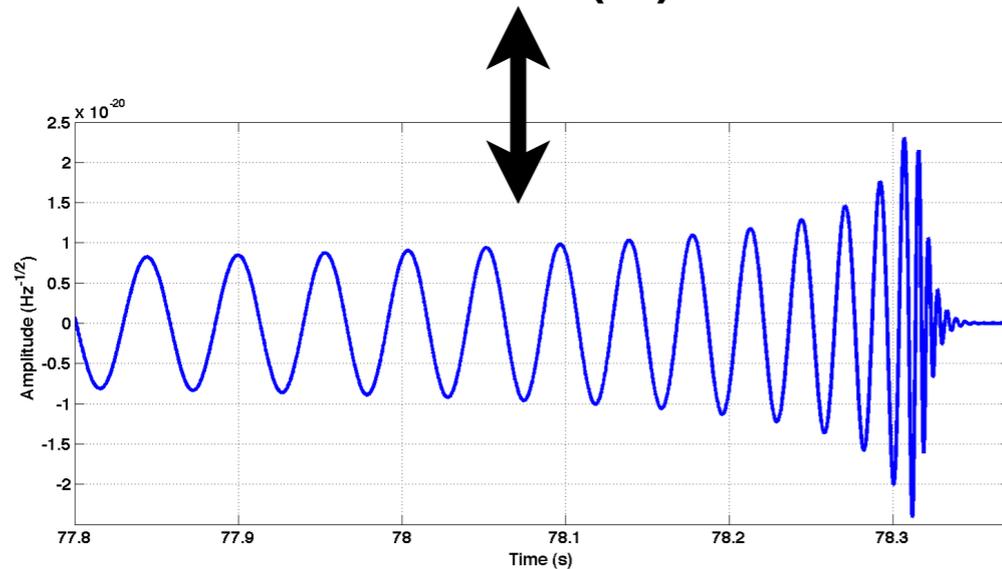
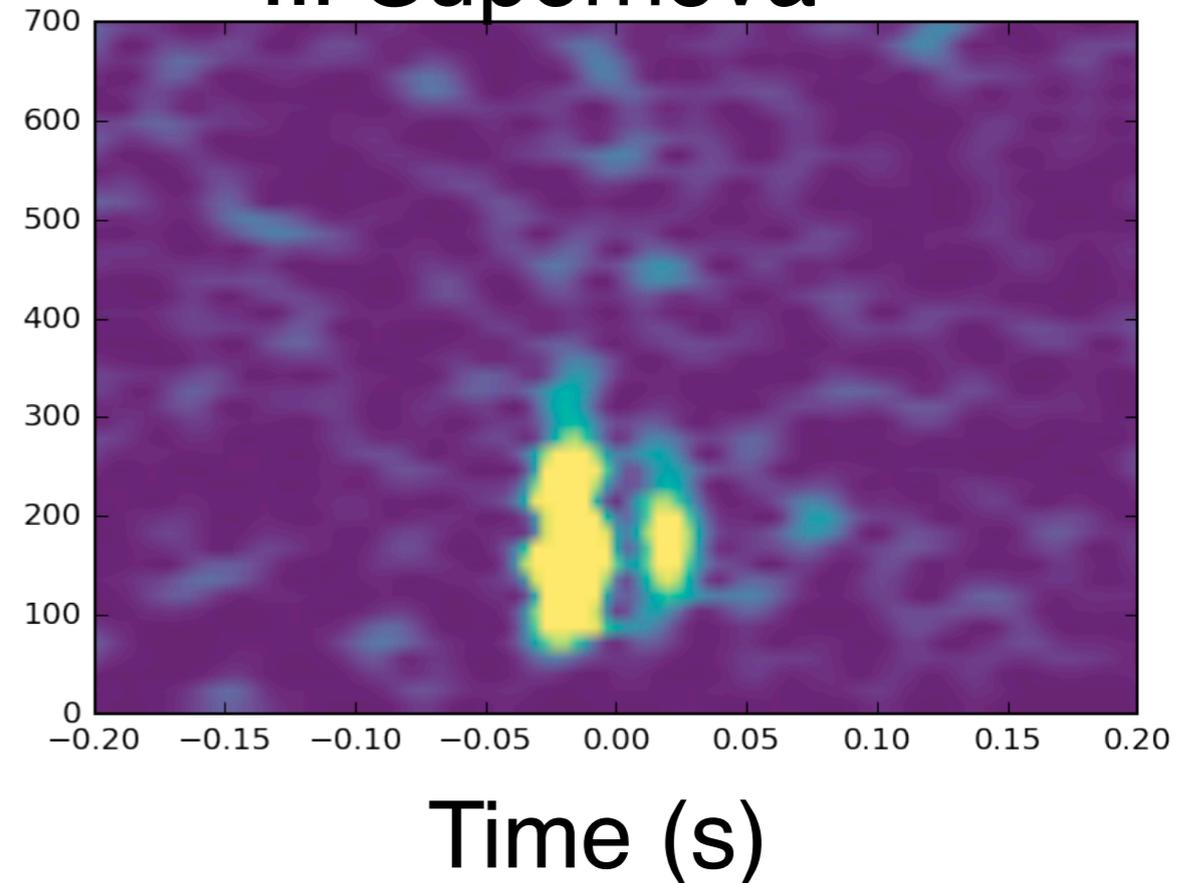
Spectrogram

Visual representation of power at certain frequencies as a function of time.

I. Binary black hole



II. Supernova



Also known as a
“chirp” signal.

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Sept 12, 2015

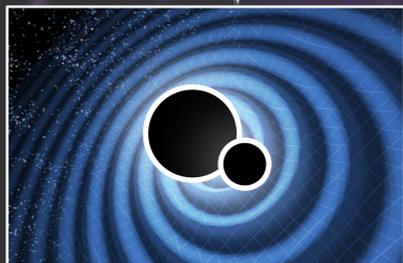


Advanced LIGO's First
Observing Run (O1)

Sep 2015	Oct 2015	Nov 2015	Dec 2015	Jan 2016
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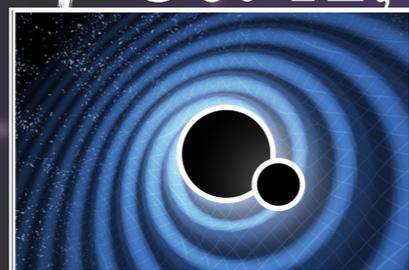
GW150914

Sept 14, 2015



GW151012

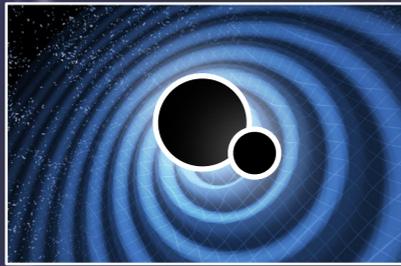
Oct 12, 2015



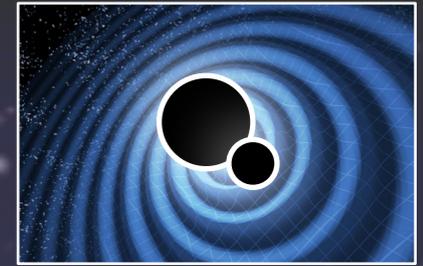
GW151226
Dec 26, 2015



GW170104
Jan 4, 2017



GW170814
Aug 14, 2017

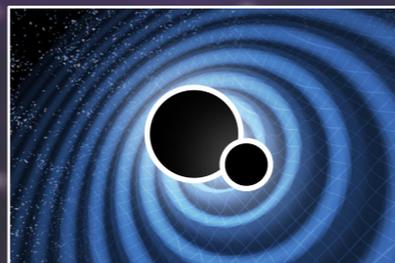


Advanced LIGO's Second
Observing Run (O2)

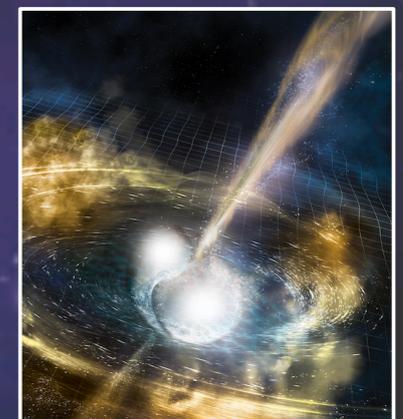
Virgo
turns
on

Nov 2016	Dec 2016	Jan 2017	Feb 2017	Mar 2017	Apr 2017	May 2017	Jun 2017	Jul 2017	Aug 2017
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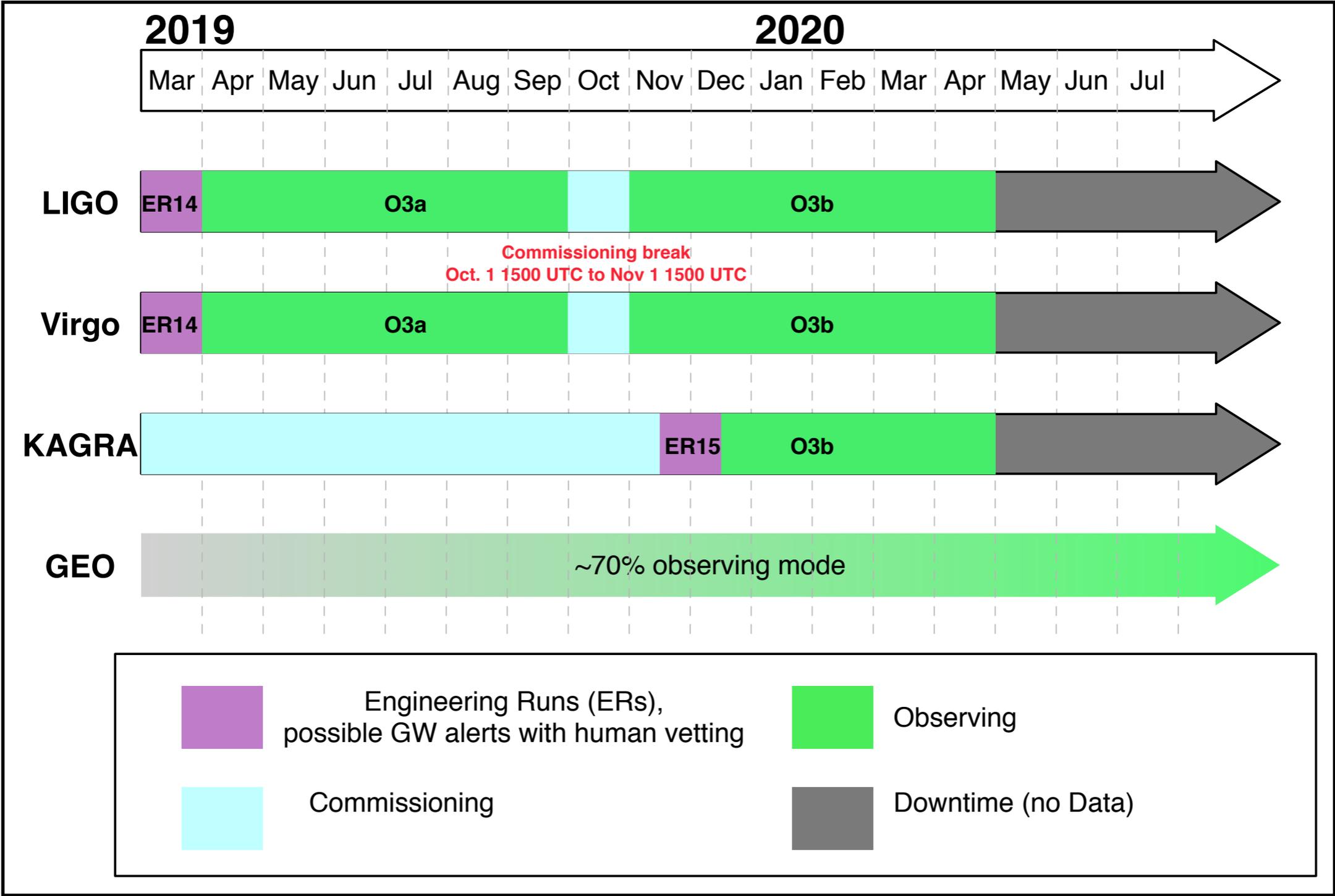
GW170608
Jun 6, 2017



GW170817
Aug 17, 2017

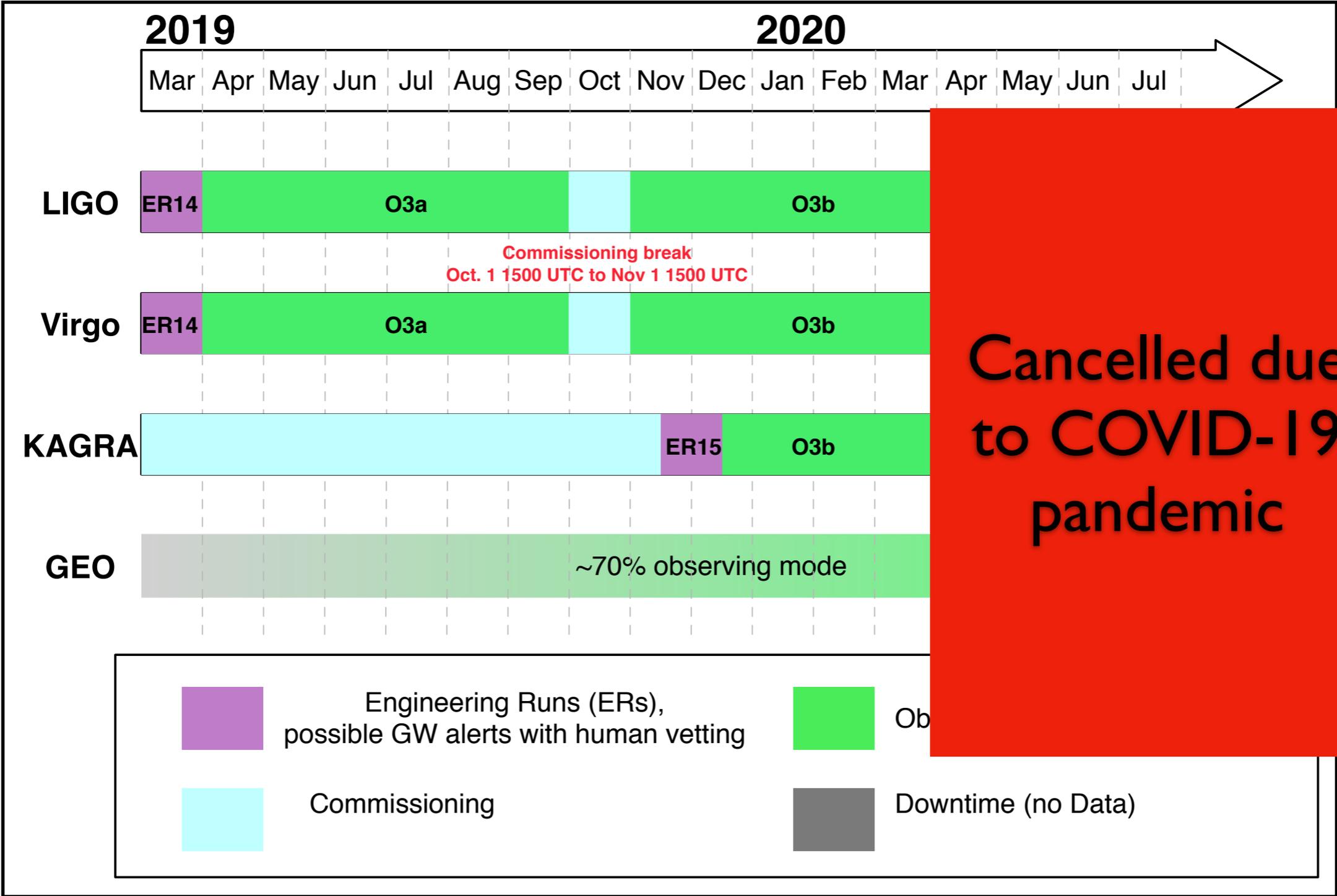


Proposed O3 Schedule



Reference: LIGO-G1901531

Proposed O3 Schedule



Reference: LIGO-G1901531

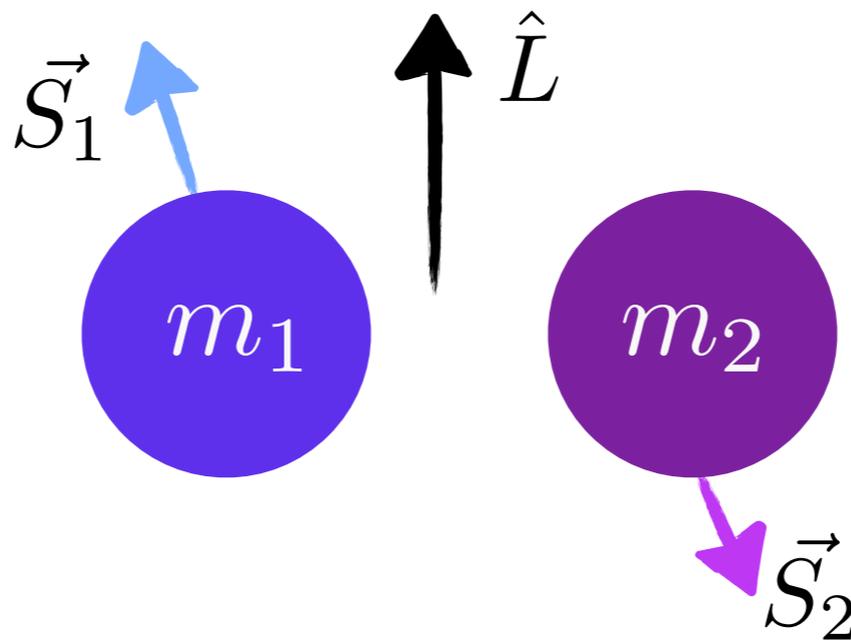
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Matched Filtering

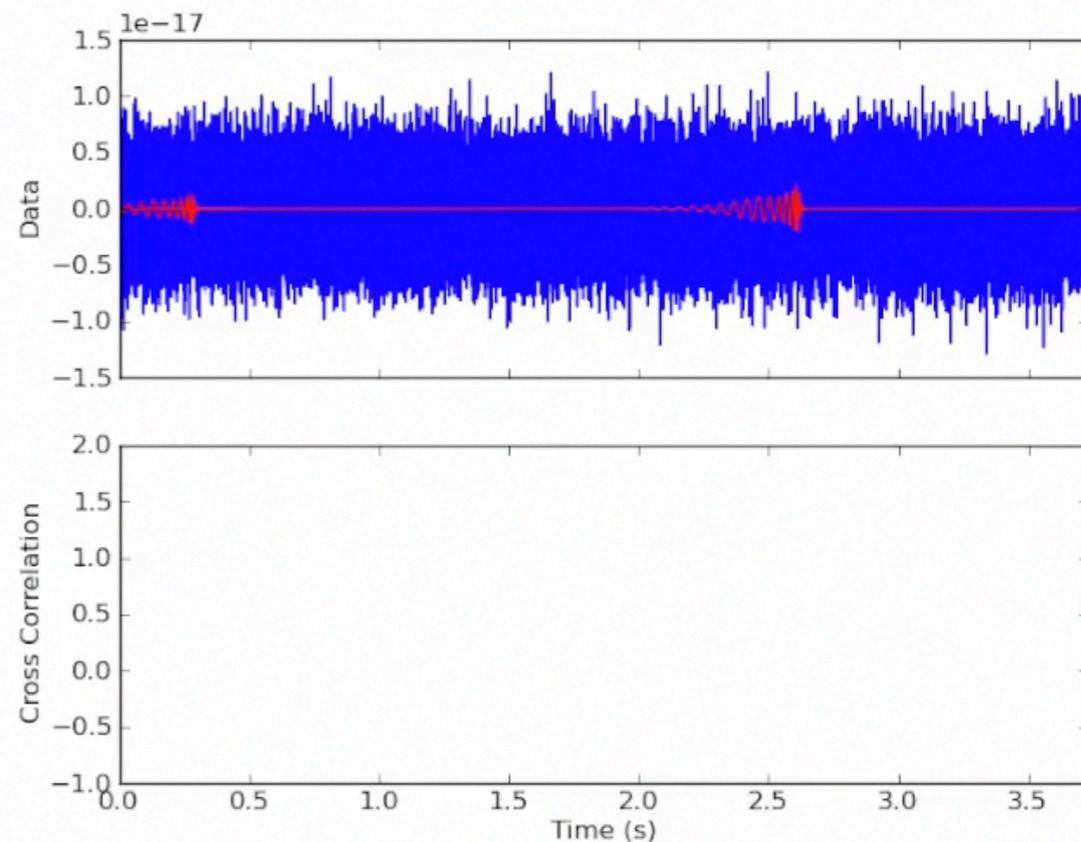
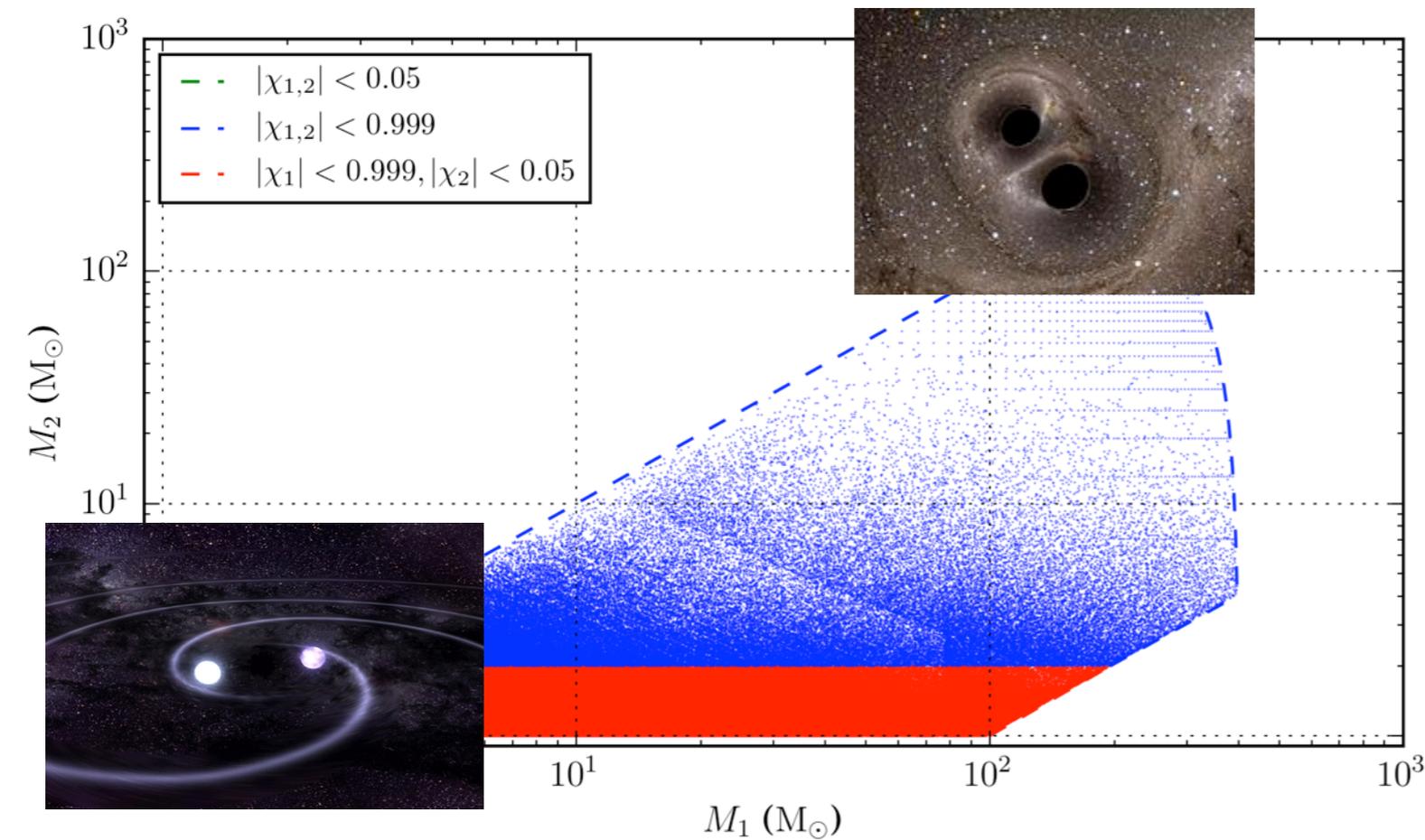
gstreamer

LALSuite

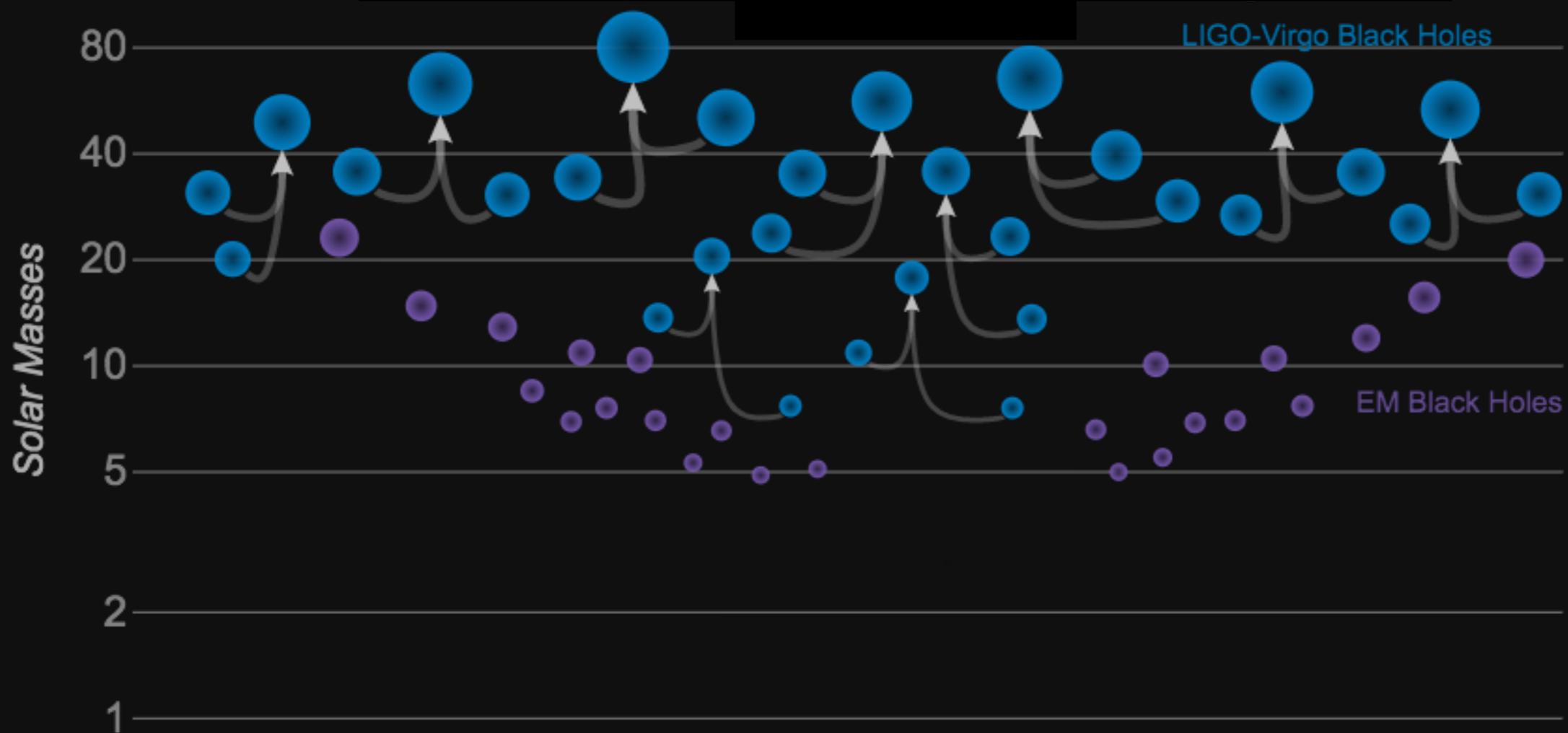
gstLAL



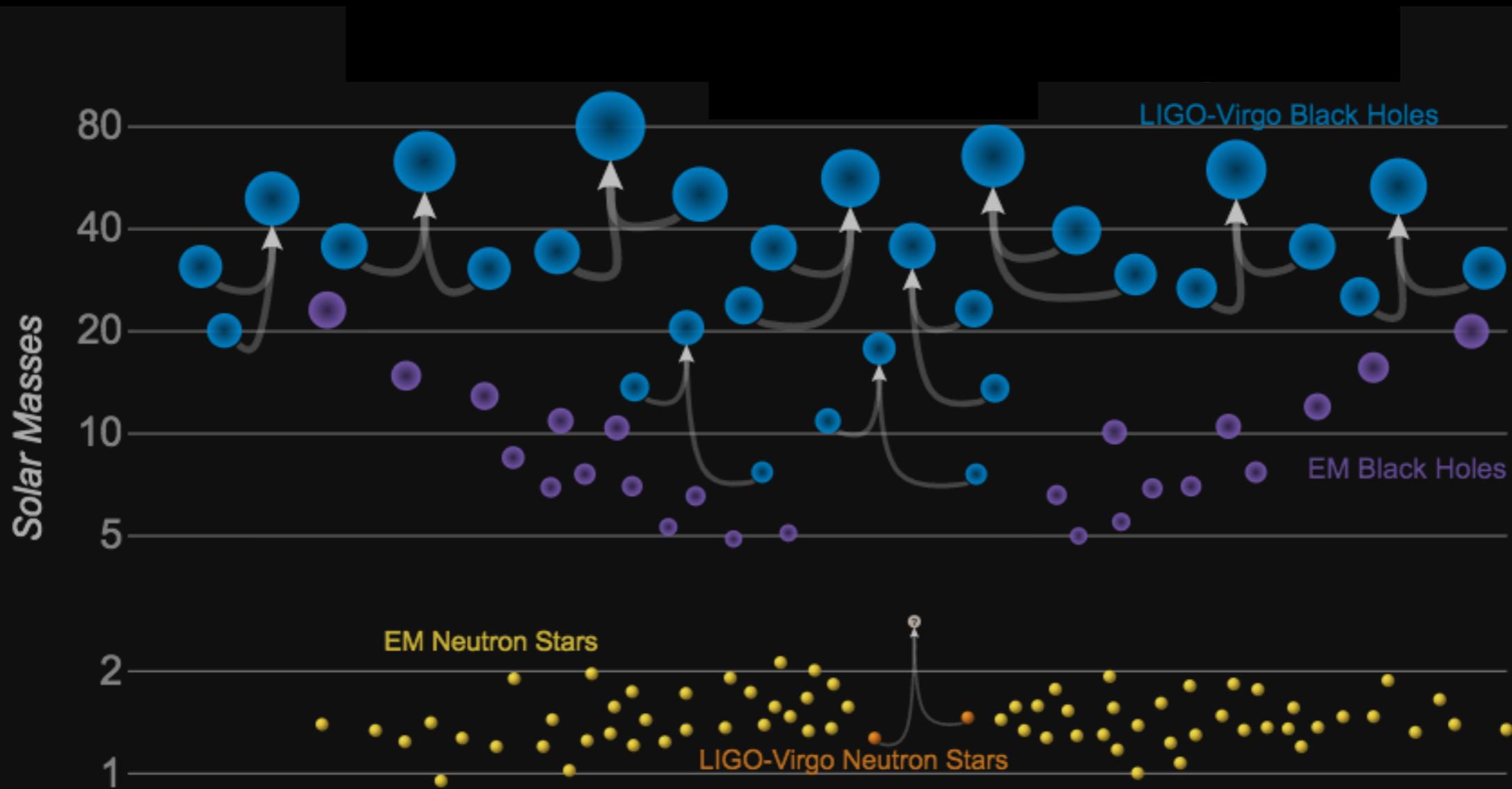
$$\chi_{1,2} \propto \vec{S}_{1,2} \cdot \hat{L}$$

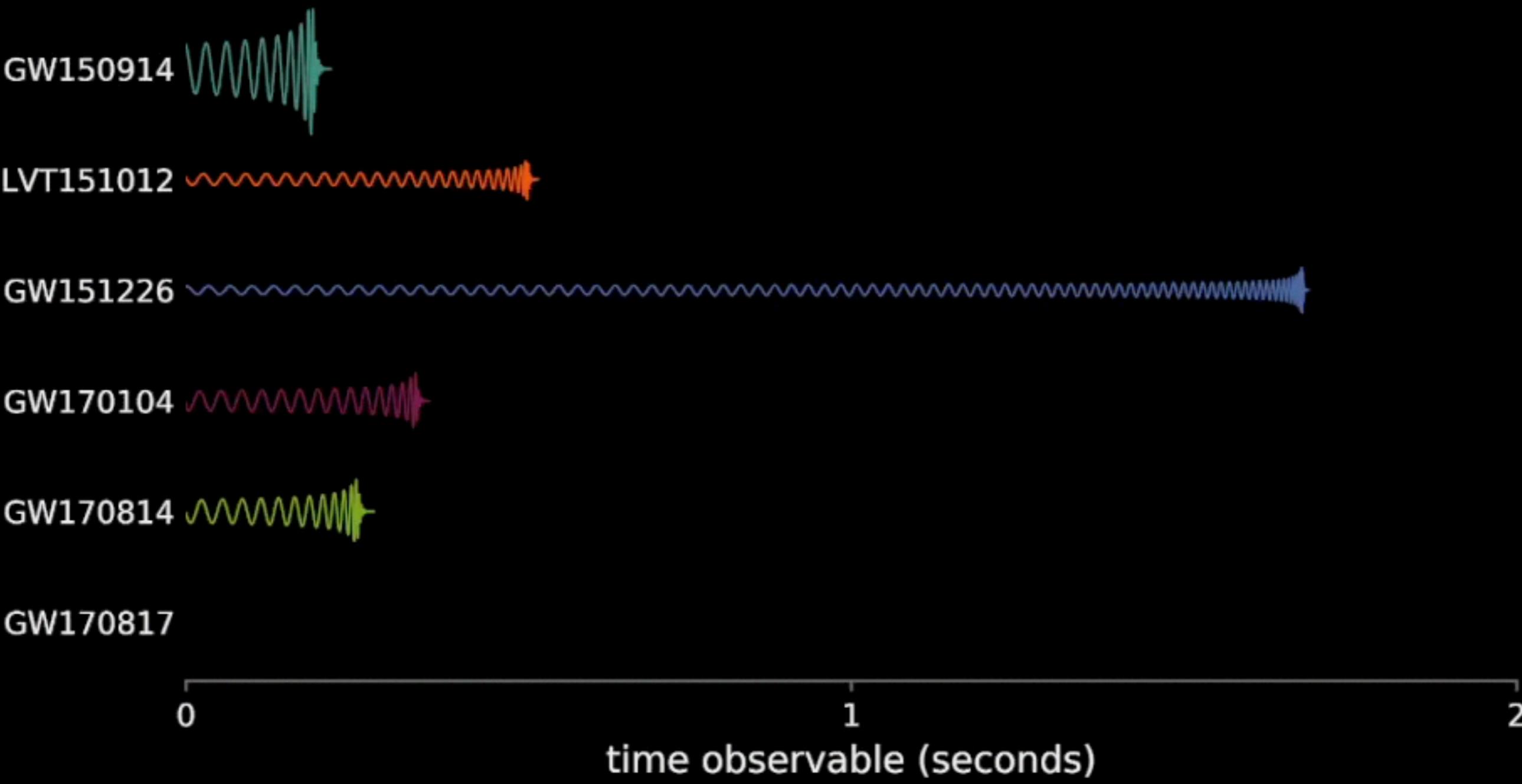


New Population of Stellar-mass Black Holes

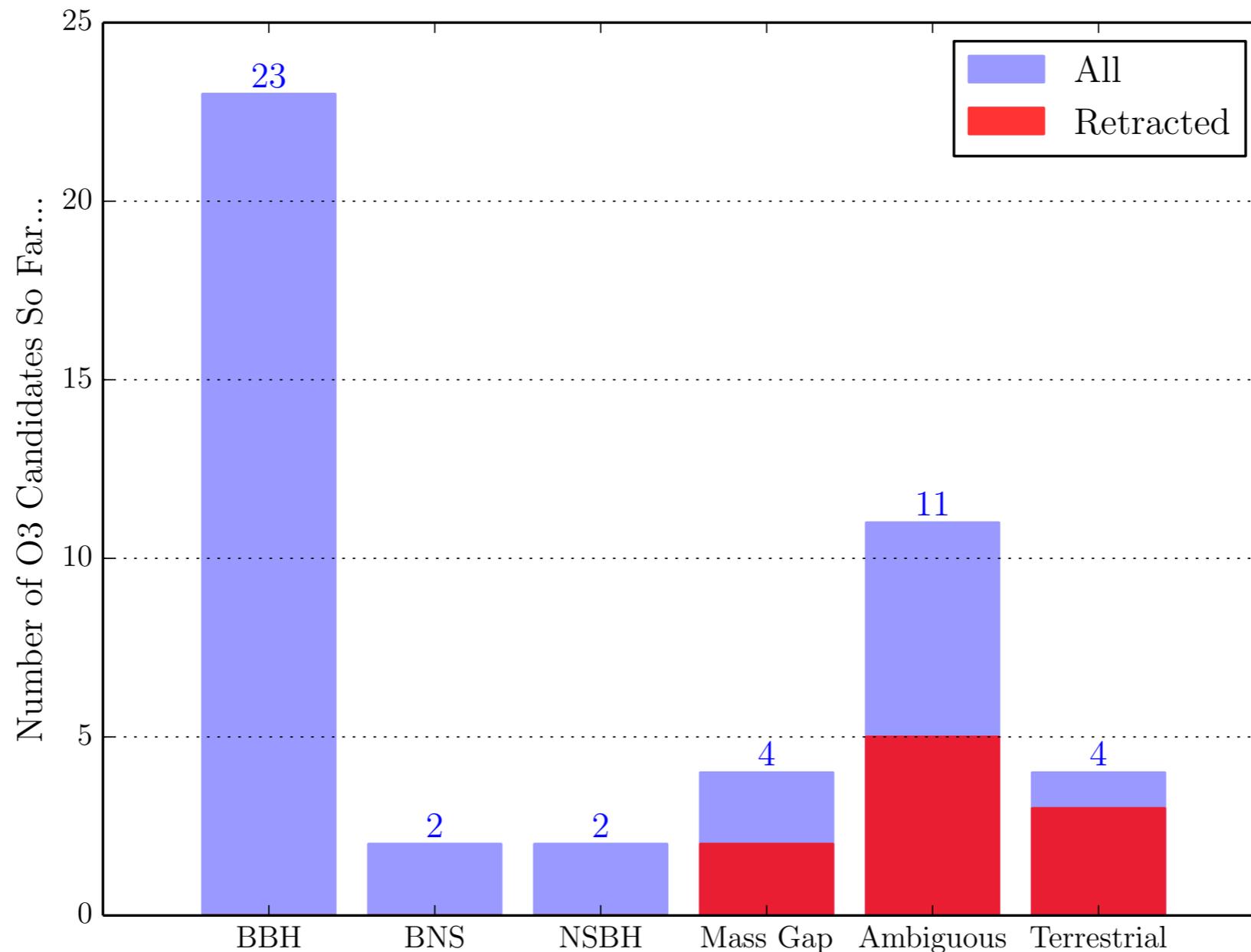


GWs from a Binary Neutron Star





O3 Candidates So Far...



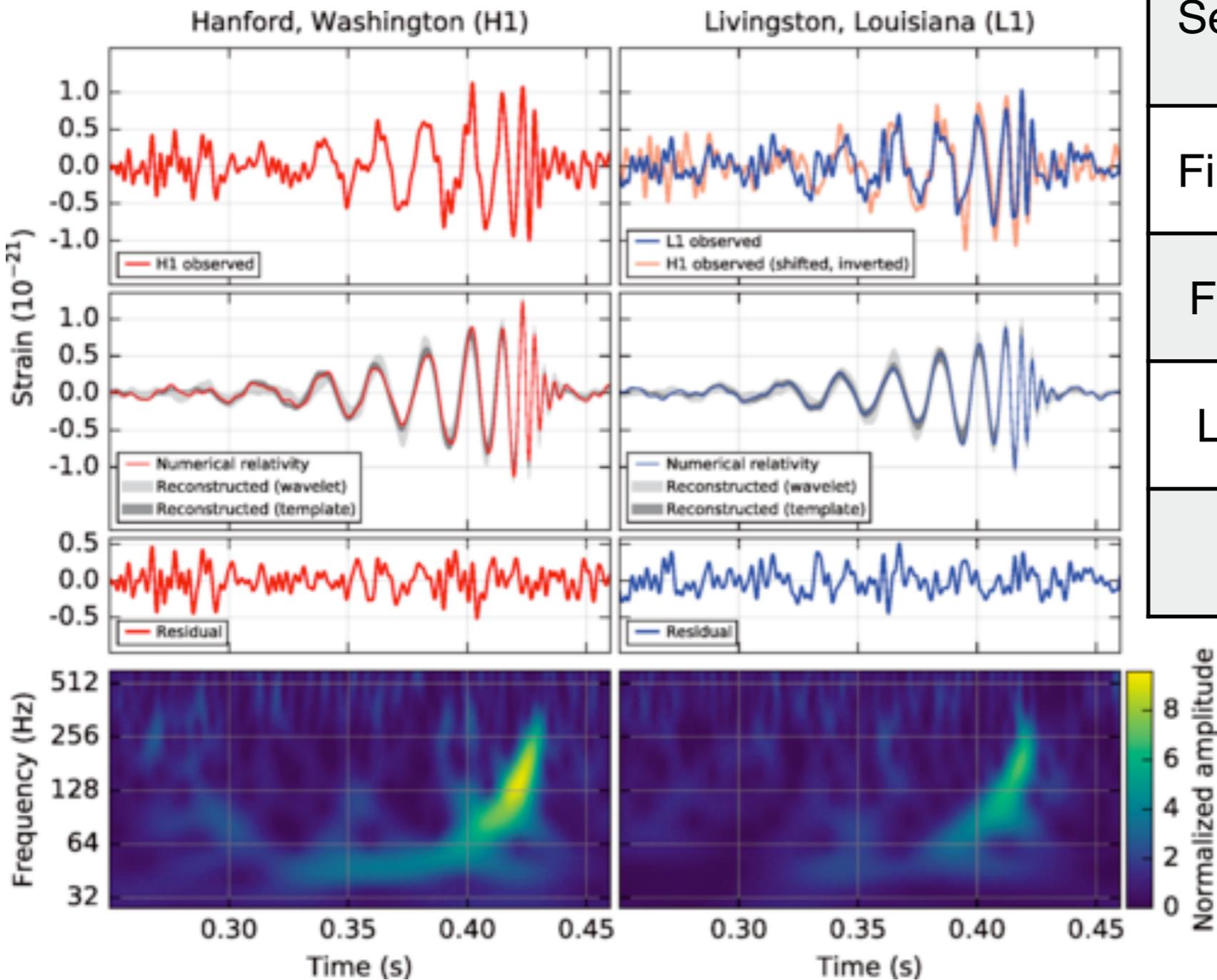
Preliminary classifications:

About 1 binary black hole candidate a week!

2 new binary neutron star candidates with no EM detections yet.

Two new classes: NSBH and Mass Gap.

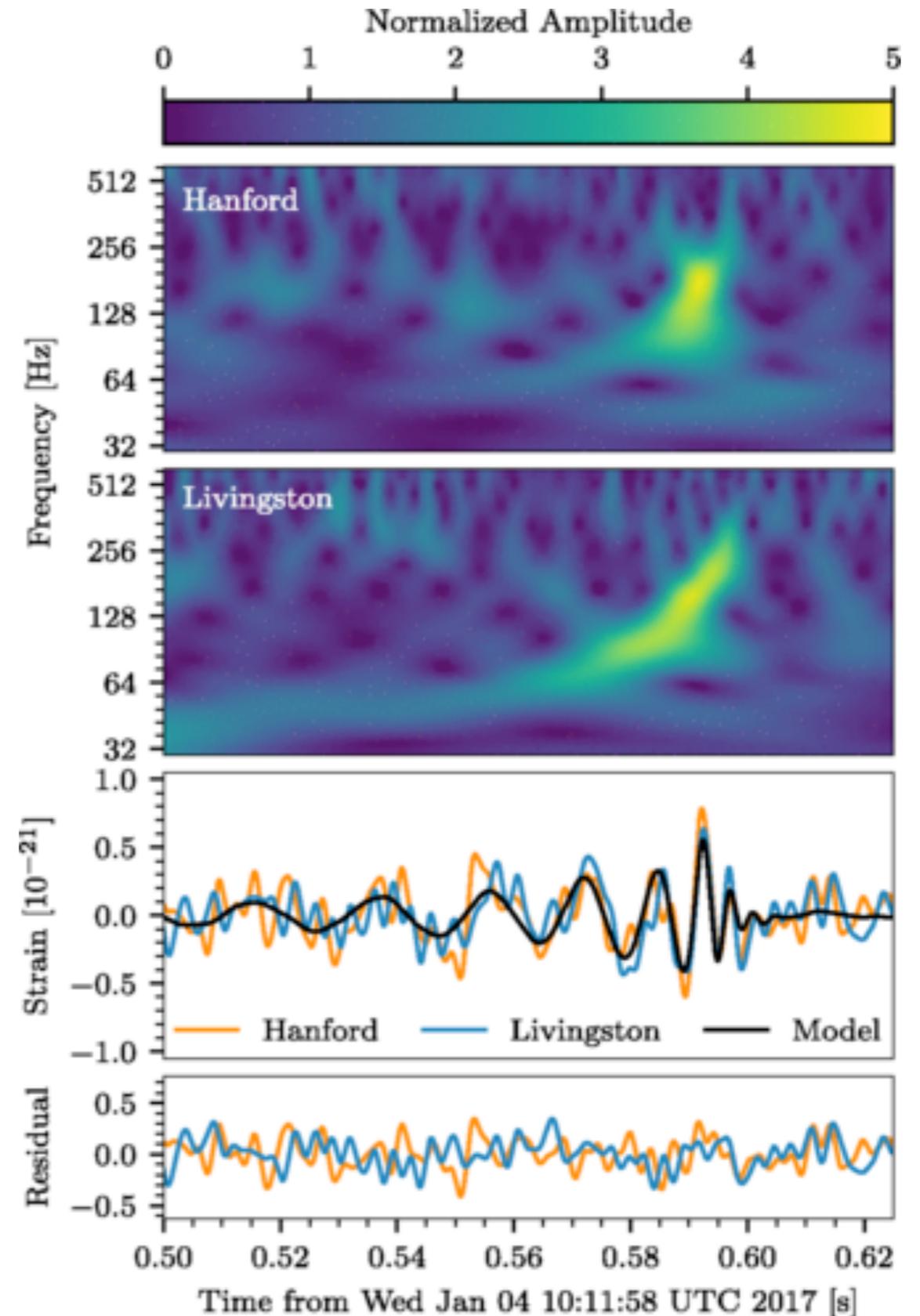
GW150914: The First Direct Detection of Gravitational Waves



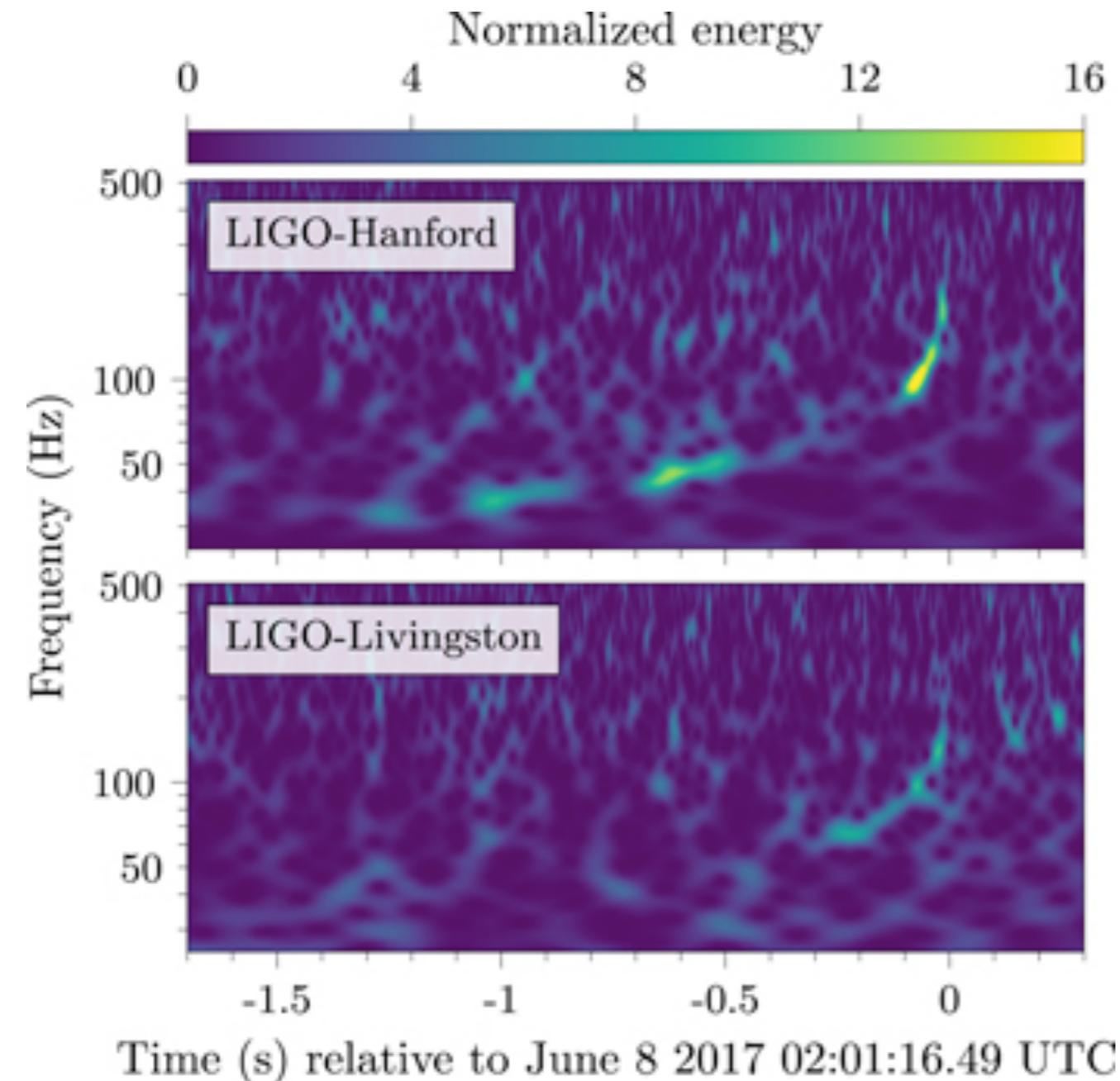
Primary black hole mass	$36^{+5}_{-4} M_{\odot}$
Secondary black hole mass	$29^{+4}_{-4} M_{\odot}$
Final black hole mass	$62^{+4}_{-4} M_{\odot}$
Final black hole spin	$0.67^{+0.05}_{-0.07}$
Luminosity distance	$410^{+160}_{-180} \text{Mpc}$
Source redshift z	$0.09^{+0.03}_{-0.04}$

GW170104: The Farthest Confident Detection of Gravitational Waves

Primary black hole mass	$31.2^{+8.4}_{-6.0} M_{\odot}$
Secondary black hole mass	$19.4^{+5.3}_{-5.9} M_{\odot}$
Final black hole mass	$48.7^{+5.7}_{-4.6} M_{\odot}$
Final black hole spin	$0.64^{+0.09}_{-0.20}$
Luminosity distance	$880^{+450}_{-390} \text{Mpc}$
Source redshift z	$0.18^{+0.08}_{-0.07}$

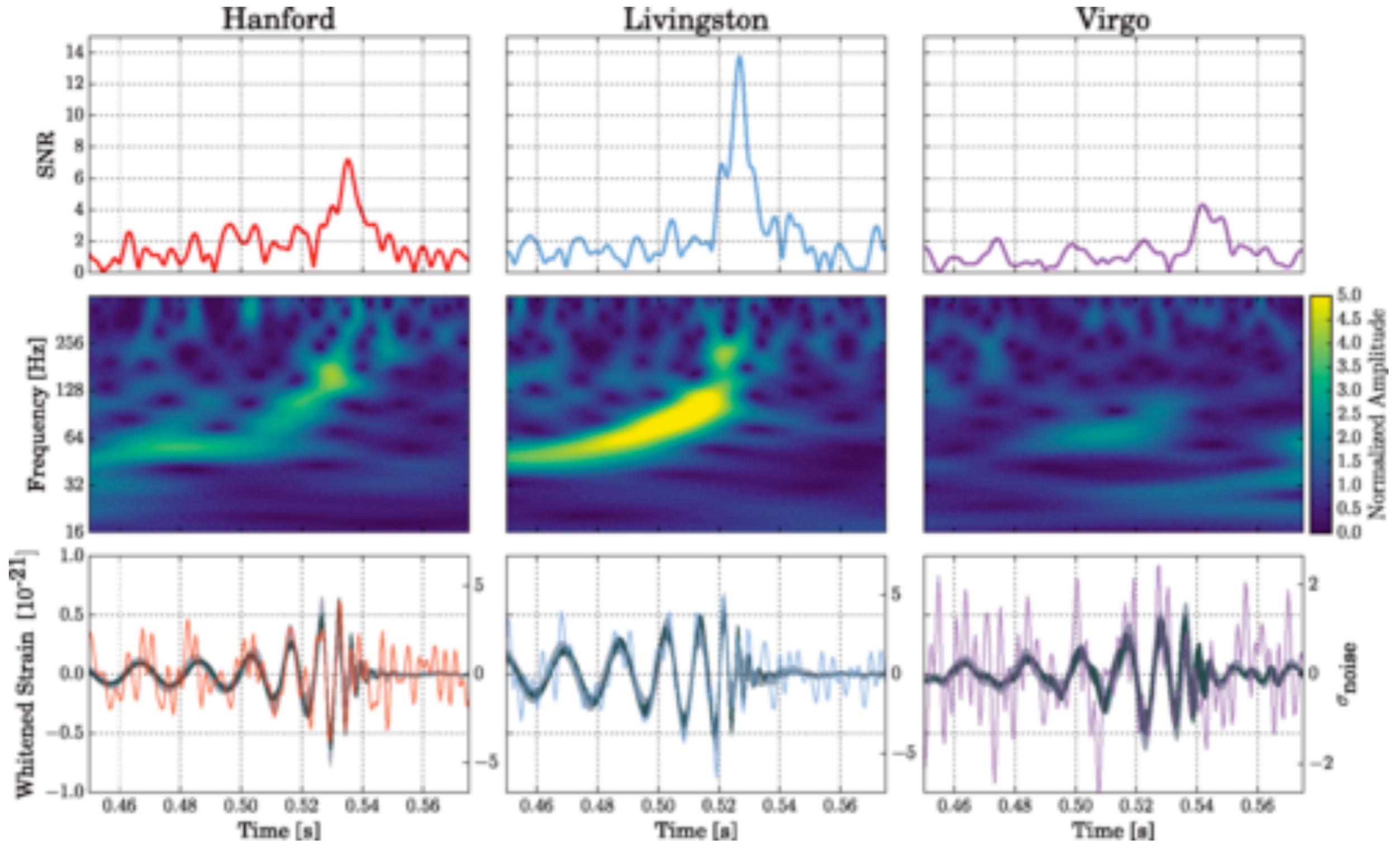


GW170608: Least Massive Binary Black Hole Observed So Far



Primary black hole mass	$12^{+7}_{-2} M_{\odot}$
Secondary black hole mass	$7^{+2}_{-2} M_{\odot}$
Final black hole mass	$18.0^{+4.8}_{-0.9} M_{\odot}$
Final black hole spin	$0.69^{+0.04}_{-0.05}$
Luminosity distance	$340^{+140}_{-140} \text{Mpc}$
Source redshift z	$0.07^{+0.03}_{-0.03}$

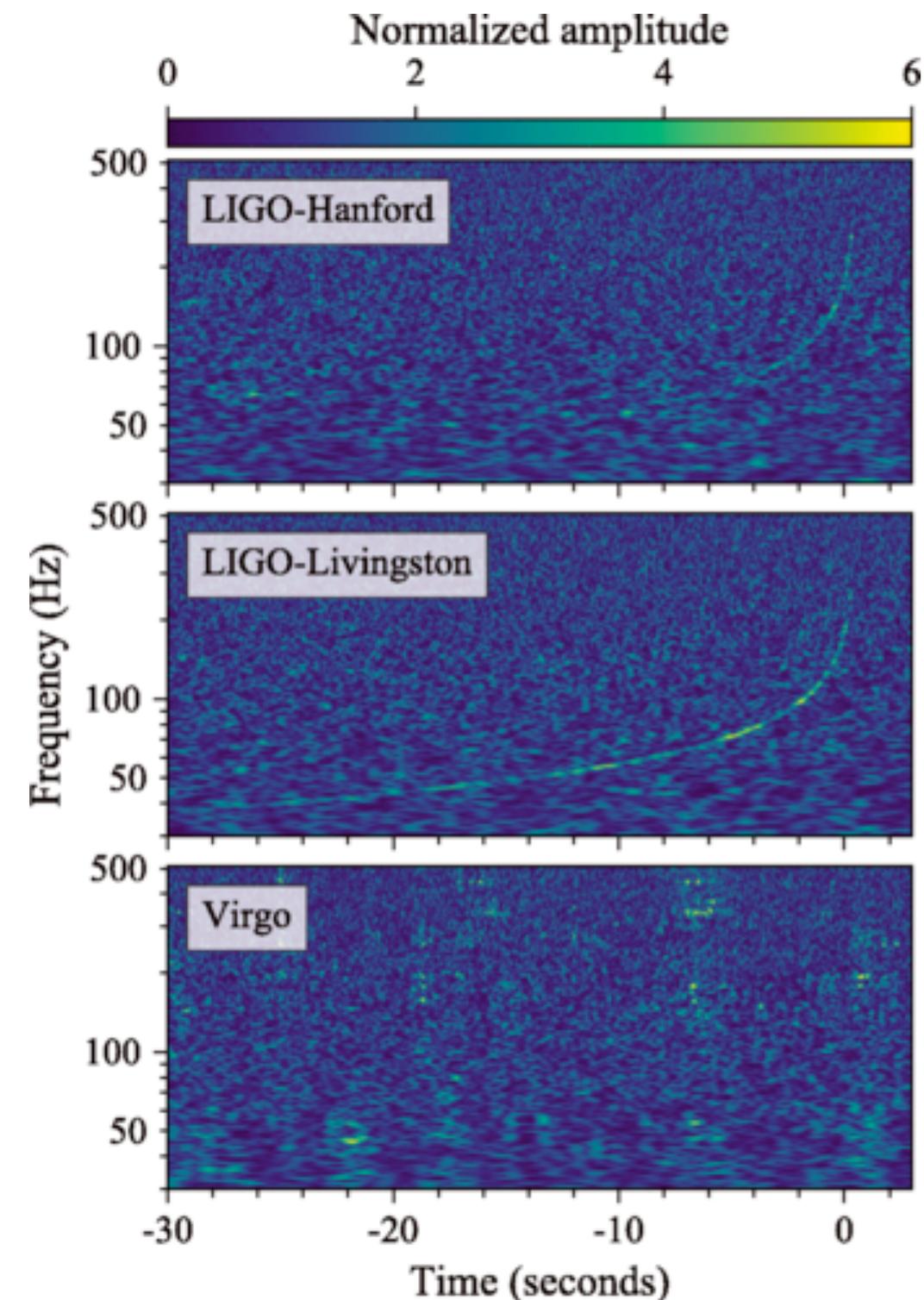
GW170814: First Event Seen By LIGO and Virgo Detectors



GW170814: First Event Seen By LIGO and Virgo Detectors

Primary black hole mass	$30.5^{+5.7}_{-3.0} M_{\odot}$
Secondary black hole mass	$25.3^{+2.8}_{-4.2} M_{\odot}$
Final black hole mass	$53.2^{+3.2}_{-2.5} M_{\odot}$
Final black hole spin	$0.07^{+0.07}_{-0.05}$
Luminosity distance	$540^{+130}_{-210} \text{Mpc}$
Source redshift z	$0.11^{+0.03}_{-0.04}$

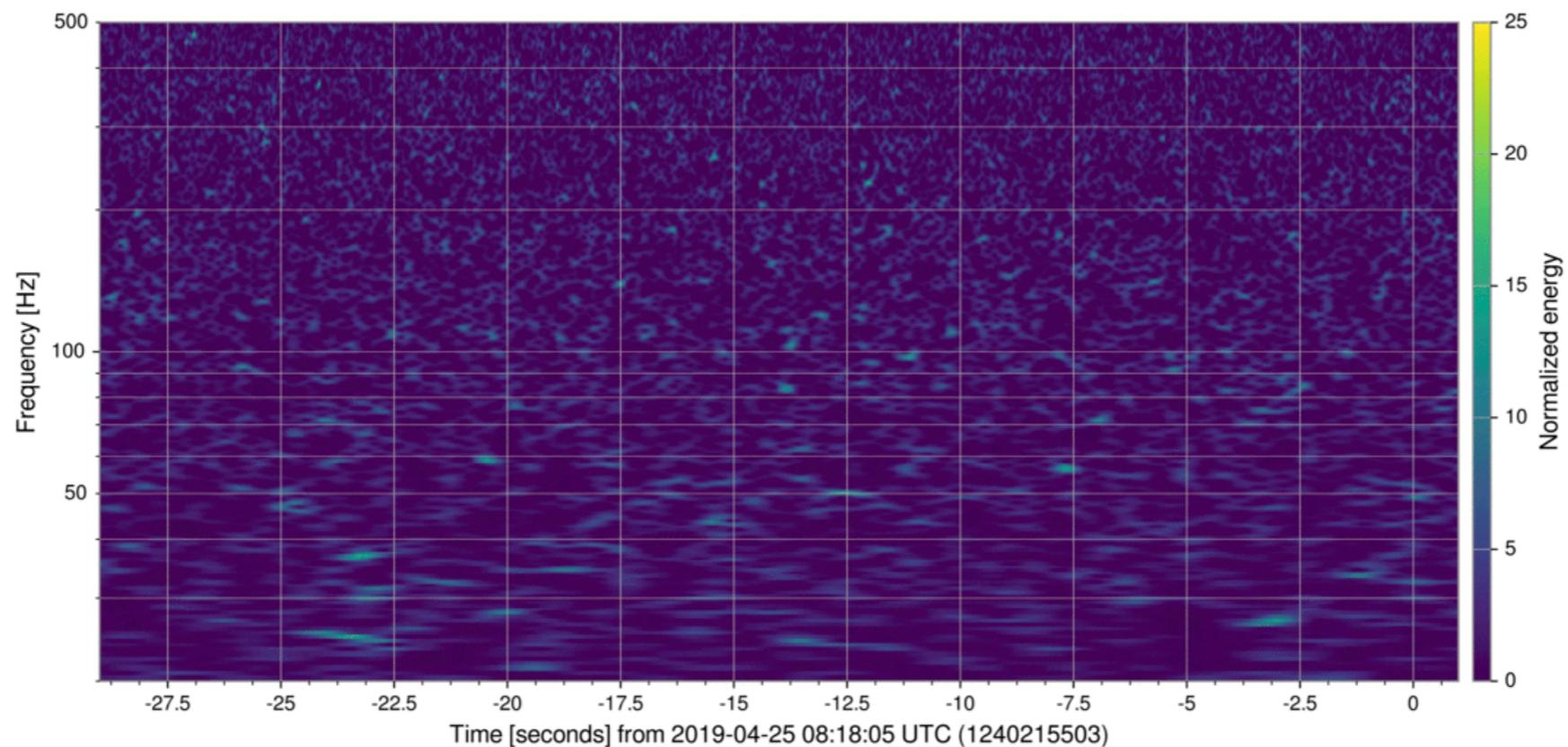
GW170817: The First Detection of Gravitational Waves from a Binary Neutron Star Coalescence



	Assuming low spin	Allowing for high spin
Primary mass	$1.36 - 1.60 M_{\odot}$	$1.36 - 2.26 M_{\odot}$
Secondary mass	$1.17 - 1.36 M_{\odot}$	$0.86 - 1.36 M_{\odot}$
Luminosity distance	40^{+8}_{-14}Mpc	

And the closest GW signal so far.

GW190425: The Second Detection of Gravitational Waves from a Binary Neutron Star Coalescence



	Assuming low spin	Allowing for high spin
Primary mass	$1.60 - 1.87 M_{\odot}$	$1.61 - 2.52 M_{\odot}$
Secondary mass	$1.46 - 1.69 M_{\odot}$	$1.12 - 1.68 M_{\odot}$
Luminosity distance	$159^{+69}_{-72} \text{Mpc}$	

More massive than any other known binary neutron star system.

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- Homework

BBH Science: Measuring a Mass Distribution

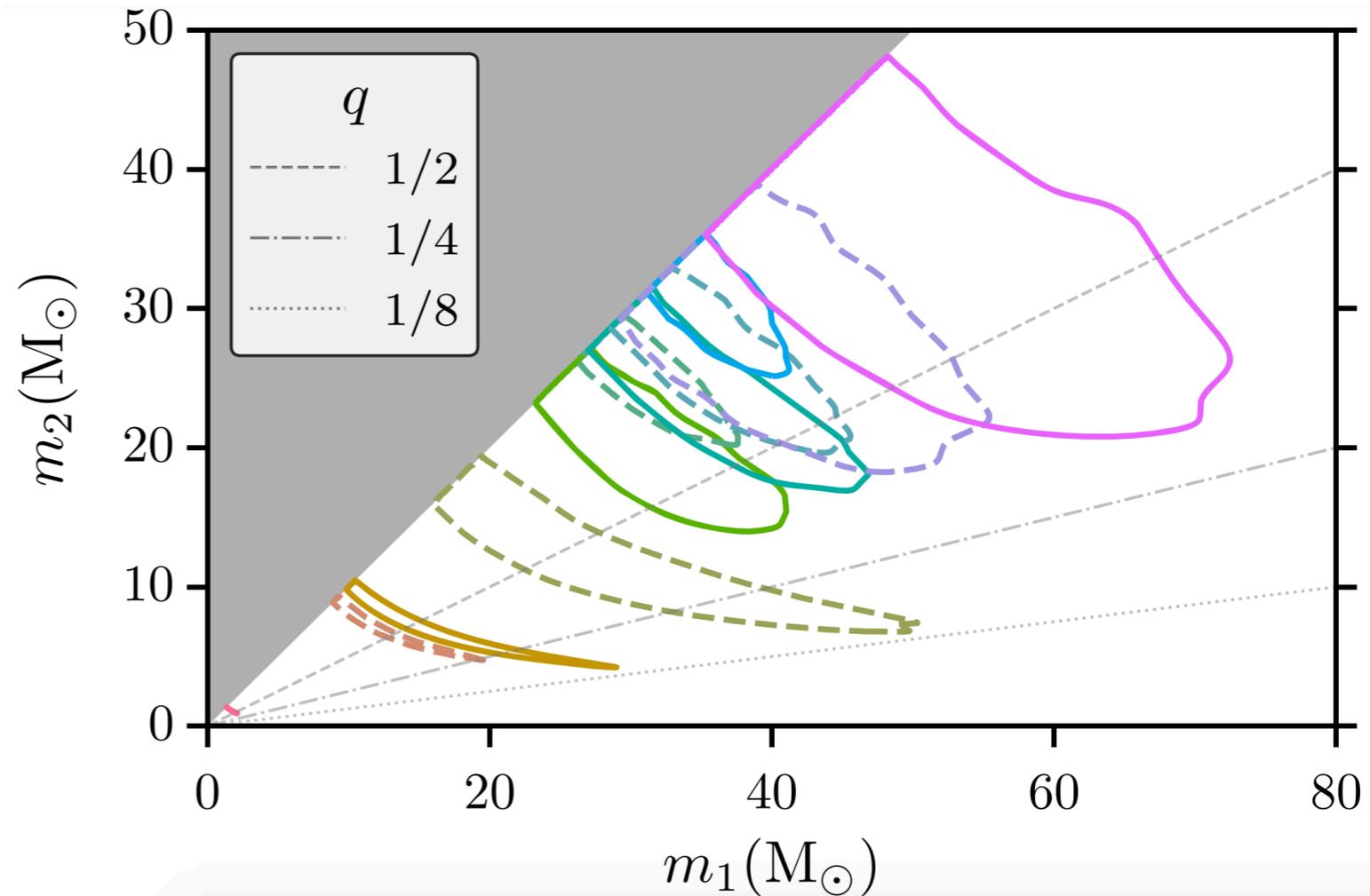
From early inspiral:

$$\mathcal{M}_c = \frac{(m_1 m_2)^{3/5}}{M^{1/5}}$$

$$\approx \frac{c^3}{G} \left[\frac{5}{96} \pi^{-8/3} f^{-11/3} \dot{f} \right]^{3/5}$$

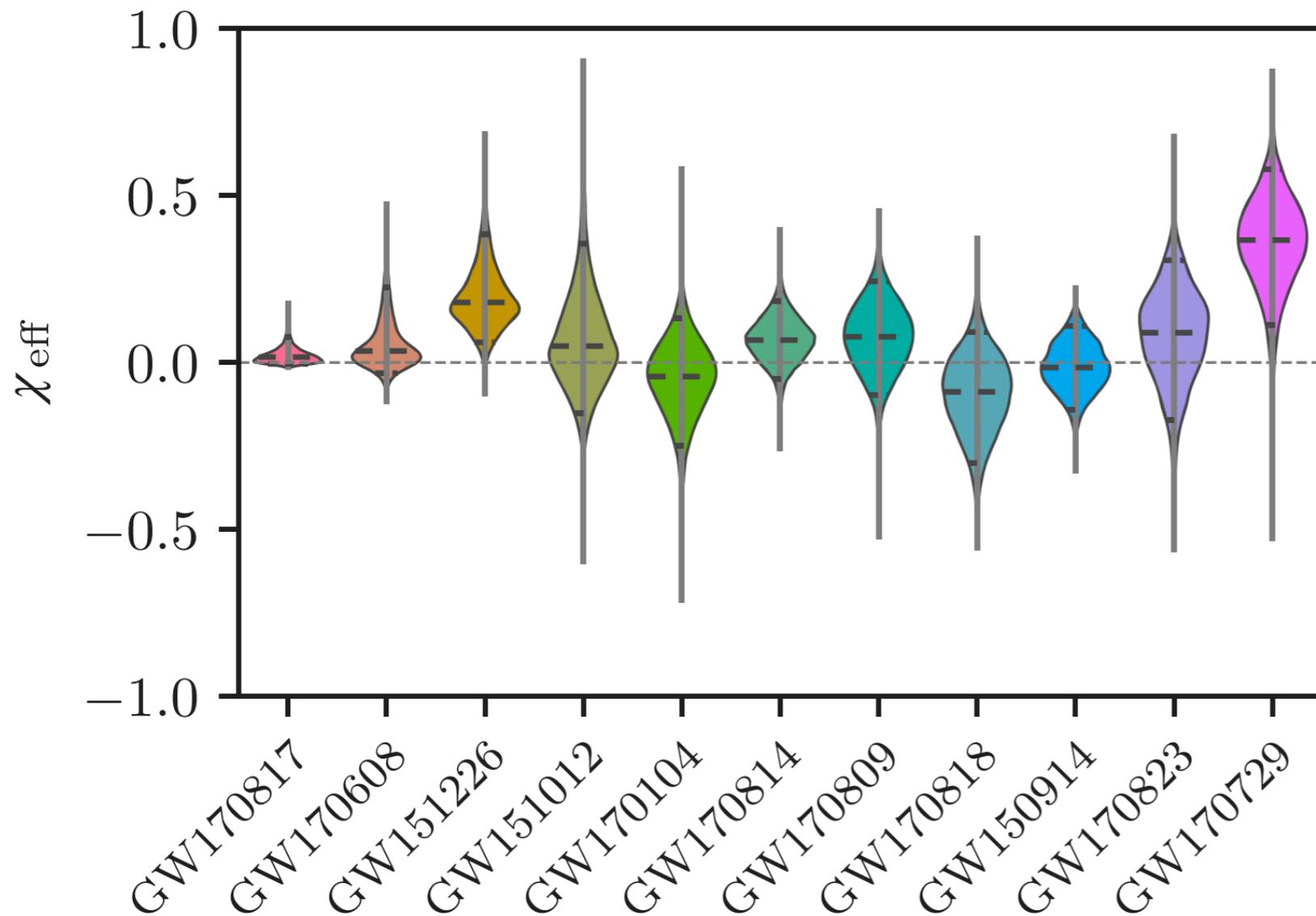
From late inspiral:

$$M_{\text{total}} = m_1 + m_2$$

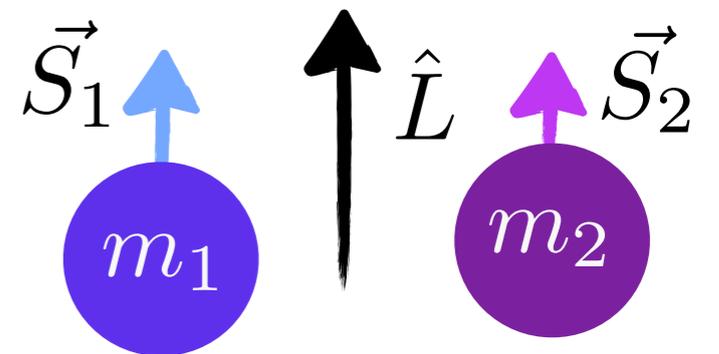


Revealing a new population of heavy stellar-mass black holes.

BBH Science: Measuring Effective Spins

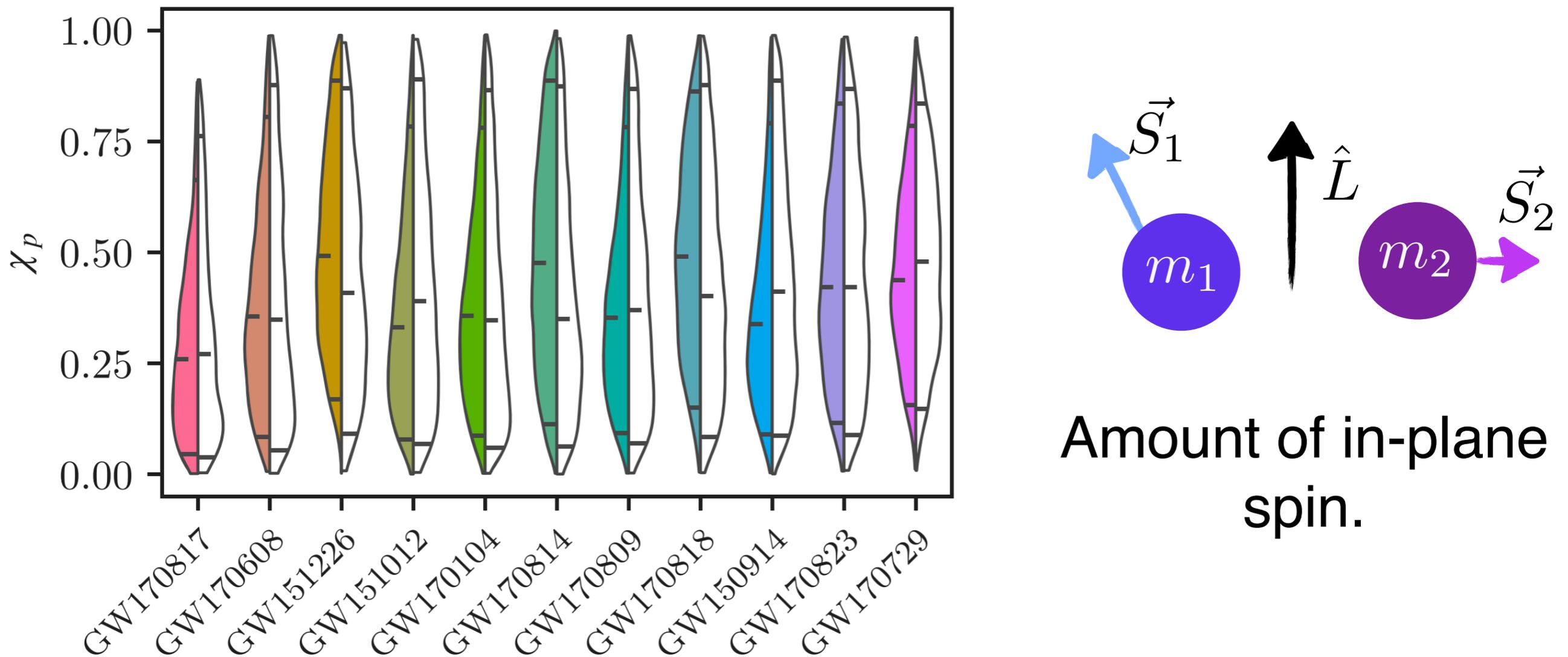


$$\chi_{\text{eff}} = \frac{c}{GM} \left(\frac{\vec{S}_1}{m_1} + \frac{\vec{S}_2}{m_2} \right)$$



Helping to distinguish between binary formation channels.

BBH Science: Measuring Precessing Spins



Helping to distinguish between binary formation channels.
Improving accuracy of other measured parameters.

BBH Science: Do the signals agree with General Relativity?

These systems give us empirical access to genuinely strong-field dynamics of spacetime.

	Solar system	Binary pulsars	Compact binary
$\frac{GM}{c^2 R}$	$\sim 10^{-8}$	$\sim 10^{-6}$	~ 0.2
$\frac{v}{c}$	$\sim 10^{-5}$	$\sim 10^{-3}$	~ 0.4

BBH Science: Parameterized Tests of General Relativity?

Allow for fractional changes with respect to the General Relativity value.

For example, the inspiral phase can be expressed in terms of characteristic speed $v(t)$

$$\Phi(v(t)) = \left(\frac{v}{c}\right)^{-5} \sum_{n=0}^7 \left[\phi_n + \phi_n^{(l)} \ln\left(\frac{v}{c}\right) \right] \left(\frac{v}{c}\right)^n$$

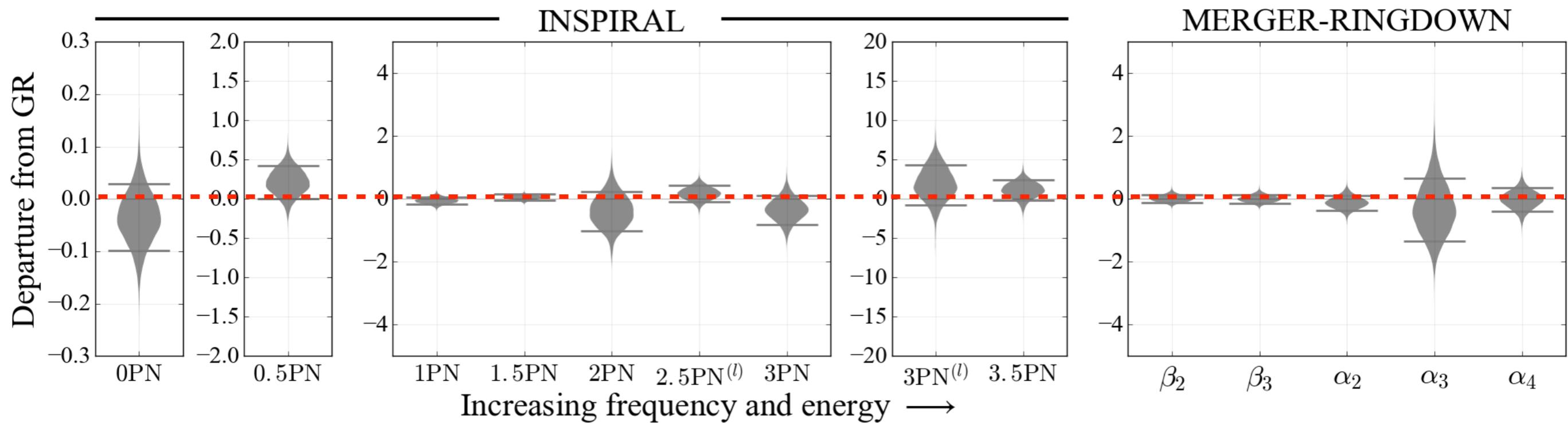
Fractional changes of the form: $p_i \rightarrow (1 + \delta\hat{p}_i) p_i$

Inspiral $\left\{ \delta\hat{\phi}_i \right\}$

Intermediate $\left\{ \delta\hat{\beta}_i \right\}$

Merger-
Ringdown $\left\{ \delta\hat{\alpha}_i \right\}$

BBH Science: Parameterized Tests of General Relativity?



No evidence for disagreement with the predictions of General Relativity.

BBH Science: Testing for Lorentz Invariance

Graviton energy:

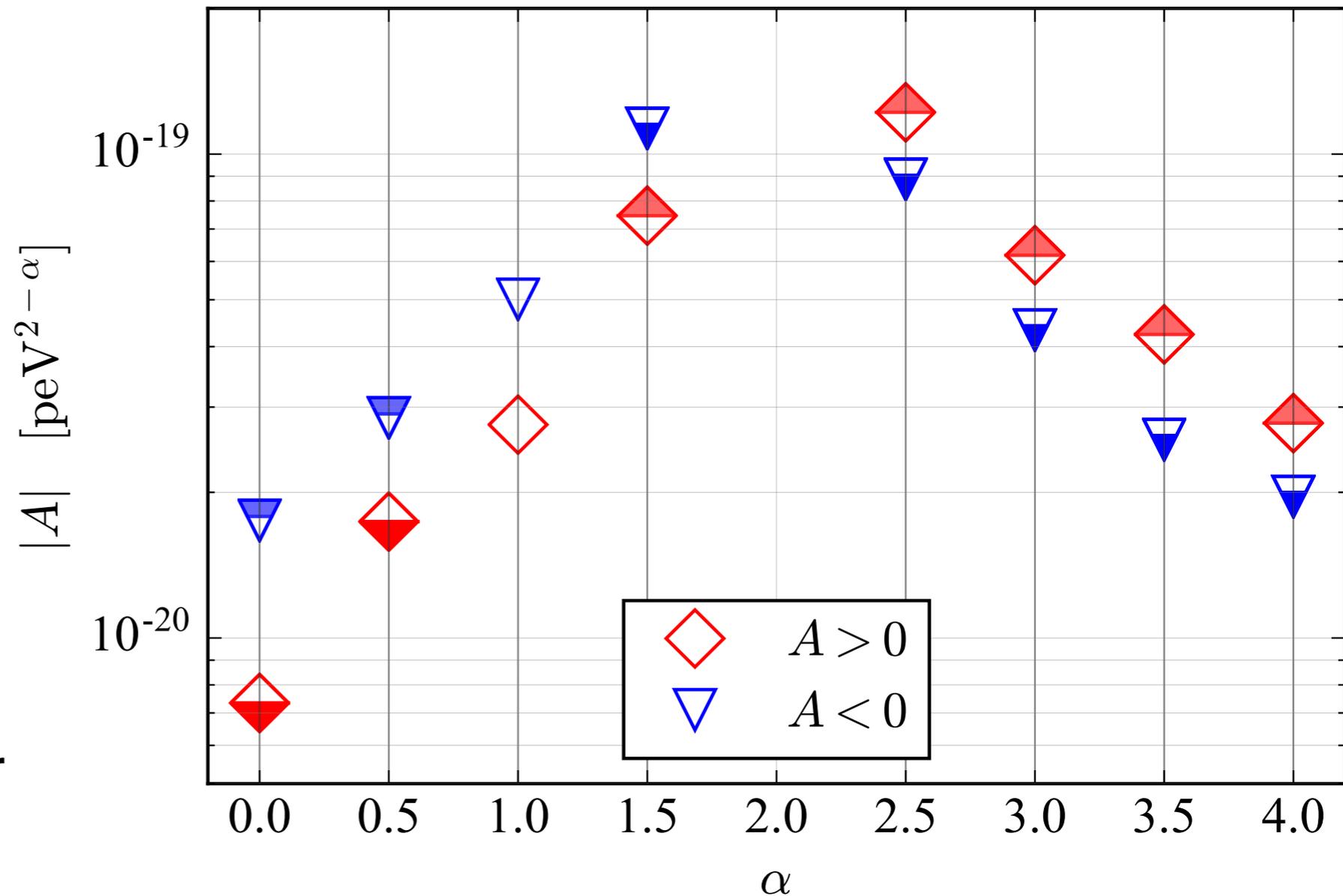
$$E^2 = p^2 c^2 + m_g^2 c^4 + A p^\alpha c^\alpha$$

Graviton's Compton wavelength:

$$\lambda_g = \frac{h}{m_g c}$$

Massive graviton propagates at energy (or frequency) dependent speed:

$$\frac{v_g^2}{c^2} \equiv \frac{c^2 p^2}{E^2} \simeq 1 - \frac{h^2 c^2}{\lambda_g^2 E^2}$$

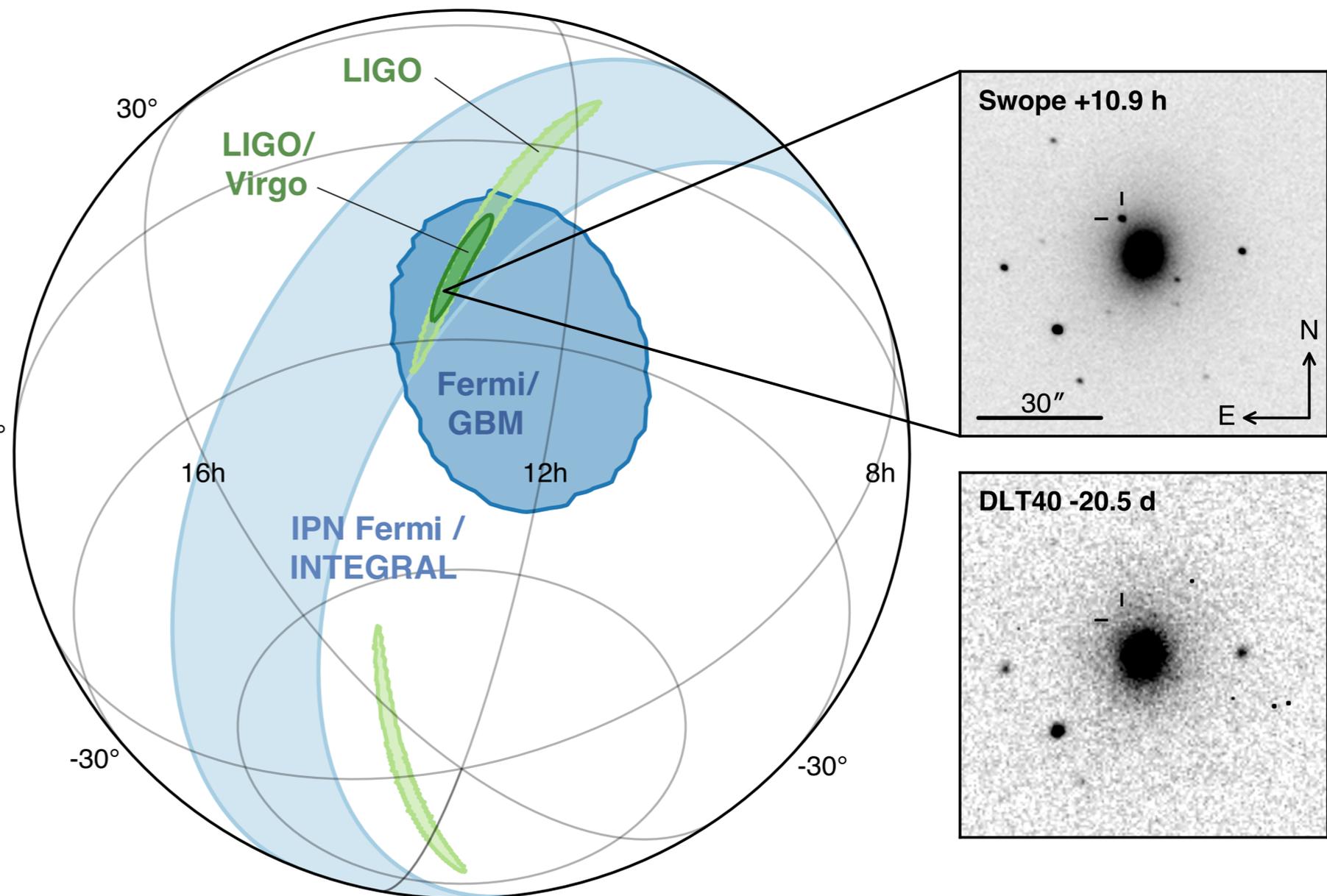


$$m_g \leq 7.7 \times 10^{-23} \text{ eV}/c^2$$

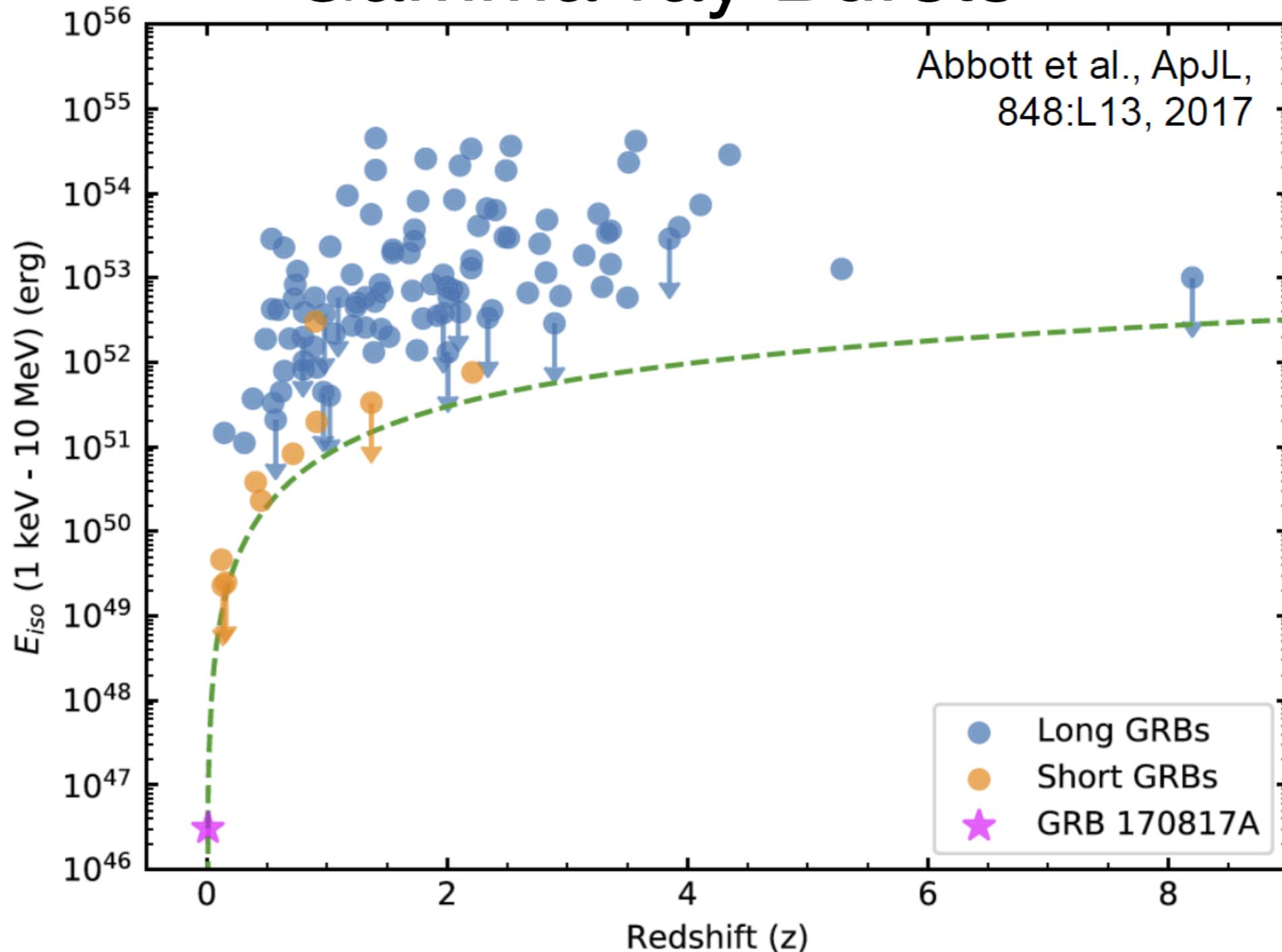
$$\lambda_g > 10^{13} \text{ km}$$

BNS Science: Start of Multi-messenger Astronomy with Gravitational Waves

Around 70 observatories worldwide observed the event by using space and ground-based telescopes.



BNS Science: Association with Short Gamma-ray Bursts



GWs identified the progenitor of the short gamma-ray burst. This was found to be the closest by and weakest short gamma-ray burst with redshift measured.

BNS Science: In-depth Study of Kilonova

Element Origins

1 H																	2 He	
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne	
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar	
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr	
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe	
55 Cs	56 Ba			72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra																	
				57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu
				89 Ac	90 Th	91 Pa	92 U											

Merging Neutron Stars
Dying Low Mass Stars

Exploding Massive Stars
Exploding White Dwarfs

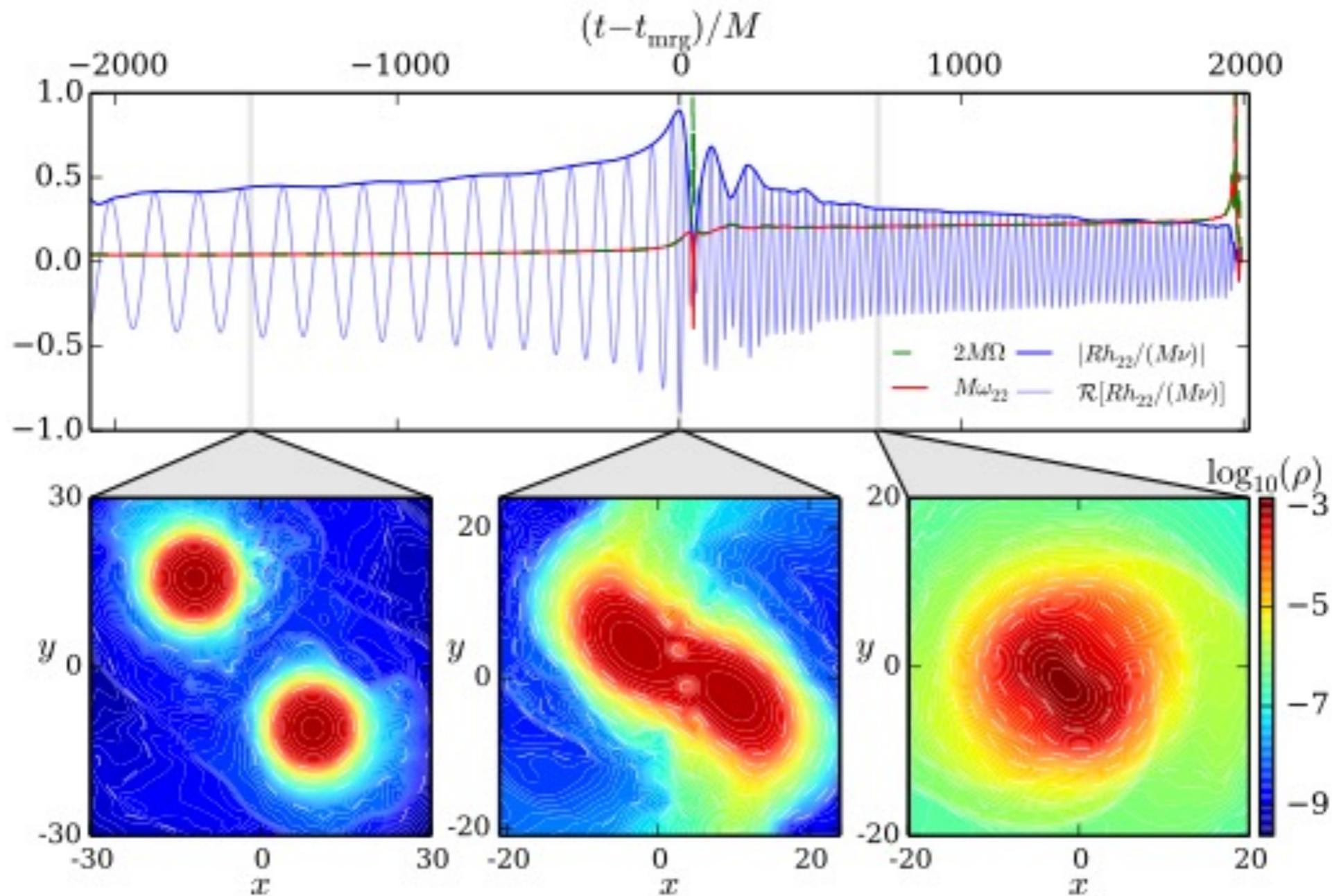
Big Bang
Cosmic Ray Fission

Based on graphic created by Jennifer Johnson

Astronomers performed in-depth study of a kilonova and witnessed the creation of heavy elements.

BNS Science: Neutron Star Equation of State (EOS)

Neutron stars are similar density to atomic nuclei.



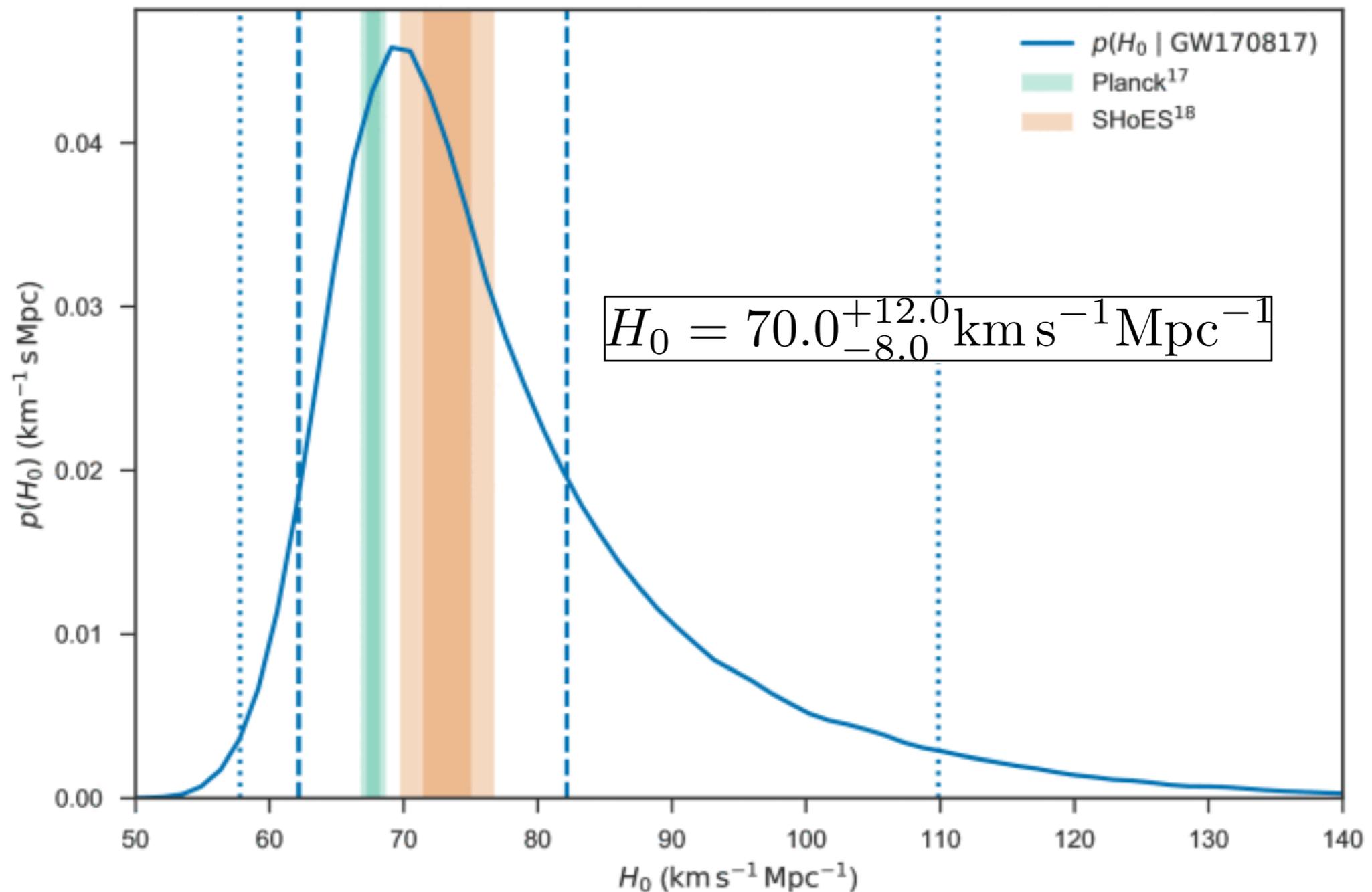
Tidal effects leave their imprint on GW signal from binary neutron stars.

BNS Science: Measuring the Hubble Constant

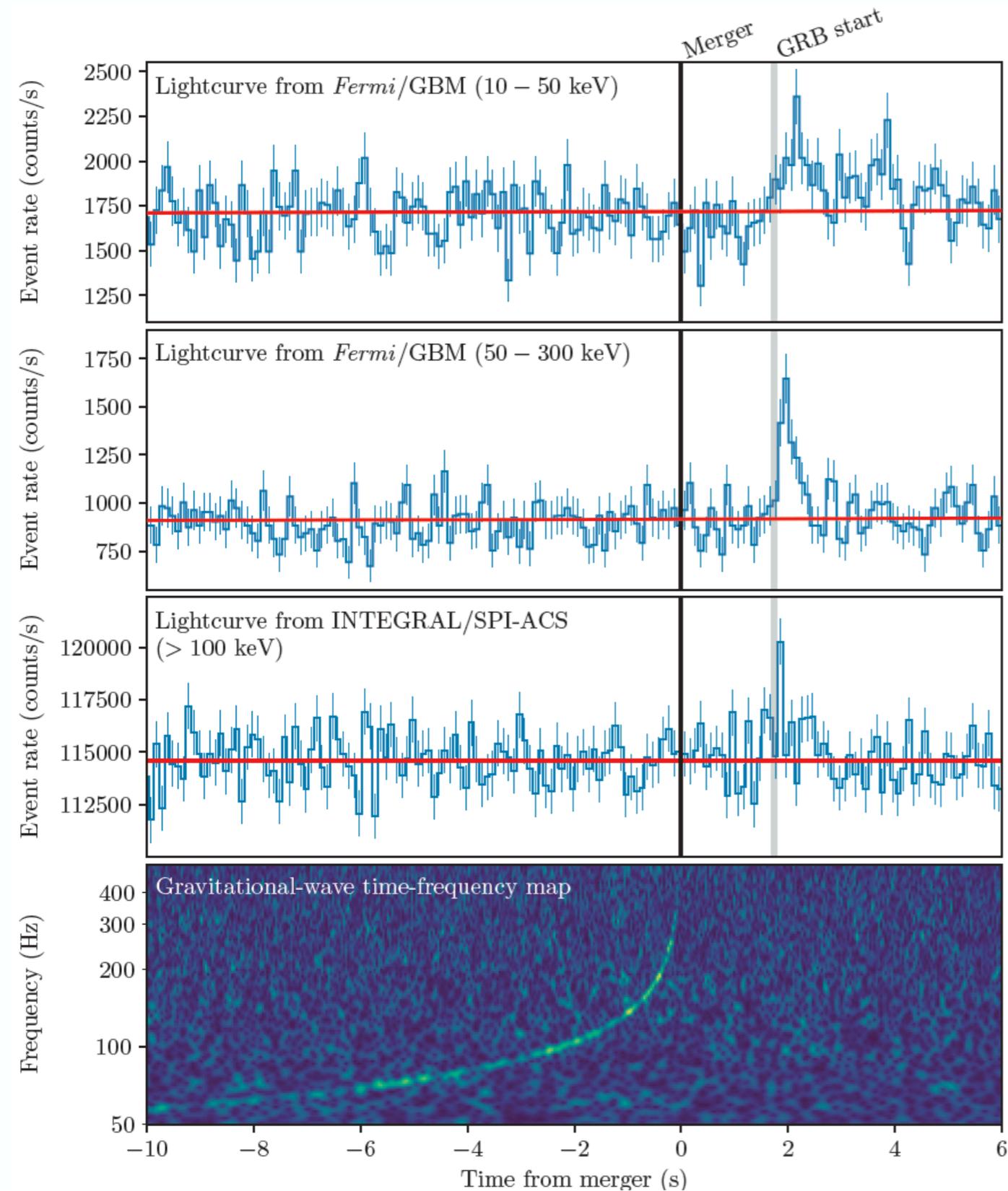
$$v_H = H_0 d$$

Distance from GW signal: $d = 44 \text{ Mpc}$

Recessional velocity from galaxy: $v_H = 3000 \text{ km/s}$



BNS Science: Measuring the speed of gravity



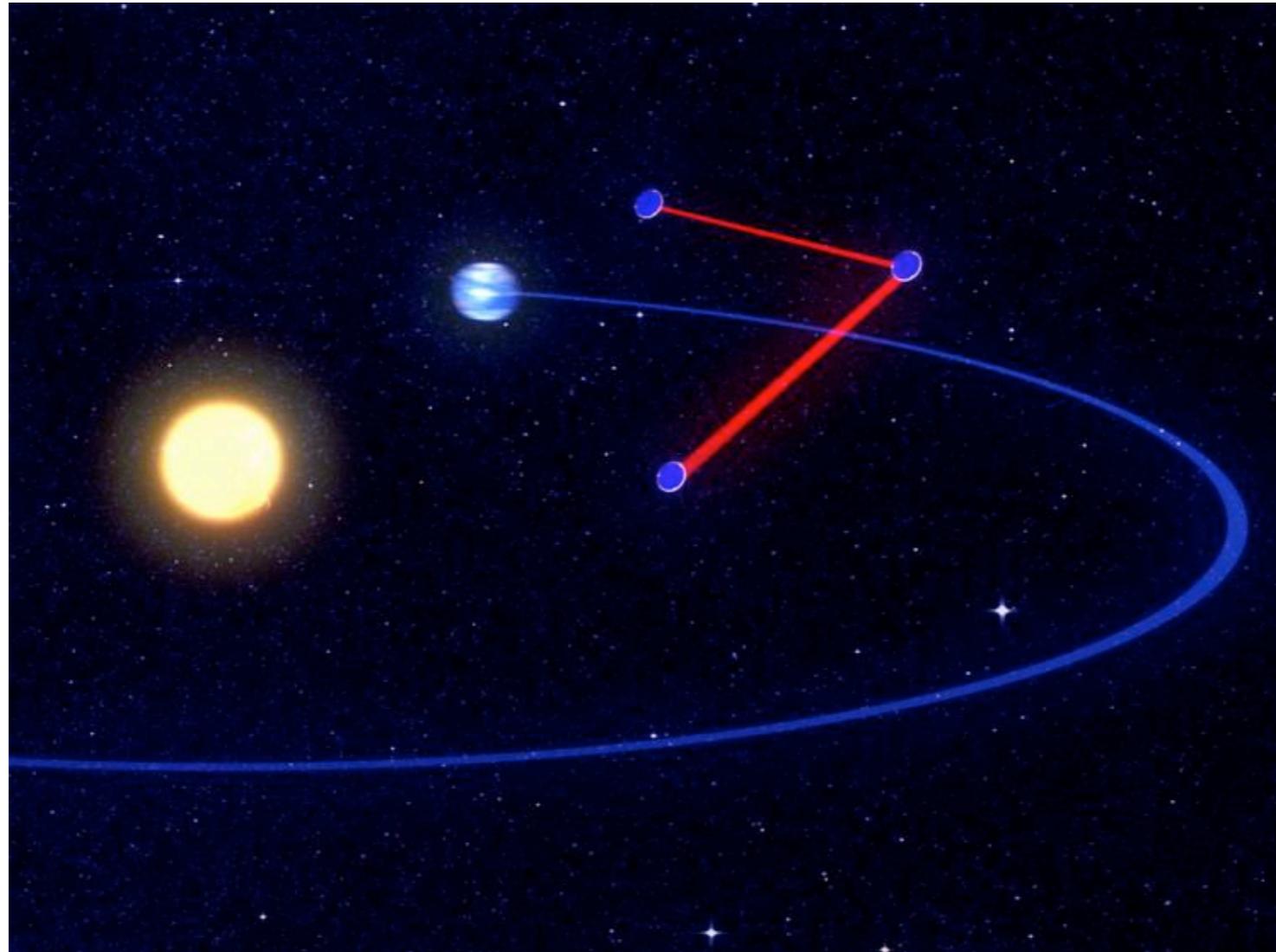
Gamma-rays reached Earth 1.7s after GW event. Nearly identical speeds of GWs and light propagation.

$$c_{\text{gw}} = c_{\text{light}} \pm \frac{(\sim 2 \text{ light seconds})}{(\sim 100\,000\,000 \text{ years})}$$

$$= 299\,792\,458.000\,000\,0 \pm_{-0.000\,000\,9}^{+0.000\,000\,2} \text{ms}^{-1}$$

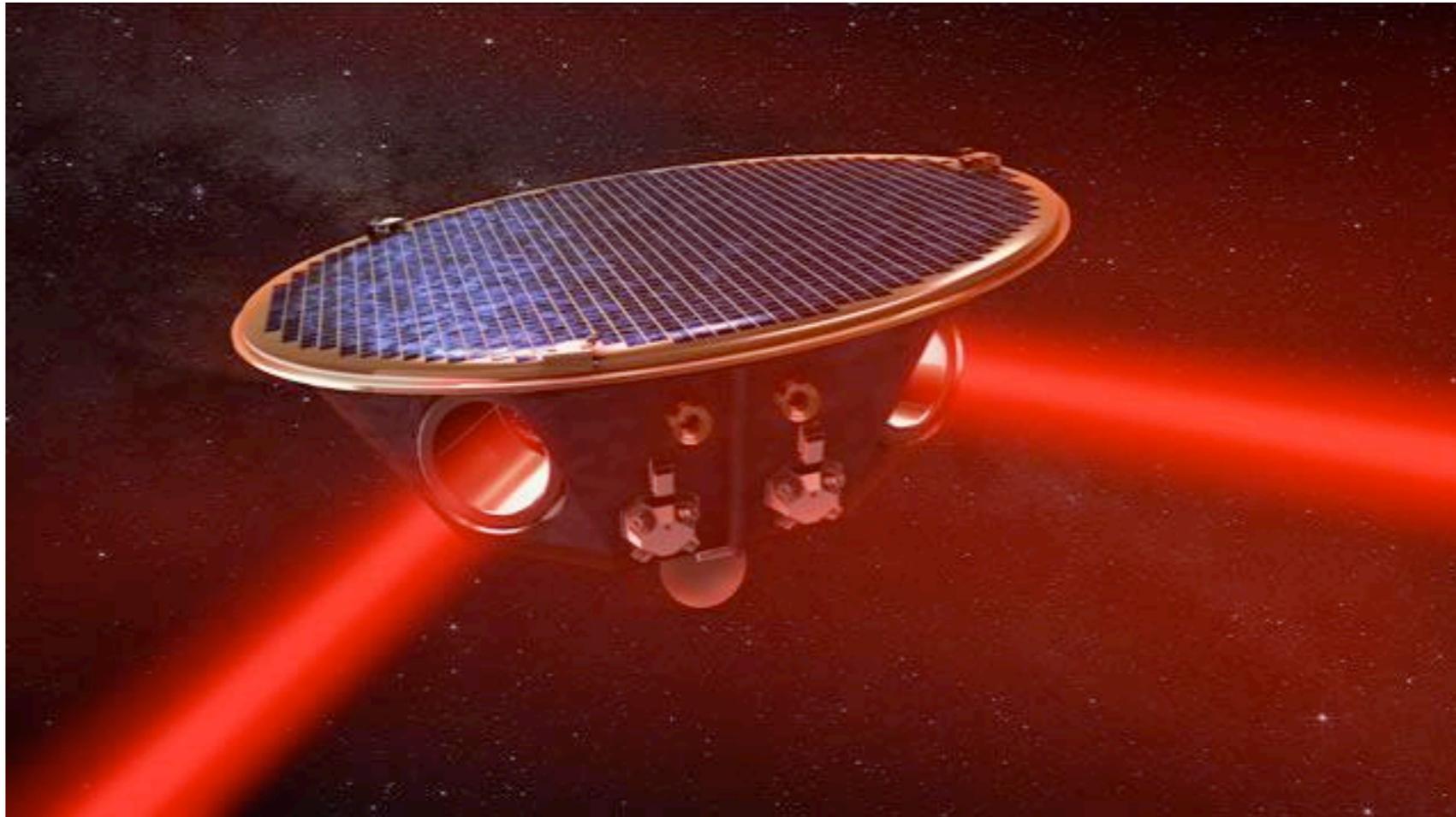
- What are gravitational waves?
 - Detectors
 - Sources
- A brief history of the field
- Some terminology
- Current state of the field
 - Notable detections
 - Science results
- Next steps
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Space-based Detectors: eLISA



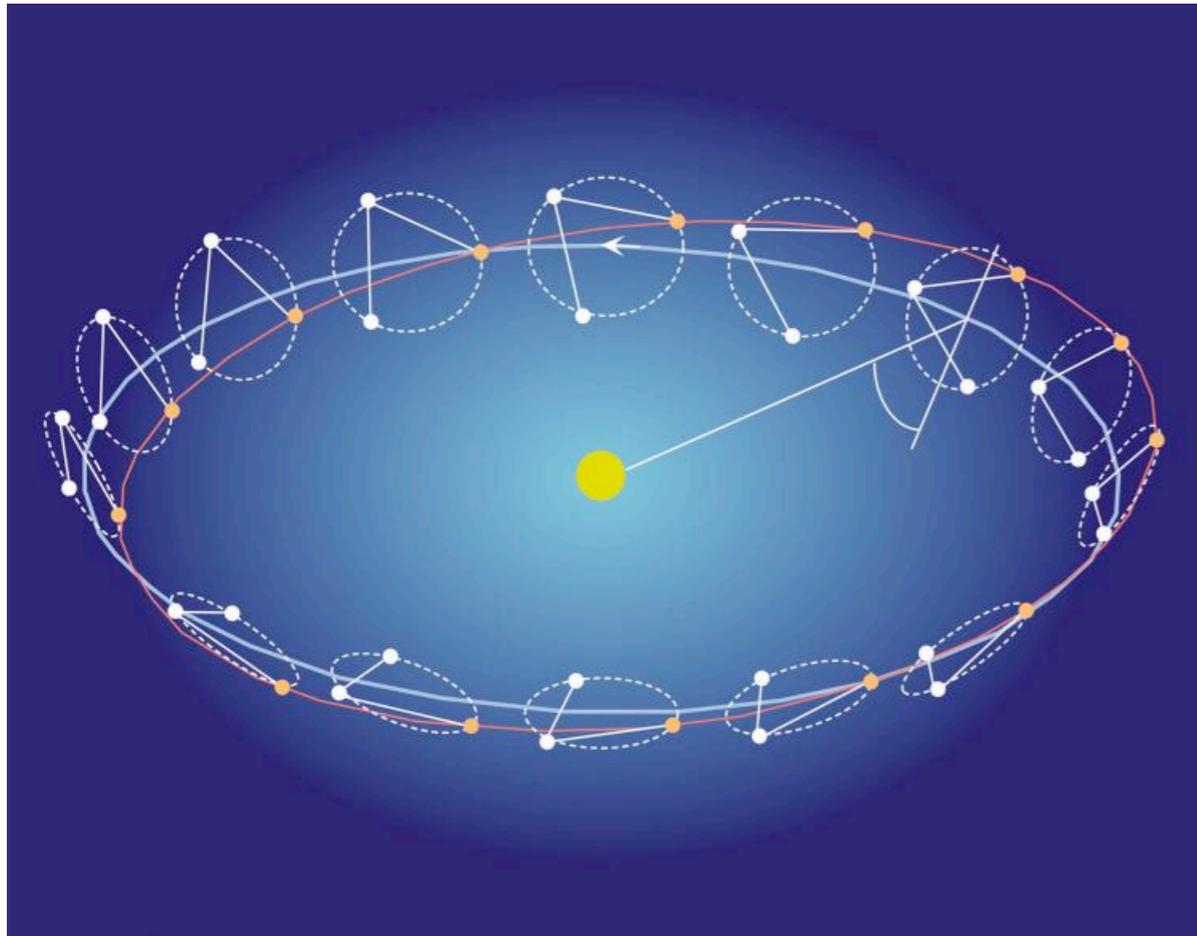
Three spacecraft in Earth-like orbits around Sun.
Arm length of 1 million km, equilateral triangle formation.

Space-based Detectors: eLISA



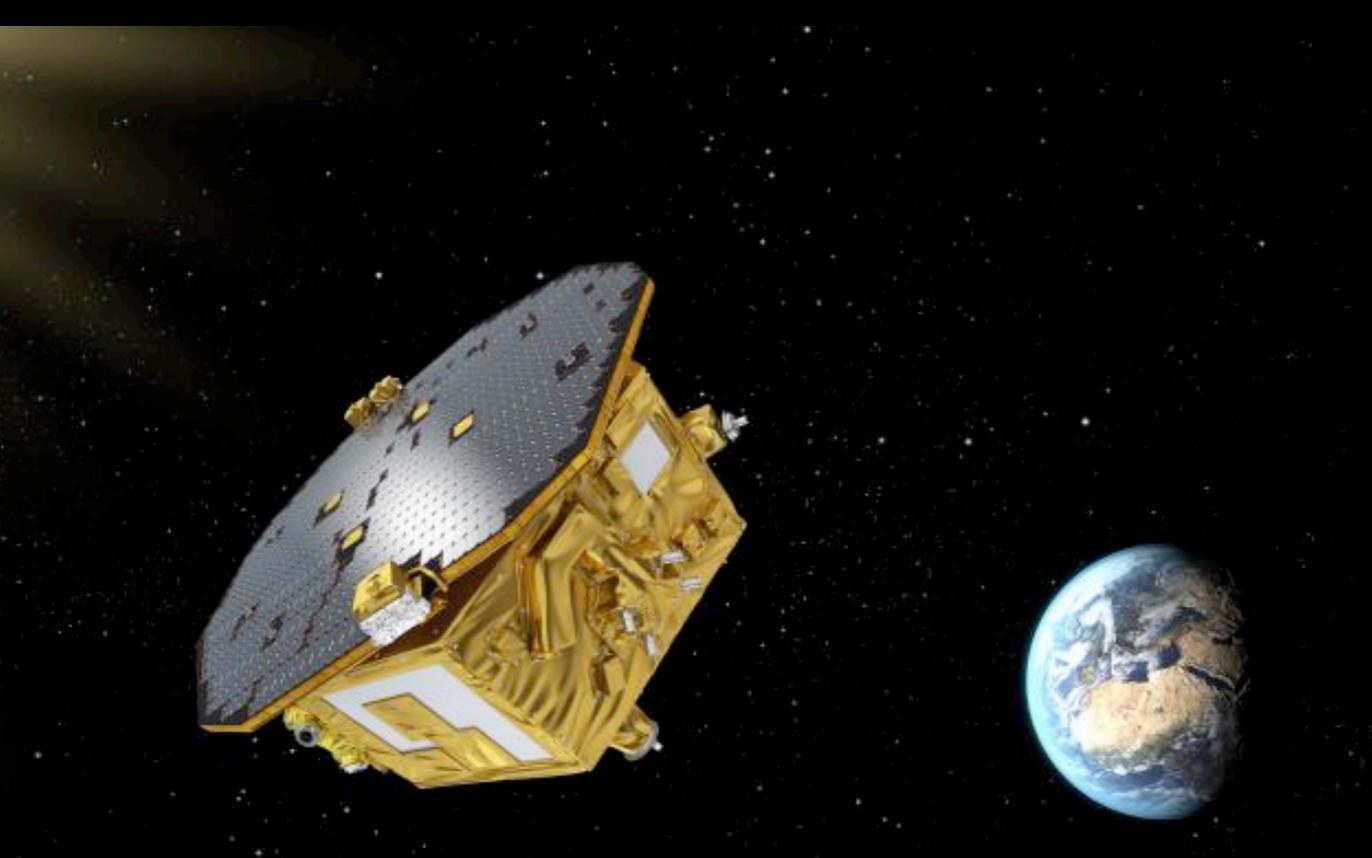
Each spacecraft carries lasers. They're too far apart for the laser light to reflect back and forth. Instead, when light from one arrives at the other, the onboard laser of receiving craft amplifies and returns it (like reflection): active mirrors.

Space-based Detectors: eLISA

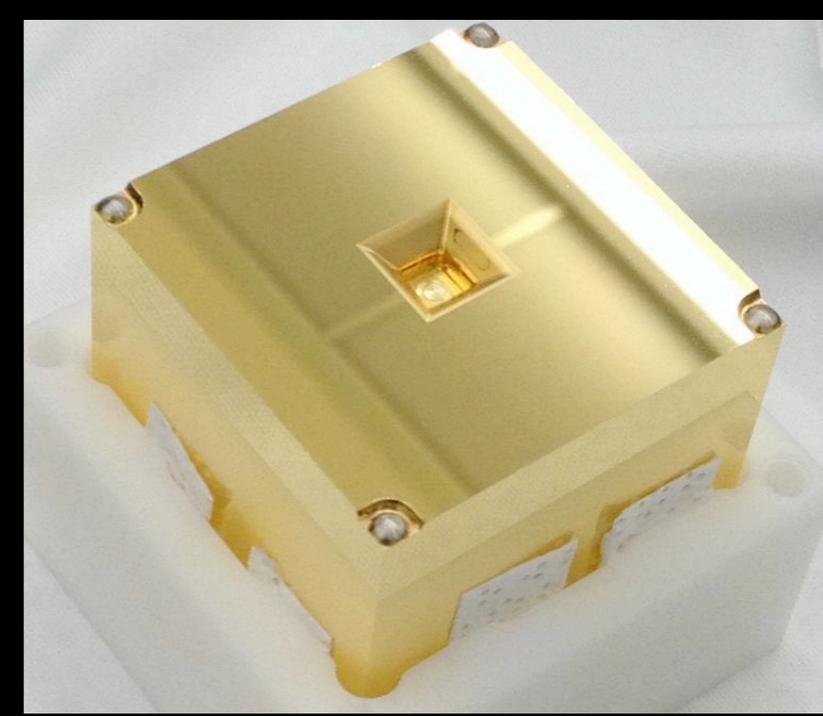
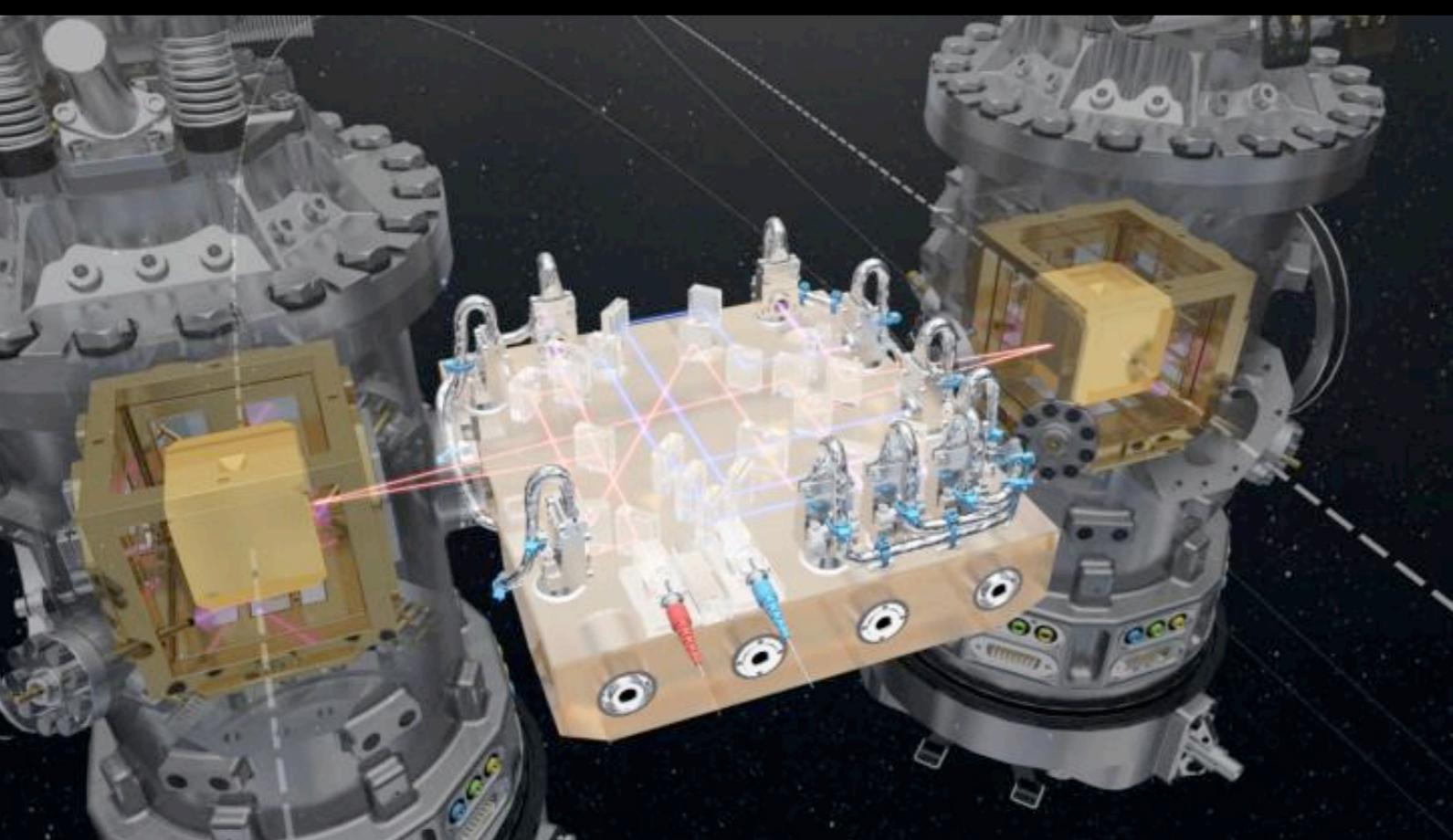


Changing orientation of triangle through one orbit will allow observers to determine direction of sources.

LISA pathfinder operates 1.5 mill km from Earth toward the Sun, orbiting the Sun-Earth langrangian point L1.



Two inertial sensors surrounding independent test masses and an optical bench in-between monitor test masses as they free-fall through space



One of LISA Pathfinder's two test masses, cube of solid gold-platinum alloy, 1.96kg.

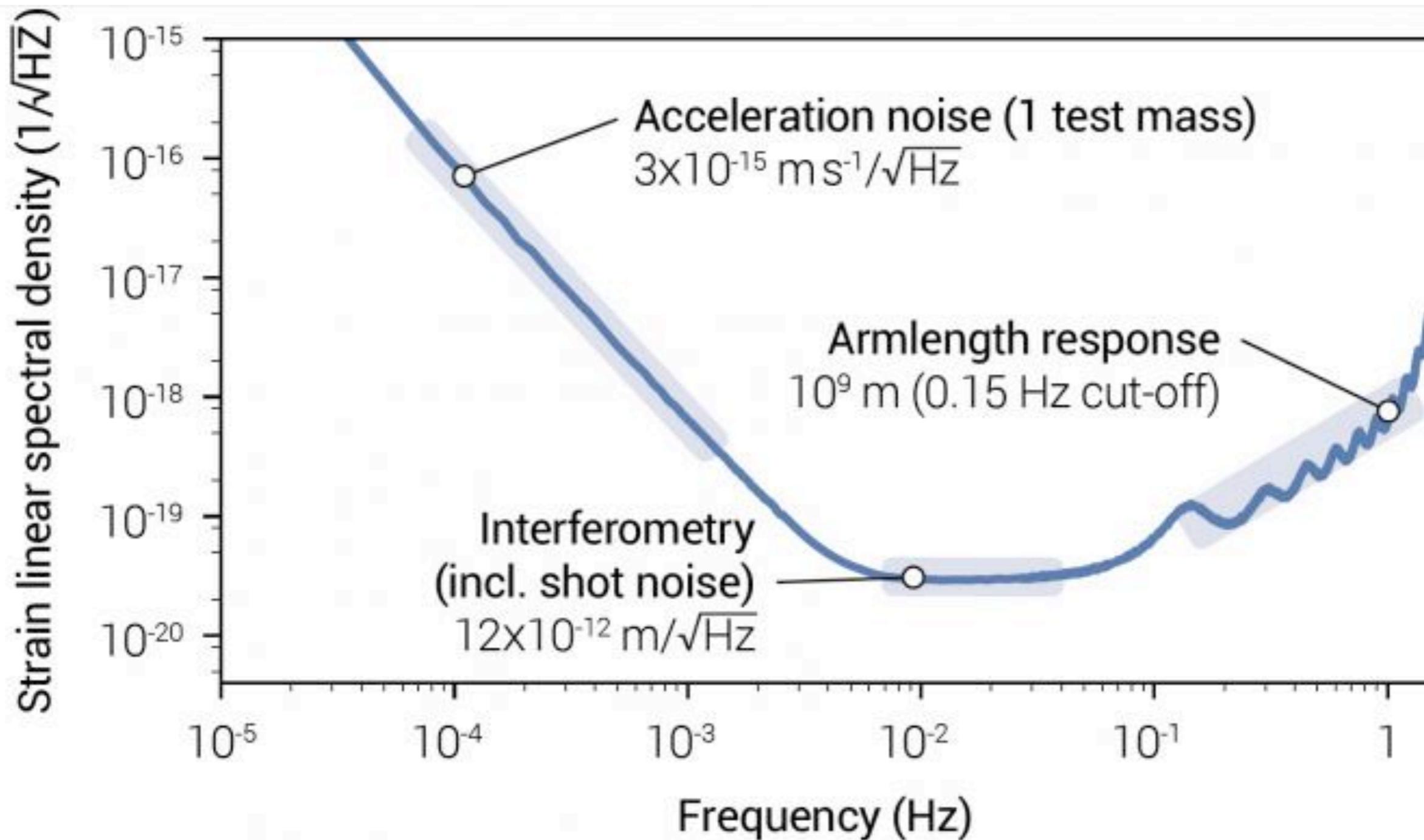
Space-based Detectors: eLISA



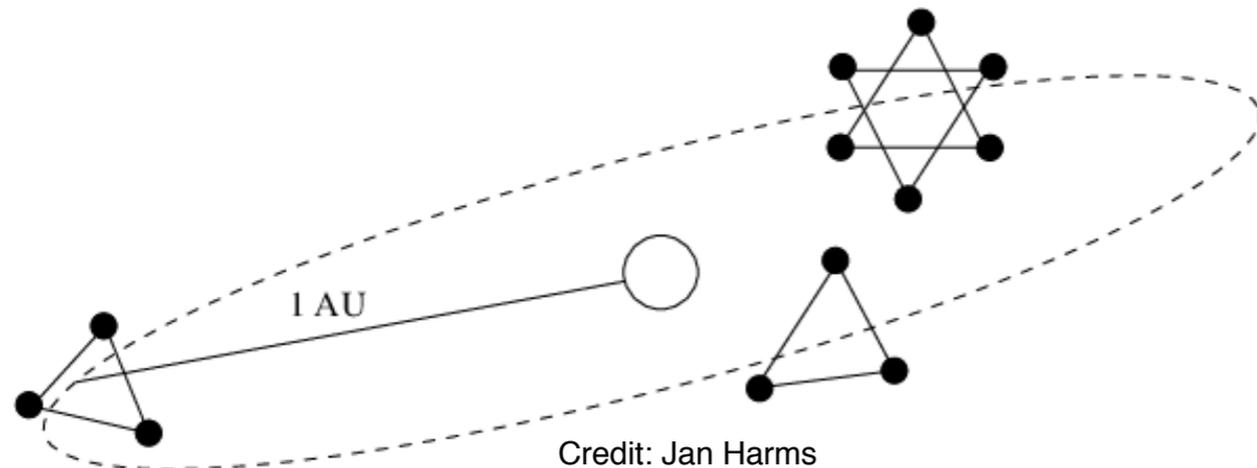
Microthrusters will make minuscule shifts in order to keep the spacecraft centered on one of the masses. This will isolate the two cubes from all external and internal forces except gravity, placing them in the most precise freefall ever obtained.



LISA Sensitivity



Big-Bang Observer Concept



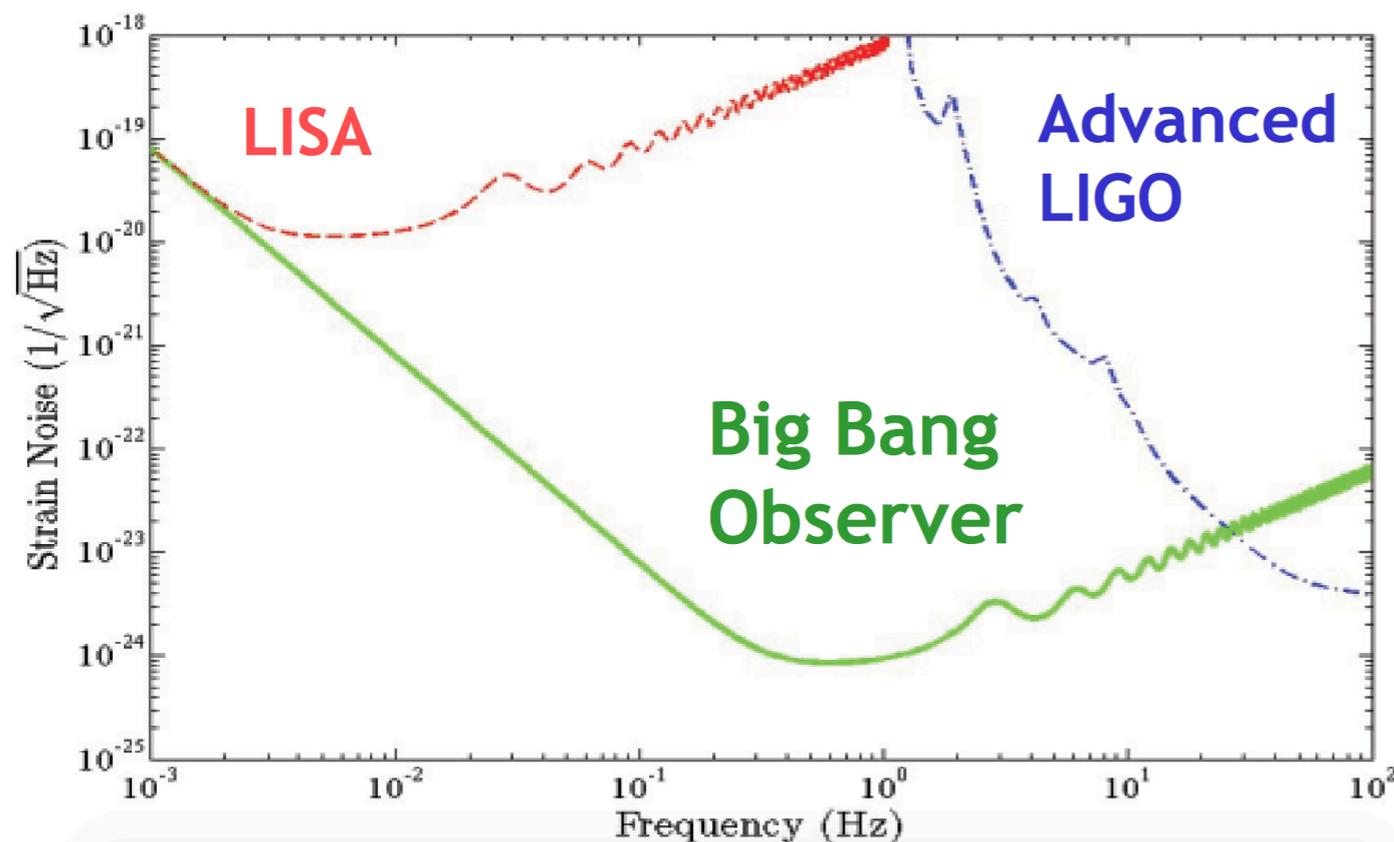
Proposed successor to LISA.

Four LISA-like triangles with more powerful lasers.

If funding obtained, development is many decades away.

Goal: detect GW relics from inflation

Would be sensitive to all LIGO and LISA sources.



DECIGO Concept

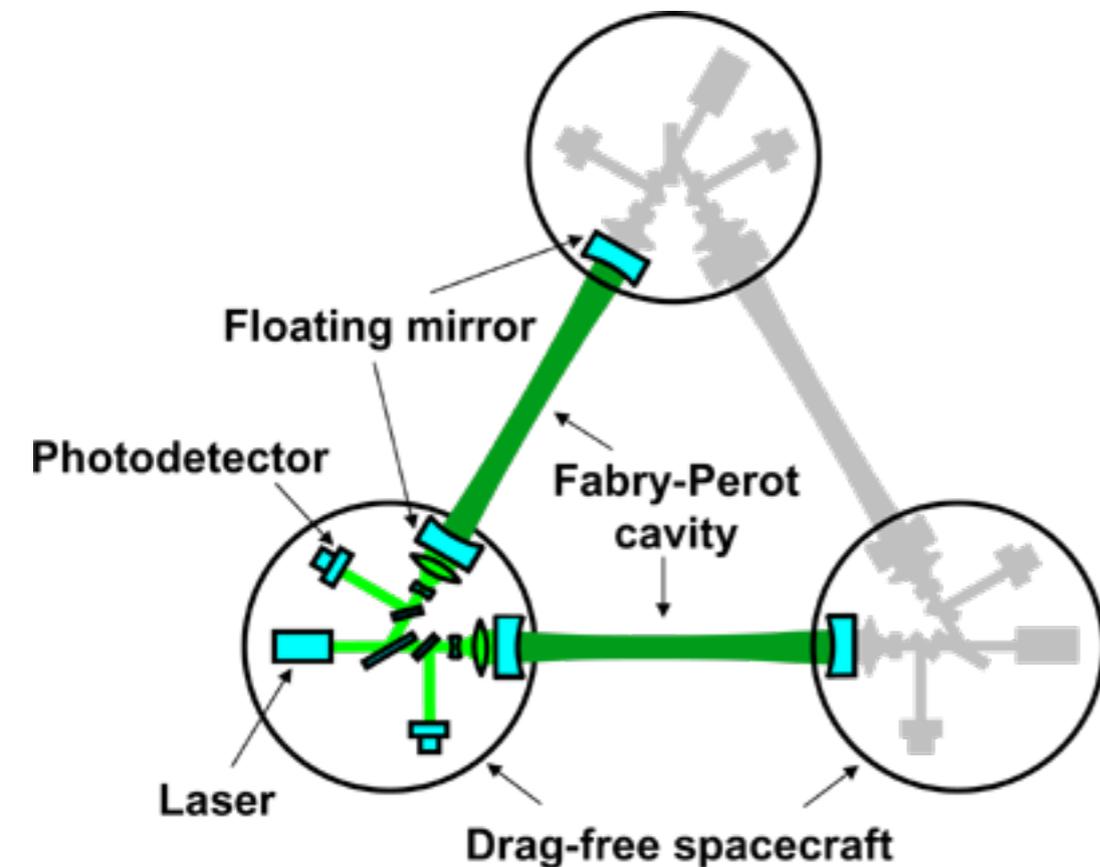
DECIGO - DECI-hertz
Interferometer Gravitational-wave
Observatory.

Fill in gap between LIGO and
LISA (0.1-10 Hz)

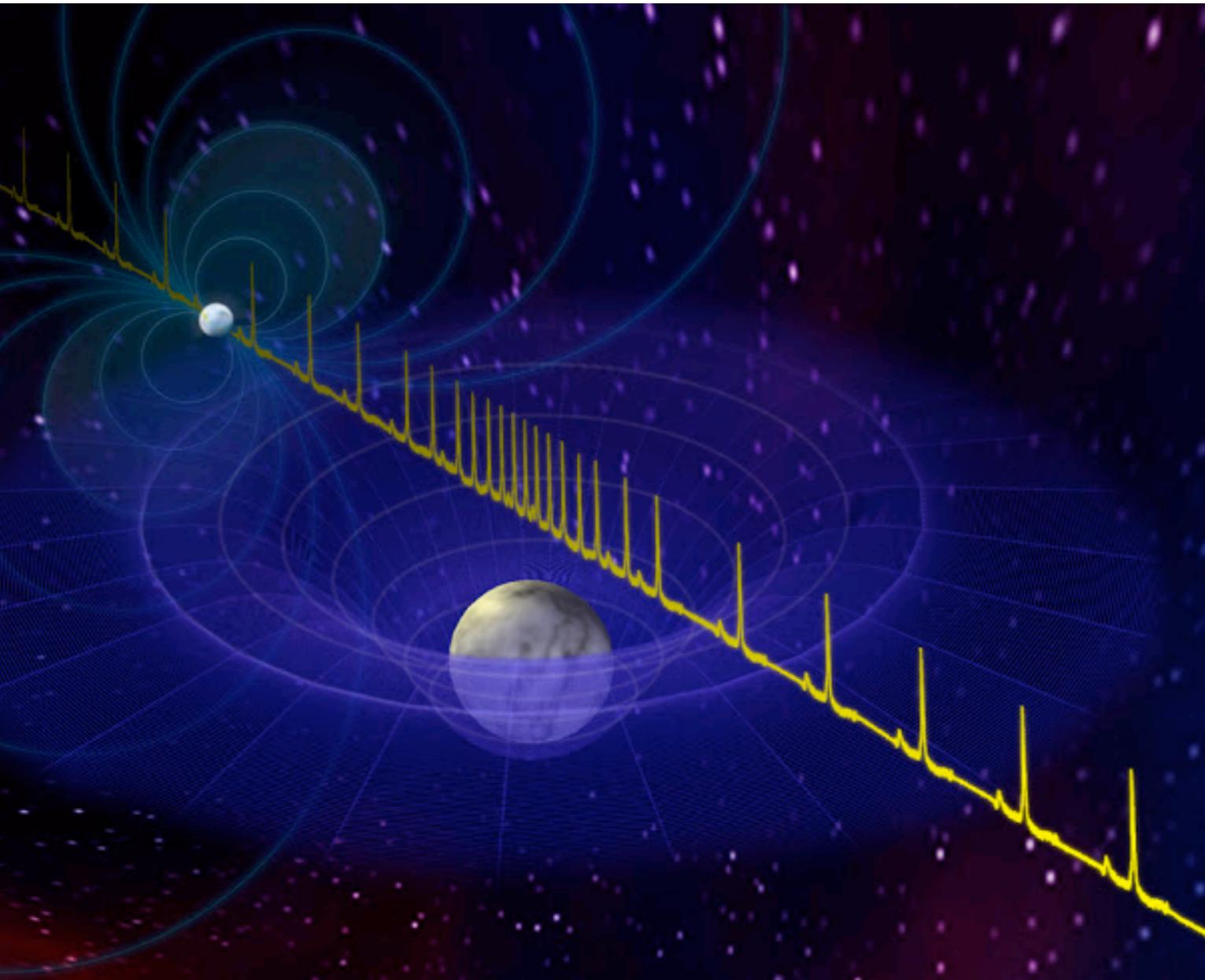
Similar to LISA but smaller
separation.

Precursor mission planned for late
2020s

Goal: detect GW relics from
inflation



Pulsar Timing Arrays



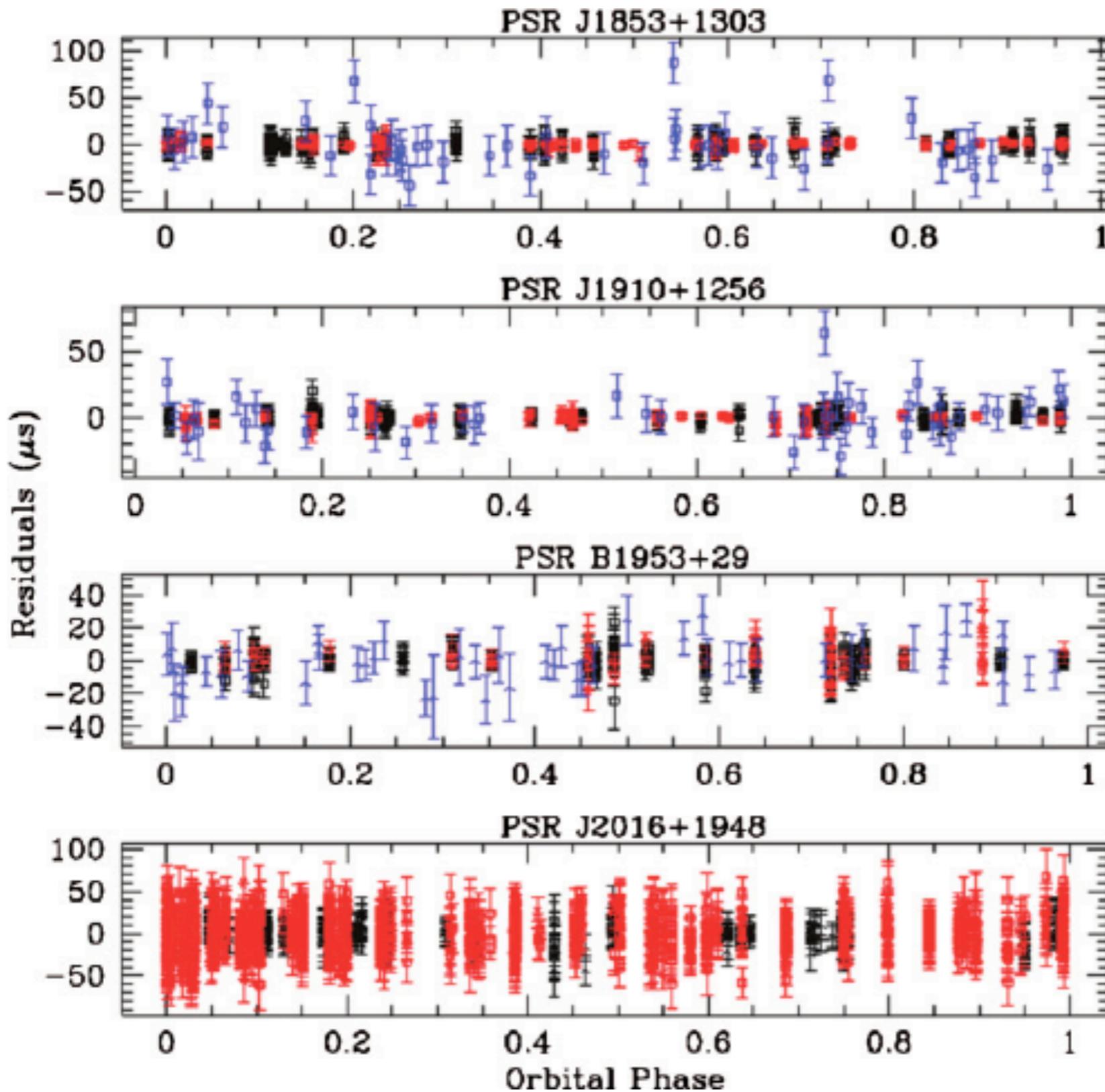
Millisecond pulsars - rotate with incredible stability, can be used as precise clocks.

The time of arrival (TOA) of pulse can be usually measured to:

$$\frac{\delta}{\text{SNR}}$$

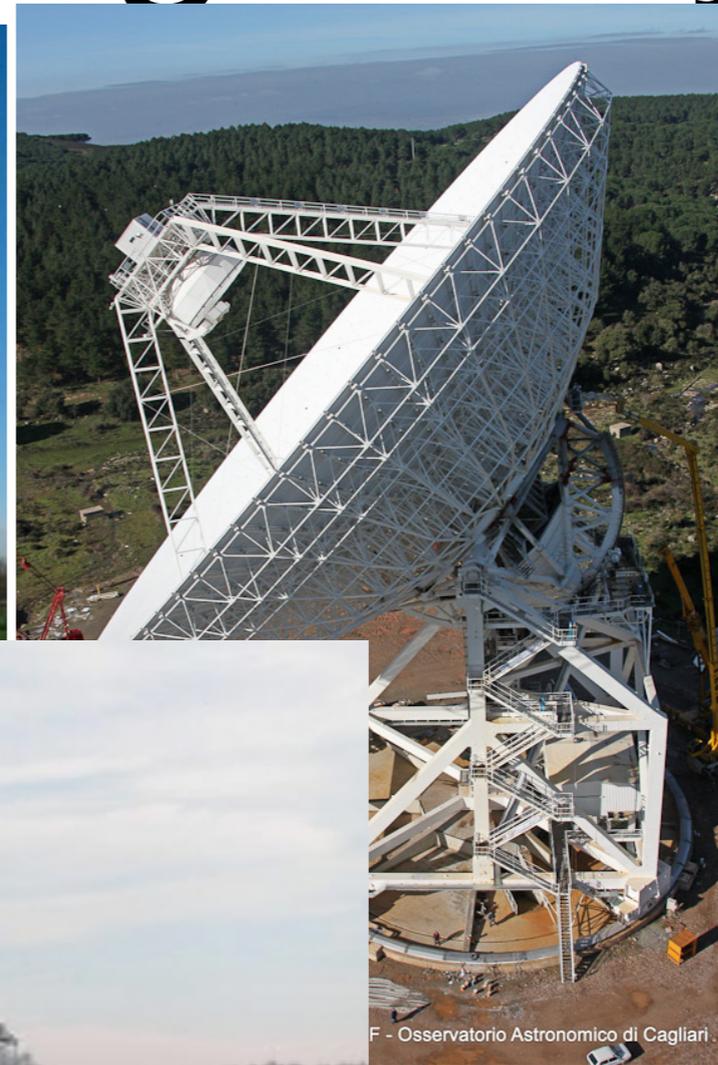
δ - pulse width; few hundred microseconds

Pulsar Timing Arrays

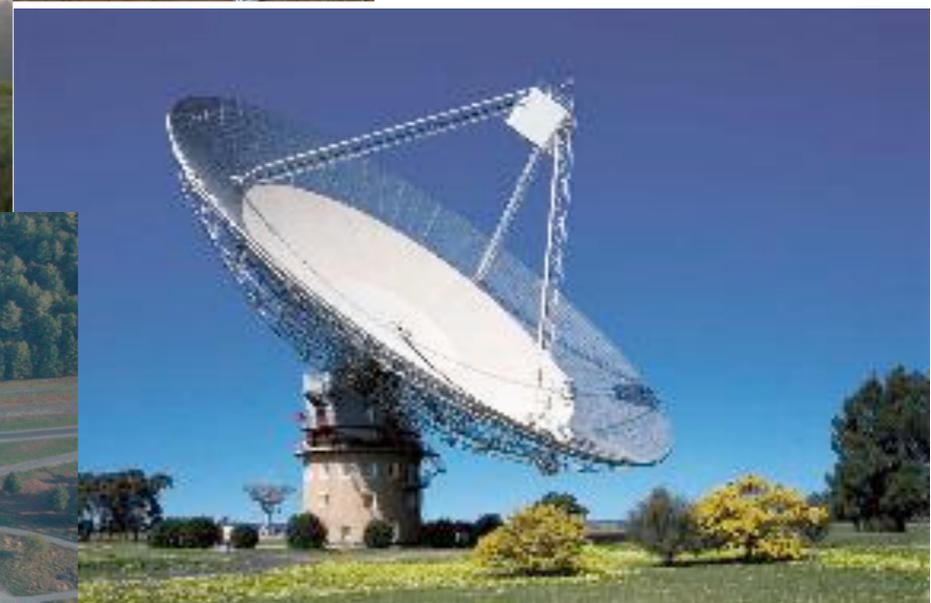


Difference between predicted and measured TOAs can have RMS scatter as little as 100 nanoseconds over timescales of many years.

Pulsar Timing Arrays



Germany
UK
France
Italy
Netherlands
US
Puerto Rico
Australia



Pulsar Timing Arrays



Pulsar timing brings together astronomers who use the world's most sensitive radio telescopes to observe and discover millisecond pulsars.

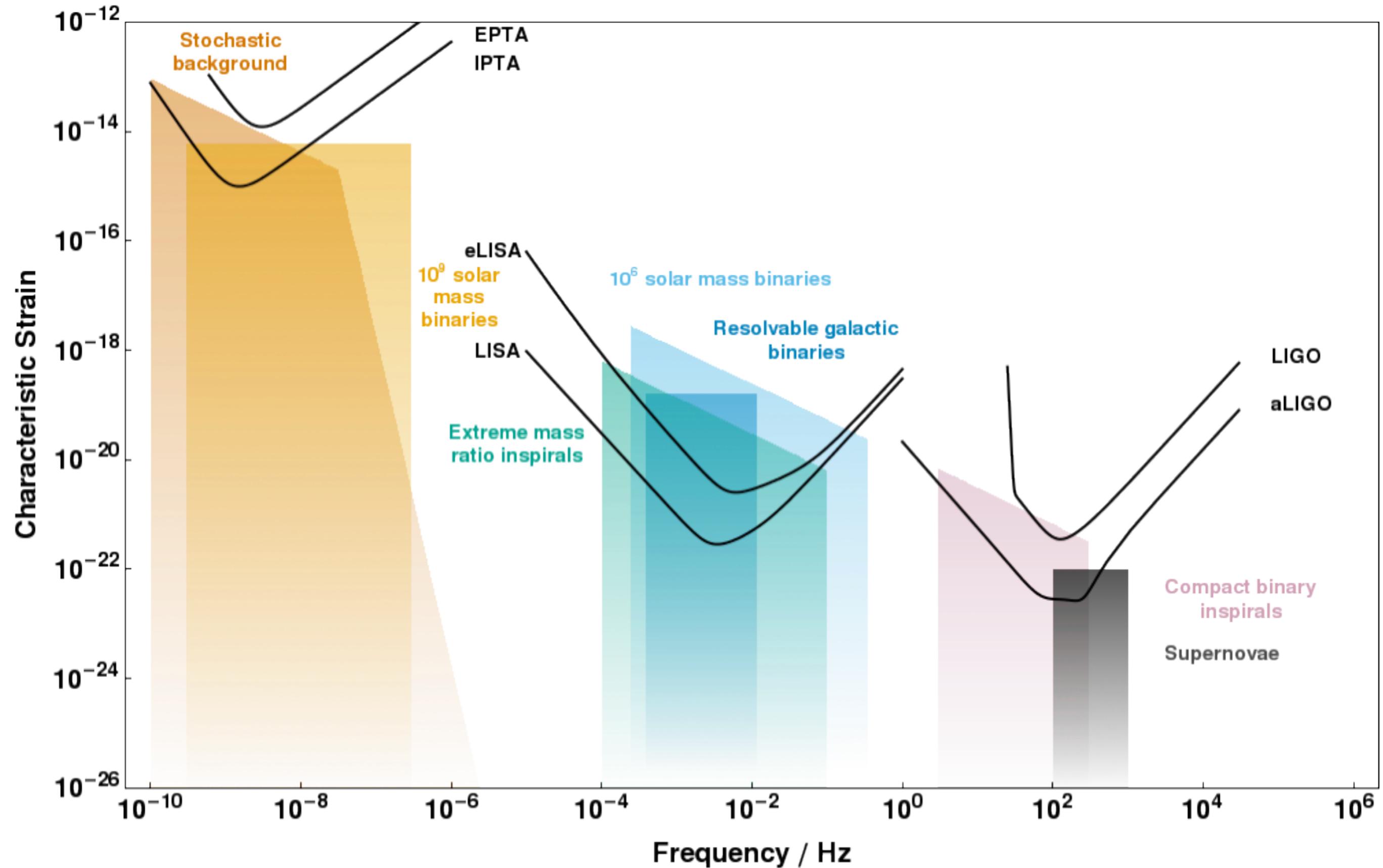
They look for the influence of GWs on timing residuals from ultra-precise millisecond pulsars, in particular for correlations in timing residuals between pairs of pulsars

Pulsar Timing Arrays



Sensitive to GWs with periods between the cadence of pulsar timing observations (weeks) and the span of the dataset (years), ie nanohertz frequencies.

Gravitational-wave Spectrum

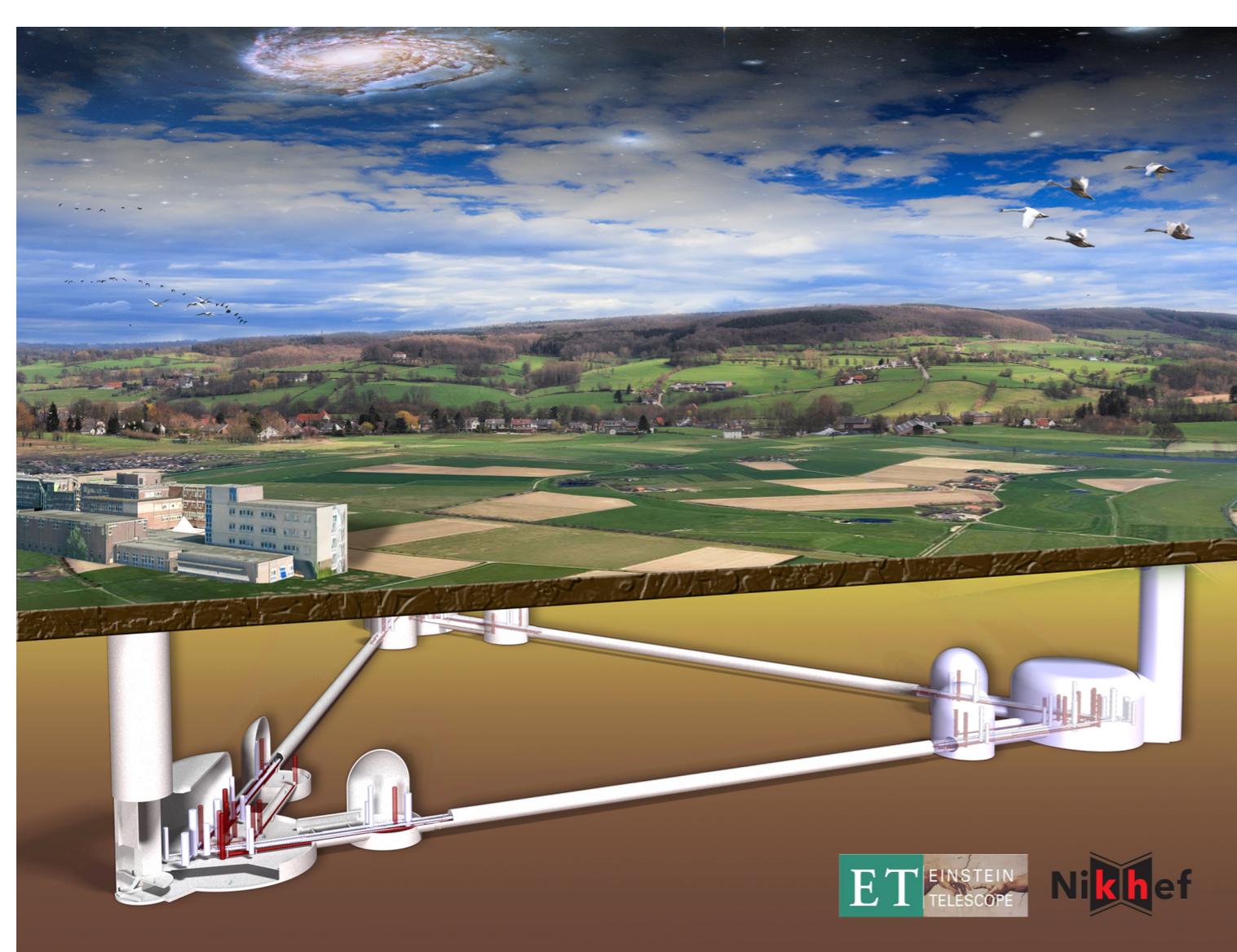


Einstein Telescope

Proposed 3G ground-based GW detector.

10-km arms, equilateral triangle design, two detectors, underground, cryogenic.

Proposed site could be south Netherlands or Sardinia



Einstein Telescope Pathfinder

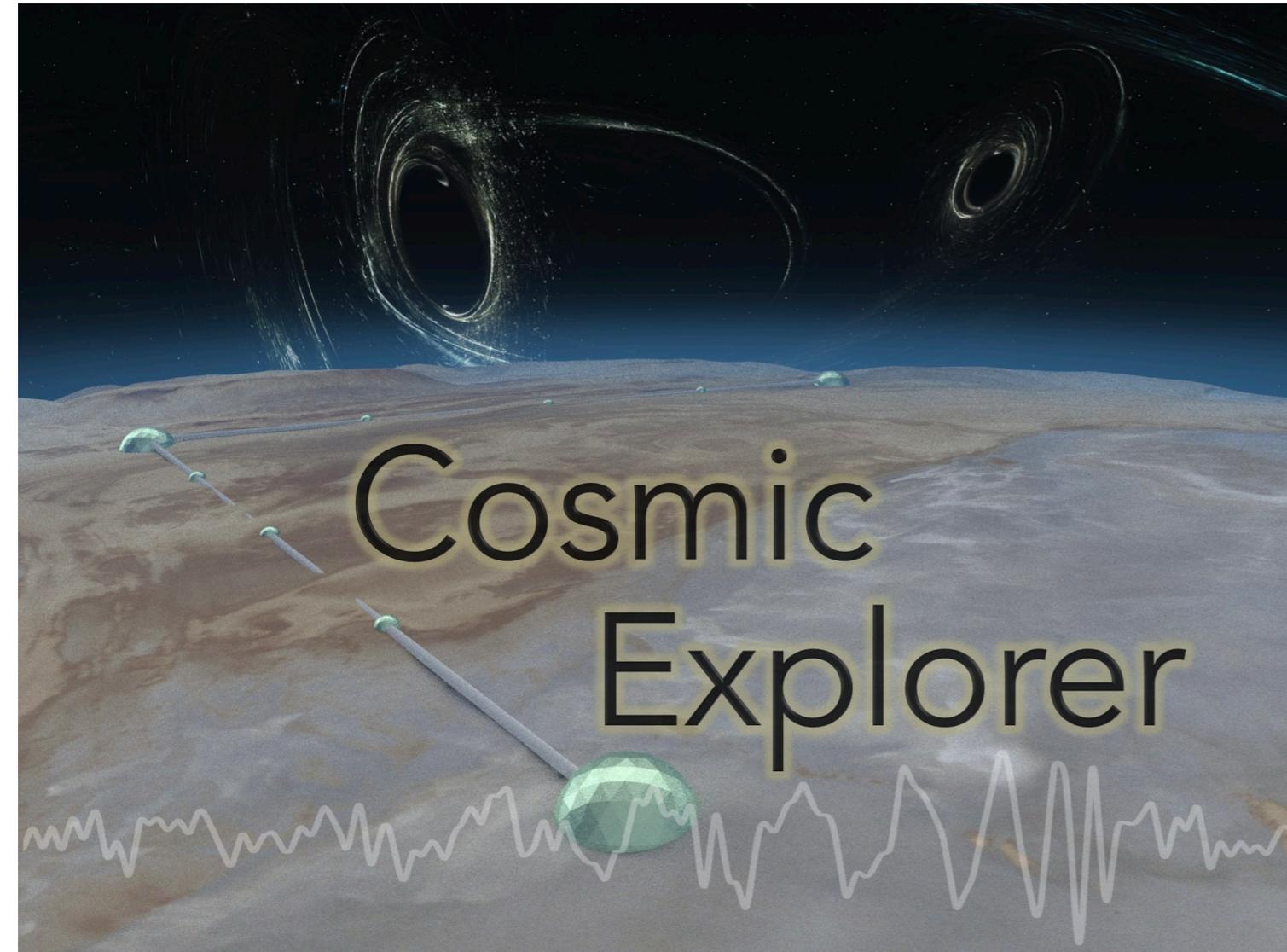


Housed in Maastricht University building.

Construction begins soon.

Allows for research into technologies needed by ET.

Cosmic Explorer

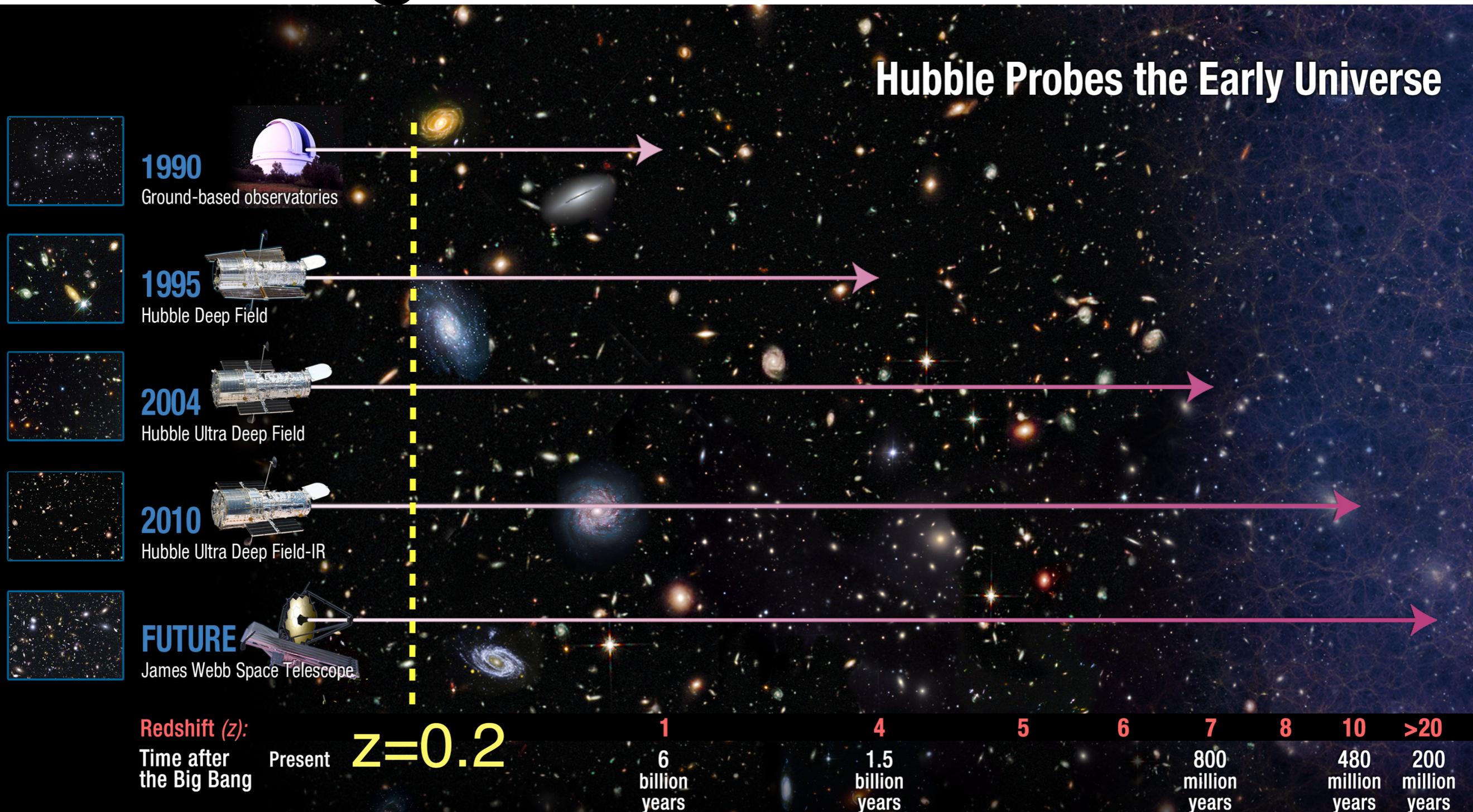


US-proposed 3G
ground-based GW
detector.

40-km arms, L-shape,
cryogenic.

Need very flat and
seismically quiet land in
US.

3G Goal: observing all BBH mergers in the Universe



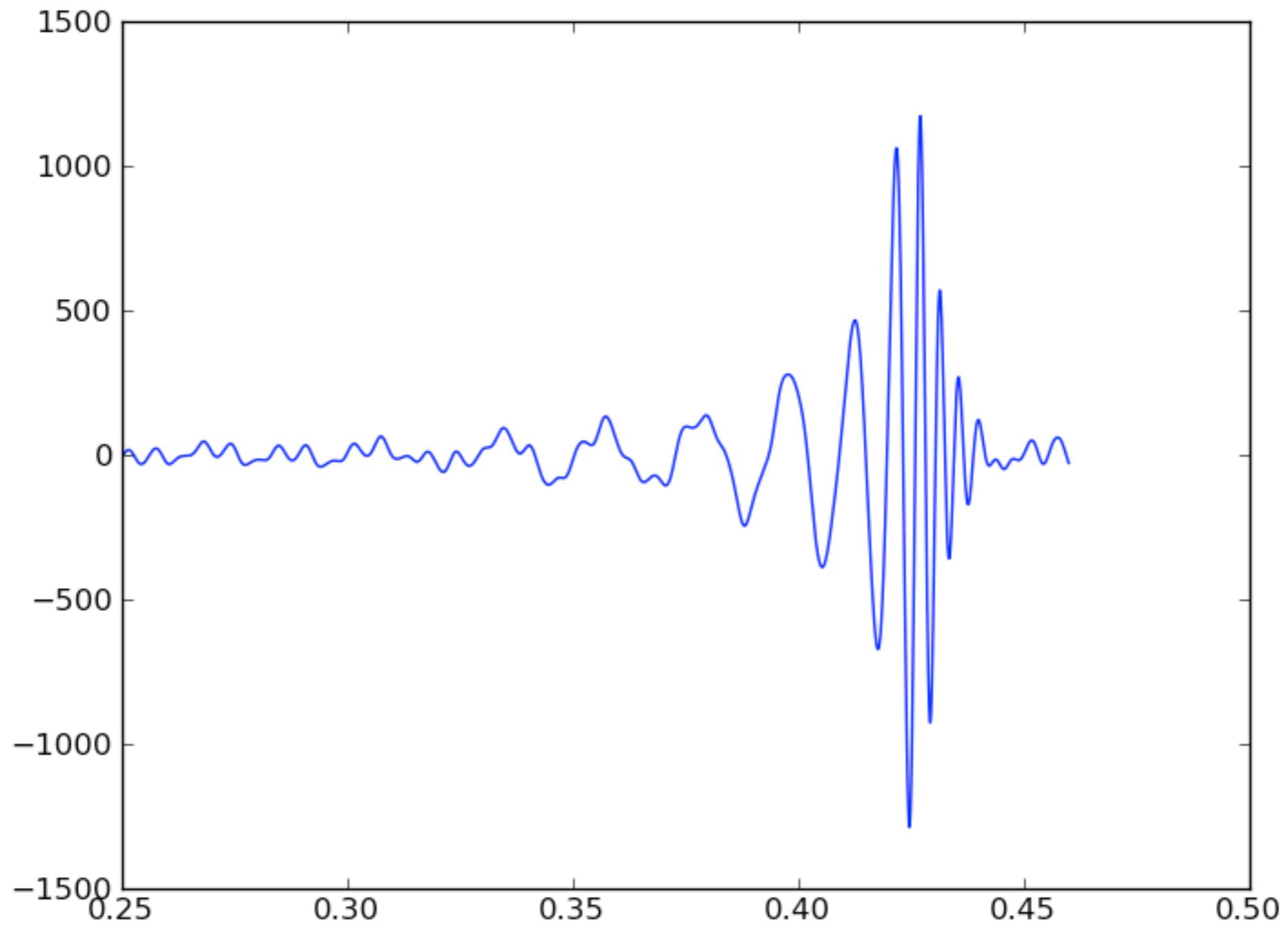
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Python Tutorials:

- <https://wiki.python.org/moin/BeginnersGuide/Programmers>
- Also check out Coursera

This week's homework is based on paper here about the basic physics of binary black hole merger GW150914:
<https://arxiv.org/abs/1608.01940>

Problem 2: Waveform should look like:



Problem 2: Determine zero crossings:

