Announcements

Please turn in Assignment 2 and pick up Assignment 3
You can also email assignments to the TAs:

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Pick up Visualization Project description

Last week's big announcement: first image of black hole

Event Horizon Telescope

Gravitational Wave Derivation and Astrophysical Sources

Lecture 3: Gravitational Waves MSc Course

- Solving the Einstein Equations
    - Linearized Theory
    - Vacuum Solution
    - Solution with Source Term
- Generation of Gravitational Waves
- Effect of Gravitational Waves on Matter
- LIGO & Virgo Astrophysical Sources
    - Coalescing Binaries
    - Continuous Waves
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    - Stochastic Background
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The Einstein Equations

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$$

Given the source distribution $T_{\mu\nu}$, one can solve this set of 10 coupled nonlinear partial differential equations for the metric $g_{\mu\nu}(x)$

Methods

Solving Einstein's equations is difficult. They're non-linear. In fact, the equations of motion are impossible to solve unless there is some symmetry present.

In the absence of symmetry, there are two methods:

1. Numerical relativity (next time)
2. Approximation techniques

For the approximation technique, we consider a metric very close to flat space with a small perturbation. And we consider only first order perturbations.

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Linearized Theory of Metric Field

Consider the Minkowski metric - a combination of three dimensional Euclidean space and time into four dimensions.

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2 = \eta_{\mu\nu} dx^\mu dx^\nu$$

$$\eta_{\mu\nu} = \begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Consider a small perturbation $h_{\mu\nu}$ on flat space:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad |h_{\mu\nu}| \ll 1$$

so that higher orders of $h_{\mu\nu}$ can be neglected when substituting in Einstein Field Equations (EFE)

Linearized Theory of Metric Field

Can we make coordinate transformations under such systems? Yes, from one slightly curved one to another, aka “Background Lorentz transformation”

So EFE are invariant under general coordinate transformations **but** invariance is broken as a result of background.

$h_{\mu\nu}$ is an as yet unknown perturbation on flat space. We can make small changes in coordinates that leave $\eta_{\mu\nu}$ unchanged but make small changes in $h_{\mu\nu}$

We can only consider a sufficiently large specific reference frame where $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ holds.

In other words, we’re restricted in how much we can change the coordinates.

Linearized Theory of Metric Field

We are restricted to a limited set of coordinate transformations called “gauge transformations”

$$x^{\mu} \to x'^{\mu} + \xi\left(x^{\mu}\right)$$

If we transform the metric under this change of coordinates we find that the metric has the same form but with new perturbations given by

$$h_{\mu\nu}\left(x\right) \to h'_{\mu\nu}\left(x'\right) = h_{\mu\nu}\left(x\right) - \left(\partial_{\mu}\xi_{\nu} + \partial_{\nu}\xi_{\mu}\right)$$

Linearized Theory of Metric Field

We can stream line some calculations by an appropriate choice of gauge conditions.

We require a coordinate system in which **Lorentz gauge** (or harmonic gauge) holds

$$\partial^\mu \bar{h}_{\mu\nu} = 0$$

where we've defined the trace-reversed perturbation:

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{h}{2}\eta_{\mu\nu}$$

such that the trace has opposite sign:

$$\bar{h}^\mu_\mu \equiv \bar{h}_{\mu\nu} = -h$$

Linearized Theory of Metric Field

The Riemann curvature tensor

$$R_{\mu\nu\alpha\beta} = \frac{1}{2}\left(\partial_\mu\partial_\alpha g_{\nu\beta} - \partial_\nu\partial_\alpha g_{\mu\beta} + \partial_\nu\partial_\beta g_{\mu\alpha} - \partial_\mu\partial_\beta g_{\nu\alpha}\right)$$

for a flat metric with a perturbation will become

$$R_{\mu\nu\rho\sigma} = \frac{1}{2}\left(\partial_\nu\partial_\rho h_{\mu\sigma} + \partial_\mu\partial_\sigma h_{\nu\rho} - \partial_\mu\partial_\rho h_{\nu\sigma} - \partial_\nu\partial_\sigma h_{\mu\rho}\right)$$

Then substituting the trace-reversed perturbation, EFE takes form:

$$\partial_\mu\partial^\mu \bar{h}_{\mu\nu} + \eta_{\mu\nu}\partial^\rho\partial^\sigma \bar{h}_{\rho\sigma} - \partial^\rho\partial_\nu \bar{h}_{\mu\rho} - \partial^\rho\partial_\mu \bar{h}_{\nu\rho} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

If we define the d'Alembertian operator: $\Box \equiv \partial_\mu\partial^\mu$

$$\Box \bar{h}_{\mu\nu} + \eta_{\mu\nu}\partial^\rho\partial^\sigma \bar{h}_{\rho\sigma} - \partial^\rho\partial_\nu \bar{h}_{\mu\rho} - \partial^\rho\partial_\mu \bar{h}_{\nu\rho} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

Linearized Theory of Metric Field

And impose the harmonic gauge, then the last three terms in previous equation vanish and we end up with the **Linearized Einstein Equations**

$$\boxed{\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}}$$

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Solution in a Vacuum

What happens outside the source, where $T_{\mu\nu} = 0$?

Then, the EFE reduces to

$$\Box \bar{h}_{\mu\nu} = 0$$
$$\left(-\frac{1}{c^2}\partial t^2 + \nabla^2\right)\bar{h}_{\mu\nu} = 0$$

Wave equation for waves propagating at speed of light c!

Solutions to wave equation can be written as superpositions of plane waves traveling with wave vectors $\vec{k}$ and frequency

$$\omega = c\left|\vec{k}\right|$$

Solution in a Vacuum

Plane wave solution:

$$h(t) = A_{\mu\nu} \cos\left(\omega t - \vec{k} \cdot \vec{x}\right)$$

Implications: Spacetime has dynamics of its own, independent of matter. Even though matter generated the solution, it can still exist far away from the source where $T_{\mu\nu} = 0$

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Solution with Source

Now allow for source. What would cause the waves to be generated?

$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

Solve using retarded Green's function assuming no incoming radiation from infinity. The solution is

$$\bar{h}_{\mu\nu}\left(t, \vec{x}\right) = \frac{4G}{c^4} \int d^3x' \frac{1}{|\vec{x} - \vec{x}'|} T_{\mu\nu} \left(t - \frac{|\vec{x} - \vec{x}'|}{c}, \vec{x}'\right)$$

Solution with Source

We can utilize an additional gauge freedom by imposing the radiation gauge:

$$h = 0, \;\; h_{0i} = 0$$

Combining the harmonic gauge and this radiation gauge, we can write the solution in the **transverse traceless** (TT) gauge

$$h_{ij}^{\mathrm{TT}}\,(t, \vec{x}) = \frac{4G}{c^4}\Lambda_{ij,kl}(\hat{n}) \int d^3x' \frac{1}{|\vec{x} - \vec{x}'|} T_{kl}\left(t - \frac{|\vec{x} - \vec{x}'|}{c}, \vec{x}'\right)$$

$\vec{n}$ - direction of propagation of GW

$\Lambda_{ij,kl}(\hat{n})$ is a tool to bring $h_{\mu\nu}$ outside the source in the TT gauge.

Solution with Source

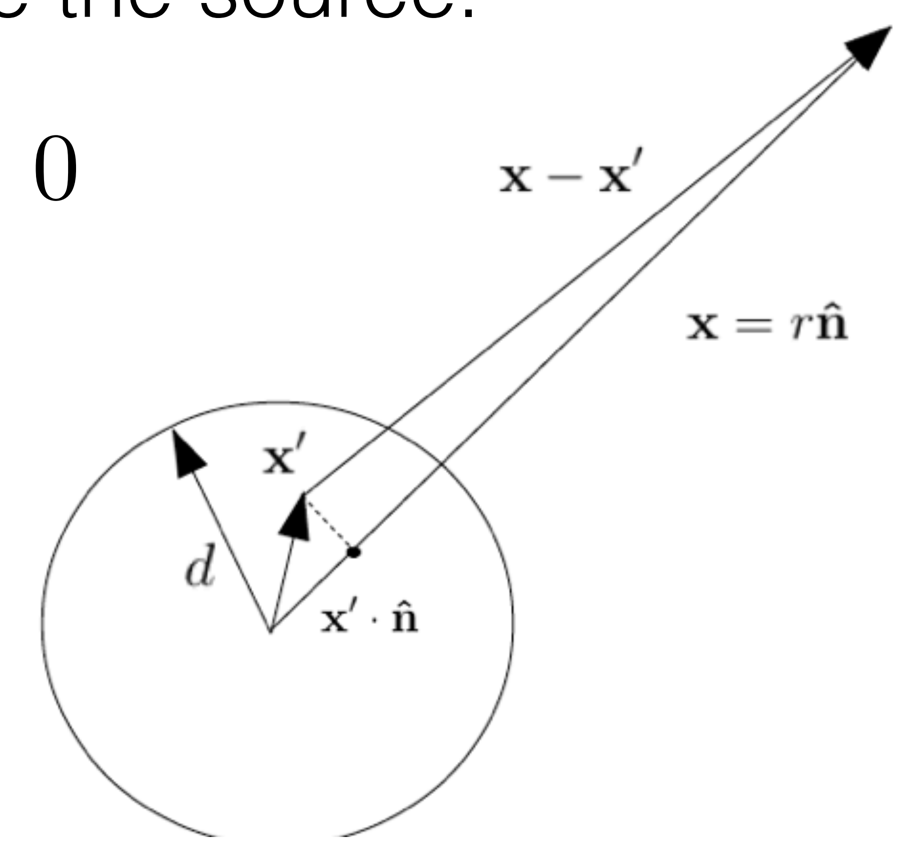
$\Lambda_{ij,kl}(\hat{n})$ is a tool to bring $h_{\mu\nu}$ outside the source in the TT gauge.

$$\Lambda_{ij,kl}(\hat{n}) = P_{ik}P_{jl} - \frac{1}{2}P_{ij}P_{kl}$$

$$P_{ij} \equiv \delta_{ij} - n_i n_j$$

Then the perturbation $h_{ij}^{TT}(t,\vec{x})$ can be evaluated outside the source at $\vec{x}$ while $\vec{x}'$ is a point inside the source.

$$T_{kl}\left(t - \left|\vec{x} - \vec{x}'\right|/c, \vec{x}'\right) \neq 0$$



We're looking at a distance r that is much larger than the size of the source d. Then we can expand

$$\Delta\vec{x} = r - \vec{x}' \cdot \hat{n} + \mathcal{O}\left(d^2/r\right)$$

Solution with Source

Then we can write the TT solution as

$$h_{ij}^{\rm TT}\,(t,\vec{x}) = \frac{4G}{c^4}\Lambda_{ij,kl}(\hat{n})\int d^3x'\,\frac{1}{|r-\vec{x}'\cdot\hat{n}|}T_{kl}\left(t-\frac{r}{c}+\frac{\vec{x}'\cdot\hat{n}}{c},\vec{x}'\right)$$

If the source is non-relativistic, $\mathrm{v/c} << 1$, then we can expand

$$T_{kl}\left(t-\frac{r}{c}+\frac{\vec{x}'\cdot\hat{n}}{c},\vec{x}'\right) = T_{kl}\left(t-\frac{r}{c},\vec{x}'\right)+\frac{x'^i n^i}{c}\partial_0 T_{kl}+\frac{1}{2c^2}x'^i x'^j n^i n^j \partial_0^2 T_{kl}+...$$

We can substitute this for T_{kl} in the TT solution to get the **multipole expansion**

$$h_{ij}^{\rm TT}\,(t,\vec{x}) = \frac{1}{r}\frac{4G}{c^4}\Lambda_{ij,kl}(\hat{n})\left[S^{kl}+\frac{1}{c}n_m\dot{S}^{kl,m}+\frac{1}{2c^2}n_m n_p\ddot{S}^{kl,mp}+\dots\right]_{\rm ret}$$

where ret is the retarded time $t-r/c$

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Generation of Gravitational Waves

Multipole moments of stress tensor T^{ij}

$$S^{ij} = \int d^3x T^{ij}\left(t, \vec{x}\right)$$

$$S^{ij,k} = \int d^3x T^{ij}\left(t, \vec{x}\right) x^k$$

$$S^{ij,kl} = \int d^3x T^{ij}\left(t, \vec{x}\right) x^k x^l$$

...

Multipole moments of the stress energy tensor are not physically intuitive.

Generation of Gravitational Waves

We can express the multipole moments in terms of the mass moments and the momentum multipoles.

Mass moments: momenta of energy density T^{00}/c^2

$$M = \frac{1}{c^2}\int d^3x T^{00}\left(t,\vec{x}\right)$$

$$M^i = \frac{1}{c^2}\int d^3x T^{00}\left(t,\vec{x}\right)x^i$$

$$M^{ij} = \frac{1}{c^2}\int d^3x T^{00}\left(t,\vec{x}\right)x^i x^j$$

...

Generation of Gravitational Waves

We can express the multipole moments in terms of the mass moments and the momentum multipoles.

Momenta of momentum density T^{0i}/c

$$P^i = \frac{1}{c}\int d^3x T^{0i}\left(t, \vec{x}\right)$$

$$P^{i,j} = \frac{1}{c}\int d^3x T^{0i}\left(t, \vec{x}\right) x^j$$

$$P^{i,jk} = \frac{1}{c}\int d^3x T^{0i}\left(t, \vec{x}\right) x^j x^k$$

...

Generation of Gravitational Waves

To leading order in *v/c*, we can eliminate the multipole moments in favor of the mass moments to get a solution of the form:

$$\left[h_{ij}^{\mathrm{TT}}(t,\vec{x})\right]_{\mathrm{quad}} = \frac{1}{r}\frac{2G}{c^4}\Lambda_{ij,kl}(\hat{n})\ddot{M}^{kl}\,(t-r/c)$$

where we have used: $S^{ij} = \frac{1}{2}\ddot{M}^{ij}$

Mass quadrupole radiation!

Generation of Gravitational Waves

$$\boxed{\left[h_{ij}^{\mathrm{TT}}(t,\vec{x})\right]_{\mathrm{quad}} = \frac{1}{r}\frac{2G}{c^4}\Lambda_{ij,kl}(\hat{n})\ddot{M}^{kl}\,(t-r/c)}$$

No Monopole Radiation

$$\begin{aligned}\dot{M} &= \frac{1}{c}\int_V d^3x\partial_0 T^{00}\\ &= -\frac{1}{c}\int_V d^3x\partial_i T^{0i}\\ &= -\frac{1}{c}r^2\int_S d\Omega T^{0i}\\ &= 0\end{aligned}$$

No Dipole Radiation

Mass dipole M^i zero (i.e. constant) in center of mass frame

No momentum monopole contribution

$$\dot{P}^i = 0$$

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Effect of Gravitational Waves on Matter

The best way to understand the effect of gravitational waves on matter is to consider two neighboring free-falling particles at $x^\mu(\tau)$ and $x^\mu(\tau) + \zeta^\mu(\tau)$

Consider the geodesic equations for each particle:

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\rho}(x)\frac{dx^\nu}{d\tau}\frac{dx^\rho}{d\tau} = 0$$

$$\frac{d^2(x^\mu + \zeta^\mu)}{d\tau^2} + \Gamma^\mu_{\nu\rho}(x+\zeta)\frac{d(x^\mu + \zeta^\mu)}{d\tau}\frac{d(x^\mu + \zeta^\mu)}{d\tau} = 0$$

Take the difference of the two and expand to leading order in ζ^μ:

$$\frac{d^2\zeta^\mu}{d\tau^2} + 2\Gamma^\mu_{\nu\rho}(x)\frac{dx^\nu}{d\tau}\frac{d\zeta^\rho}{d\tau} + \zeta^\sigma\partial_\sigma\Gamma^\mu_{\nu\rho}(x)\frac{dx^\nu}{d\tau}\frac{dx^\rho}{d\tau} = 0$$

Effect of Gravitational Waves on Matter

$$\frac{d^2\zeta^\mu}{d\tau^2} + \cancel{2\Gamma^\mu_{\nu\rho}(x)\frac{dx^\nu}{d\tau}}\frac{d\zeta^\rho}{d\tau} + \zeta^\sigma\partial_\sigma\Gamma^\mu_{\nu\rho}(x)\frac{dx^\nu}{d\tau}\frac{dx^\rho}{d\tau} = 0$$

Transform into a Local Lorentz Frame such that:

$$g_{\mu\nu}(\mathcal{P}) = \eta_{\mu\nu}\,,\ \ \partial_\rho g_{\mu\nu} = 0 \longrightarrow \Gamma^\rho_{\mu\nu} = 0$$

Assume the particles are moving non-relativistically:

$$\frac{dx^i}{d\tau} \ll \frac{dx^0}{d\tau}\,, \quad \frac{dx^0}{d\tau} \simeq c$$

Relate $\partial_\sigma\Gamma^\sigma_{00}$ to the Riemann tensor: $\displaystyle\frac{d^2\zeta^i}{d\tau^2} = -c^2R^i_{0j0}\zeta^j$

Effect of Gravitational Waves on Matter

The components of the Riemann tensor may be calculated in any frame due to its invariance in linearized theory. We can use the TT frame:

$$R^{i}_{0j0} = R_{i0j0} = -\frac{1}{2c^2}\ddot{h}^{\mathrm{TT}}_{ij}$$

Now we see how the geodesic deviation between two particles is related to the perturbation caused by a passing GW:

$$\ddot{\zeta}^{i} = \frac{1}{2}\ddot{h}^{\mathrm{TT}}_{ij}\zeta^{j}$$

A tidal effect!

Effect of Gravitational Waves on Matter

Gravitational wave in the z-direction:

$$h_{ij}^{\mathrm{TT}} = \begin{bmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{bmatrix}_{ij} \cos\left(\omega t - zt/c\right), \qquad \omega = c|\vec{k}|$$

Relative displacements of particles in (x, y) plane:

$$\boxed{h_\times = 0}$$

$$\delta\ddot{x} = -\frac{h_+}{2}\left(x_0 + \delta x\right)\omega^2\cos(\omega t) \qquad\qquad \delta x(t) = \frac{h_+}{2} x_0 \cos(\omega t)$$

$$\longrightarrow$$

$$\delta\ddot{y} = \frac{h_+}{2}\left(y_0 + \delta y\right)\omega^2\cos(\omega t) \qquad\qquad \delta y(t) = -\frac{h_+}{2} y_0 \cos(\omega t)$$

$$\boxed{h_+ = 0}$$

$$\delta\ddot{x} = \frac{h_\times}{2}\left(y_0 + \delta y\right)\omega^2\cos(\omega t) \qquad\qquad \delta x(t) = -\frac{h_\times}{2} y_0 \cos(\omega t)$$

$$\longrightarrow$$

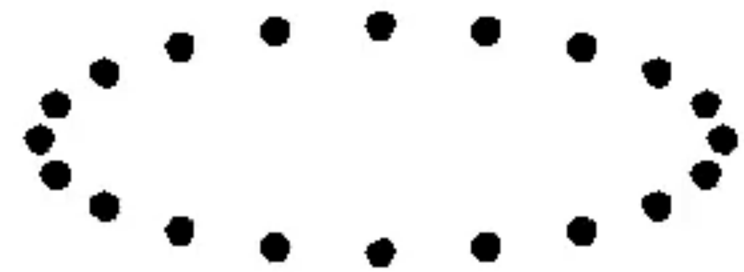
$$\delta\ddot{y} = \frac{h_\times}{2}\left(x_0 + \delta x\right)\omega^2\cos(\omega t) \qquad\qquad \delta y(t) = -\frac{h_\times}{2} x_0 \cos(\omega t)$$

Effect of Gravitational Waves on Matter

h_+ polarization

$$\delta x(t) = \frac{h_+}{2} x_0 \cos(\omega t)$$

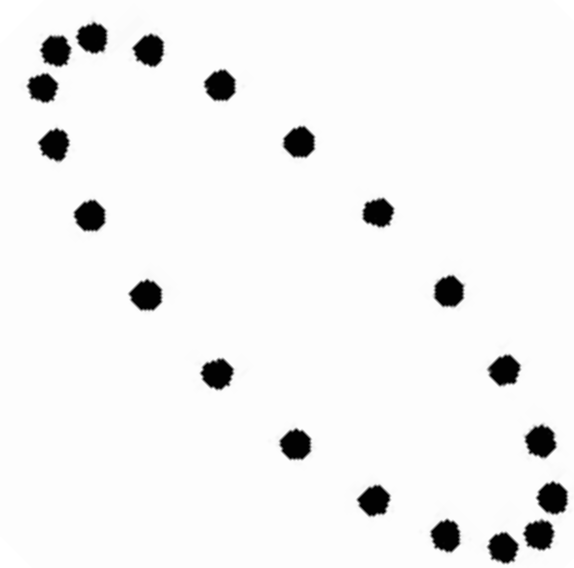
$$\delta y(t) = -\frac{h_+}{2} y_0 \cos(\omega t)$$



$h_\times$ polarization

$$\delta x(t) = -\frac{h_\times}{2} y_0 \cos(\omega t)$$

$$\delta y(t) = -\frac{h_\times}{2} x_0 \cos(\omega t)$$



Review: Generation of Gravitational Waves

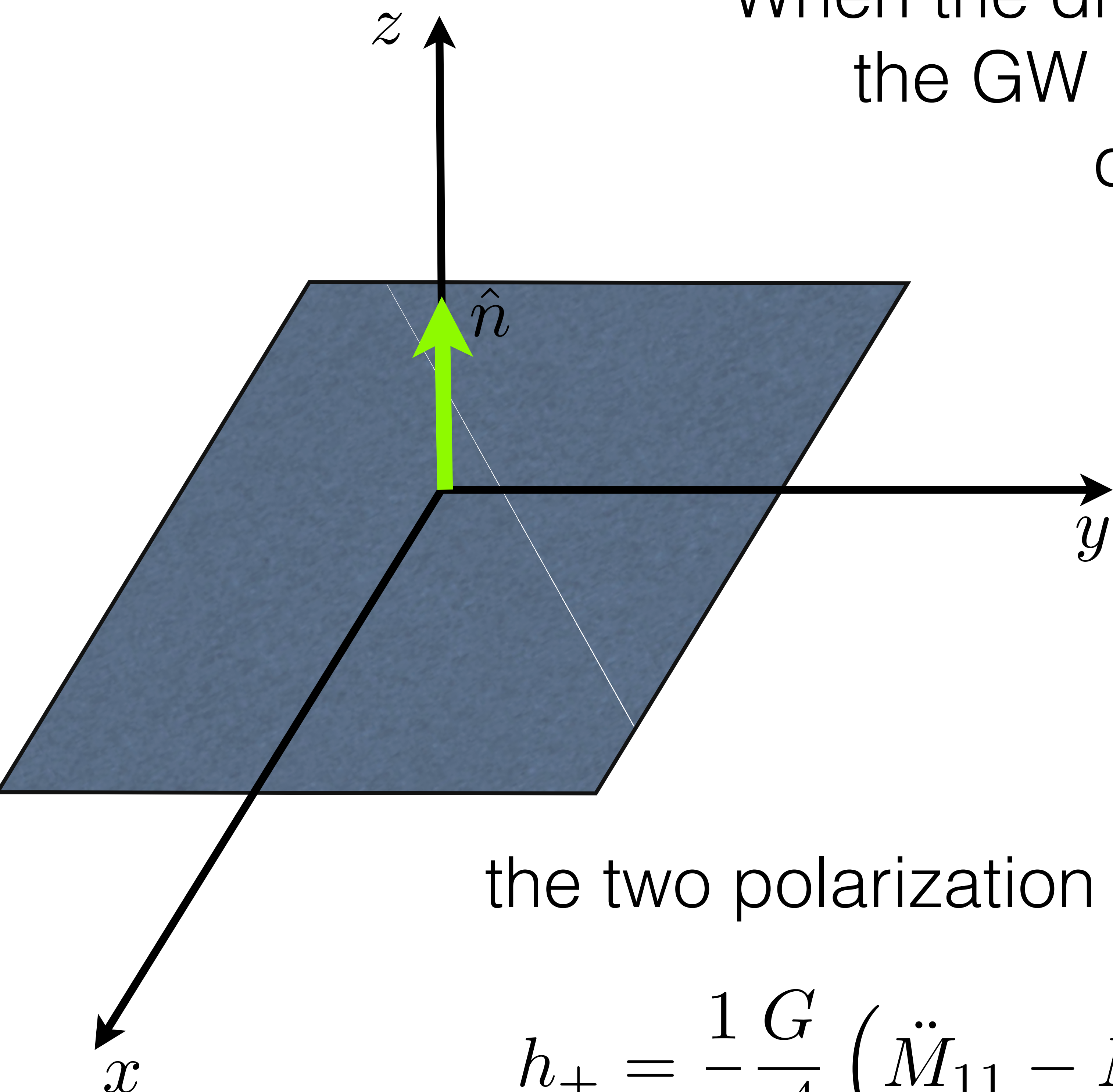
To leading order in v/c, we can eliminate the multipole moments in favor of the mass moments to get a solution of the form:

$$\left[h_{ij}^{\mathrm{TT}}(t,\vec{x})\right]_{\mathrm{quad}} = \frac{1}{r}\frac{2G}{c^4}\Lambda_{ij,kl}(\hat{n})\ddot{M}^{kl}\left(t - r/c\right)$$

where we have used: $S^{ij} = \frac{1}{2}\ddot{M}^{ij}$

Mass quadrupole radiation!

Case I: Propagation in ẑ



When the direction of propagation $\hat{n}$ of the GW is equal to $\hat{z}$, P_{ij} is the diagonal matrix:

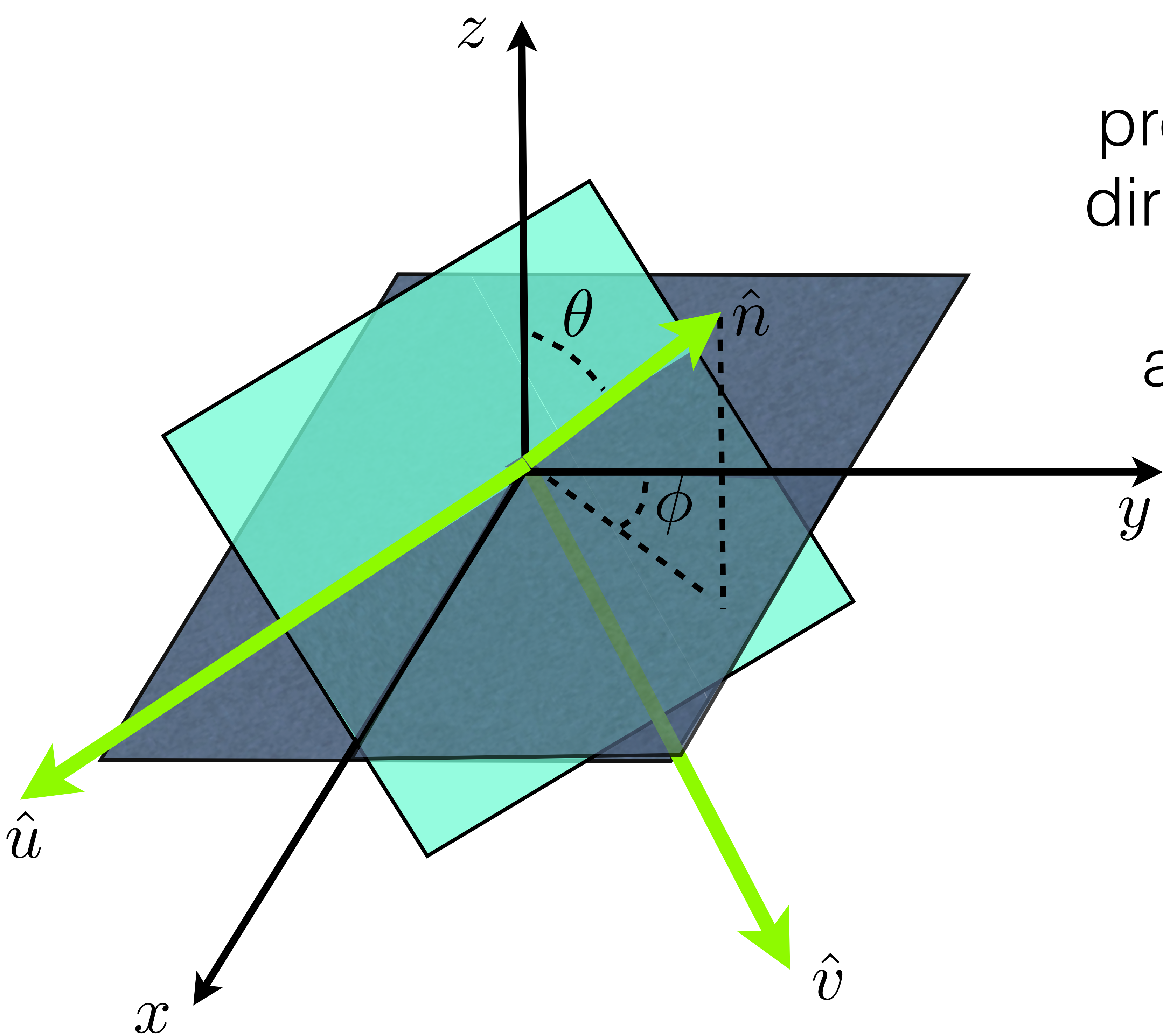
$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

i.e., a projector on the (x, y) plane,

the two polarization amplitudes have the form

$$h_+ = \frac{1}{r}\frac{G}{c^4}\left(\ddot{M}_{11} - \ddot{M}_{22}\right) \qquad h_\times = \frac{2}{r}\frac{G}{c^4}\ddot{M}_{12}$$

Case II: Propagation in $\hat{n}$



When the wave propagates in a **generic** direction $\hat{n}$, we introduce two unit vectors $\hat{u}$ and $\hat{v}$, orthogonal to $\hat{n}$

The vector $\hat{u}$ is in the $(\hat{x}, \hat{y})$ plane while $\hat{v}$ points downward with respect to the $(\hat{x}, \hat{y})$ plane.

Case II: Propagation in $\hat{n}$

For a generic propagation direction, the two polarization amplitudes have the form:

$$h_+(t;\theta,\phi) = \frac{1}{r}\frac{G}{c^4}[\ddot{M}_{11}\left(\cos^2\phi - \sin^2\phi\cos^2\theta\right)$$
$$+ \ddot{M}_{22}\left(\sin^2\phi - \cos^2\phi\cos^2\theta\right)$$
$$- \ddot{M}_{33}\sin^2\theta$$
$$- \ddot{M}_{12}\sin 2\phi\left(1+\cos^2\theta\right)$$
$$+ \ddot{M}_{13}\sin\phi\sin 2\theta$$
$$+ \ddot{M}_{23}\cos\phi\sin 2\theta]$$

$$h_\times(t;\theta,\phi) = \frac{1}{r}\frac{G}{c^4}[(\ddot{M}_{11} - \ddot{M}_{22})\sin 2\phi\cos\theta$$
$$+ 2\ddot{M}_{12}\cos 2\phi\cos\theta$$
$$- 2\ddot{M}_{13}\cos\phi\sin\theta$$
$$+ 2\ddot{M}_{23}\sin\phi\sin\theta]$$

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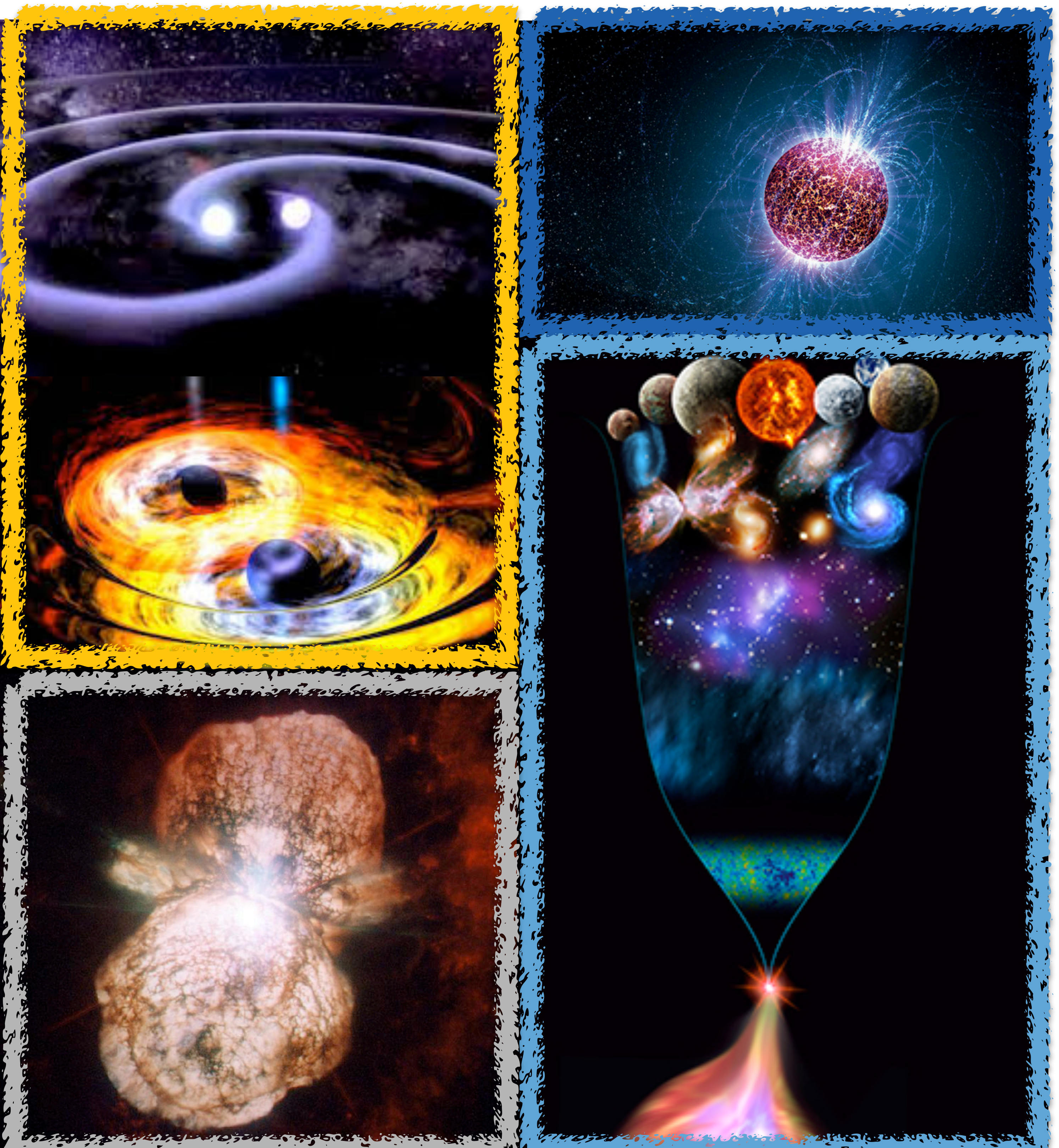
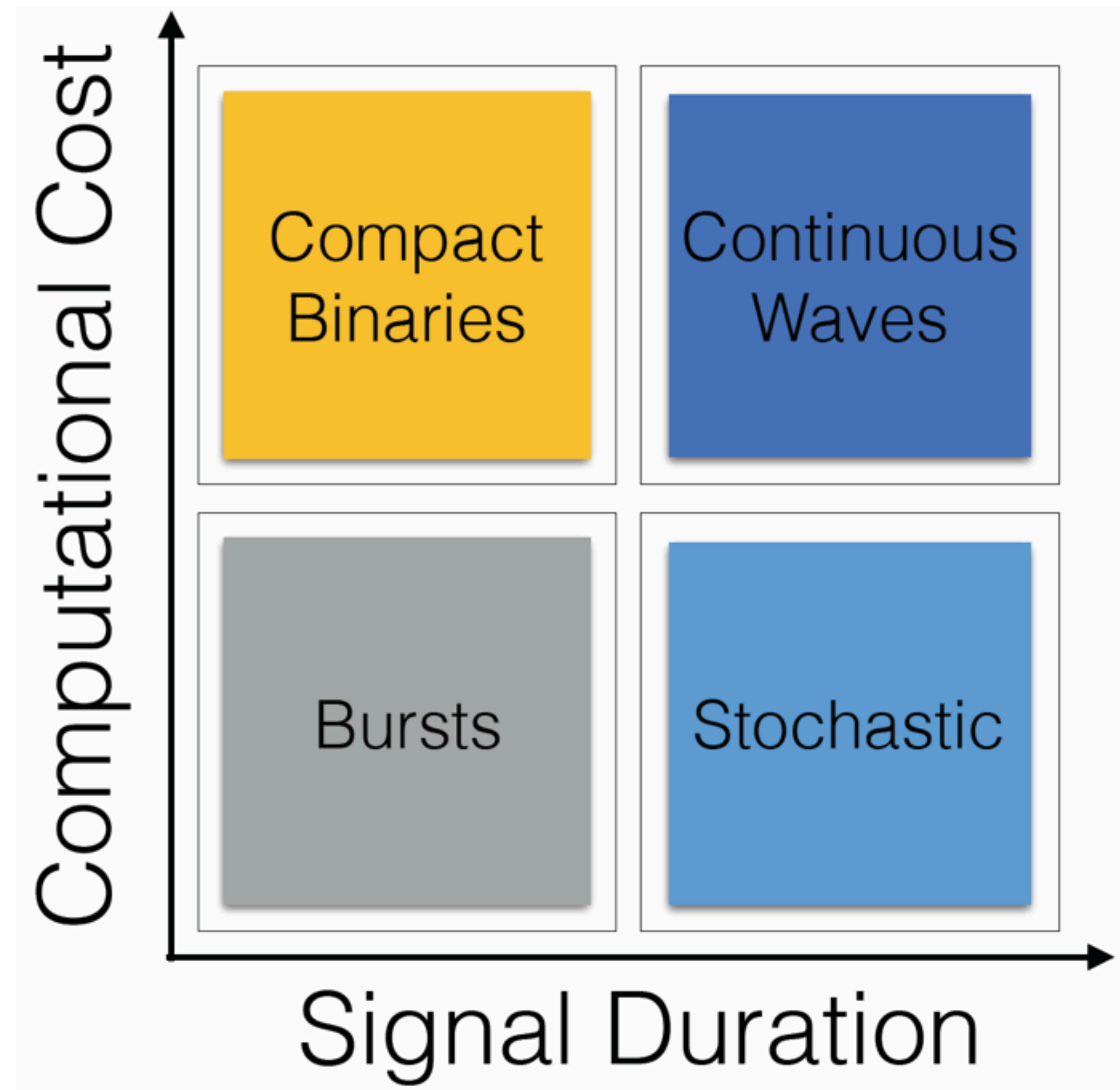
LIGO/Virgo Astrophysical Sources





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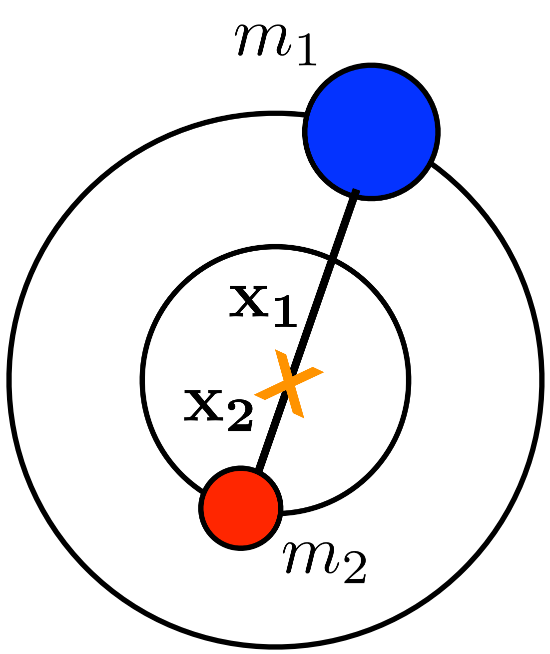
Example I: Quadrupole radiation from a mass in circular orbit



The usual center-of-mass coordinate is:

$$\mathbf{x}_{\mathrm{CM}} = \frac{m_1\mathbf{x_1} + m_2\mathbf{x_2}}{m_1 + m_2}$$

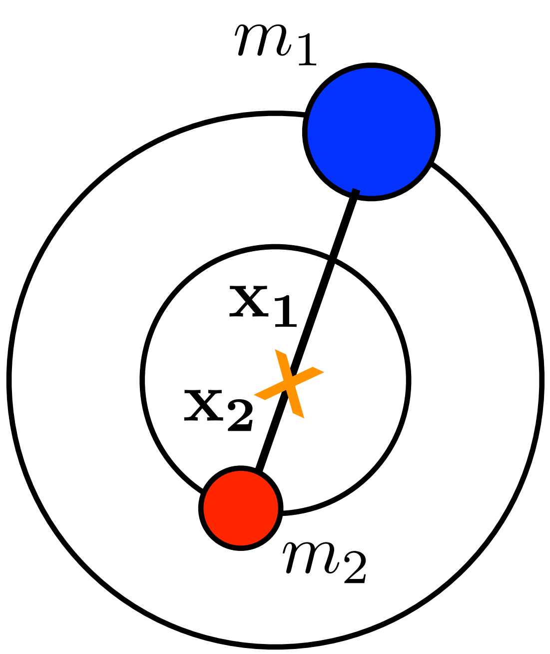
$\mathbf{x}_0 = \mathbf{x}_1 - \mathbf{x}_2$ is the relative coordinate of an isolated two-body system in the center-of-mass frame.

If we chose the origin of the coordinate system at $\mathbf{x}_{\mathrm{CM}} = 0$,

then the second mass moment is: $M^{ij}(t) = \mu x_0^i(t) x_0^j(t)$

where $\mu = \dfrac{m_1 m_2}{m_1 + m_2}$ is the reduced mass.

Example I: Quadrupole radiation from a mass in circular orbit



$$\mathbf{x}_0 = \mathbf{x}_1 - \mathbf{x}_2$$

Choose (x, y, z) frame so orbit is in (x, y) plane.

Orbit is given by:

$$x_0(t) = R\cos(\omega_s t + \pi/2)$$

$$y_0(t) = R\sin(\omega_s t + \pi/2)$$

$$z_0(t) = 0$$

The only non-vanishing second mass moment components are:

$$M_{11} = \mu R^2 \frac{1 - \cos 2\omega_s t}{2}$$

$$M_{22} = \mu R^2 \frac{1 + \cos 2\omega_s t}{2}$$

$$M_{12} = -\frac{1}{2}\mu R^2 \sin 2\omega_s t$$

Compute $\ddot{M}_{ij}$. Plug into generic expressions for polarization amplitudes to get:

$$h_+(t;\theta,\phi) = \frac{1}{r}\frac{4G\mu\omega_s^2 R^2}{c^4}\left(\frac{1+\cos^2\theta}{2}\right)\cos(2\omega_s t_{\rm ret} + 2\phi)$$

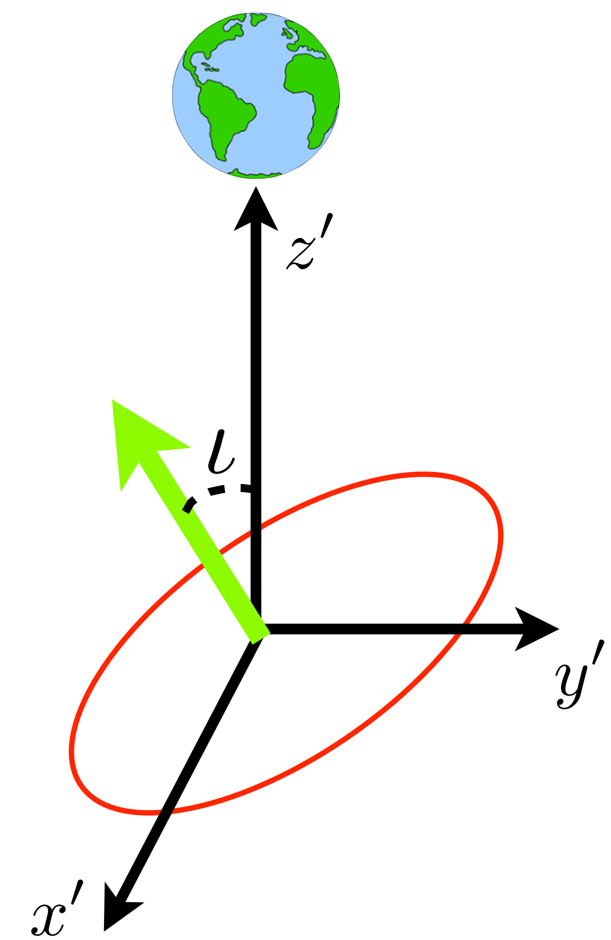
$$h_\times(t;\theta,\phi) = \frac{1}{r}\frac{4G\mu\omega_s^2 R^2}{c^4}\cos\theta\sin(2\omega_s t_{\rm ret} + 2\phi)$$

Example I: Quadrupole radiation from a mass in circular orbit

$$h_+(t;\theta,\phi) = \frac{1}{r}\frac{4G\mu\omega_s^2R^2}{c^4}\left(\frac{1+\cos^2\theta}{2}\right)\cos(2\omega_s t_{\rm ret}+2\phi)$$

$$h_\times(t;\theta,\phi) = \frac{1}{r}\frac{4G\mu\omega_s^2R^2}{c^4}\cos\theta\sin(2\omega_s t_{\rm ret}+2\phi)$$

Quadrupole radiation is at **twice** the frequency ω_s of the source: $\omega_{\rm gw} = 2\omega_s$



A rotation of the source by $\Delta\phi$ is the same as a time translation so that

$$\omega_s \Delta t = \Delta\phi$$

The angle θ is equal to the angle ι between the normal to the orbit and the line-of-site.

Example I: Quadrupole radiation from a mass in circular orbit

Use Kepler's law, the chirp mass, and the GW frequency to rewrite the solutions.

$$\omega_s^2 = \frac{GM}{R^3} \qquad M_c = \mu^{3/5} M^{2/5} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} \qquad \omega_{\mathrm{gw}} = 2\omega_s$$

$$\omega_{\mathrm{gw}} = 2\pi f_{\mathrm{gw}}$$

$$h_+(t) = \frac{4}{r}\left(\frac{GM_c}{c^2}\right)^{5/3}\left(\frac{\pi f_{\mathrm{gw}}}{c}\right)^{2/3}\frac{1+\cos^2\theta}{2}\cos(2\pi f_{\mathrm{gw}} t_{\mathrm{ret}} + 2\phi)$$

$$h_\times(t) = \frac{4}{r}\left(\frac{GM_c}{c^2}\right)^{5/3}\left(\frac{\pi f_{\mathrm{gw}}}{c}\right)^{2/3}\cos\theta\sin(2\pi f_{\mathrm{gw}} t_{\mathrm{ret}} + 2\phi)$$

The amplitudes of the GWs emitted depend on the masses m_1 and m_2 only through the combination M_c.

Example I: Quadrupole radiation from a mass in circular orbit

Angular distribution of the radiated power in quadrupole approximation:

$$\boxed{\left(\frac{dP}{d\Omega}\right)_{\text{quad}} = \frac{r^2 c^3}{16\pi G}\left\langle \dot{h}_+^2 + \dot{h}_\times^2 \right\rangle}$$

For our binary system example:

$$\left(\frac{dP}{d\Omega}\right)_{\text{quad}} = \frac{2G\mu^2 R^4 \omega_s^6}{\pi c^5} g(\theta)$$

$$g(\theta) = \left(\frac{1+\cos^2\theta}{2}\right)^2 + \cos^2\theta$$

Total power radiated in quadrupole approximation

$$\boxed{P_{\text{quad}} = \left(\frac{dE_{\text{gw}}}{d\Omega}\right)_{\text{quad}} = \frac{r^2 c^3}{16\pi G}\int_{\mathcal{S}} d\Omega \left\langle \dot{h}_+^2 + \dot{h}_\times^2 \right\rangle}$$

For our binary system example:

$$P_{\text{quad}} = \frac{32}{5}\frac{G\mu^2}{c^5} R^4 \omega_s^6$$

Example I: Quadrupole radiation from a mass in circular orbit

In terms of the chirp mass M_c , the total radiated power in the binary system is

$$P = \frac{32}{5}\frac{c^5}{G}\left(\frac{GM_c\omega_{\mathrm{gw}}}{2c^3}\right)^{10/3}$$

Example I: Quadrupole radiation from a mass in circular orbit

The emission of GWs costs energy. Previous equations are only valid if sources are on fixed, circular Keplerian orbit.

$$E_{\text{orbit}} = E_{\text{kin}} + E_{\text{pot}}$$
$$= -\frac{1}{2}\frac{Gm_1 m_2}{R}$$

Kepler's law

$$\omega_s^2 = \frac{GM}{R^3}$$

$$P = \frac{32}{5}\frac{c^5}{G}\left(\frac{GM_c\omega_{\text{gw}}}{2c^3}\right)^{10/3}$$

To compensate for loss of energy to GWs, R must decrease in time.

If R decreases, ω_s increases.

Then power radiated in GWs increases which means R must decrease even more.

Runaway process $\Longrightarrow$ binary system must coalesce.

Example I: Quadrupole radiation from a mass in circular orbit

Changes needed to:

$$h_+(t;\theta,\phi) = \frac{1}{r}\frac{4G\mu\omega_s^2R^2}{c^4}\left(\frac{1+\cos^2\theta}{2}\right)\cos(2\omega_s t_{\rm ret}+2\phi)$$

$$h_\times(t;\theta,\phi) = \frac{1}{r}\frac{4G\mu\omega_s^2R^2}{c^4}\cos\theta\sin(2\omega_s t_{\rm ret}+2\phi)$$

In arguments of the trigonometric functions: $\omega_{\rm gw}t \to \Phi(t)$

In factors in front of trigonometric functions: $\omega_{\rm gw} \to \omega_{\rm gw}(t)$

May have contributions from derivatives of $R(t)$ and $\omega_{\rm gw}(t)$.

$\dot{R}(t)$ is negligible as long as $f_{\rm gw} \ll 13{\rm kHz}\,(1.2M_\odot/M_c)$

Example I: Quadrupole radiation from a mass in circular orbit

Time to coalescence τ measured by the observer:

$$\tau \equiv t_{\rm coal} - t \qquad -\infty < t < t_{\rm coal}$$

Evolution of GW frequency:

$$f_{\rm gw}(\tau) = \frac{1}{\pi}\left(\frac{5}{256\tau}\right)^{3/8}\left(\frac{GM_c}{c^3}\right)^{-5/8}$$

Evolution of arguments of trigonometric functions:

$$\Phi(\tau) = -2\left(\frac{5GM_c}{c^3}\right)^{-5/8}\tau^{5/8} + \Phi_0 \qquad \Phi_0 = \Phi(\tau = 0)$$

Then the GW amplitudes are

$$\boxed{h_+(t) = \frac{1}{r}\left(\frac{GM_c}{c^2}\right)^{5/4}\left(\frac{5}{c\tau}\right)^{1/4}\frac{1+\cos^2\iota}{2}\cos\left[\Phi(\tau)\right]}$$

$$\boxed{h_\times(t) = \frac{1}{r}\left(\frac{GM_c}{c^2}\right)^{5/4}\left(\frac{5}{c\tau}\right)^{1/4}\cos\iota\sin\left[\Phi(\tau)\right]}$$

Example I: Quadrupole radiation from a mass in circular orbit

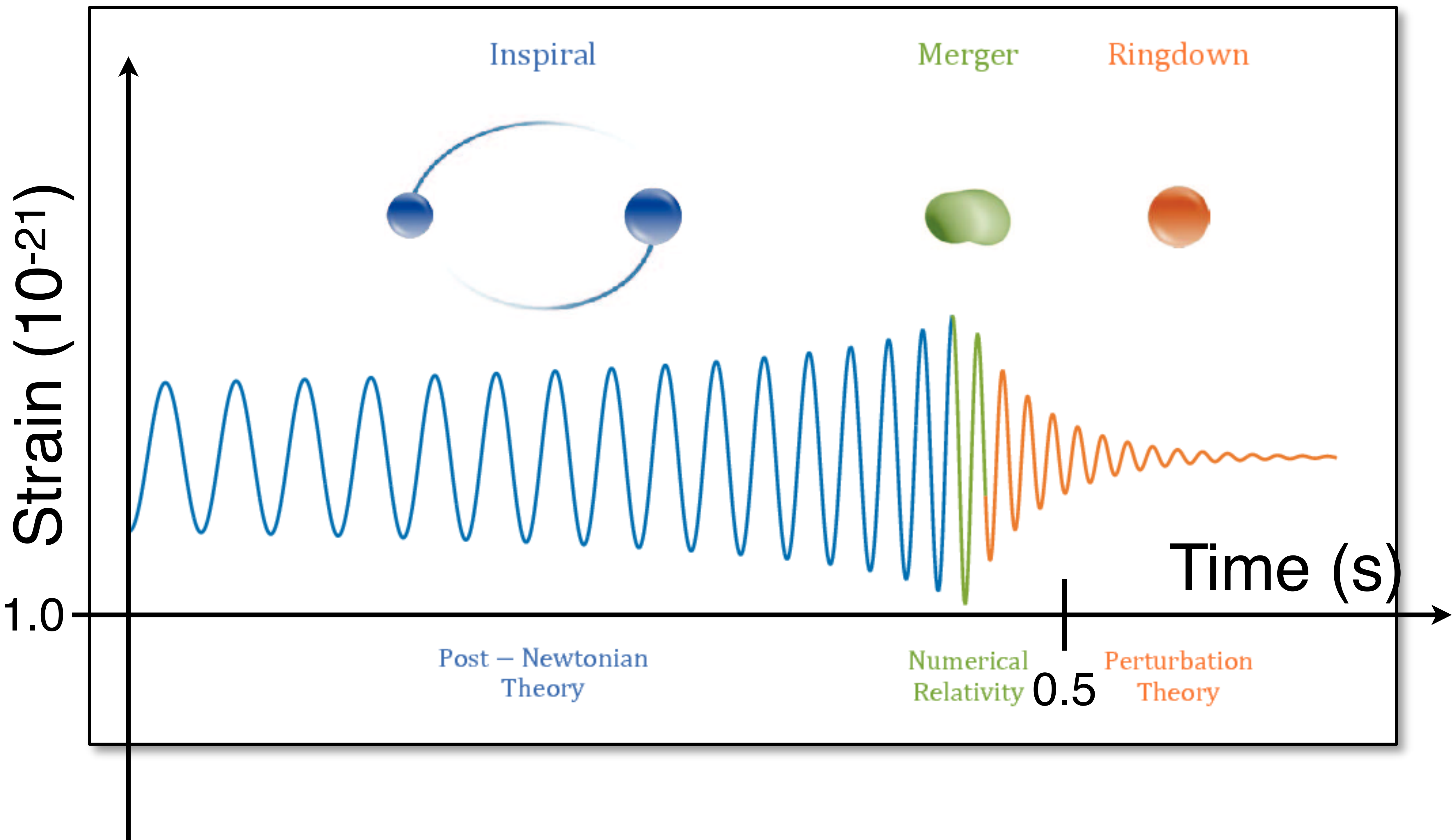
In Schwarzschild geometry, there is a minimum value of the radial distance beyond which stable circular orbits are no longer allowed, i.e. the Innermost Stable Circular Orbit (ISCO):

$$r_{\rm ISCO} = \frac{6GM}{c^2}$$

For binaries of BH or NS, a phase of slow adiabiatic inspiral, going through quasi-circular orbit and driven by emission of GWs can only take place at distances $r \gtrsim r_{\rm ISCO}$

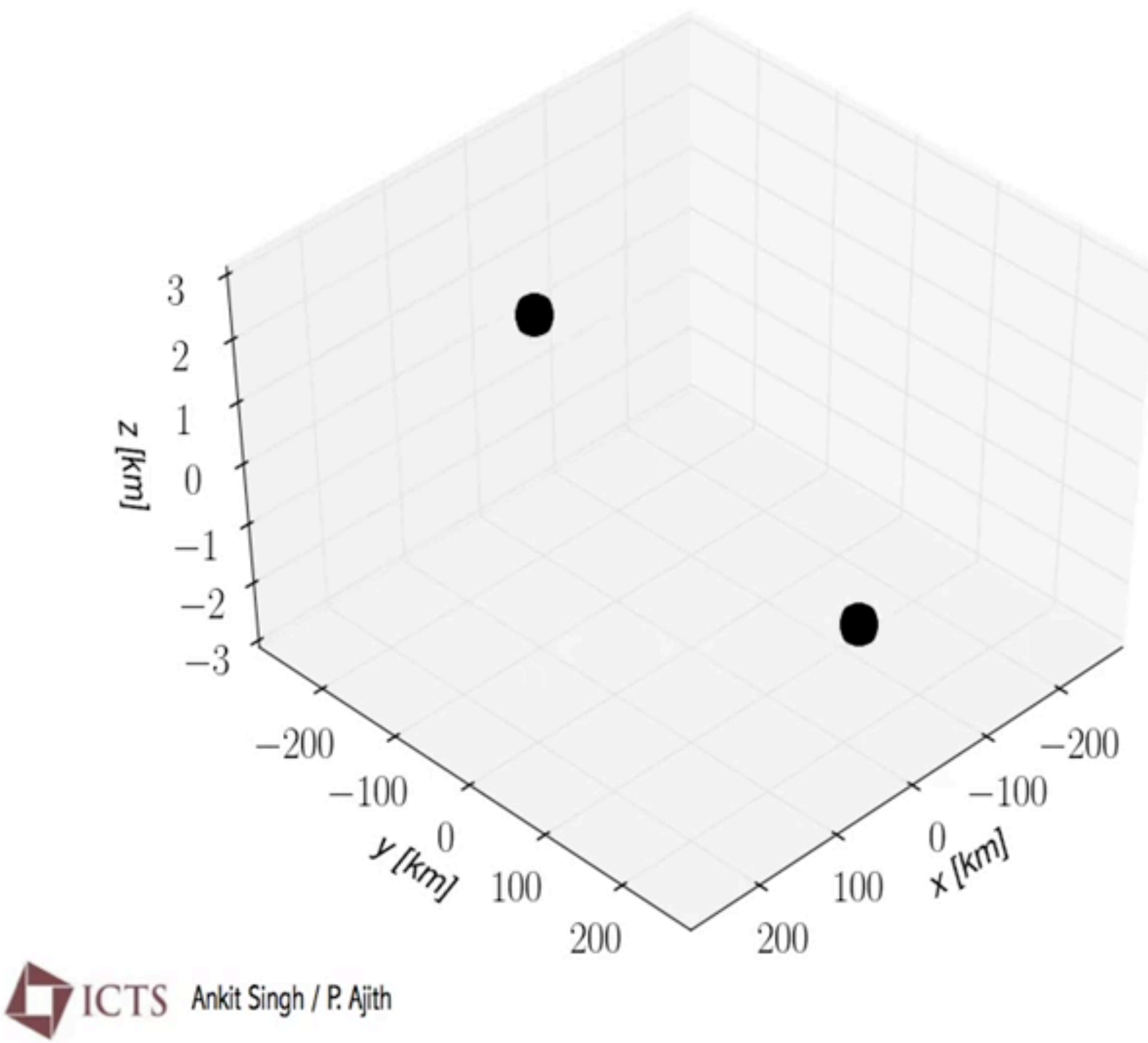
$$f_{\rm max} = (f_s)_{\rm ISCO} = \frac{1}{12\sqrt{6}\pi}\frac{c^3}{GM}$$

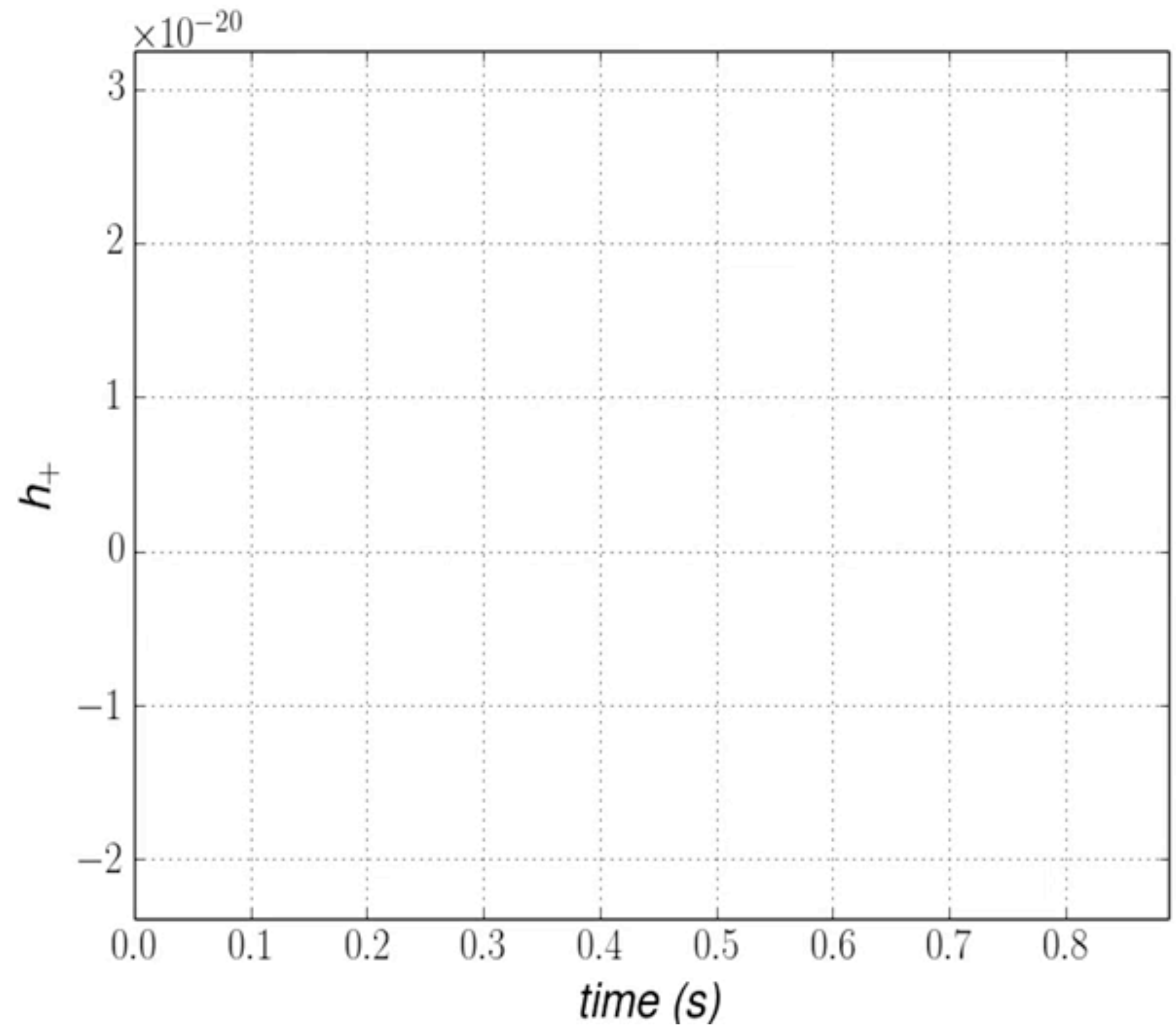
Full Coalescing Binary Signal



Coalescing Binaries

Non-spinning, equal mass black holes

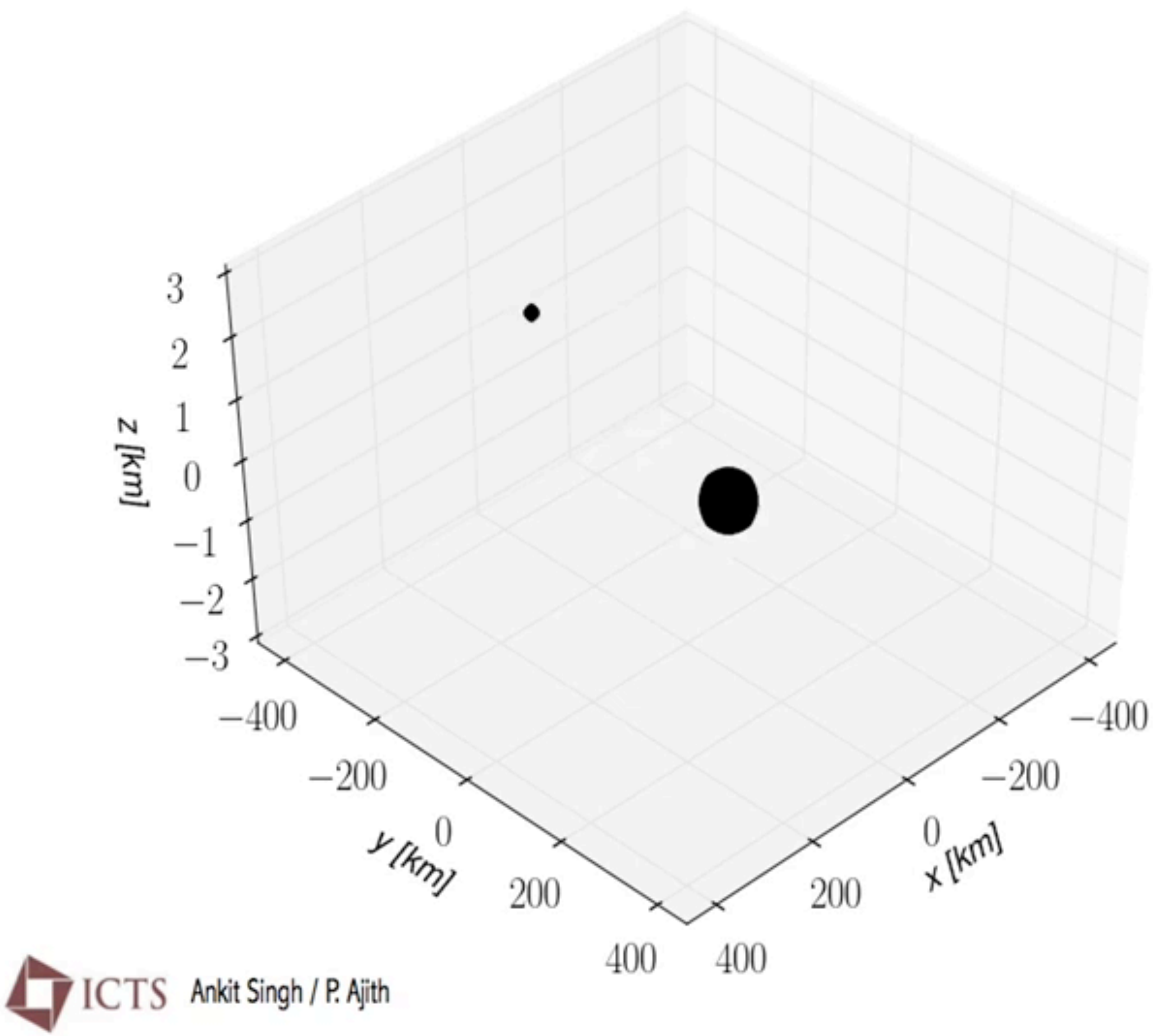


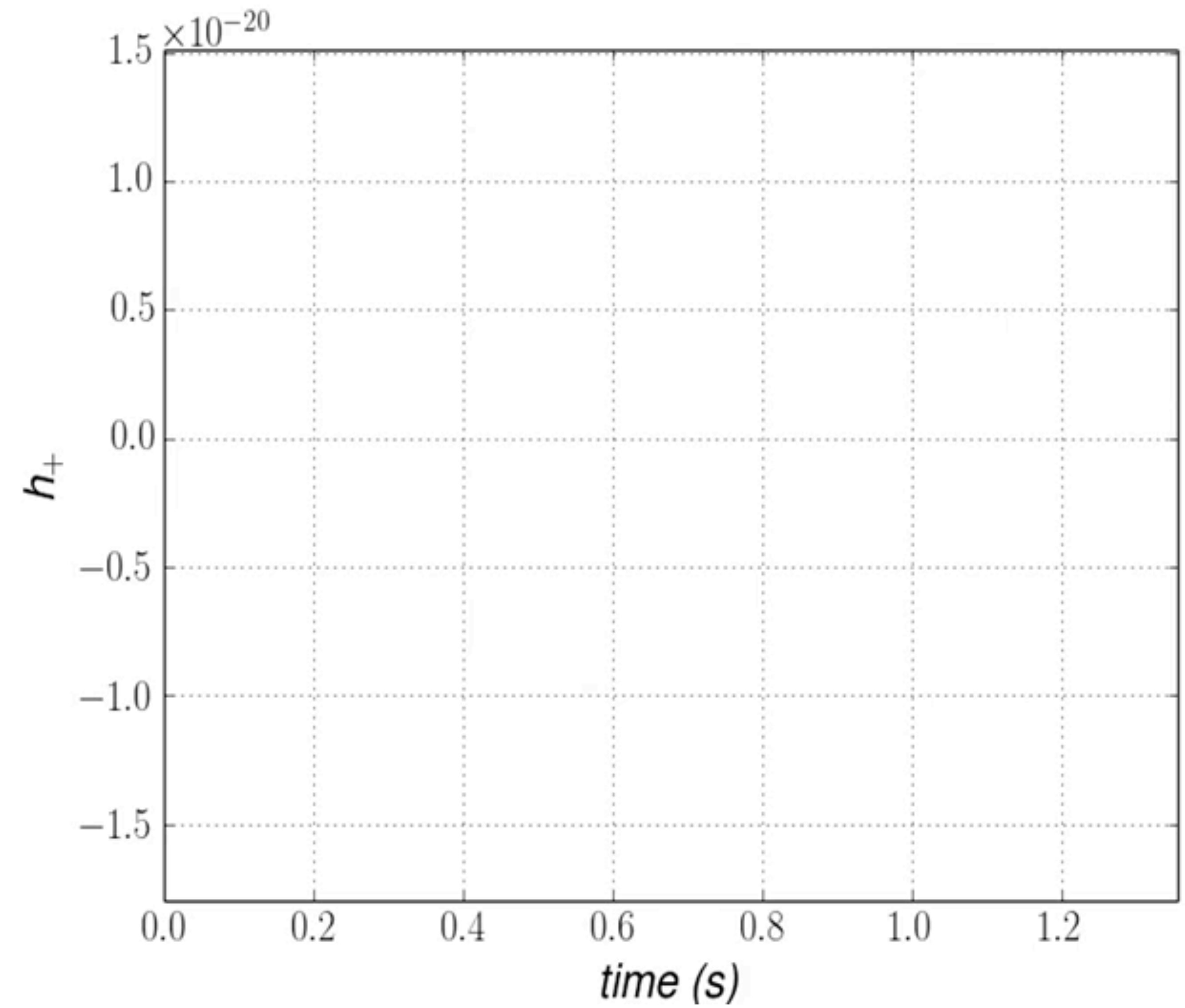


(m1, m2) = (10, 10) $M_\odot$

Coalescing Binaries

Non-spinning, unequal mass black holes





(m1, m2) = (4, 16) $M_\odot$

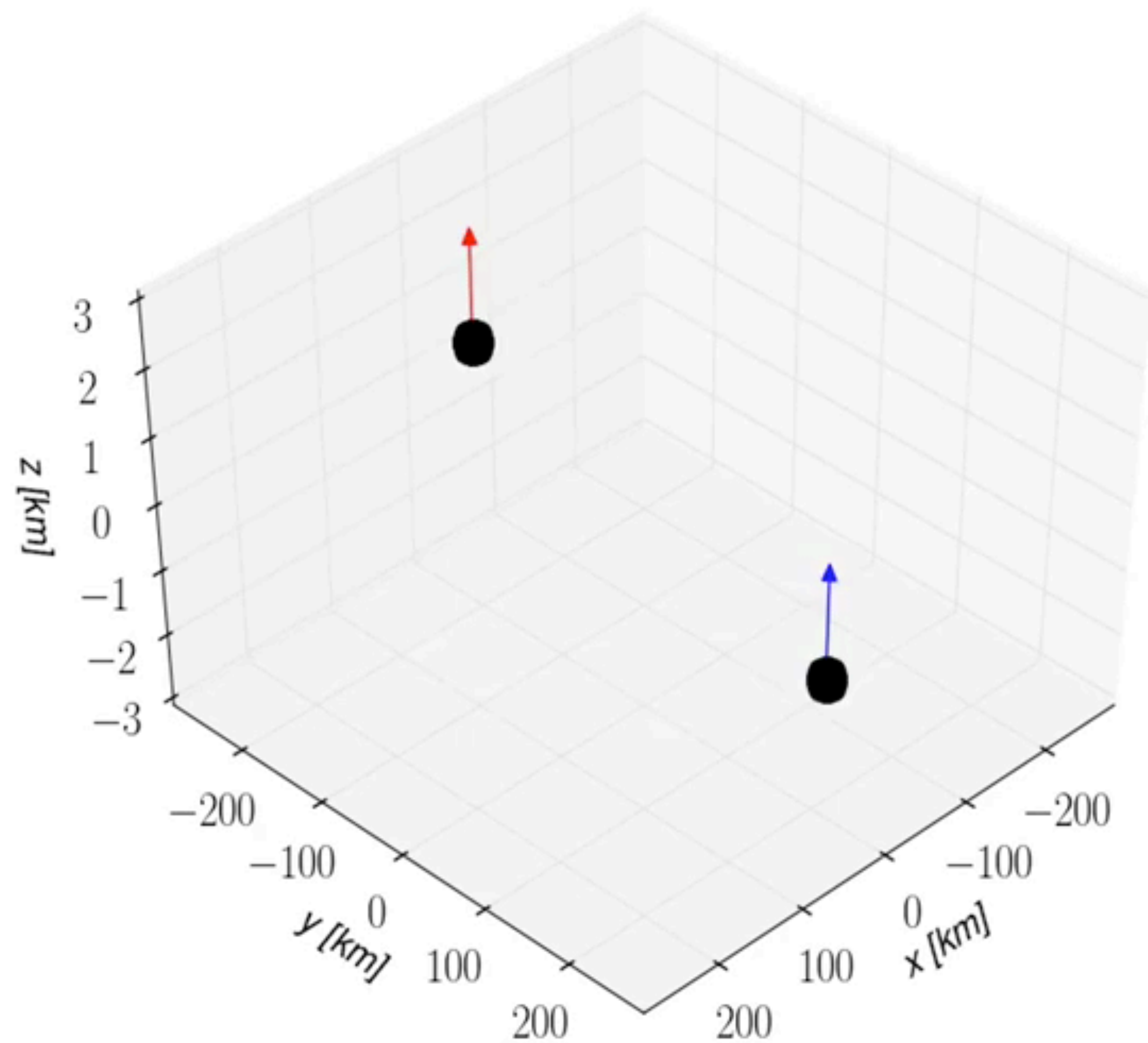
The more massive BH is closer to the center of mass.

The energy radiated is lower than an equal-mass binary.

The binary takes longer to inspiral.

Coalescing Binaries

Aligned spin, equal mass black holes



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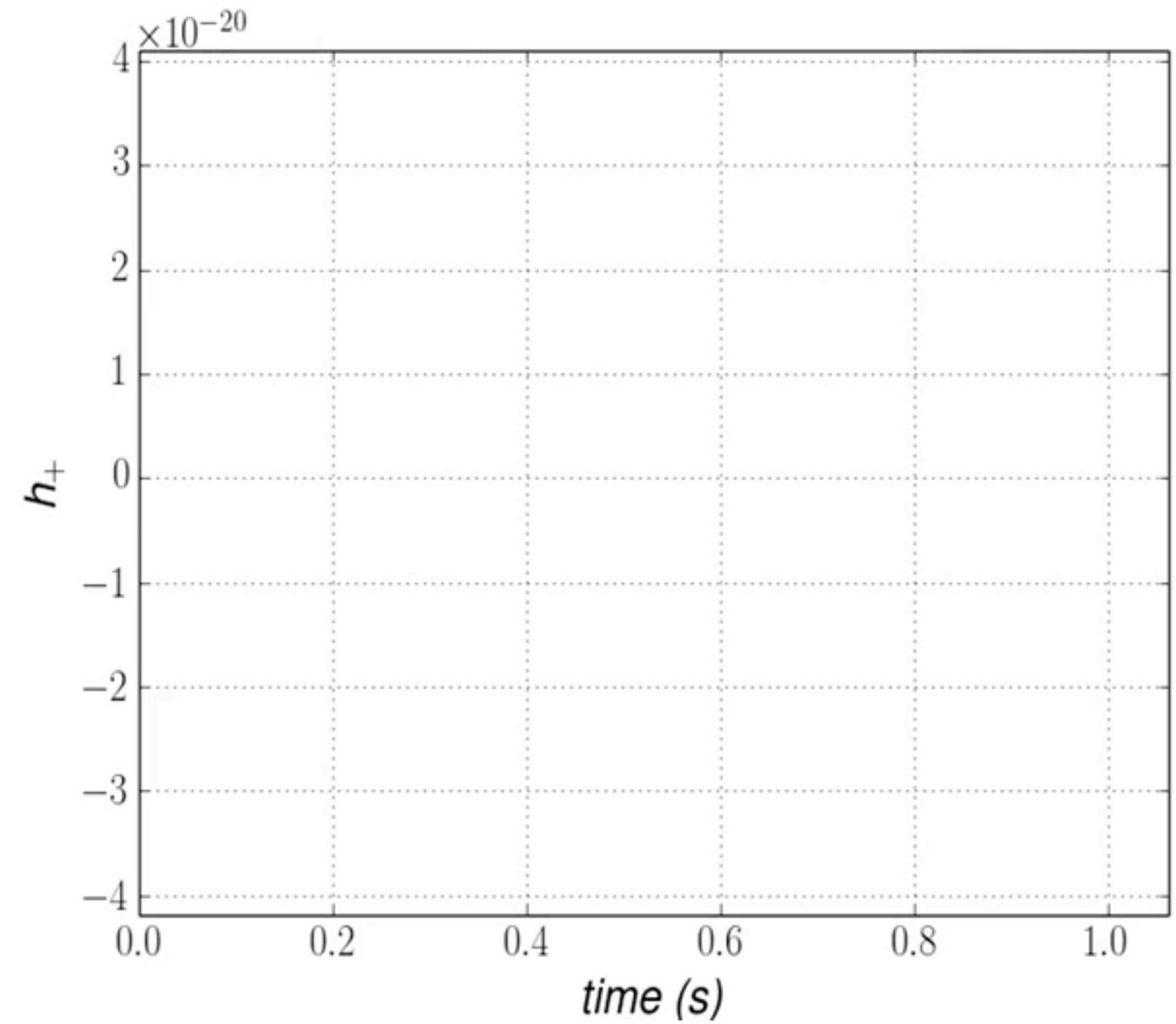
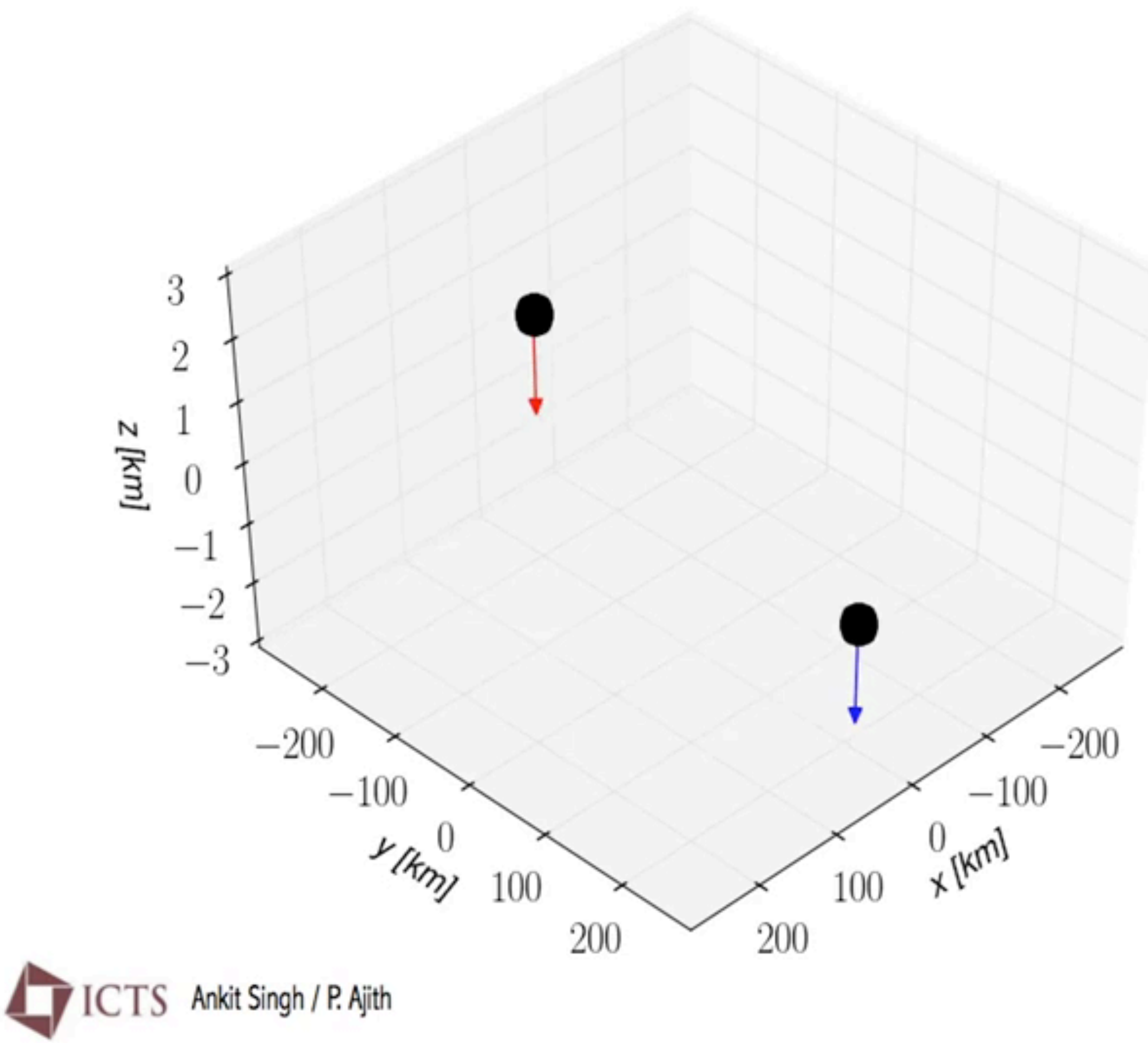


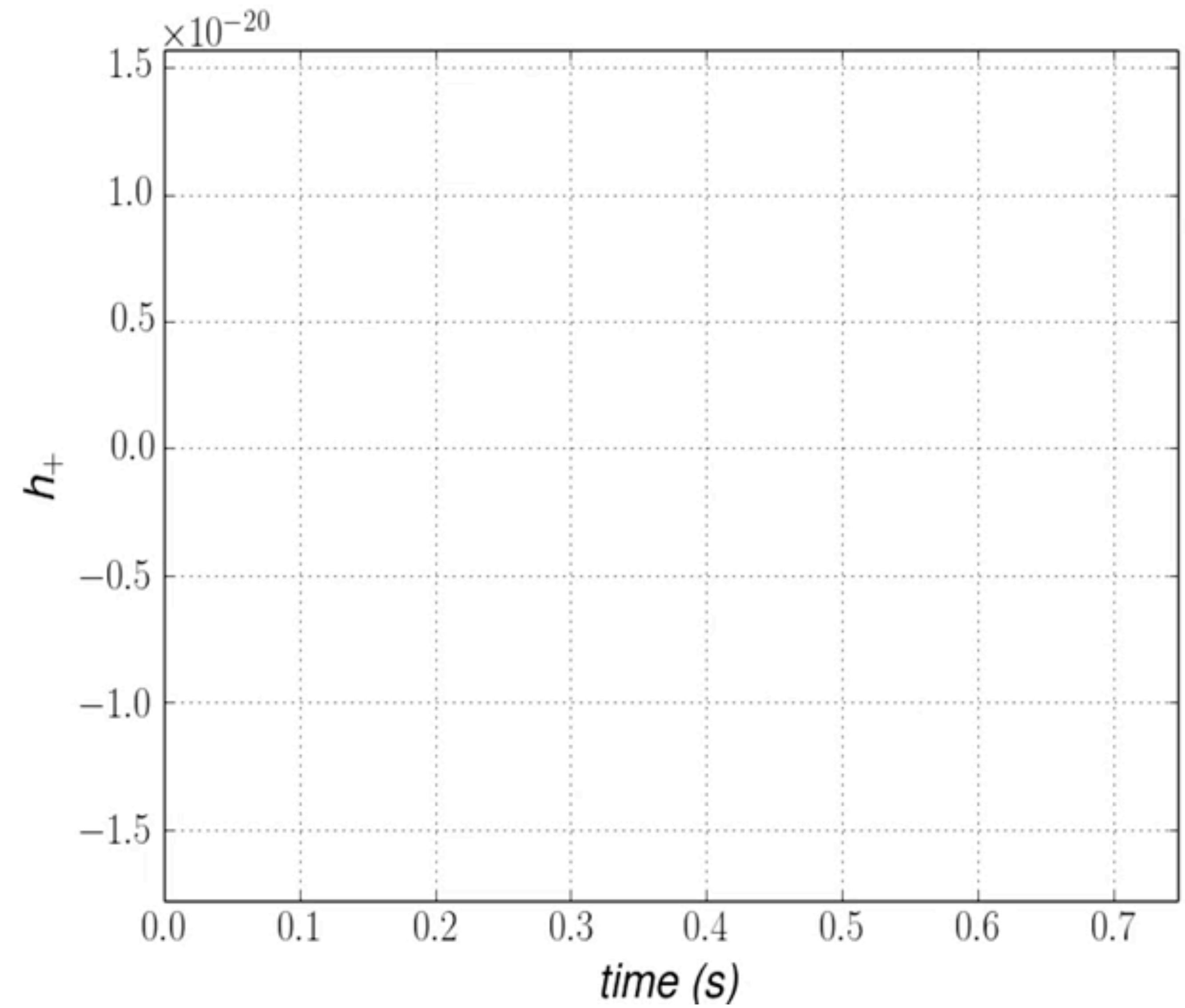
Spin vectors are aligned with orbital angular momentum.

Orbital hang-up effect: aligned-spin black holes can inspiral to much closer separations, resulting in longer and stronger GW signals, compared to non-spinning binary.

Coalescing Binaries

Anti-aligned spin, equal mass black holes

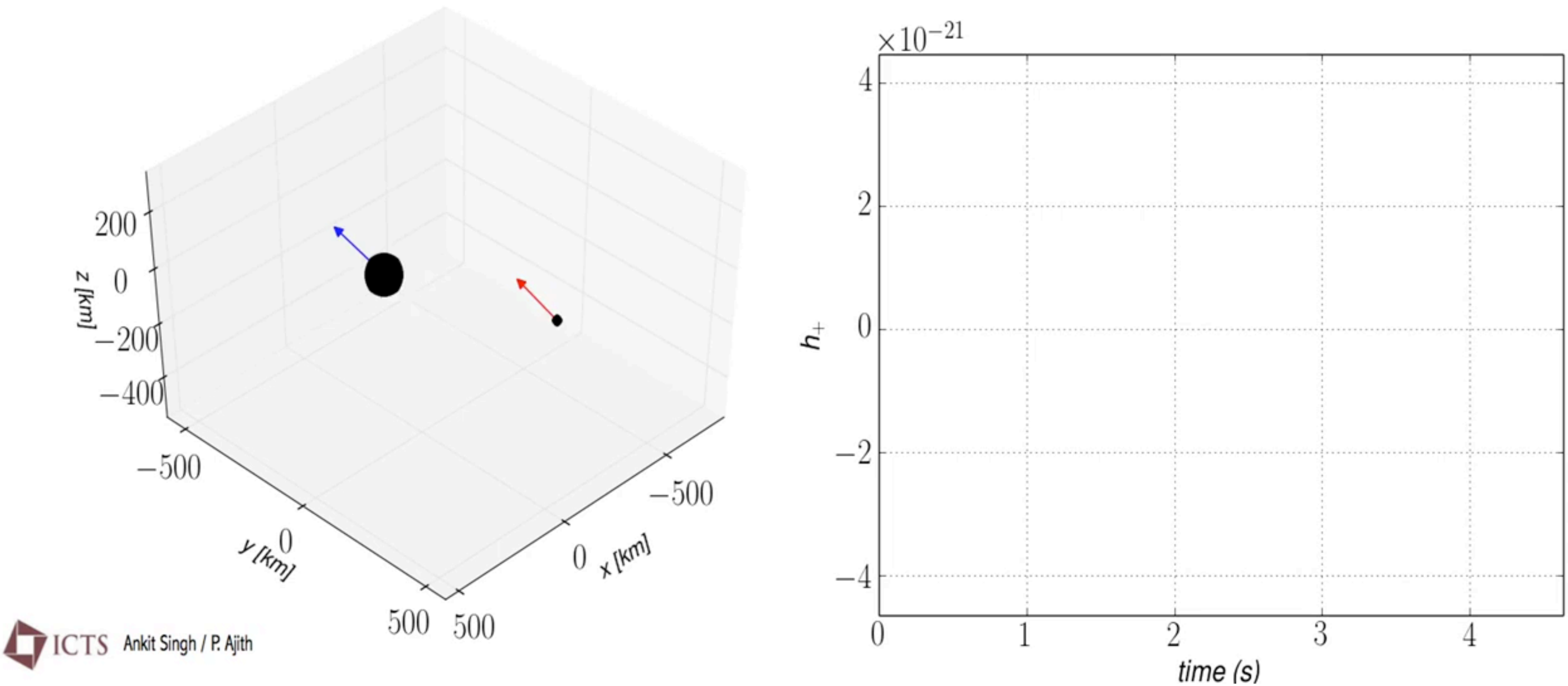




Spin vectors are aligned opposite to orbital angular momentum.
Anti-aligned-spin black holes have shorter and weaker GW signals, compared to non-spinning binary.

Coalescing Binaries

Misaligned spin, unequal mass black holes



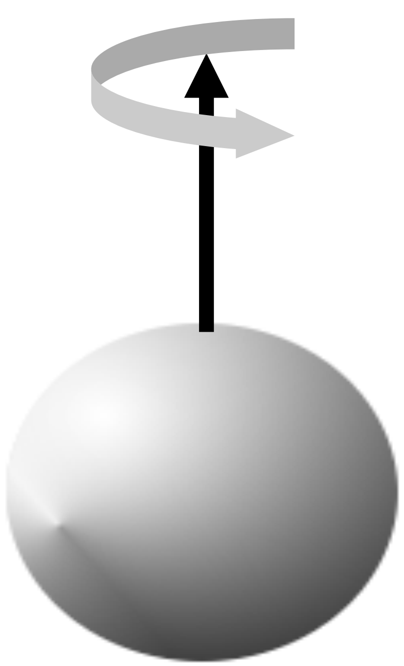
Spin vectors are misaligned with orbital angular momentum.
There are **spin-orbit and spin-spin interactions** between spins and orbital angular momentum that cause spins to precess.
Results in complicated modulations in amplitude and phase of GW signals.

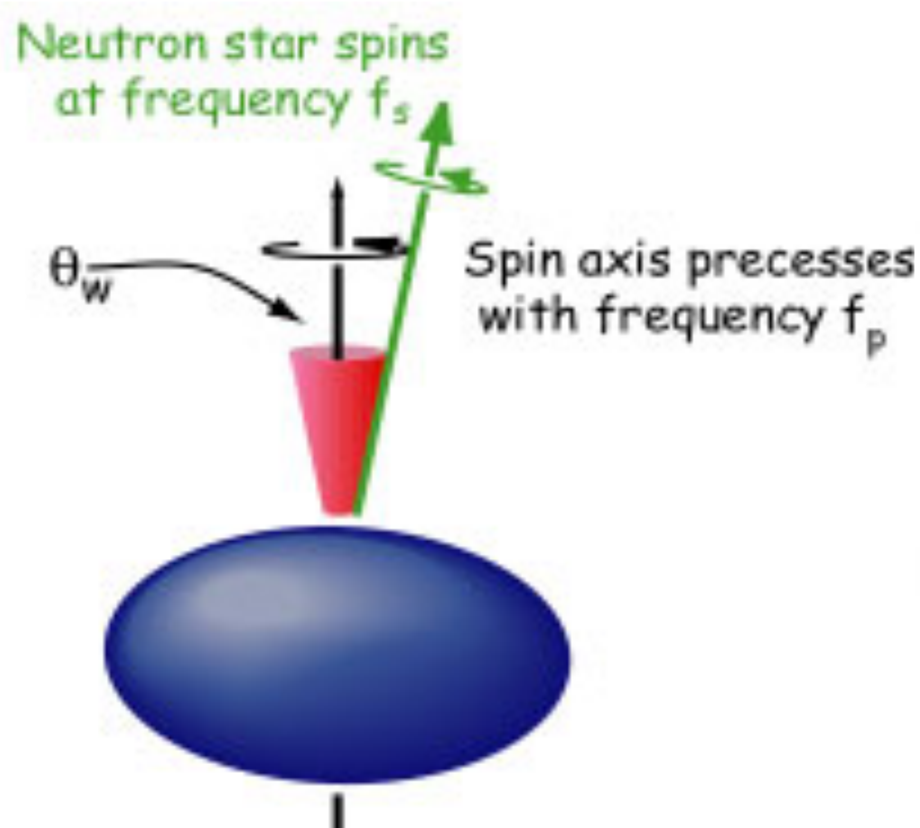
- Solving the Einstein Equations
    - Linearized Theory
    - Vacuum Solution
    - Solution with Source Term
- Generation of Gravitational Waves
- Effect of Gravitational Waves on Matter
- LIGO & Virgo Astrophysical Sources
    - Coalescing Binaries
    - Continuous Waves
    - Transient Bursts
    - Stochastic Background
- LISA & PTA Sources

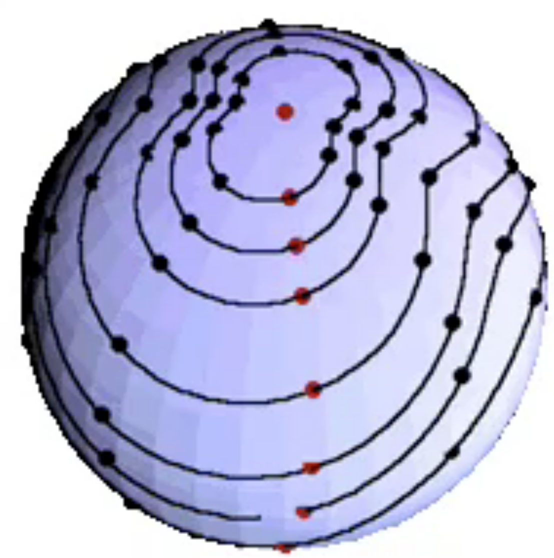
Continuous Waves

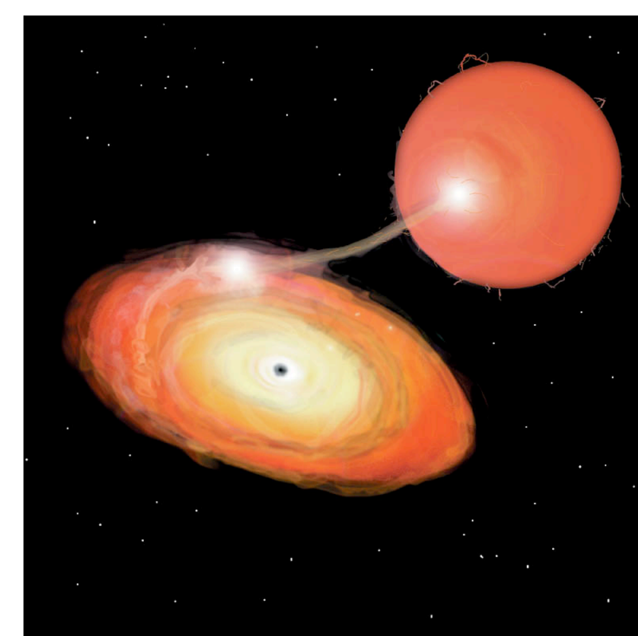
Non-axisymmetric rotating neutron stars;
asymmetry could arise from:

- equatorial ellipticity (mm-high mountain) $f_{\mathrm{GW}} = 2 f_{\mathrm{rot}}$
- free precession around rotation axis $f_{\mathrm{GW}} \sim f_{\mathrm{rot}} + f_{\mathrm{prec}}$
- excitation of long-lasting oscillations $f_{\mathrm{GW}} \sim 4/3 f_{\mathrm{rot}}$
- deformation due to matter accretion $f_{\mathrm{GW}} = 2 f_{\mathrm{rot}}$



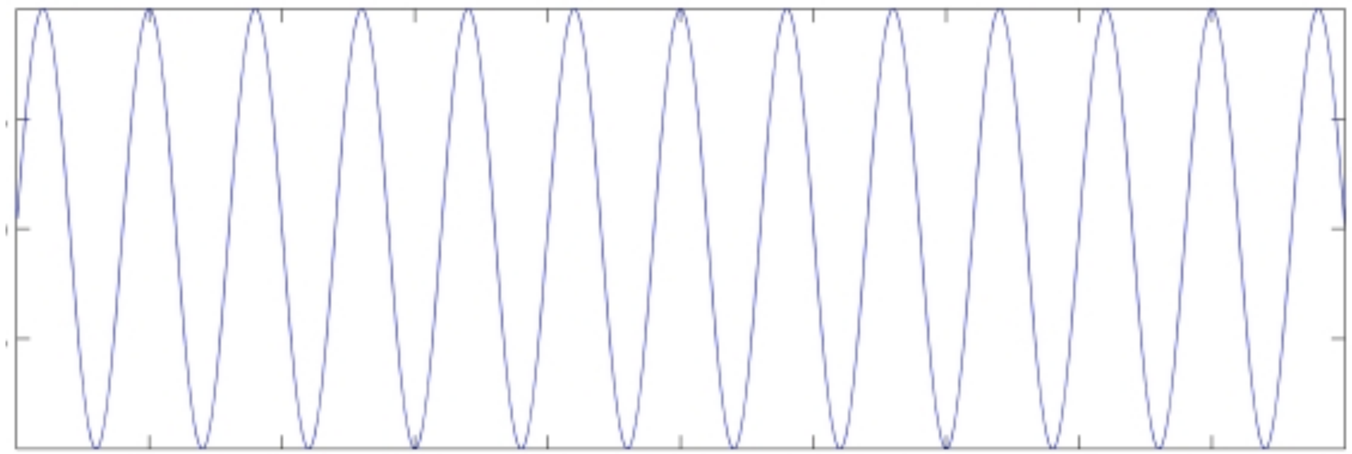




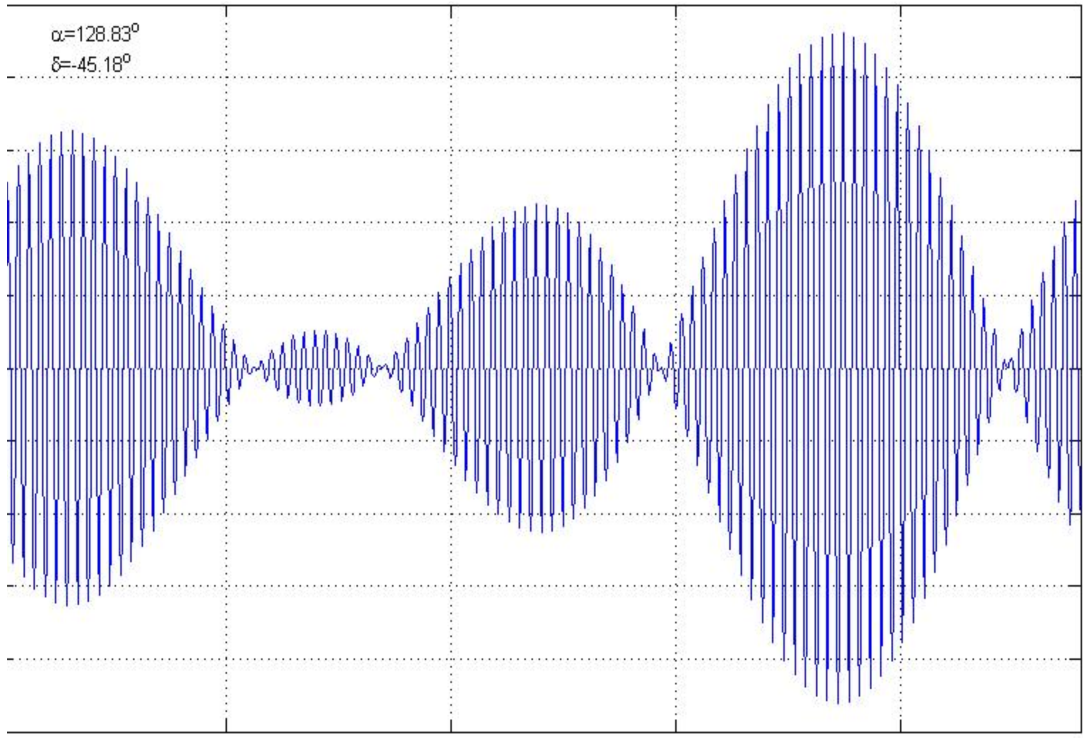


Continuous Waves

At the source



At the detector



Nearly monochromatic, continuous signal but could have:

- relative velocity between source/detector (Doppler Effect)
- amplitude modulation due to antenna sensitivity of detector
- frequency and phase evolution

Example II: Quadrupole radiation from rotating rigid body

A rigid body is characterized by its inertia tensor:

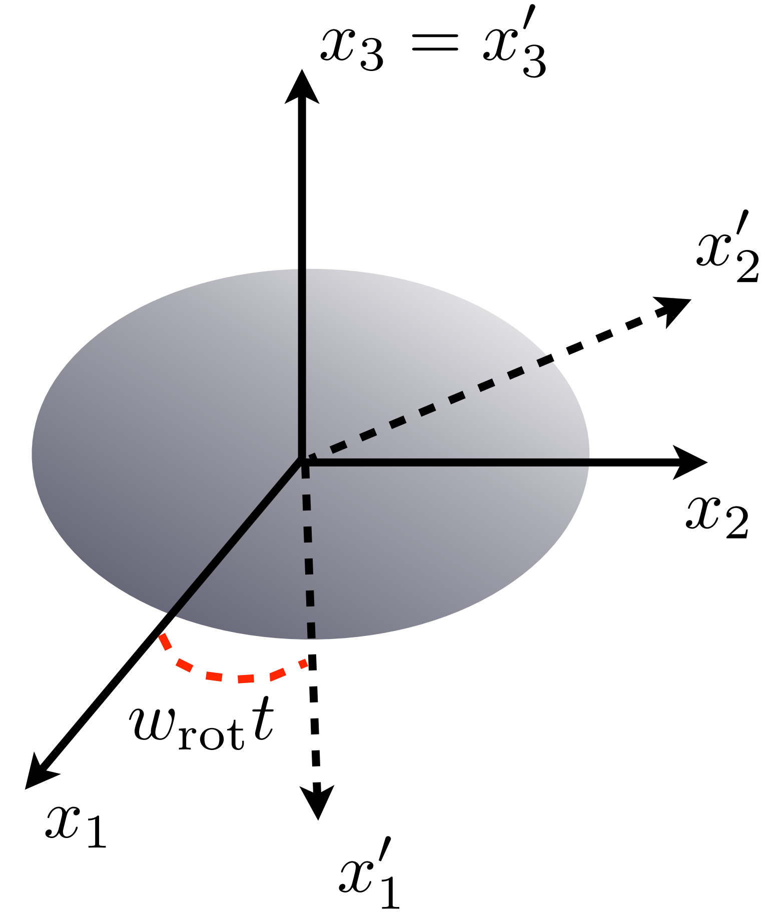
$$I^{ij} = \int d^3x\, \rho(\mathbf{x}) \left(r^2 \delta^{ij} - x^i x^j\right)$$

There is a frame where the inertia tensor is diagonal. The principal moments of inertia are

$$I_1 = \int d^3x'\, \rho(\mathbf{x}') \left(x_2'^2 + x_3'^2\right)$$

$$I_2 = \int d^3x'\, \rho(\mathbf{x}') \left(x_1'^2 + x_3'^2\right)$$

$$I_3 = \int d^3x'\, \rho(\mathbf{x}') \left(x_1'^2 + x_2'^2\right)$$

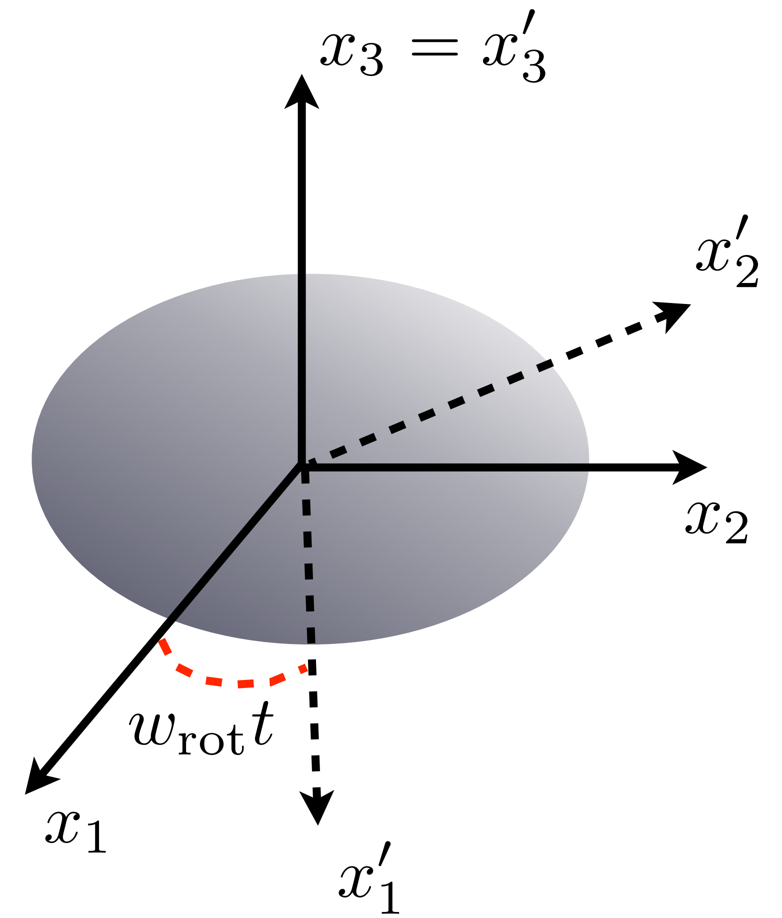


Consider a simple situation in which an ellipsoidal body rotates rigidly about one of its principle axes.

Example II: Quadrupole radiation from rotating rigid body

(x'_1, x'_2, x'_3) - attached to body and rotate with it

(x_1, x_2, x_3) - fixed reference frame



The two frames are related by time-dependent rotation matrix:

$$x'_i = \mathcal{R}_{ij} x_j$$

$$\mathcal{R}_{ij} = \begin{bmatrix} \cos\omega_{\rm rot}t & \sin\omega_{\rm rot}t & 0 \\ -\sin\omega_{\rm rot}t & \cos\omega_{\rm rot}t & 0 \\ 0 & 0 & 1 \end{bmatrix}_{ij}$$

The time-dependent inertia tensor is then given as $I = \mathcal{R}^T I' \mathcal{R}$

$$I_{11} = 1 + \frac{I_1 - I_2}{2}\cos 2\omega_{\rm rot}t \qquad I_{22} = 1 - \frac{I_1 - I_2}{2}\cos 2\omega_{\rm rot}t$$

$$I_{12} = \frac{I_1 - I_2}{2}\sin 2\omega_{\rm rot}t \qquad I_{33} = I_3 \qquad I_{13} = I_{23} = 0$$

Example II: Quadrupole radiation from rotating rigid body

Compare the inertia tensor with the second mass moment:

$$I^{ij} = \int d^3x\, \rho(\mathbf{x}) \left(r^2 \delta^{ij} - x^i x^j\right) \qquad M^{ij} = \int d^3x\, \rho(\mathbf{x}) x^i x^j$$

They differ by a minus sign and a trace term.

$$M^{ij} = -I^{ij} + \mathrm{Tr}(I)\delta^{ij}$$

But the trace is a constant :

$$\mathrm{Tr}(I) = \mathrm{Tr}(\mathcal{R}^T I' \mathcal{R}) = \mathrm{Tr}(\mathcal{R}\mathcal{R}^T I') = \mathrm{Tr}(I') = I_1 + I_2 + I_3$$

Example II: Quadrupole radiation from rotating rigid body

So when taking the second time derivative of M^{ij}, the trace terms vanish.

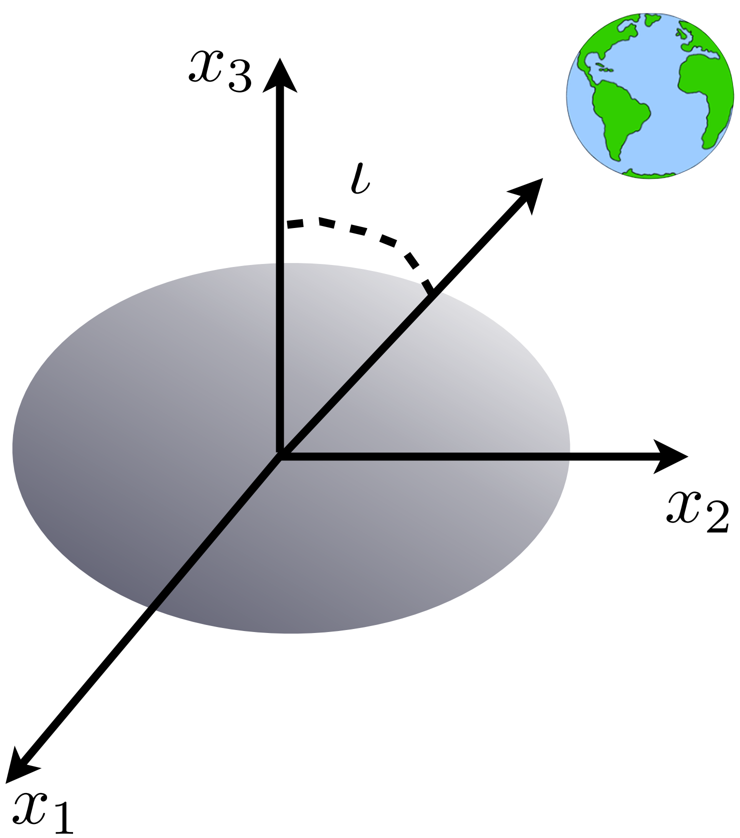
$$M_{11} = -\frac{I_1 - I_2}{2}\cos 2\omega_{\rm rot}t + \text{constant}$$
$$M_{12} = -\frac{I_1 - I_2}{2}\sin 2\omega_{\rm rot}t + \text{constant}$$
$$M_{22} = +\frac{I_1 - I_2}{2}\cos 2\omega_{\rm rot}t + \text{constant}$$
$$M_{13} = M_{23} = M_{33} = \text{constant}$$

Note, there is a time-varying second mass moment only if $I_1 \neq I_2$.

M_{ij} is a periodic function so we have production of gravitational waves with frequency:

$$\omega_{\rm gw} = 2\omega_{\rm rot}$$

Example II: Quadrupole radiation from rotating rigid body



Use equations for generic propagation.
Set $\theta = \iota$ and $\phi = 0$.

$$\boxed{h_+ = \frac{1}{r}\frac{4G\omega_{\rm rot}^2}{c^4}\left(I_1 - I_2\right)\frac{1+\cos^2\iota}{2}\cos\left(2\omega_{\rm rot}t\right)}$$

$$\boxed{h_\times = \frac{1}{r}\frac{4G\omega_{\rm rot}^2}{c^4}\left(I_1 - I_2\right)\cos\iota\sin\left(2\omega_{\rm rot}t\right)}$$

Define ellipticity by: $\epsilon \equiv \dfrac{I_1 - I_2}{I_3}$

$$h_+ = h_0\frac{1+\cos^2\iota}{2}\cos\left(2\pi f_{\rm gw}t\right)$$

$$h_\times = h_0\cos\iota\sin\left(2\pi f_{\rm gw}t\right)$$

$$h_0 = \frac{4\pi^2 G}{c^4}\frac{I_3 f_{\rm gw}^2}{r}\epsilon$$

Neutron stars that rotate more rapidly produce a stronger GW signal.

Example II: Quadrupole radiation from rotating rigid body

Angular distribution of the radiated power in quadrupole approximation:

$$P_{\rm quad} = \left(\frac{dE_{\rm gw}}{d\Omega}\right)_{\rm quad} = \frac{r^2 c^3}{16\pi G}\int_{\mathcal{S}} d\Omega \left\langle \dot{h}_+^2 + \dot{h}_\times^2 \right\rangle$$

For our NS example: $P = \frac{32G}{5c^5}\epsilon^2 I_3^2 \omega_{\rm rot}^6$

Then we can say that the rotational energy of the star decreases because of GW emission as

$$\frac{dE_{\rm rot}}{dt} = -\frac{32G}{5c^5}\epsilon^2 I_3^2 \omega_{\rm rot}^6$$

Rotational energy of star rotating around its principal axis is

$$E_{\rm rot} = (1/2) I_3 \omega_{\rm rot}^2$$

Then rotational frequency of neutron star should decrease as

$$\dot{\omega}_{\rm rot} = -\frac{32G}{5c^5}\epsilon^2 I_3 \omega_{\rm rot}^5$$

Example II: Quadrupole radiation from rotating rigid body

$$\dot{\omega}_{\mathrm{rot}} \sim -\omega_{\mathrm{rot}}^{n}$$

Table 1

Braking index measurements for six pulsars. Also given are the pulsar period, period derivative, period second derivative and characteristic age.

Pulsar names J2000; B1950	P (ms)	$\dot{P}$ (10^{-15})	$\ddot{P}$ (s^{-1})	τ_c (yr)	braking index n
Crab*					
J0534+2200; B0531+21	33.085	423	-3.61×10^{-24}	1240	2.51(1)[a]
J0540−6919; B0540−69*	50.499	479	-1.6×10^{-24}	1670	2.140(9)[b]
Vela					
J0835−4510; B0833−45	89.328	125		11 300	1.4(2)[c]
J1119−6127*	407.746	4022	-8.8×10^{-24}	1160	2.91(5)[d]
J1513−5908; B1509−58*	150.658	1540	-1.312×10^{-23}	1550	2.839(3)[e]
J1846−0258*	325.684	7083		728	2.65(1)[f]

[a] Demiański & Prószyński (1983), Lyne, Pritchard & Smith (1988, 1993).

[b] Livingstone et al. (2005).

[c] Lyne et al. (1996), Dodson, McCulloch & Lewis (2002).

[d] Camilo et al. (2000).

[e] Manchester, Durdin & Newton (1985), Kaspi et al. (1994).

[f] Livingstone et al. (2006).

N. Vranesevic, D.B. Melrose, MNRAS 410, 4 (2011)

n is the braking index.

Experimentally, n ranges between 2 and 3, rather than $n = 5$ so GW emission is not main energy loss mechanism for rotating pulsars.

Other EM mechanisms dominate.

Continuous Waves

Continuous signal with $h \propto \epsilon$ $\mathrm{SNR} \propto \frac{h}{\sqrt{S_n}}\sqrt{T}$

Equatorial ellipticity $\epsilon = \frac{I_{\mathrm{XX}} - I_{\mathrm{YY}}}{I_{\mathrm{ZZ}}}$

Maximum Deformations

$\epsilon < 10^{-5}$ Normal Neutron Star

$\epsilon < 10^{-3}$ Hybrid Neutron Star

$\epsilon < 10^{-1}$ Extreme Quark Star

- Solving the Einstein Equations
    - Linearized Theory
    - Vacuum Solution
    - Solution with Source Term
- Generation of Gravitational Waves
- Effect of Gravitational Waves on Matter
- LIGO & Virgo Astrophysical Sources
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    - Stochastic Background
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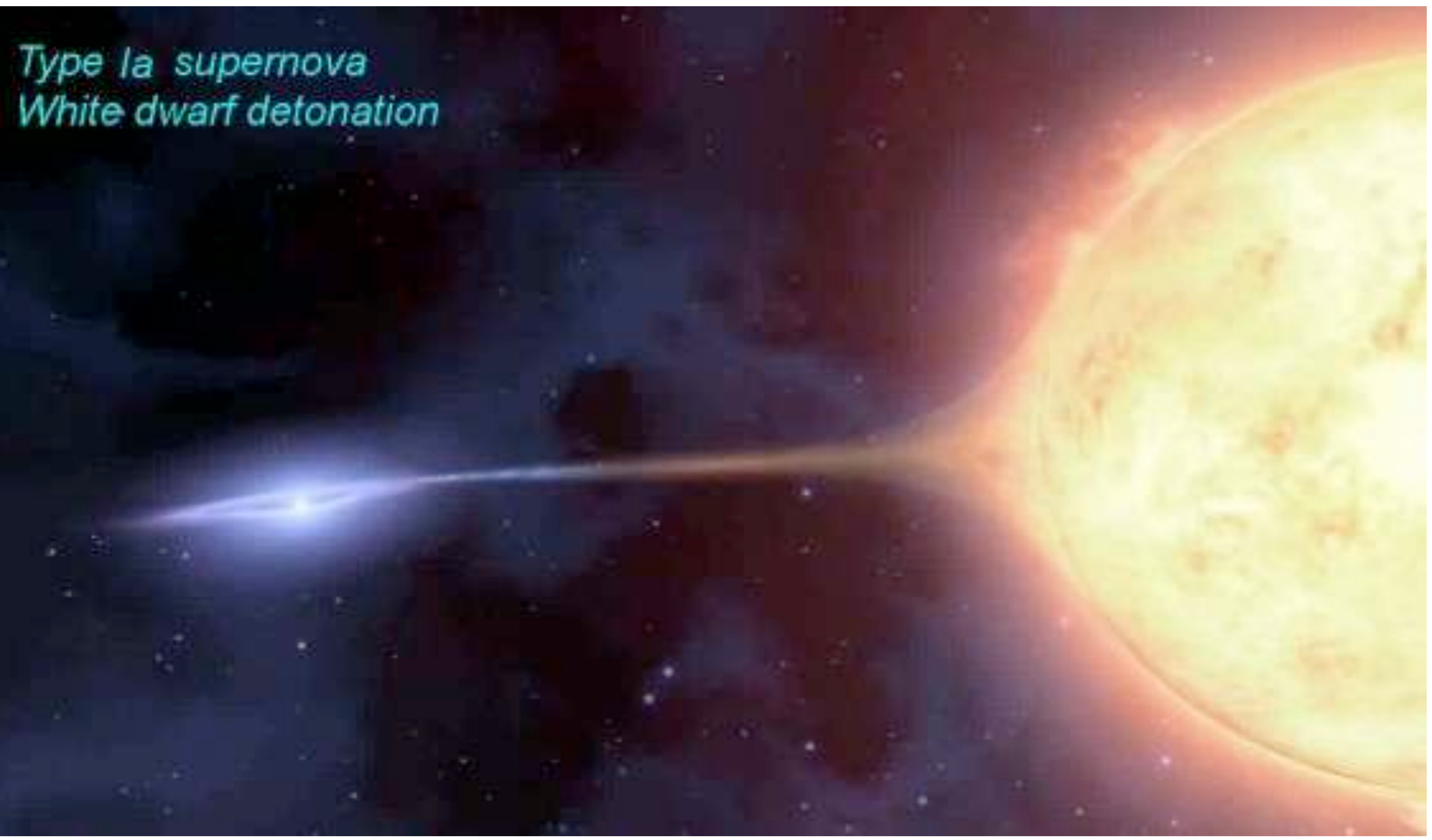
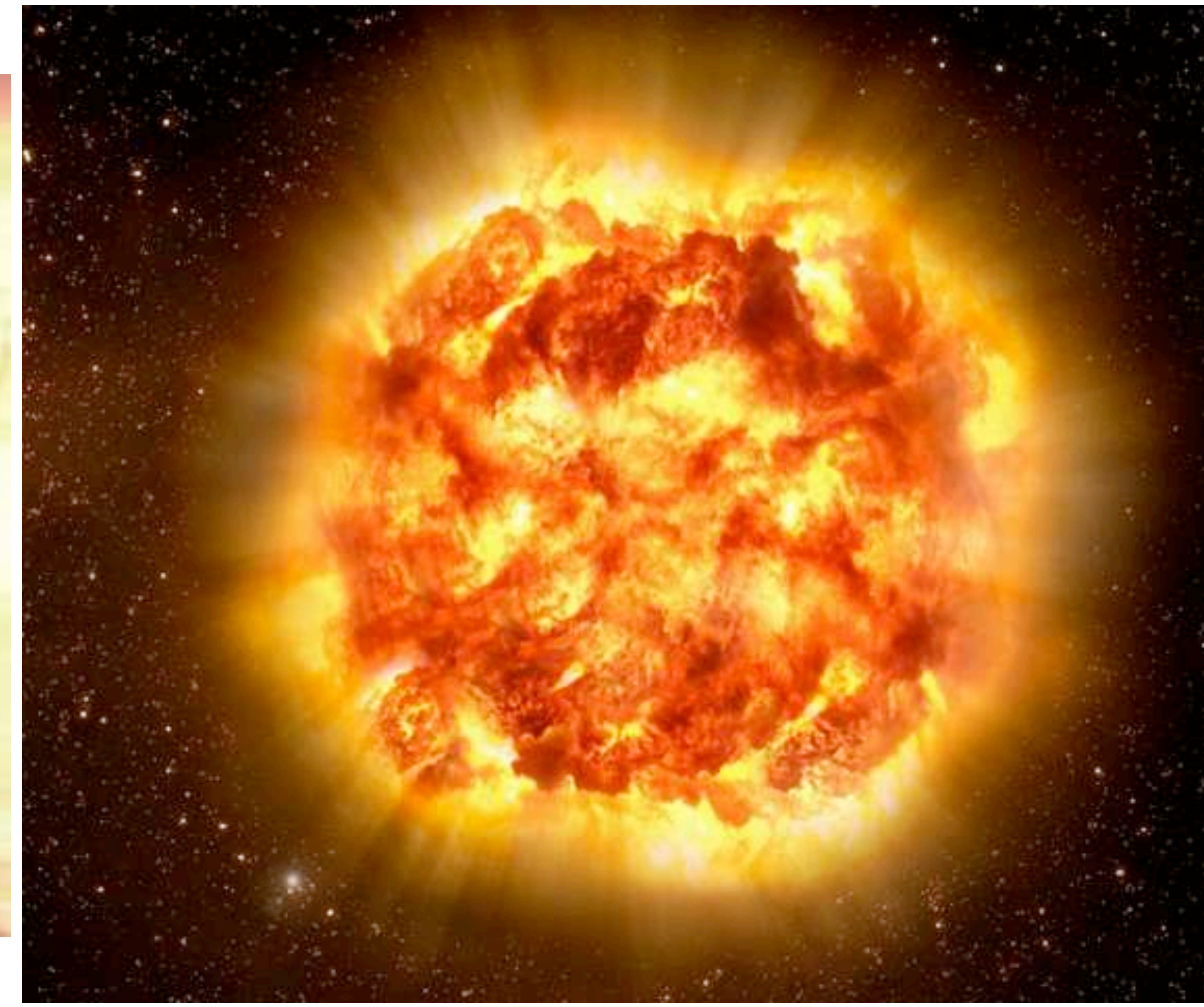
Burst Sources

Supernovae

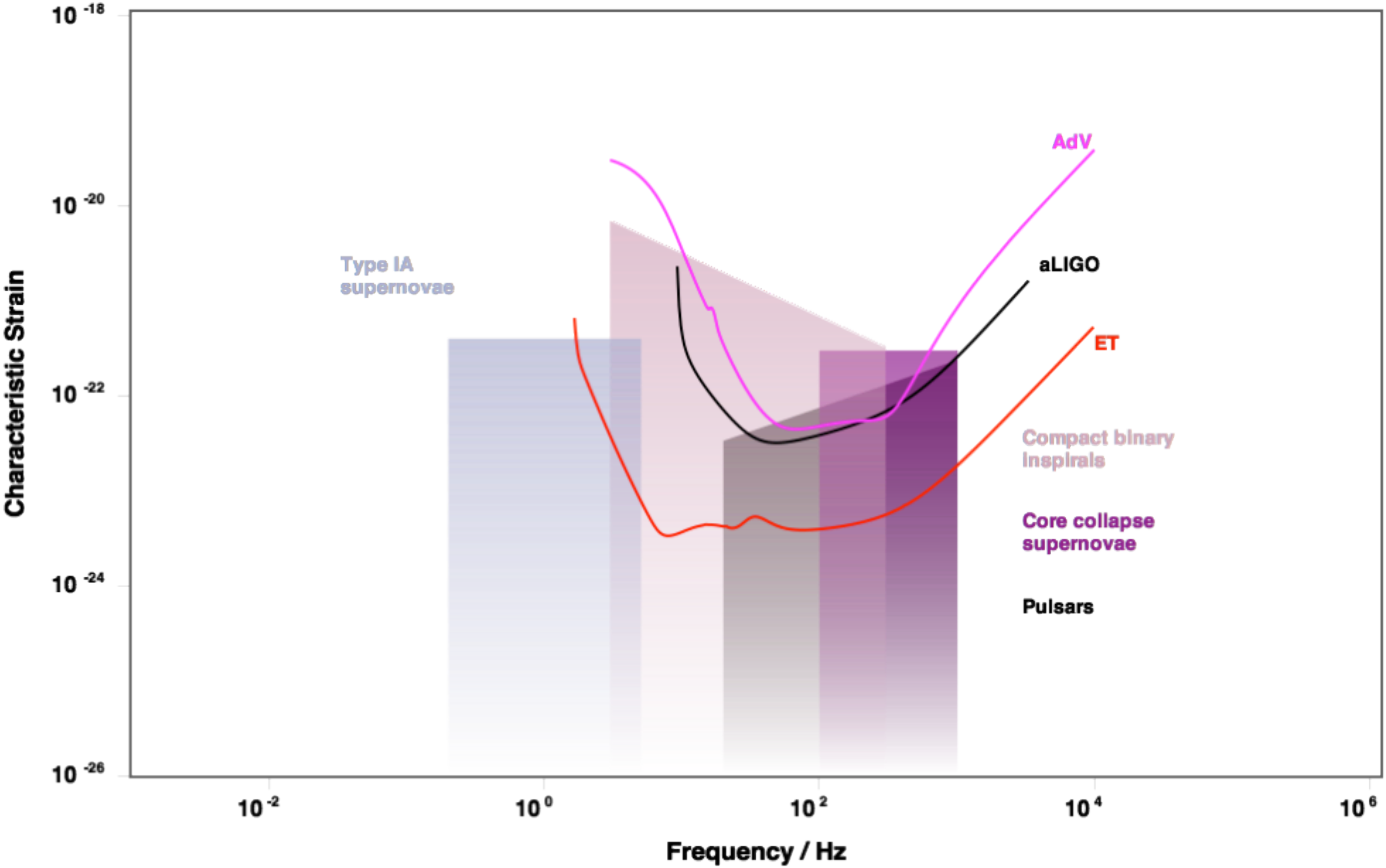


Type Ia supernovae when white dwarfs in binary detonate.

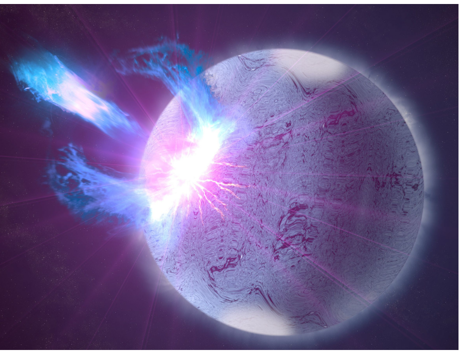


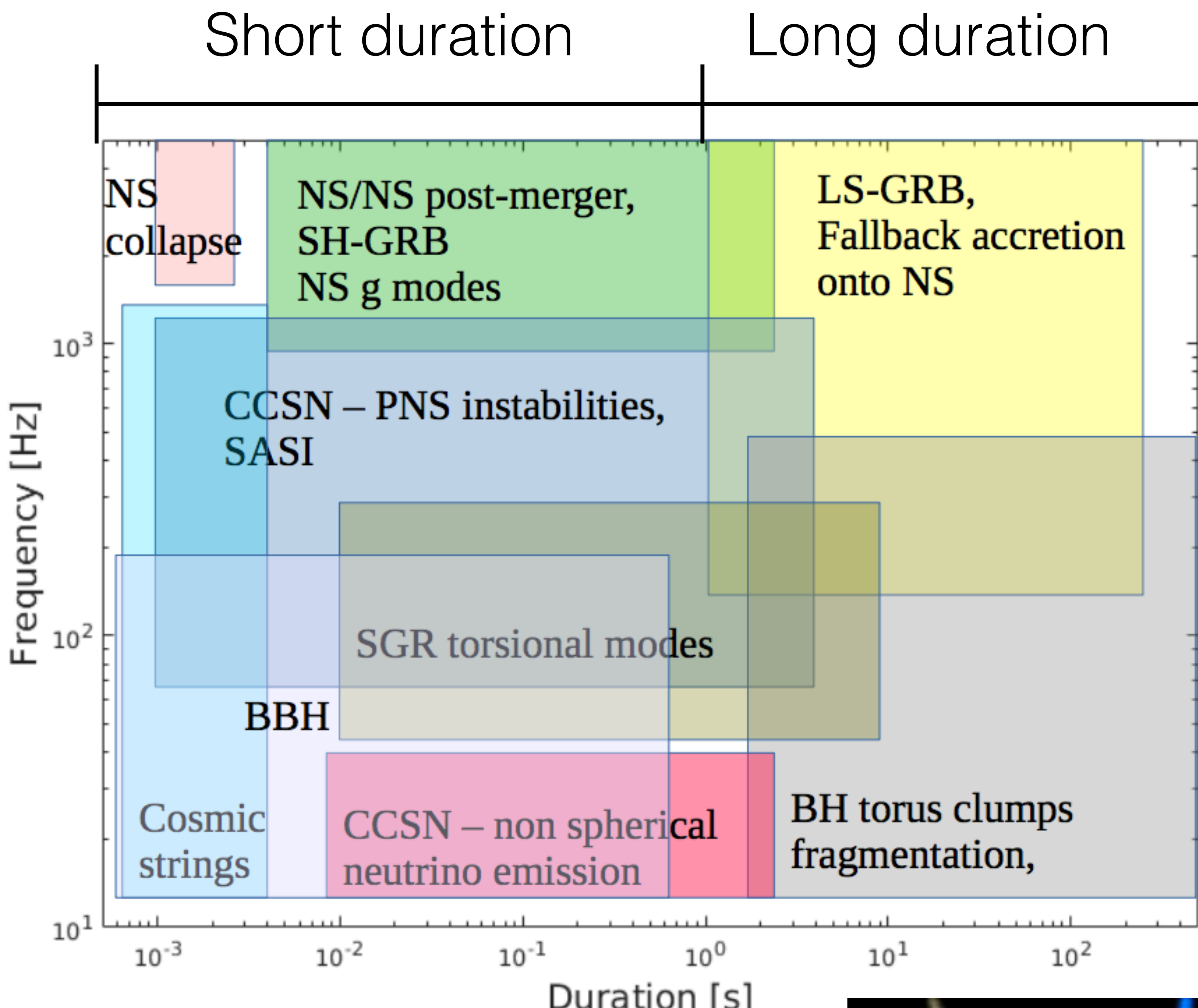
The Iconic Burst GW Source - Core collapse supernovae (Type Ib/Ic & II) when massive stars die.

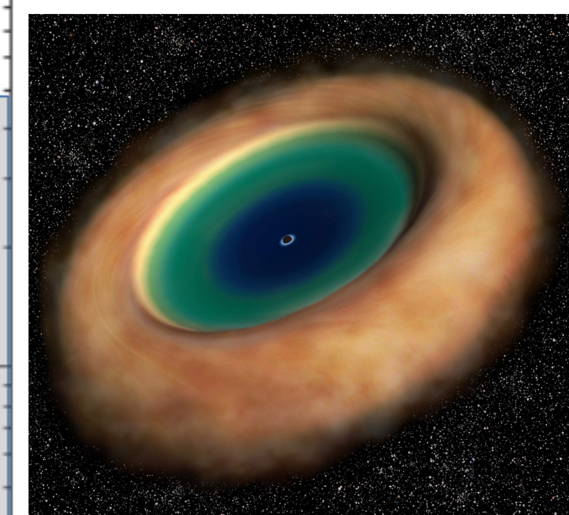
Burst Sources

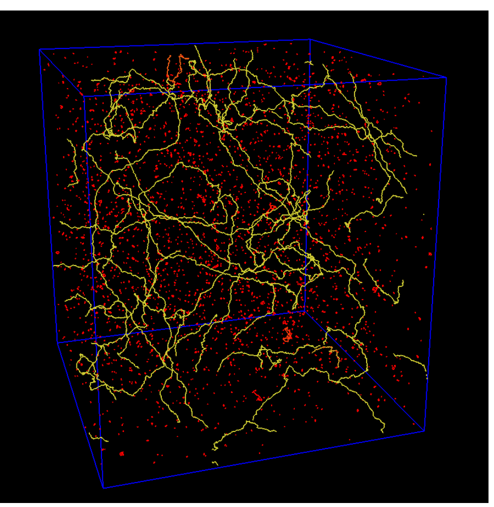


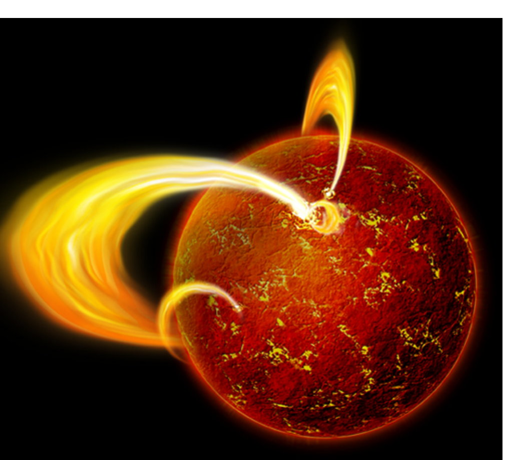
Burst Sources

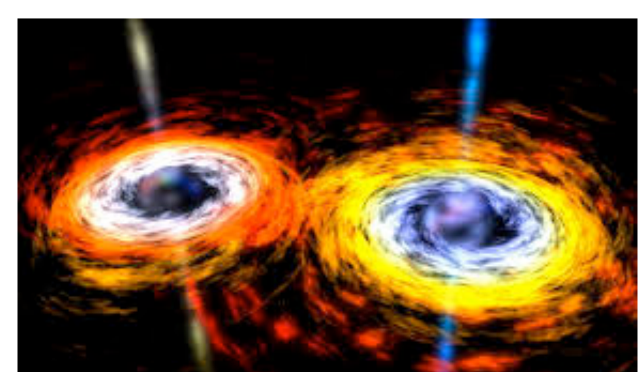












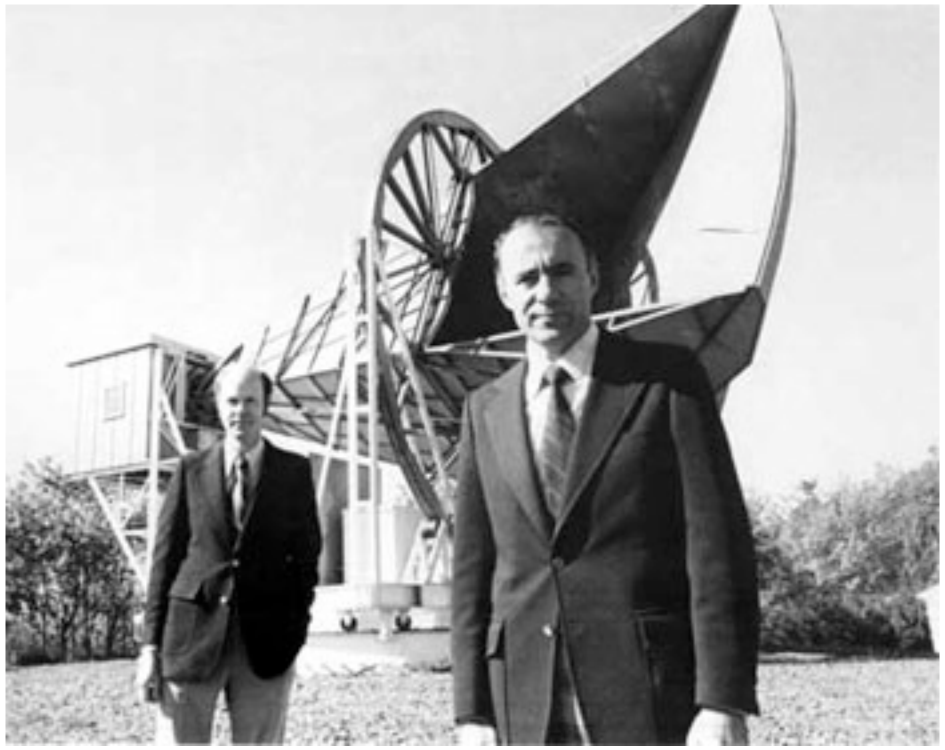
Stochastic Background

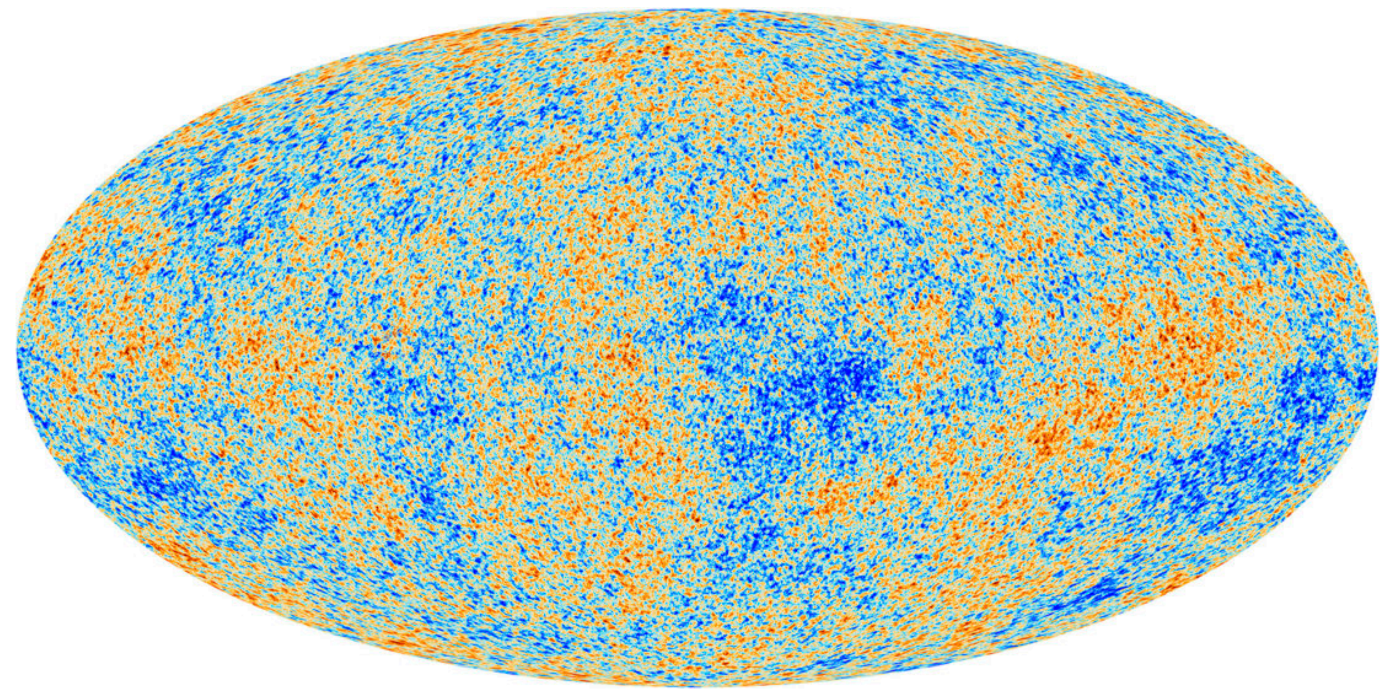
- Stochastic (random) background of gravitational radiation

- Can arise from superposition of large number of unresolved GW sources
  1. Cosmological origin
  2. Astrophysical origin

- Strength of background measured as gravitational wave energy density ρ_{GW}

Cosmic Microwave Background



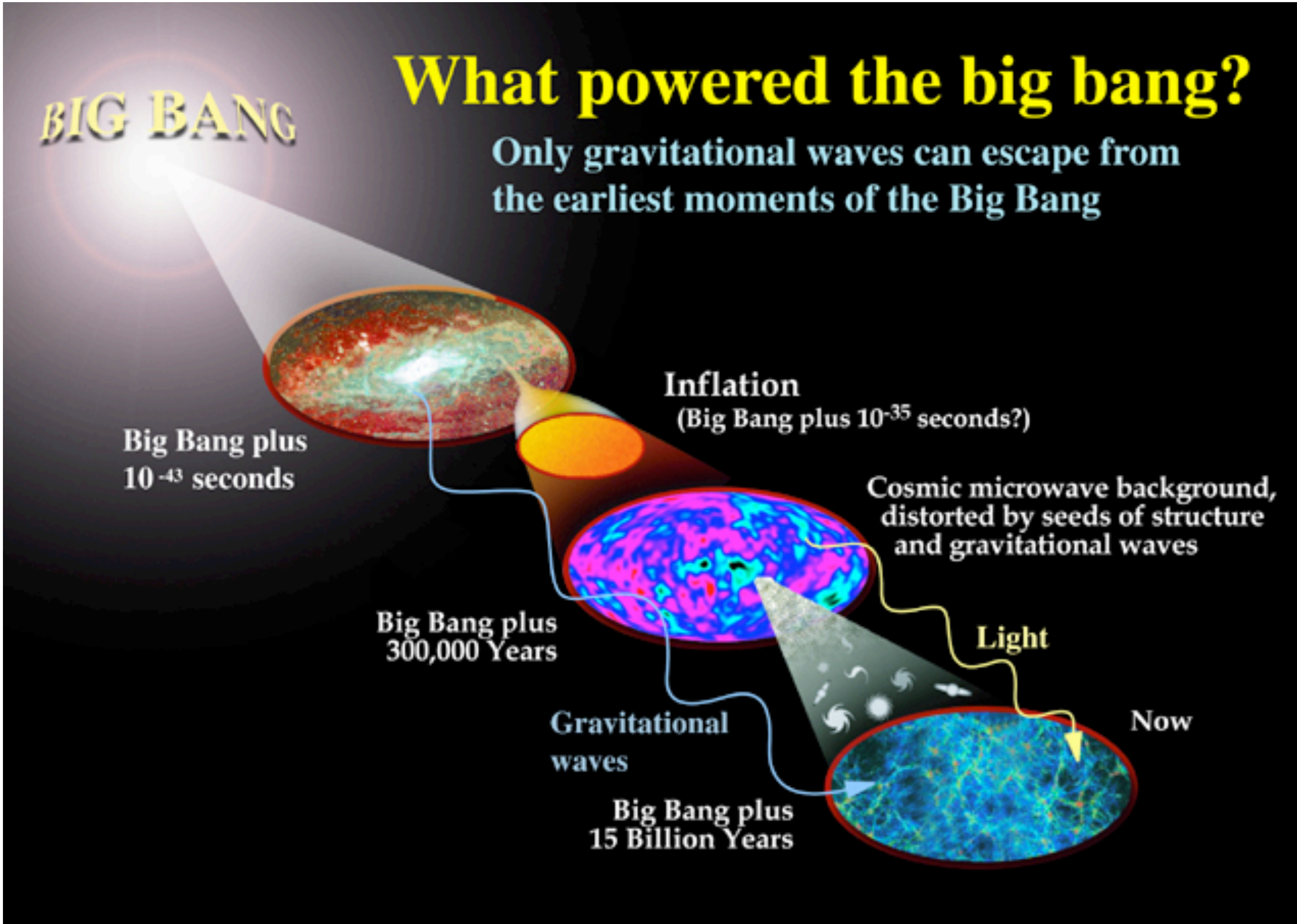


- 1965 - Penzias and Wilson accidently discovered Cosmic Microwave Background (CMB), leftover radiation from 380,000 years Big Bang

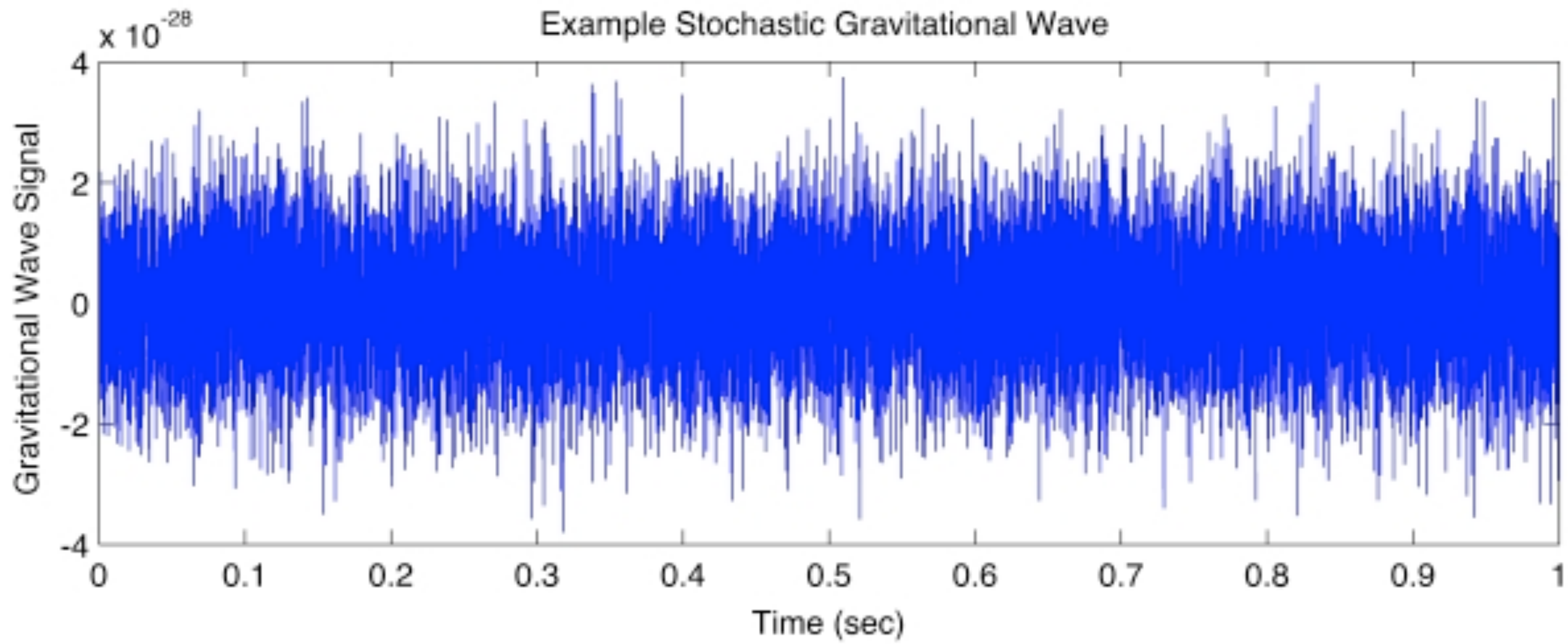
- 1978 - awarded Nobel prize

- CMB as seen by Planck, an ESA observatory
- Wavelengths of photons are greatly redshifted (1mm)
- Effective temperature ~ 2.7K
- Can be detected by far-infrared and radio telescopes

Cosmological Gravitational Wave Background



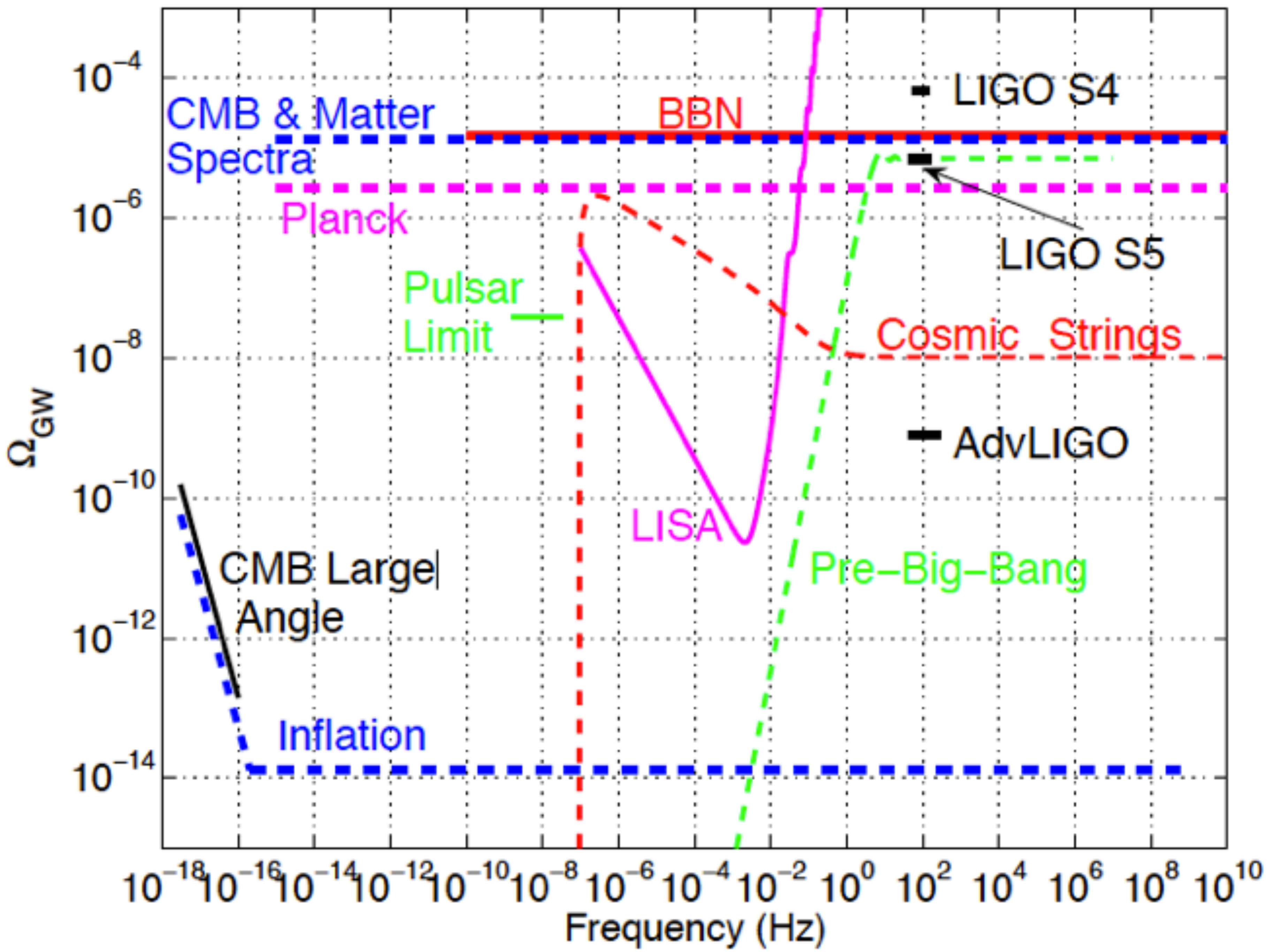
Cosmological Gravitational Wave Background



GW spectrum: $\Omega_{\mathrm{GW}}(f) = \frac{f}{\rho_c}\frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}f}$

Critical energy density of universe: $\rho_c = \frac{3c^2 H_0^2}{8\pi G}$

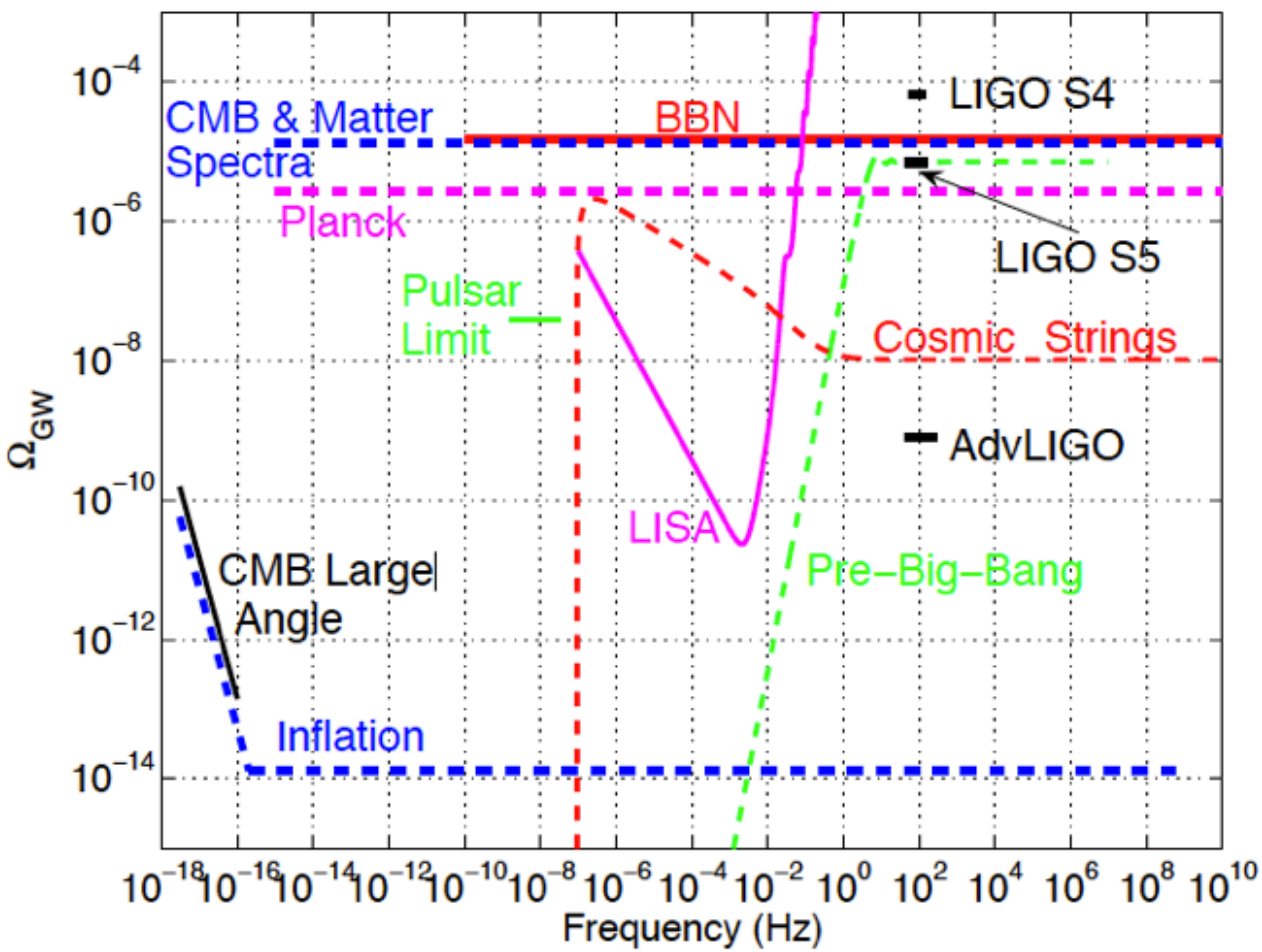
Cosmological Gravitational Wave Background



Big-Bang-Nucleosynthesis: abundances of light nuclei produced

Cosmic Microwave Background Measurements: structure of CMB and matter power spectra

Cosmological Gravitational Wave Background

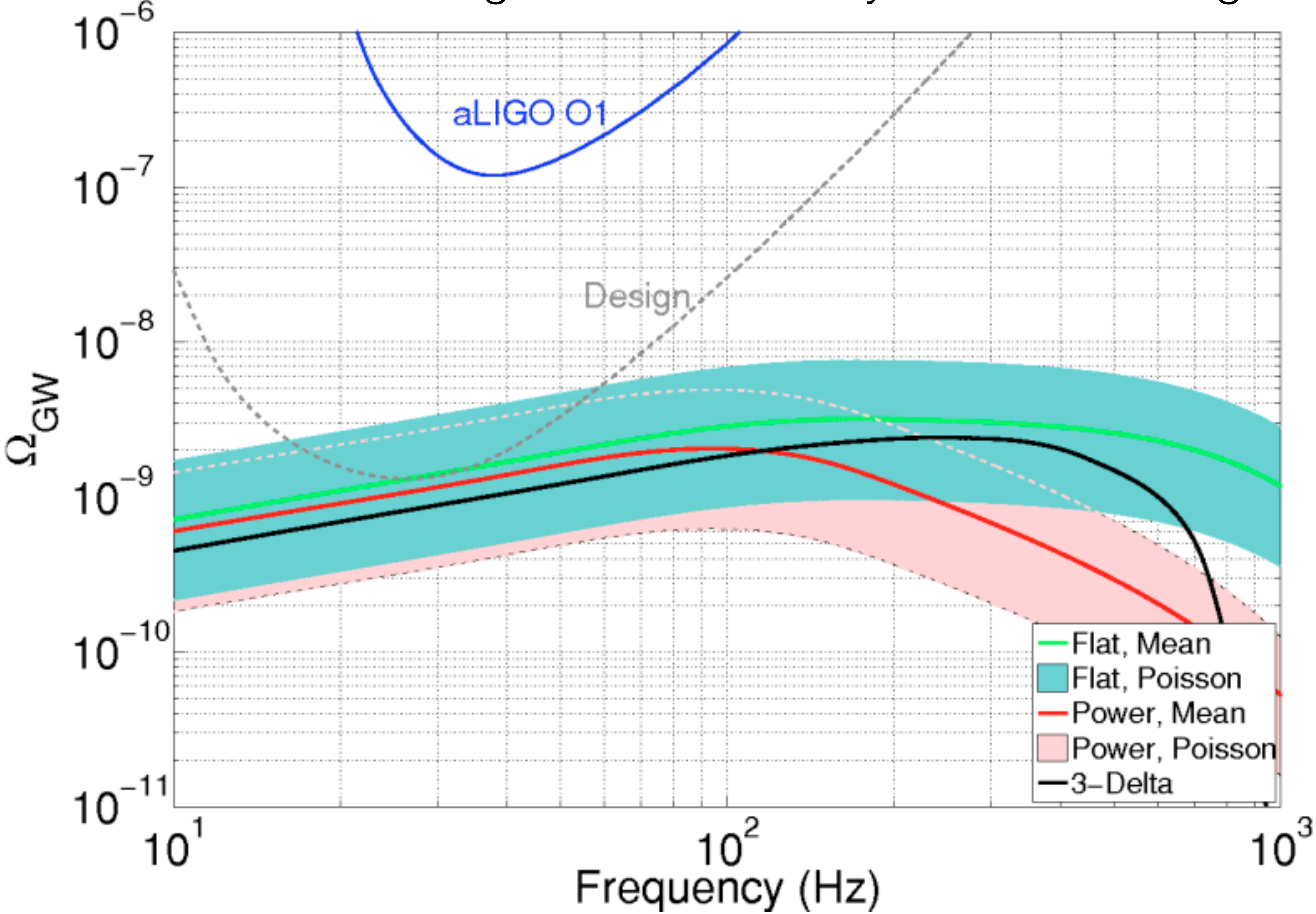


Inflation: measuring GWs can test for "stiffness" in early universe

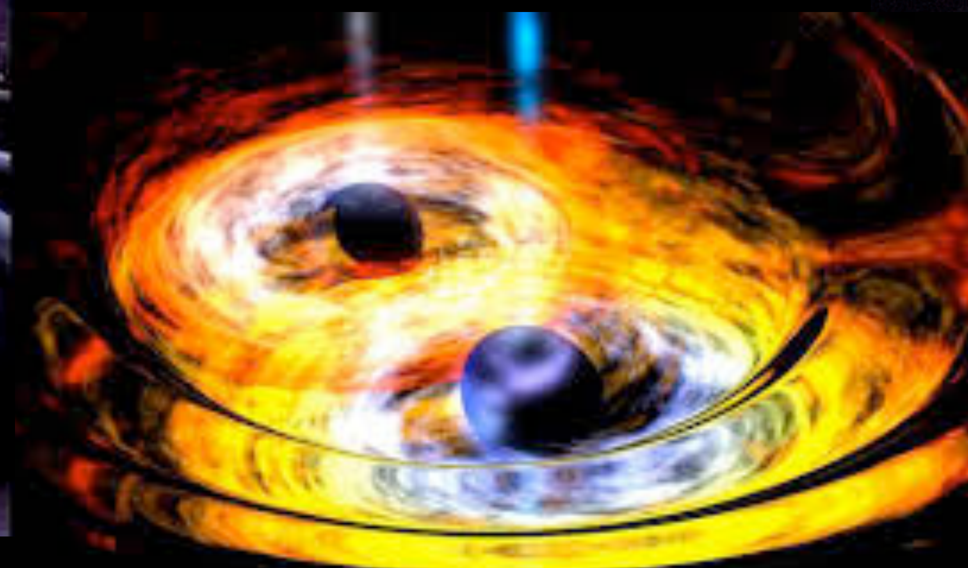
Models of Cosmic Strings: topological defects in early universe

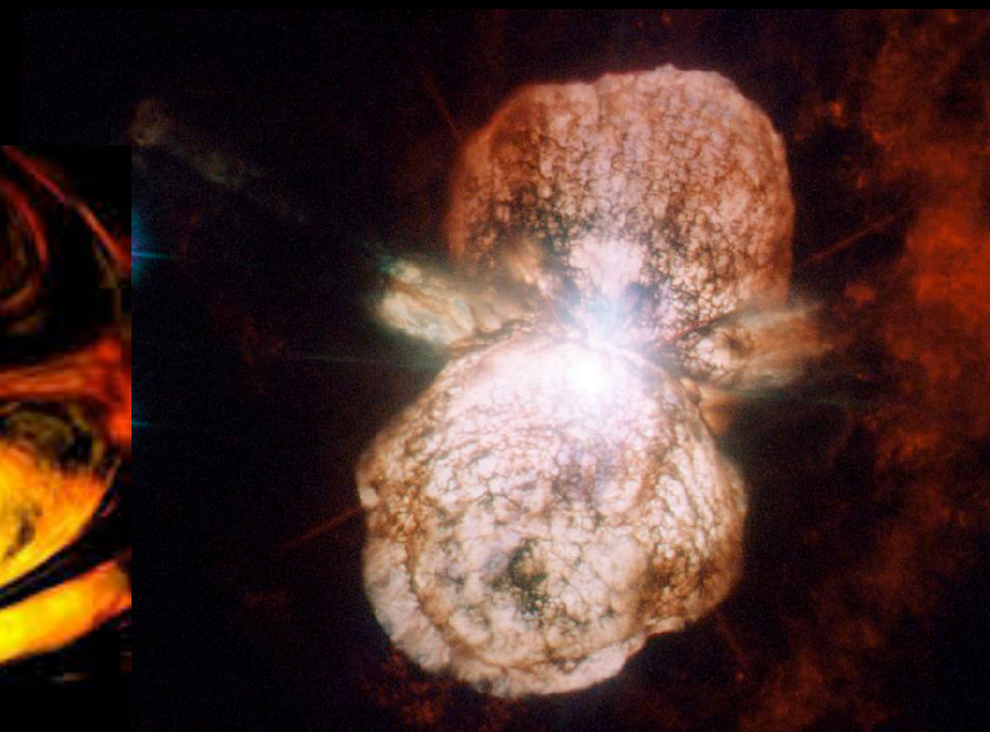
Astrophysical Gravitational Wave Backgrounds

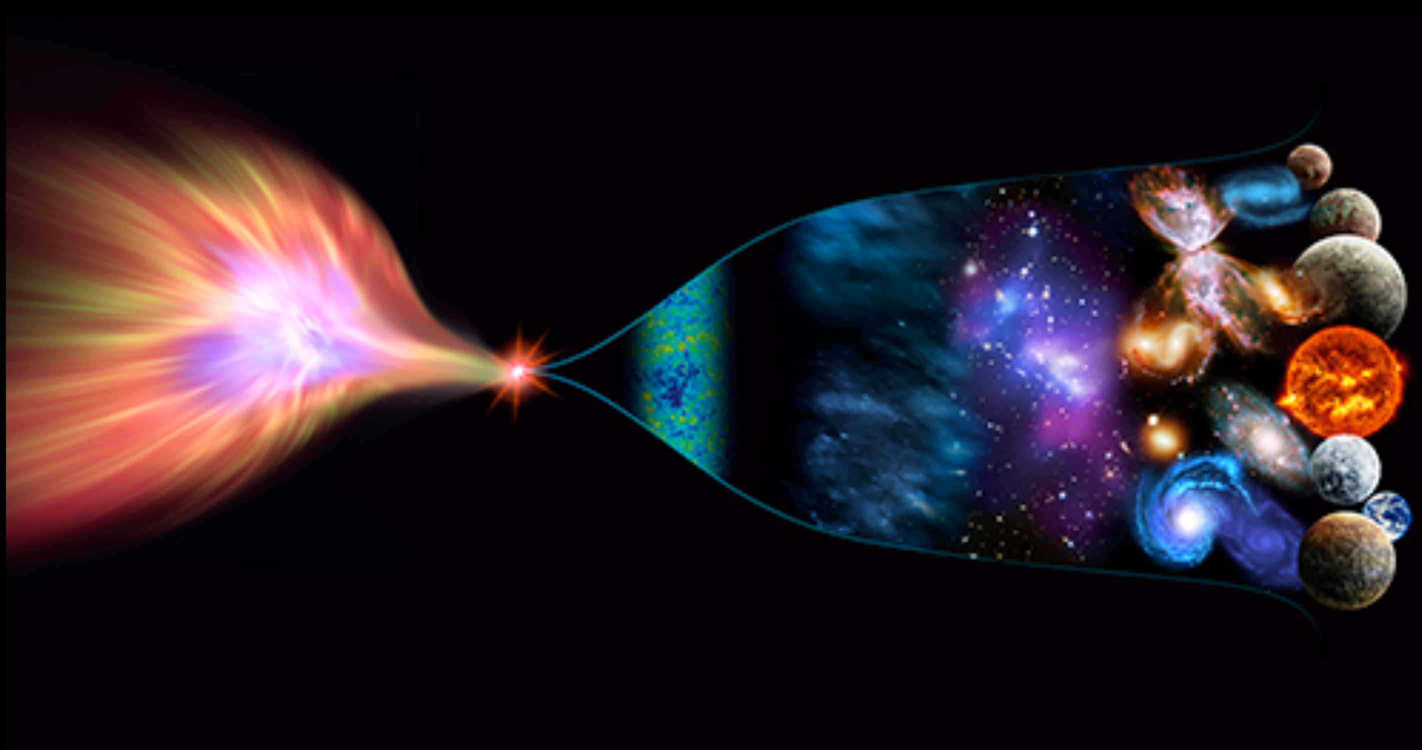
Potential background from binary black hole mergers

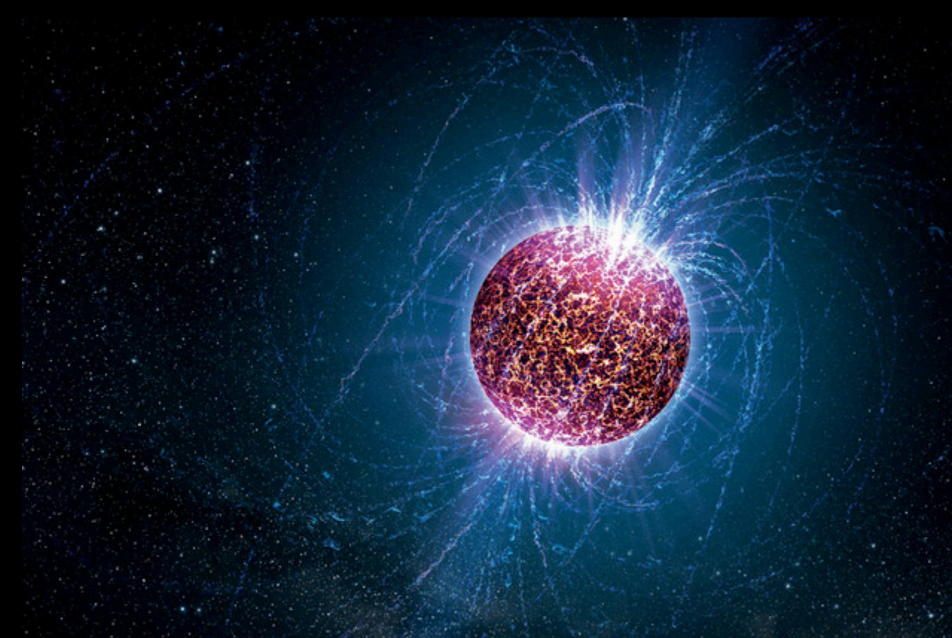


Frequencies of signals as audio





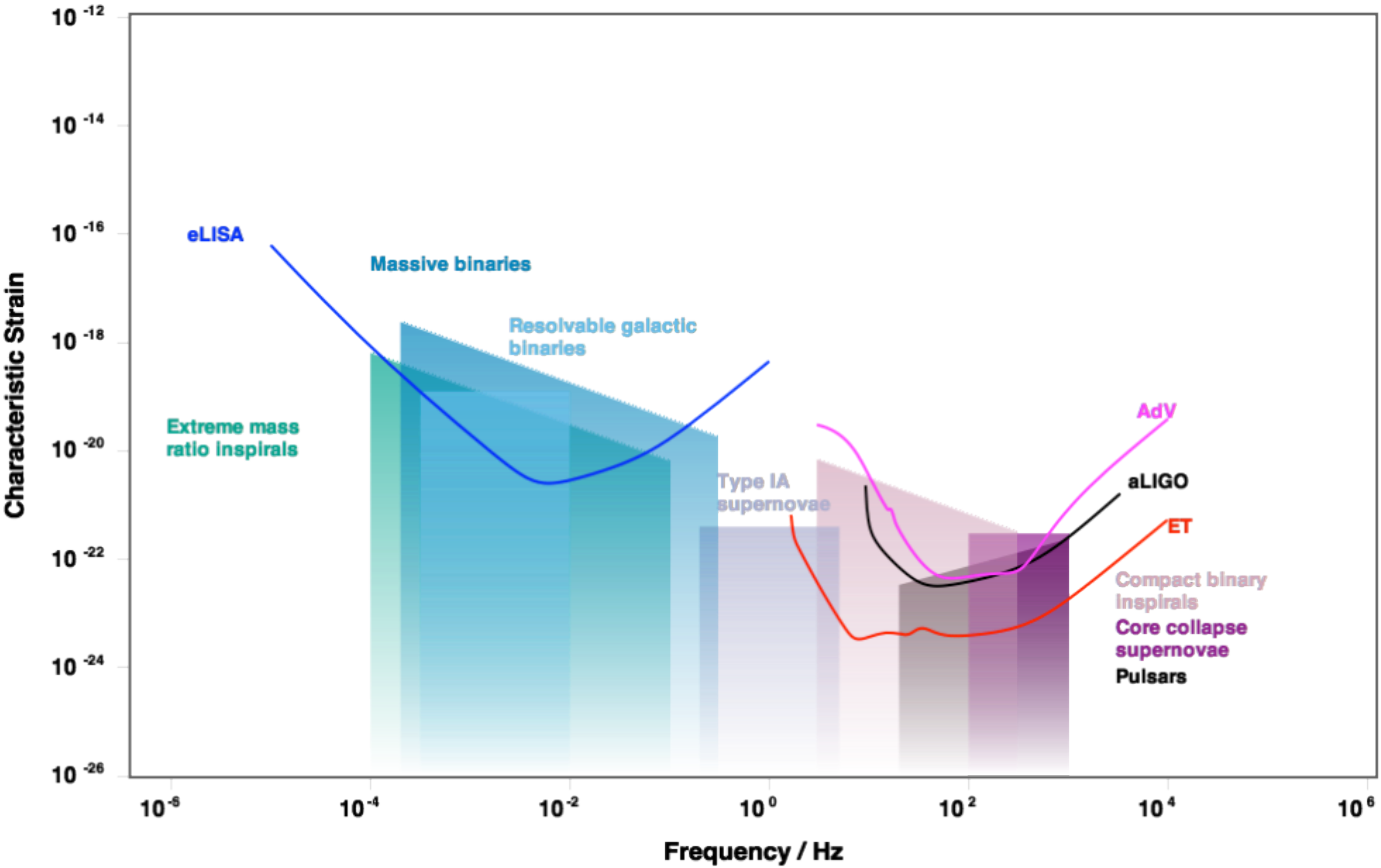




- Solving the Einstein Equations
  - Linearized Theory
  - Vacuum Solution
  - Solution with Source Term
- Generation of Gravitational Waves
- Effect of Gravitational Waves on Matter
- LIGO & Virgo Astrophysical Sources
  - Coalescing Binaries
  - Continuous Waves
  - Transient Bursts
  - Stochastic Background
- LISA & PTA Sources

LISA Sources

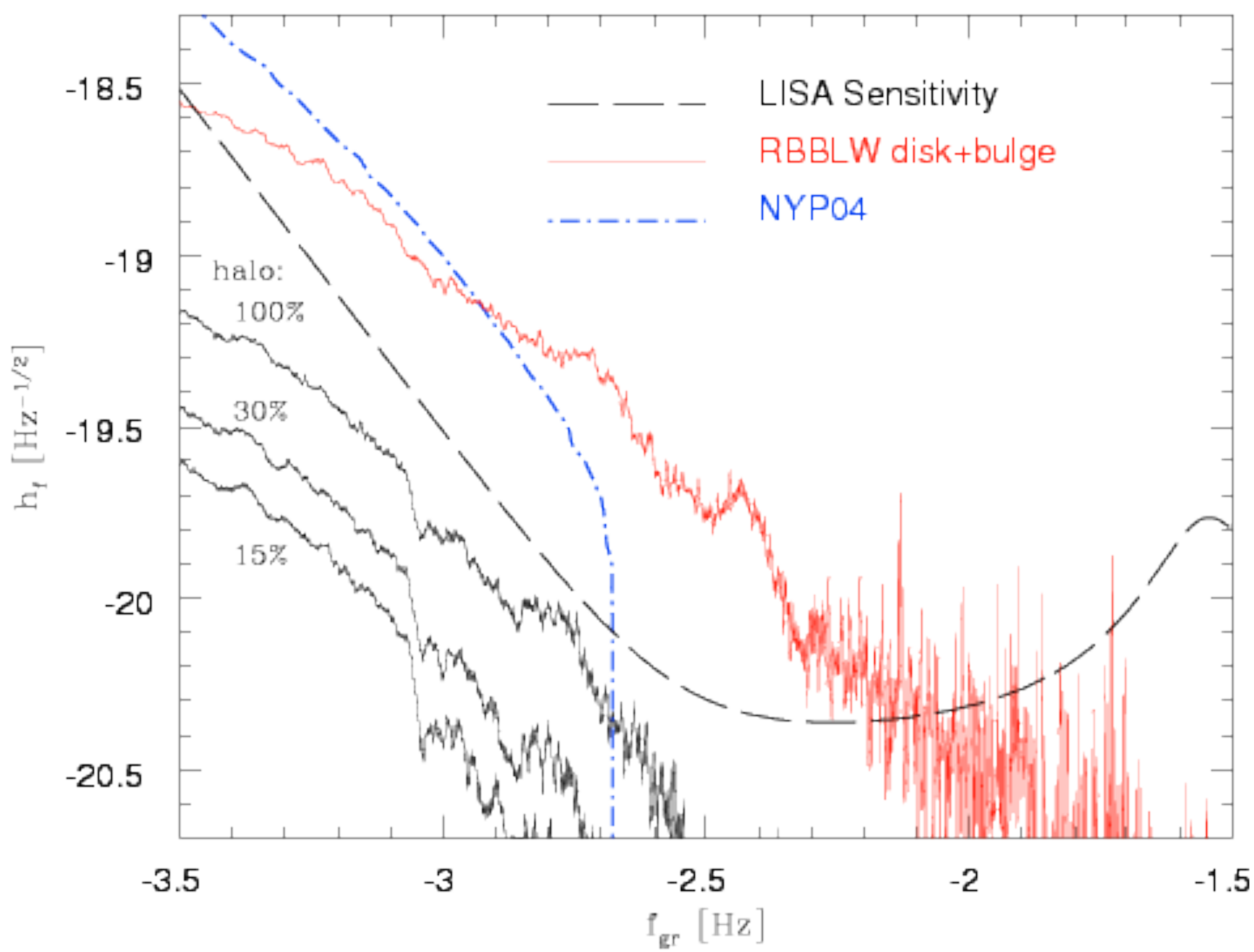
- Galactic white dwarfs
- Primordial backgrounds
- Supermassive binary black holes
- Capture orbits



Characteristic Strain
Frequency / Hz
10 -12
10 -14
10 -16
10 -18
10 -20
10 -22
10 -24
10 -26
10 -6
10 -4
10 -2
10 0
10 2
10 4
10 6
eLISA
Massive binaries
Resolvable galactic binaries
Extreme mass ratio inspirals
Type IA supernovae
AdV
aLIGO
ET
Compact binary inspirals
Core collapse supernovae
Pulsars

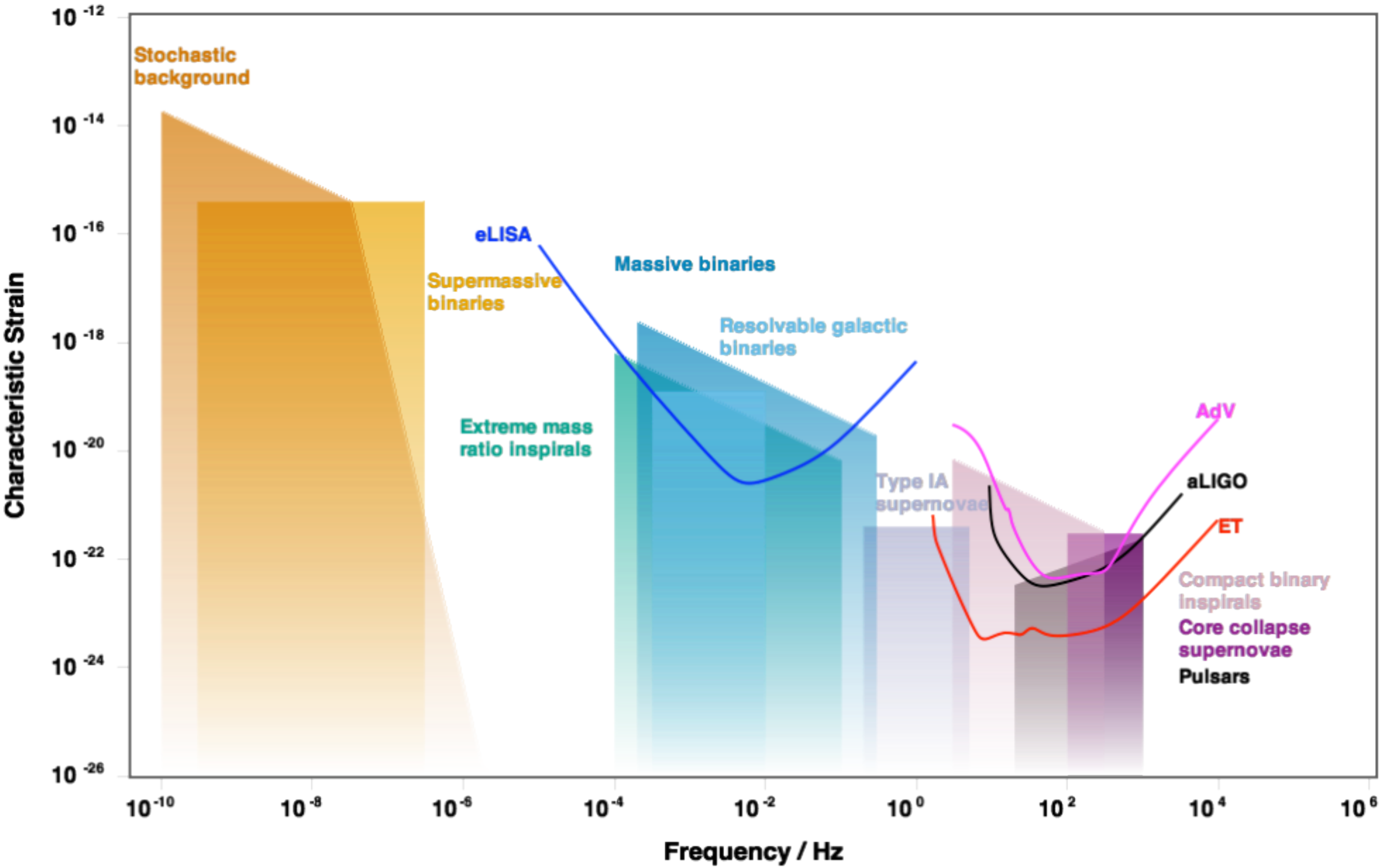
LISA Gravitational Wave Background

- Produced by an extremely large number of weak, independent, and unresolved gravitational-wave sources. For LISA, this will be white dwarf binaries.



Pulsar Timing Array Sources

- Also, supermassive binary black holes!



Stochastic background
Supermassive binaries
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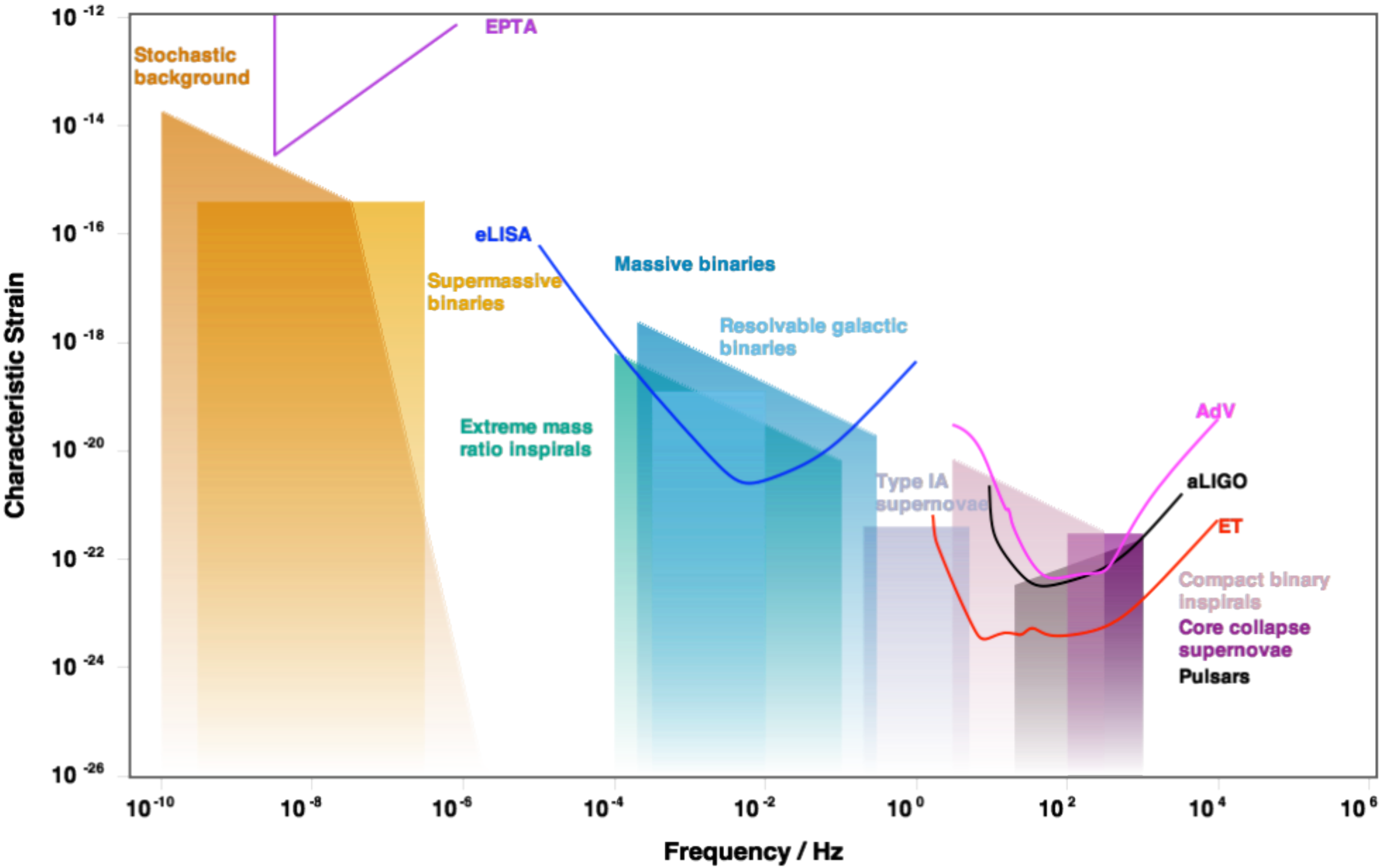

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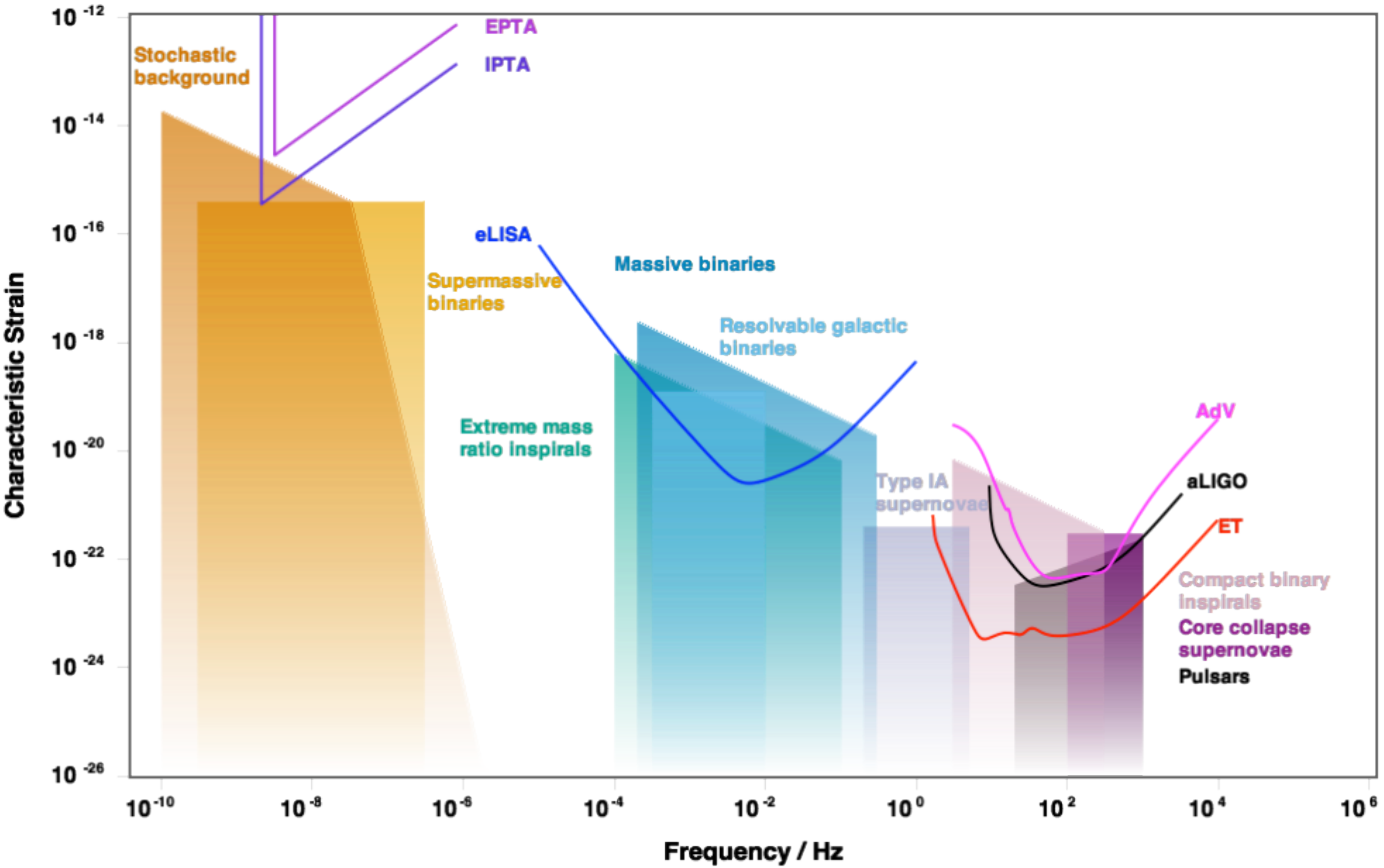
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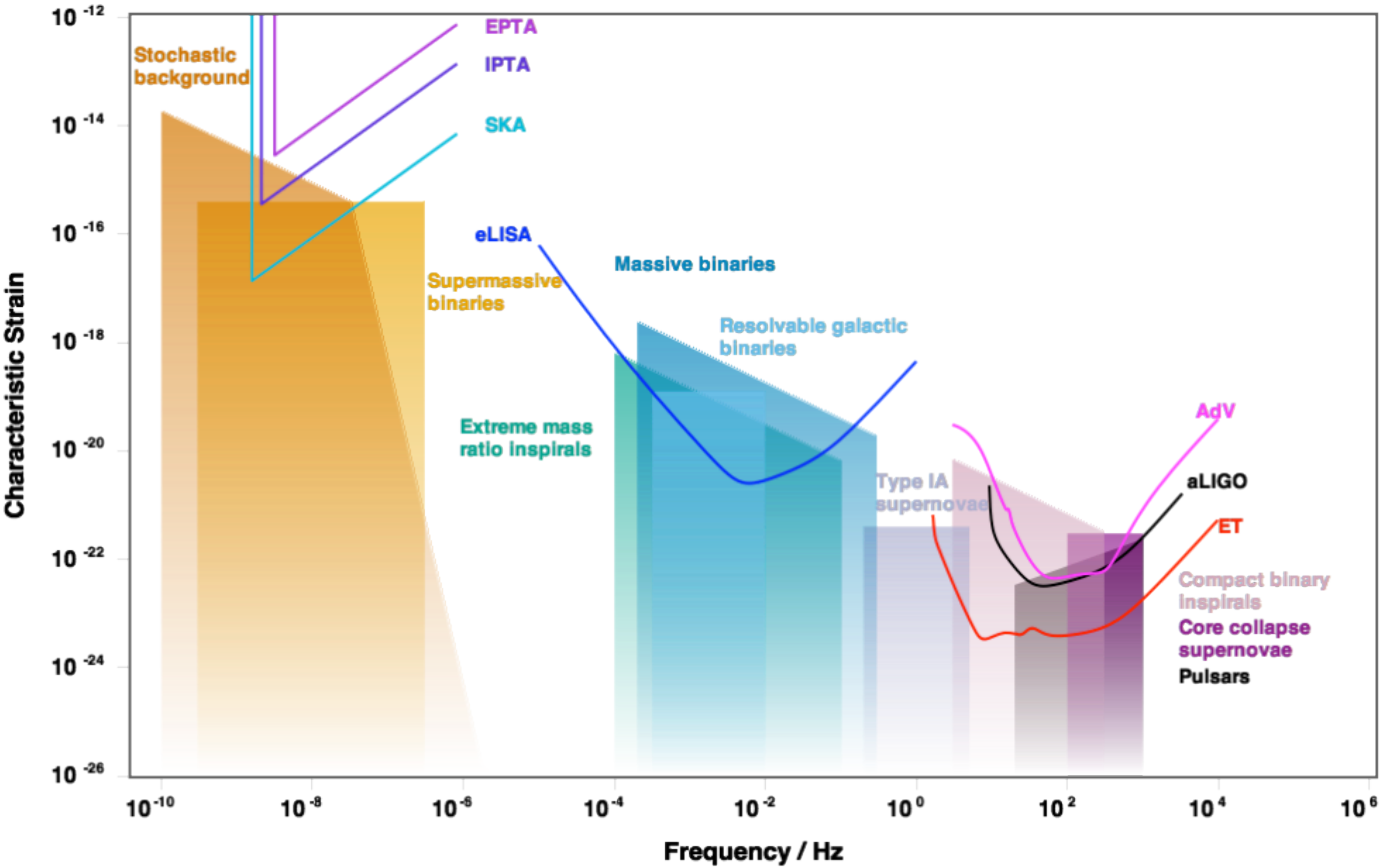
10^{4}
10^{6}



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Merging Supermassive Black Hole Binaries

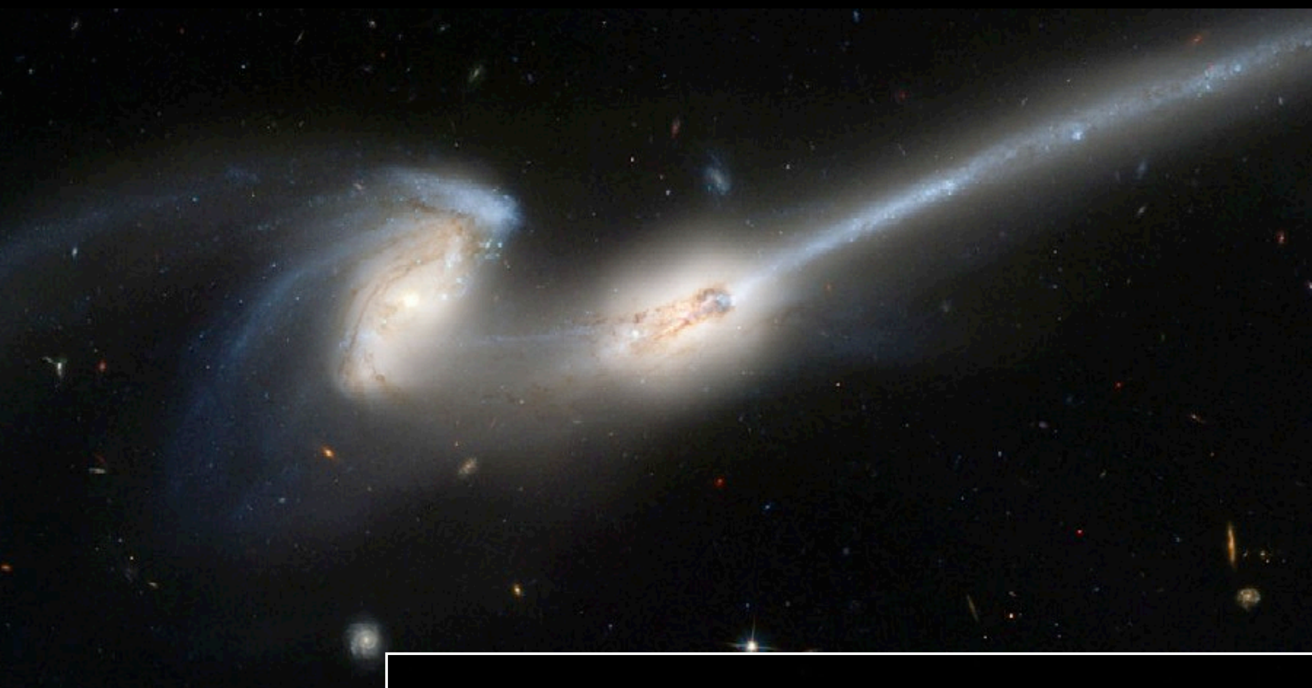

Image Credit: Debra Meloy Elmegreen (Vassar College) et al., & the Hubble Heritage Team (AURA/STScI/NASA)