Announcements

Assignment 4 due today.

Assignment 5 uploaded to website and Piazza. Will be due in two weeks, May 12th.

No class next week, May 5th.

Last lecture will be May 12th. Assignment 6 will be optional (see next slide).

We will meet again for Final Assessment on May 29.

HW Adjustments

Options for improving your grade:

1) On an assignment where you have done poorly, you can correct the problems, and receive half the points back.

AND/OR

2) You can turn in the optional assignment 6 and its grade will replace your lowest grade.

Gravitational Wave Data Analysis

Lecture 5: Gravitational Waves MSc Course

Crucial problem: How to extract a GW signal from the (typically) much larger detector noise?

Key components:

1. Characterizing detector noise
2. Understanding detectors' angular sensitivity
3. Applying filtering techniques

Output of GW detector

Signal Input of detector has the form $h(t) = D^{ij} h_{ij}(t)$

D^{ij} is the detector tensor; depends on detector geometry.

Signal Output of detector is related to input by

$$\tilde{h}_{\rm out} = \tilde{T}(f)\tilde{h}(f)$$

$T(f)$ is the transfer function.

However, output of any real detector will have noise, so output will really be given by

$$s_{\rm out}(t) = h_{\rm out}(t) + n_{\rm out}(t)$$

$$s(t) = h(t) + n(t)$$

Detector noise

To compare detector performances, we use the detectors' noise $n(t)$.

If noise is stationary, then the different Fourier components are uncorrelated. Then the ensemble average of the Fourier components of the noise is of the form:

$$\langle \tilde{n}^*(f)\tilde{n}(f')\rangle = \delta(f-f')\frac{1}{2}S_n(f)$$

Ensemble average can be replaced by time average to give the variance of the noise:

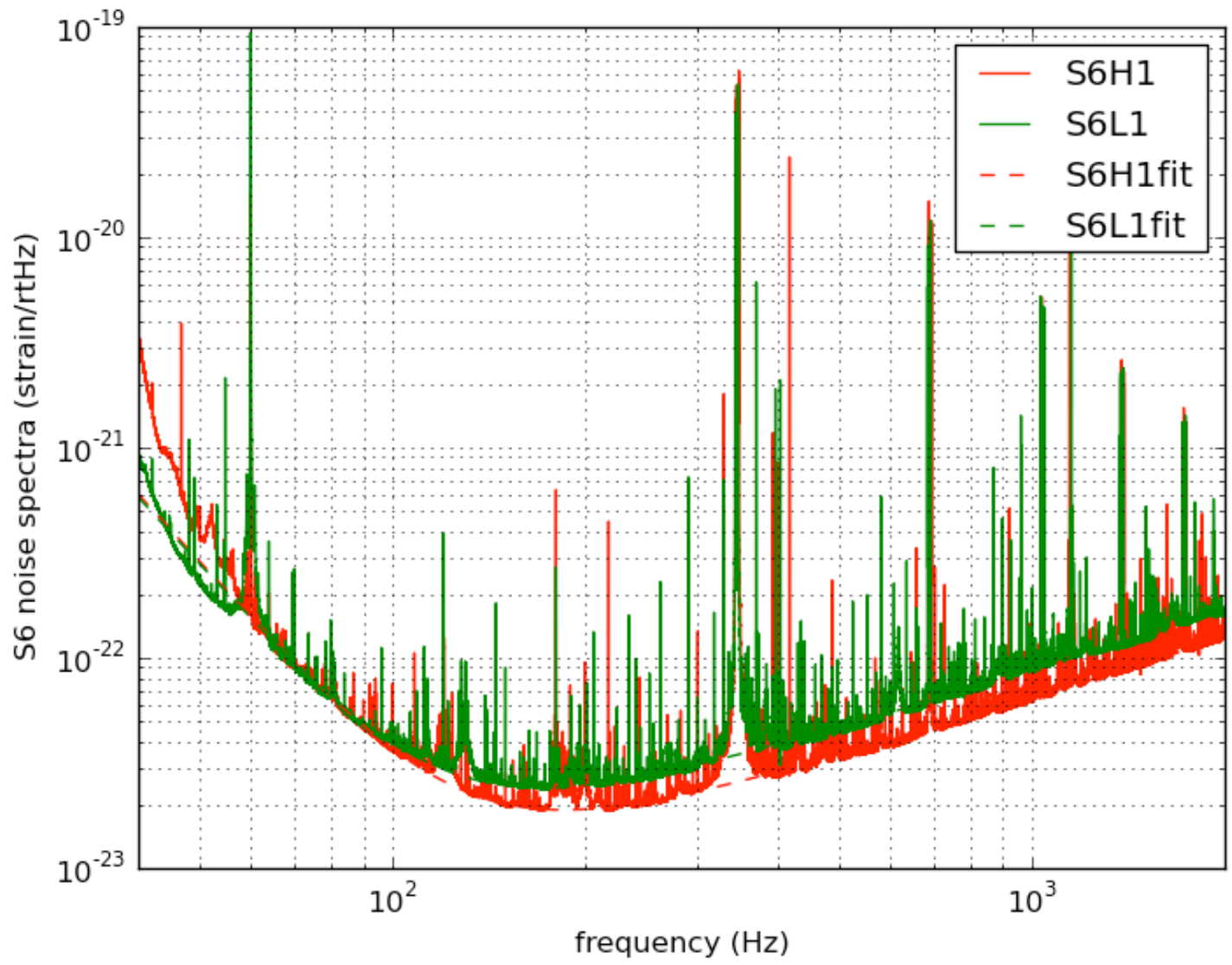
$$\left\langle |\tilde{n}(f)|^2 \right\rangle = \frac{1}{2}S_n(f)T \qquad \Delta f = \frac{1}{T}$$

T is time of experiment.

Noise spectral density

$S_n(f)$ is the noise spectral density (aka noise spectral sensitivity or noise power spectrum):

$$\langle n^2(t) \rangle = \int_0^\infty df\, S_n(f)$$



Detector Pattern Functions

Polarization tensors where $\hat{\mathbf{u}}, \hat{\mathbf{v}}$ are orthogonal to propagation direction

$$e^{+}_{ij}(\hat{\mathbf{n}}) = \hat{\mathbf{u}}_i \hat{\mathbf{u}}_j - \hat{\mathbf{v}}_i \hat{\mathbf{v}}_j$$
$$e^{\times}_{ij}(\hat{\mathbf{n}}) = \hat{\mathbf{u}}_i \hat{\mathbf{v}}_j + \hat{\mathbf{v}}_i \hat{\mathbf{u}}_j$$

In the frame where $\hat{n}$ is along the $\hat{z}$ direction, we can choose

$$\hat{u} = \hat{x} \qquad \hat{v} = \hat{y}$$

$$e^{+}_{ab} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}_{ab} \qquad e^{\times}_{ab} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}_{ab}$$

$a, b = 1, 2$ in the (x,y) plane

Detector Pattern Functions

GW with given propagation direction $\hat{n}$ is given by

$$h_{ij}(t, \mathbf{x}) = \sum_{A=+,\times} e_{ij}^{A}(\hat{n}) \int_{-\infty}^{\infty} df\, \tilde{h}_A(f) e^{-2\pi i f (t - \hat{\mathbf{n}}\cdot\mathbf{x}/c)}$$

$$h_{ij}(t) = \sum_{A=+,\times} e_{ij}^{A}(\hat{n}) \int_{-\infty}^{\infty} df\, \tilde{h}_A(f) e^{-2\pi i f t}$$

$$= \sum_{A=+,\times} e_{ij}^{A}(\hat{n}) h_A(t)$$

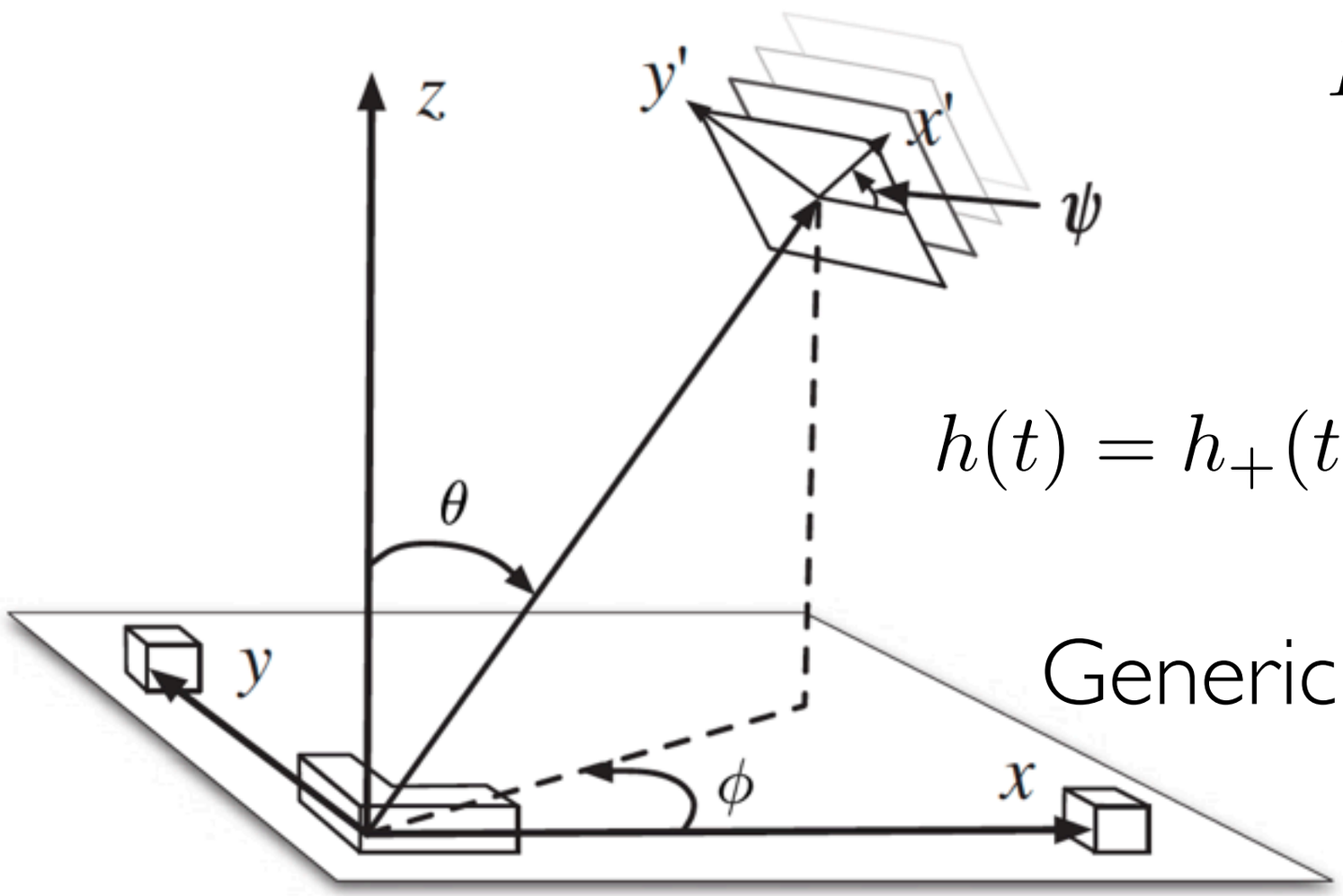
Fold in the detector tensor:

$$h(t) = D^{ij} h_{ij}(t)$$

$$h(t) = \sum_{A=+,\times} D^{ij} e_{ij}^{A}(\hat{n}) h_A(t)$$

Detector Pattern Functions

Detector pattern functions depend on the direction of propagation $\hat{n} = (\theta, \phi)$ of the wave.



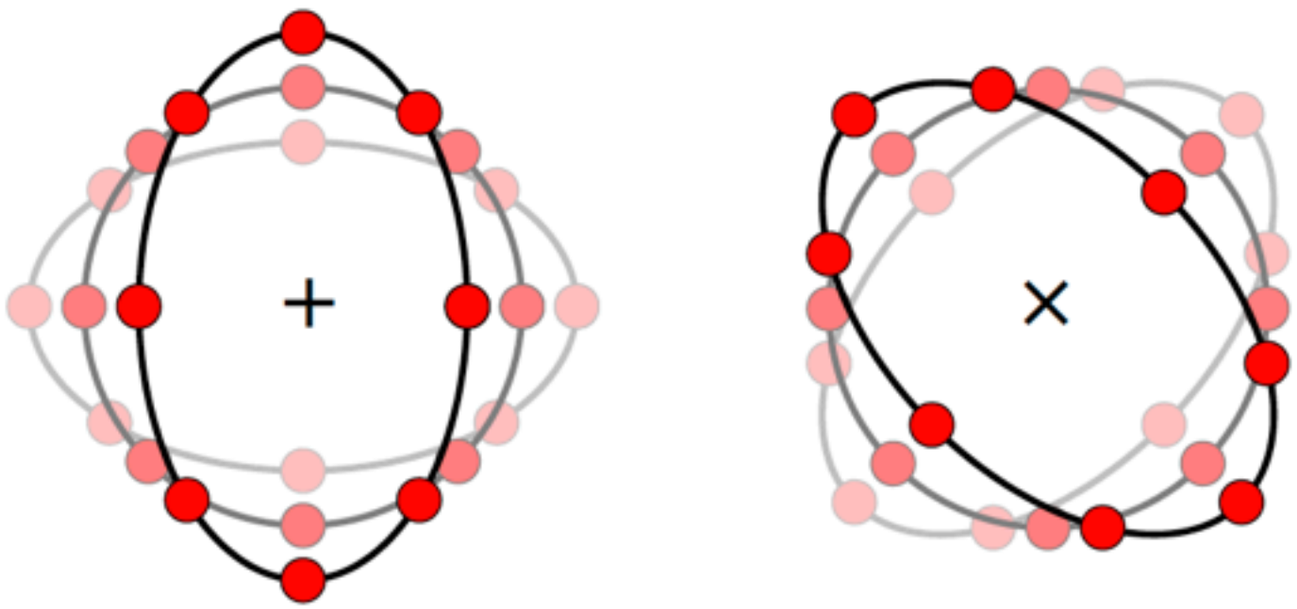
$$F_A(\hat{n}) = D^{ij} e^A_{ij}(\hat{n})$$

$$h(t) = h_+(t) F_+(\theta, \phi) + h_\times(t) F_\times(\theta, \phi)$$

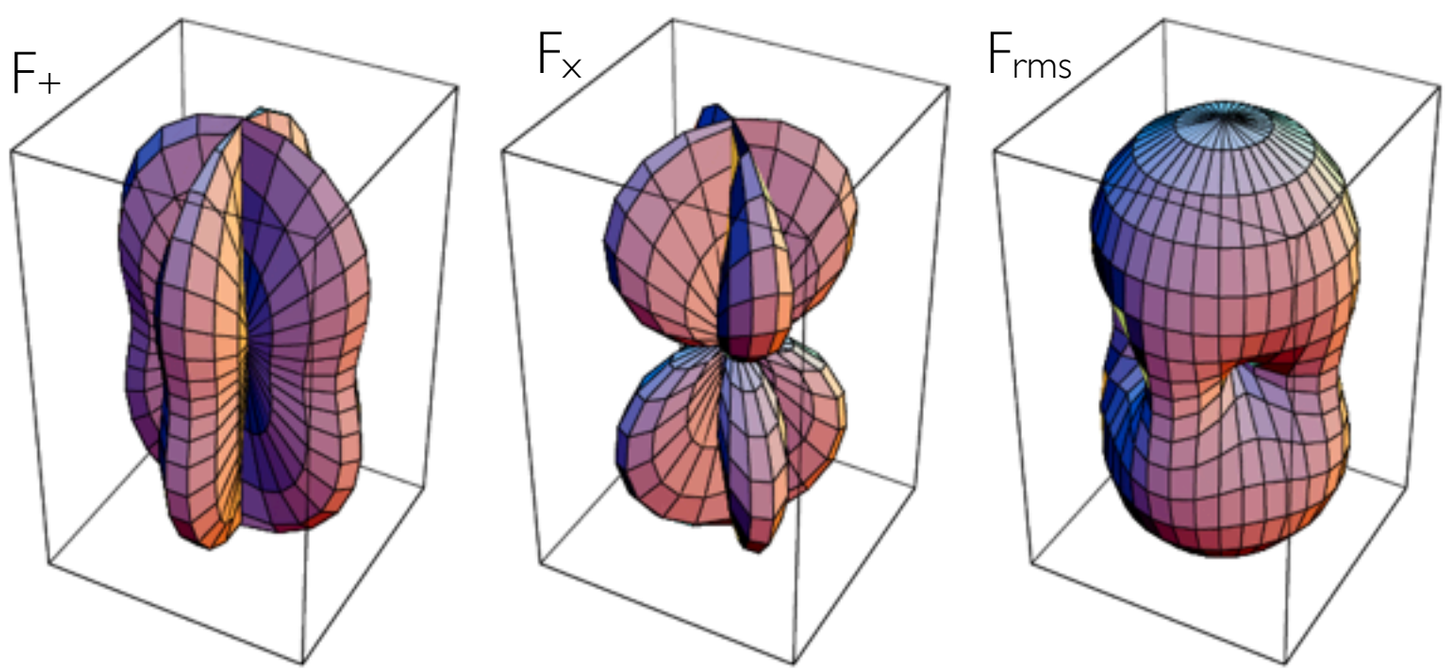
Generic case: $F_{+,\times}(\theta, \phi; \psi)$

Detector Pattern Functions

$$h(t) = h_+(t)F_+(\theta,\phi) + h_\times(t)F_\times(\theta,\phi)$$



GWs have two polarizations called + and x.



Credit: T. Fricke

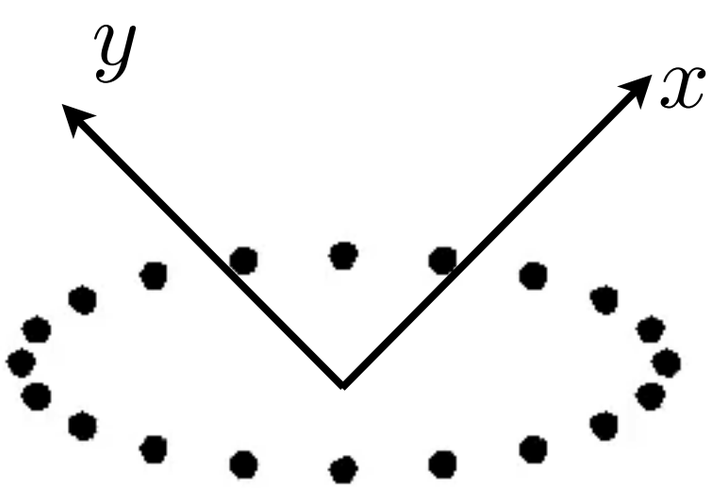
Detectors are not omnidirectional but exhibit an antenna pattern.

Interferometric GW Detector Pattern Functions

$$F_+(\theta, \phi; \psi = 0) = \frac{1}{2}\left(1 + \cos^2\theta\right)\cos 2\phi$$

$$F_\times(\theta, \phi; \psi = 0) = \cos\theta \sin 2\phi$$

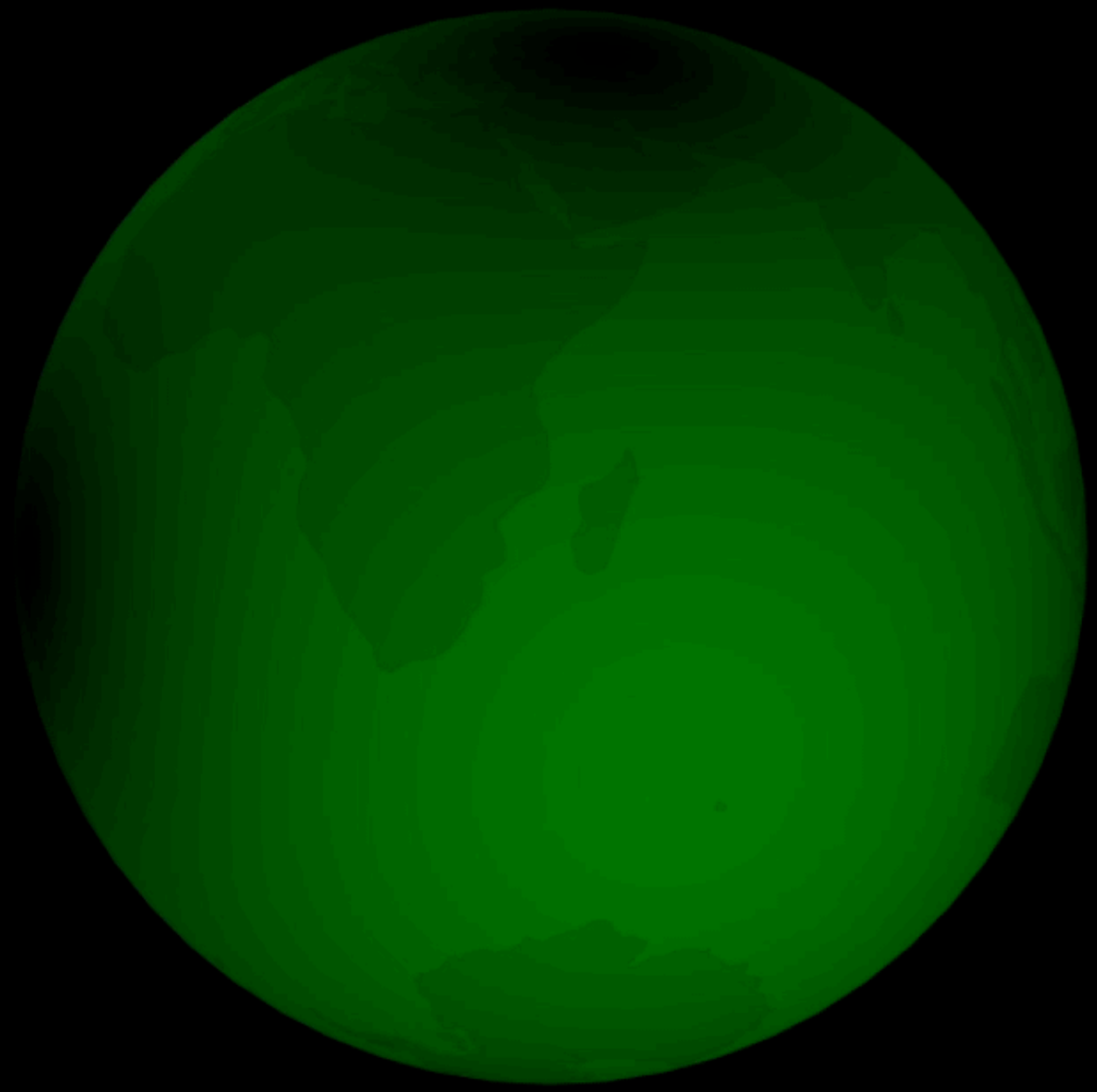
Thus GW interferometers have blind directions. For instance, for a GW with plus polarization,



$\phi = \pi/4$ and $F_+ = 0$

This wave produces the same displacement in the y and x arm.

Differential phase shift vanishes!



Finding signals

The detector output will be of the general form

$$s(t) = h(t) + n(t)$$

Naively, we can detect a GW signal only when $|h(t)|$ is larger than $|n(t)|$

But we will rather be in the situation

$$|h(t)| \ll |n(t)|$$

Fundamental question:

How can we dig out the GW signal from a much larger noise?

Answer: We can detect values of $h(t)$ much smaller than the floor of the noise if we know, at least to some level of accuracy, the form of $h(t)$.

$$s(t) = h(t) + n(t)$$

$$\frac{1}{T}\int_0^T dt\, s(t)\, h(t) = \frac{1}{T}\int_0^T dt\, h^2(t) + \frac{1}{T}\int_0^T dt\, n(t) h(t)$$

$$\frac{1}{T}\int_0^T dt\, h^2(t) \sim \underline{h_0^2} \qquad \frac{1}{T}\int_0^T dt\, n(t) h(t) \sim \underline{\left(\frac{\tau_0}{T}\right)^{1/2} n_0 h_0}$$

Fundamental question:

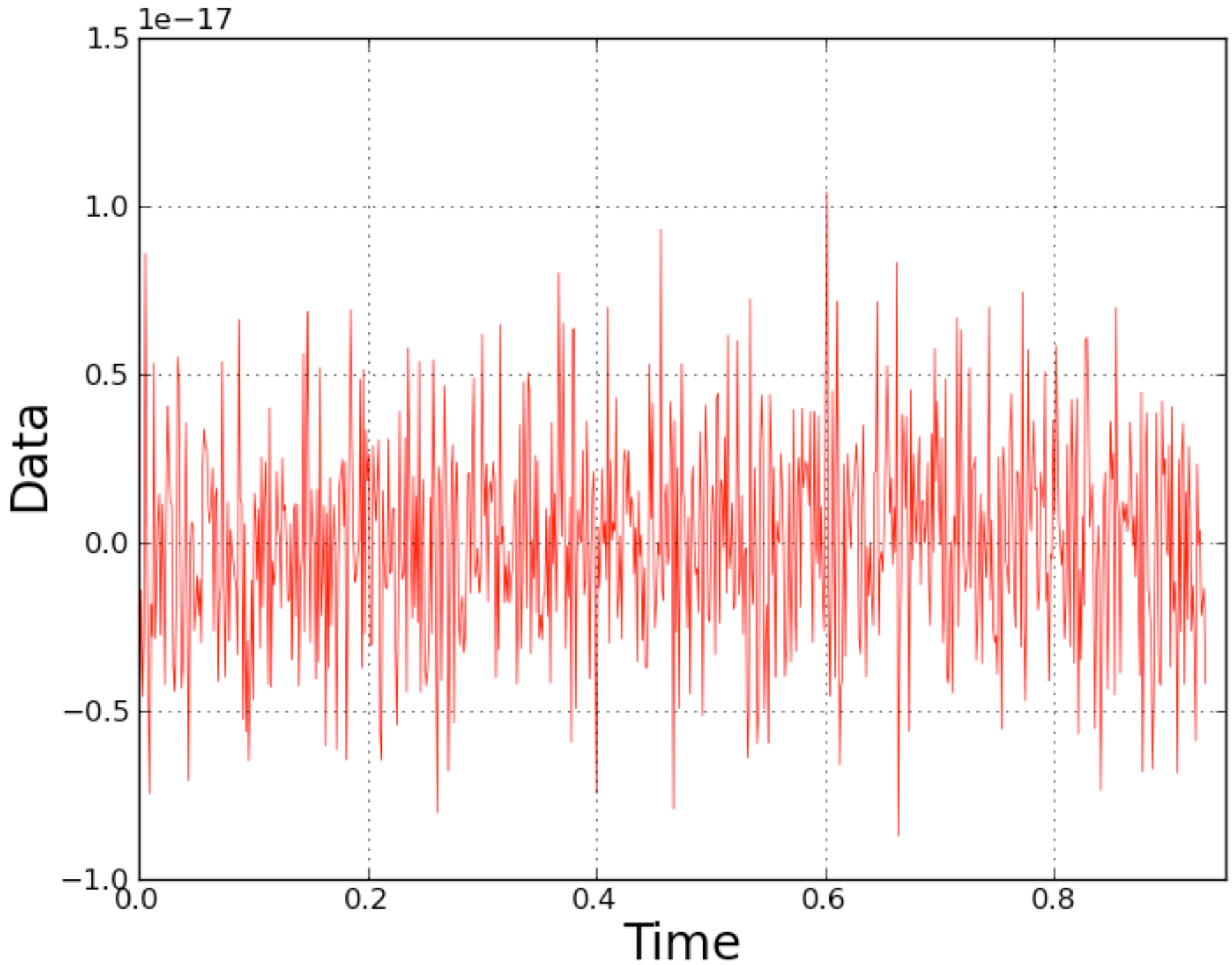
How can we dig out the GW signal from a much larger noise?

$$\frac{1}{T}\int_0^T dt\, n(t)h(t) \sim \left(\frac{\tau_0}{T}\right)^{1/2} n_0 h_0$$

Goes to zero for large T!

Thus, it is not necessary to have $h_0 > n_0$.

It is sufficient to have $h_0 > (\tau_0/T)^{1/2} n_0$.


1e−17
1.5
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Data
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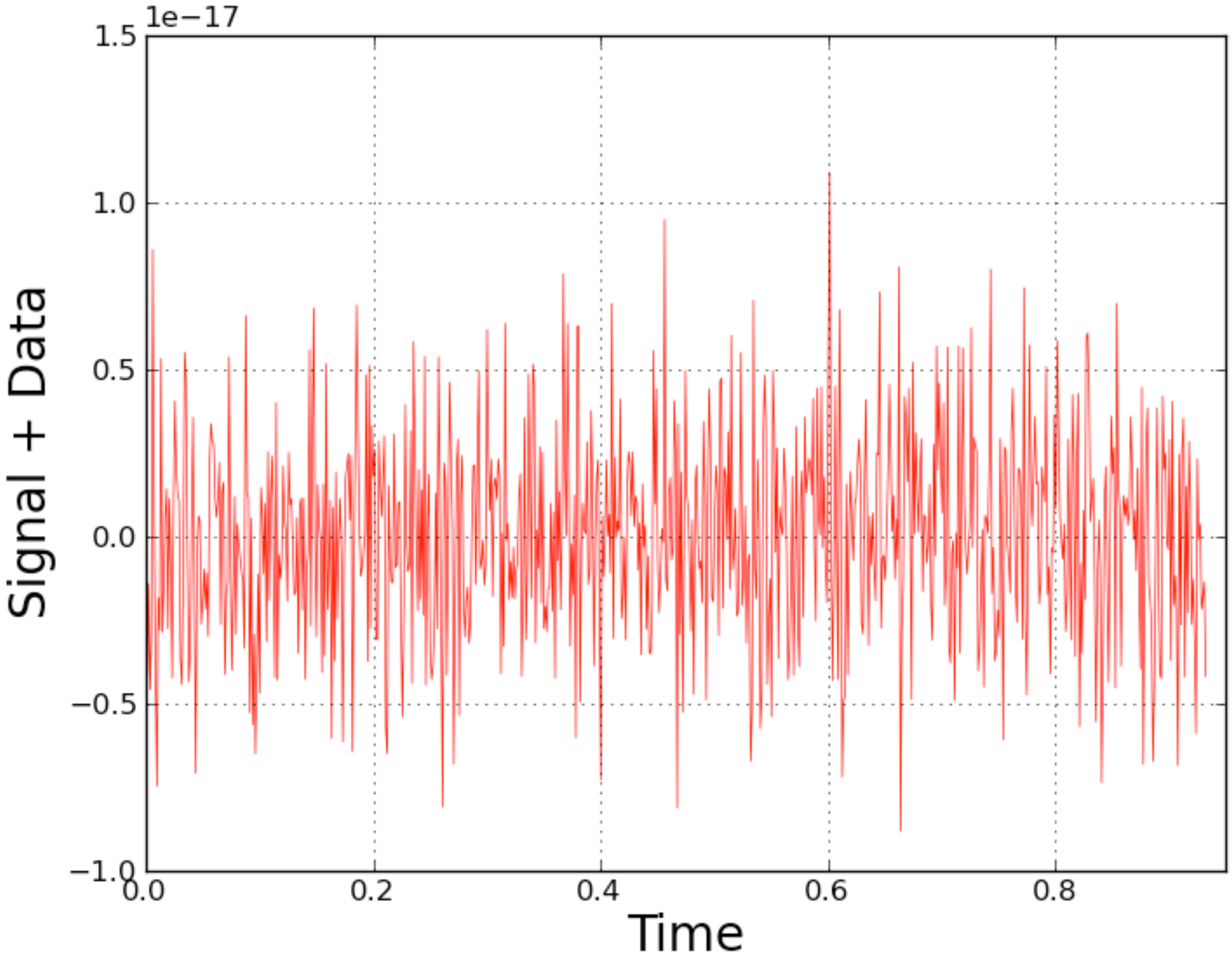
0.8
Time


1e−17
1.5
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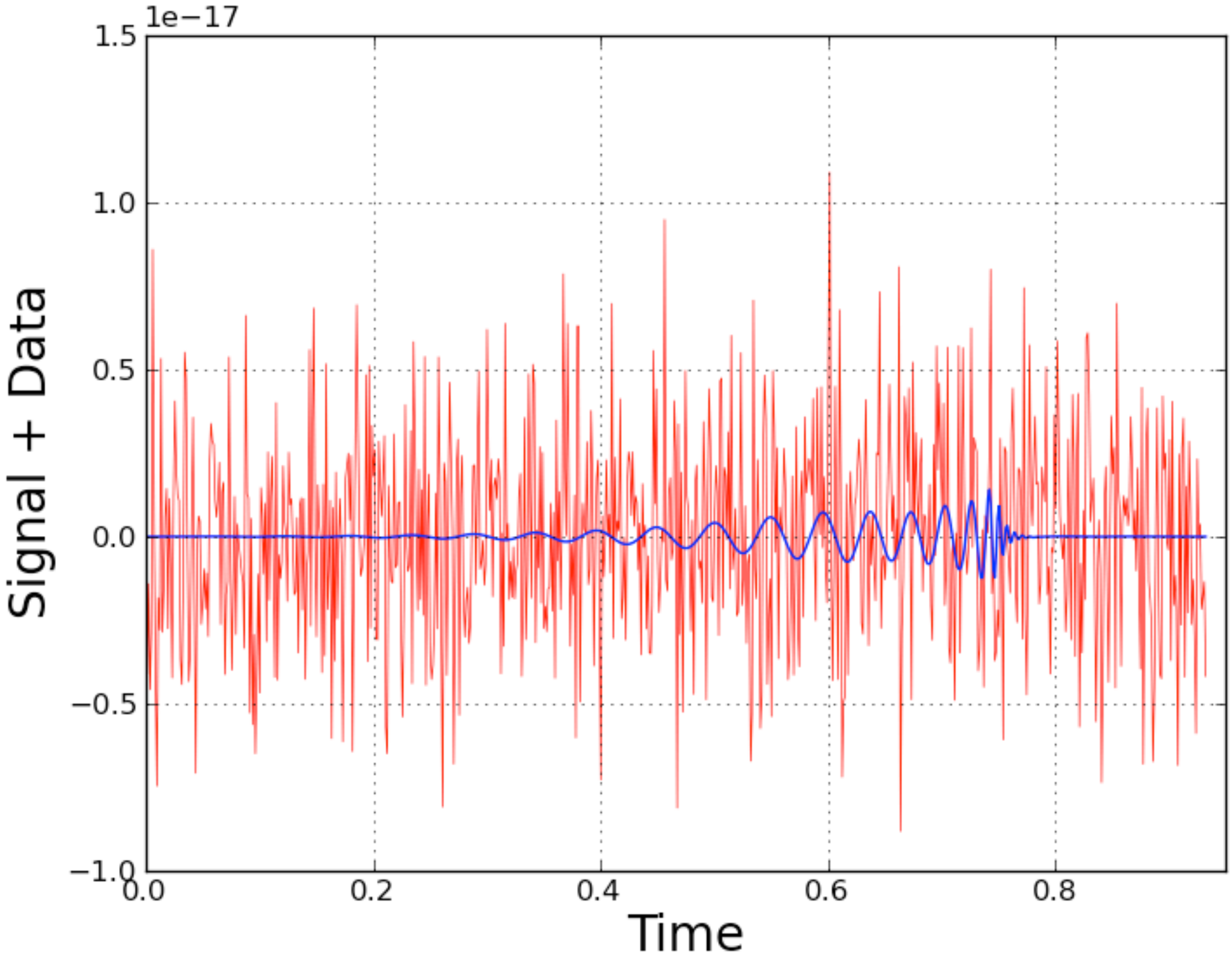
Signal + Data
Time


1e−17
1.5
1.0
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−1.0
Signal + Data
0.0
0.2
0.4
0.6
0.8
Time

Let's turn the previous procedure into a method we can use...

Goal: Obtain the highest possible signal-to-noise ratio (SNR).

$$\hat{s} = \int_{-\infty}^{\infty} dt\, s(t) K(t)$$

Consider $K(t)$ as a generic filter function.

If we know the form of $h(t)$, what is the filter function $K(t)$ that maximizes the SNR?

Aka matched filtering

Define the signal-to-noise ratio...

$$\langle s(t) \rangle = \langle n(t) \rangle + \langle h(t) \rangle$$

$$\langle n(t) \rangle = 0 \;\; \text{(if noise is stationary)}$$

Signal: expected value if signal is present

$$S = \int_{-\infty}^{\infty} dt \, \langle s(t) \rangle \, K(t) = \int_{-\infty}^{\infty} dt \, h(t) K(t) = \int_{-\infty}^{\infty} df \, \tilde{h}(f) \tilde{K}^*(f)$$

Noise: root-mean-square value if no signal present

$$N^2 = \left[\left\langle \hat{s}^2(t) \right\rangle - \cancelto{0}{\langle \hat{s}(t) \rangle^2} \right]_{\mathrm{h}=0} = \int_{-\infty}^{\infty} dt \, dt' \, K(t) K(t') \, \langle n(t) n(t') \rangle$$

$$= \int_{-\infty}^{\infty} df \left(\frac{1}{2} \right) S_n(f) \left| \tilde{K}(f) \right|^2$$

Define the signal-to-noise ratio...

$$\frac{S}{N} = \frac{\int_{-\infty}^{\infty} df \tilde{h}(f) \tilde{K}^*(f)}{\left[\int_{-\infty}^{\infty} df \left(\frac{1}{2}\right) S_n(f) \left|\tilde{K}(f)\right|^2\right]^{1/2}}$$

Convenient definition: noise-weighted scalar product between two real functions $A(t)$ and $B(t)$

$$(A|B) = \mathrm{Re} \int_{-\infty}^{\infty} df \frac{\tilde{A}^*(f)\tilde{B}(f)}{(1/2)S_n(f)}$$

$$= 4\mathrm{Re} \int_{0}^{\infty} df \frac{\tilde{A}^*(f)\tilde{B}(f)}{S_n(f)}$$

Define the signal-to-noise ratio...

Using this scalar product definition, we have:

$$\frac{S}{N} = \frac{(u|h)}{(u|u)^{1/2}} \quad \text{where} \quad \tilde{u}(f) = \frac{1}{2} S_n(f) \tilde{K}(f)$$

We are searching for vector $u/(u|u)^{1/2}$ such that its scalar product with vector $\boldsymbol{h}$ is maximum.

They should be parallel (i.e. proportional): $\boxed{\tilde{K}(f) = \mathrm{const.} \frac{\tilde{h}(f)}{S_n(f)}}$

This is the **Wiener filter** (aka matched filter).

Define the signal-to-noise ratio...

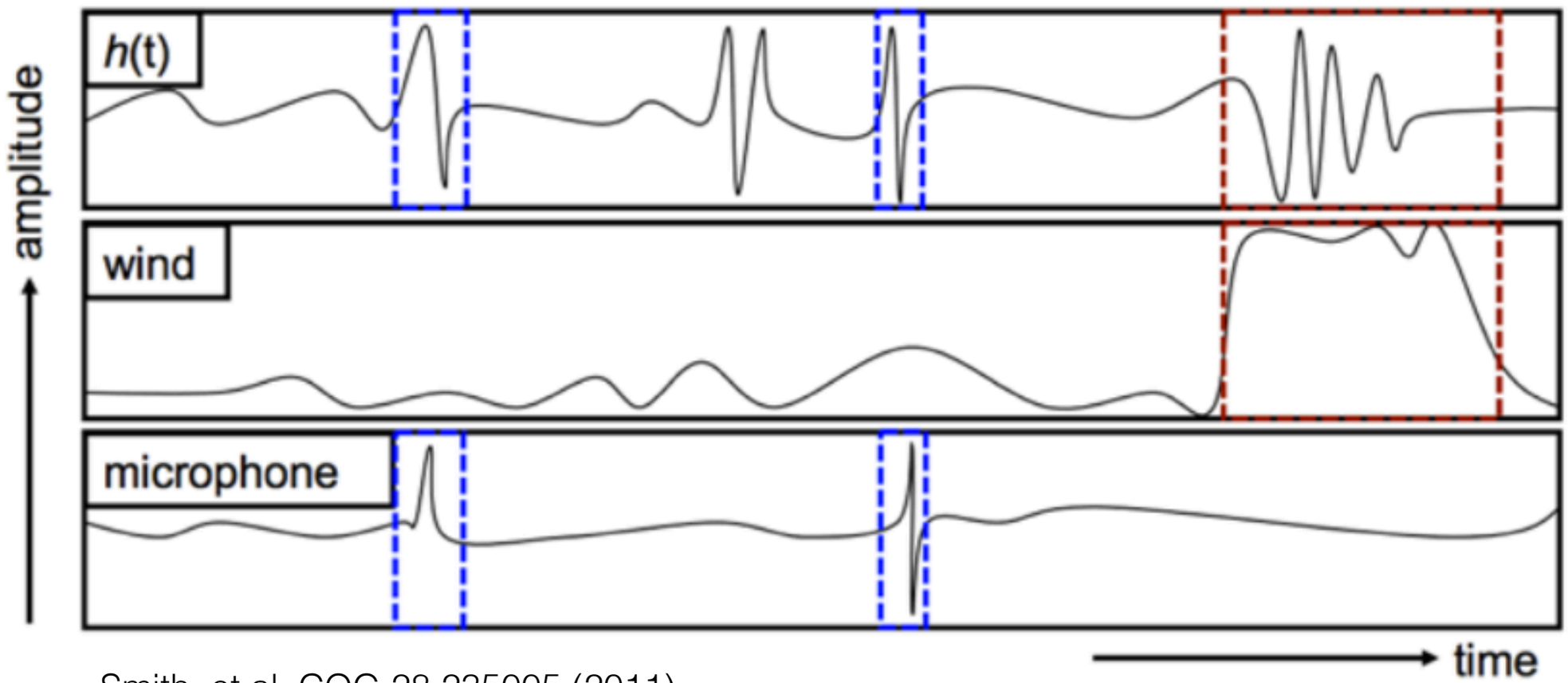
Optimal value of signal-to-noise ratio:

$$\left(\frac{S}{N}\right) = (h|h)^{1/2}$$

$$\boxed{\left(\frac{S}{N}\right)^2 = 4\int_0^\infty df\,\frac{|\tilde{h}(f)|^2}{S_n(f)}}$$

Completely generic and independent of form of $\tilde{h}(f)$

Data Quality Vetoes



Smith, et al. CQG 28 235005 (2011)

If a particularly noisy period of data is identified, and if it is correlated with a known instrument problem, then the data may be removed.

Coincidence Window

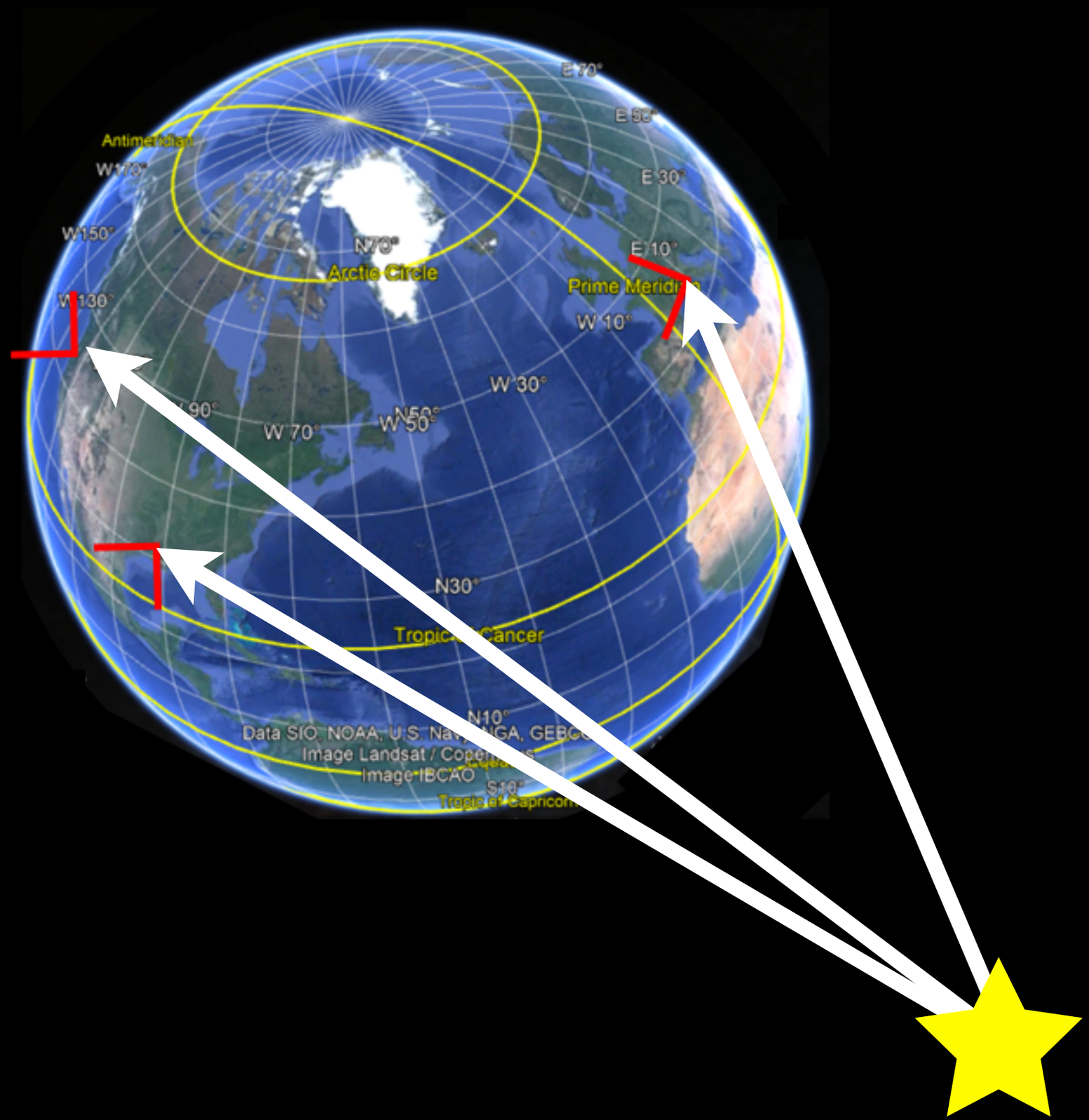
σ_1^2, σ_2^2 - variances on arrival times in two detectors.

Then, variance on coincident search is $\sigma_{1,2} = \left(\sigma_1^2 + \sigma_2^2\right)^{\frac{1}{2}}$

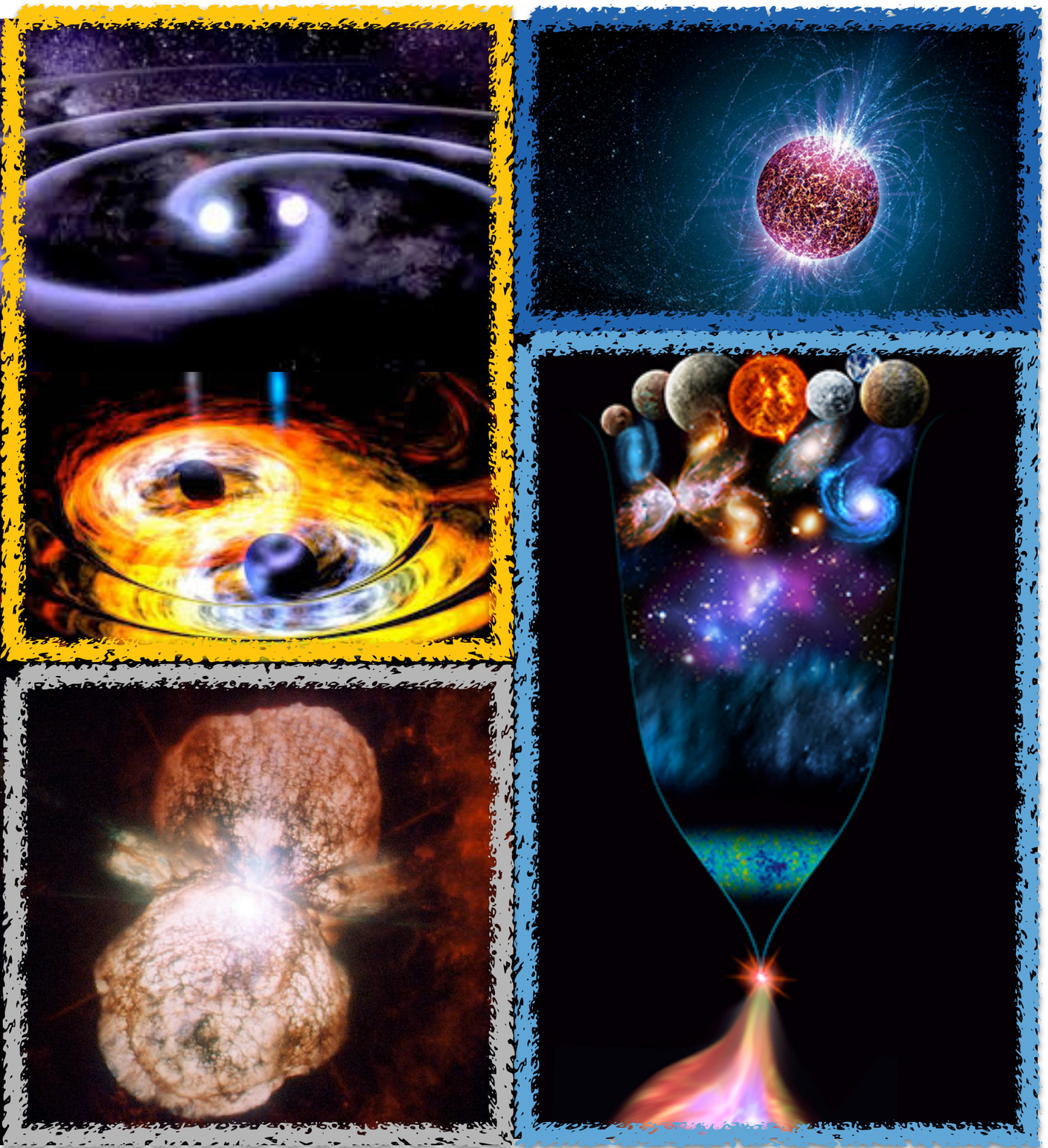
Light travel time between detectors plus k standard deviations.

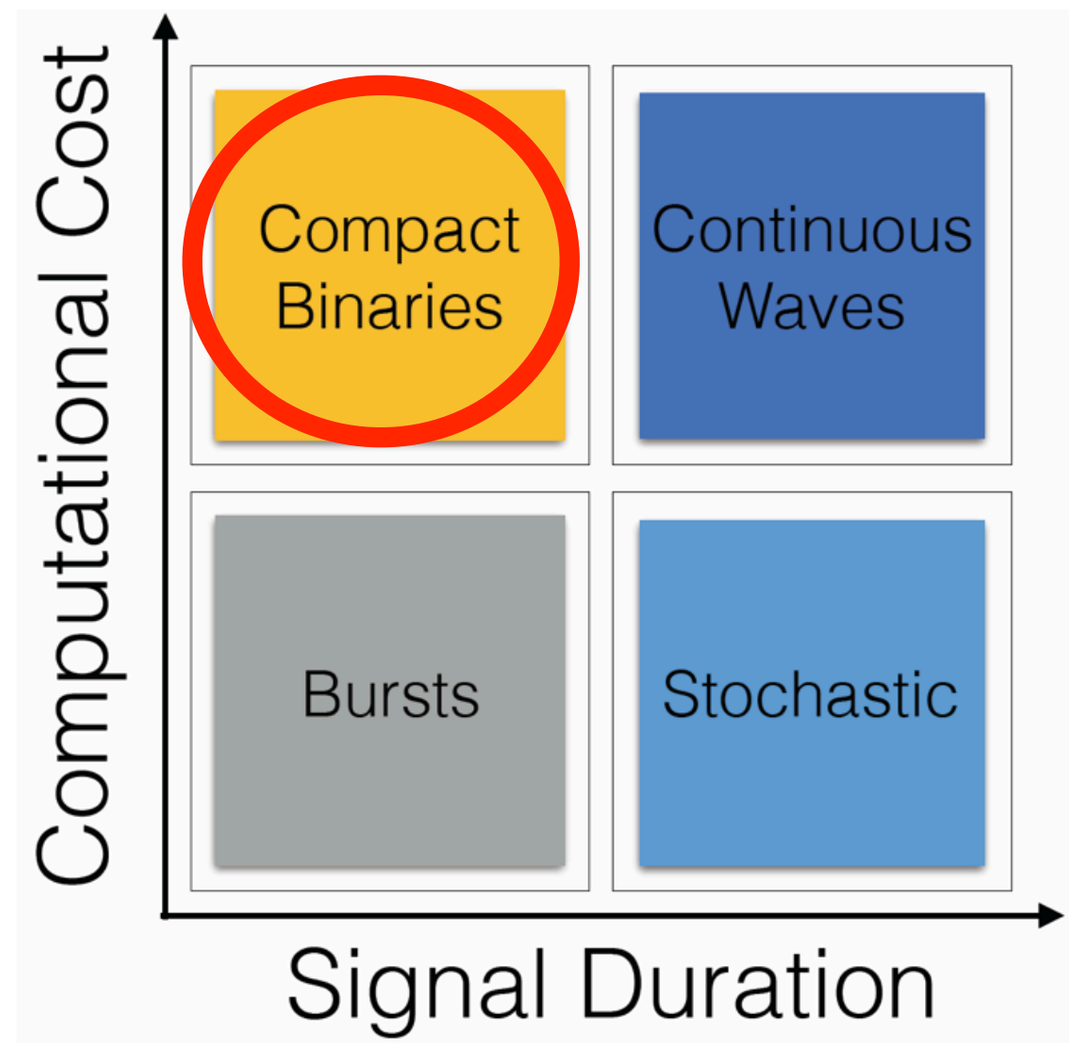
$$|t_1 - t_2| \leq (\Delta t)_{\text{light}} + k\left(\sigma_1^2 + \sigma_2^2\right)^{1/2}$$

Coincidence must be within a few tens of milliseconds.


E 70°
E 50°
E 30°
E 10°
W 10°
W 30°
W 50°
W 70°
W 90°
W 110°
W 130°
W 150°
W 170°
N70°
Arctic Circle
Prime Meridian
N30°
Tropic of Cancer
N10°
Image Landsat / Copernicus
Image IBCAO

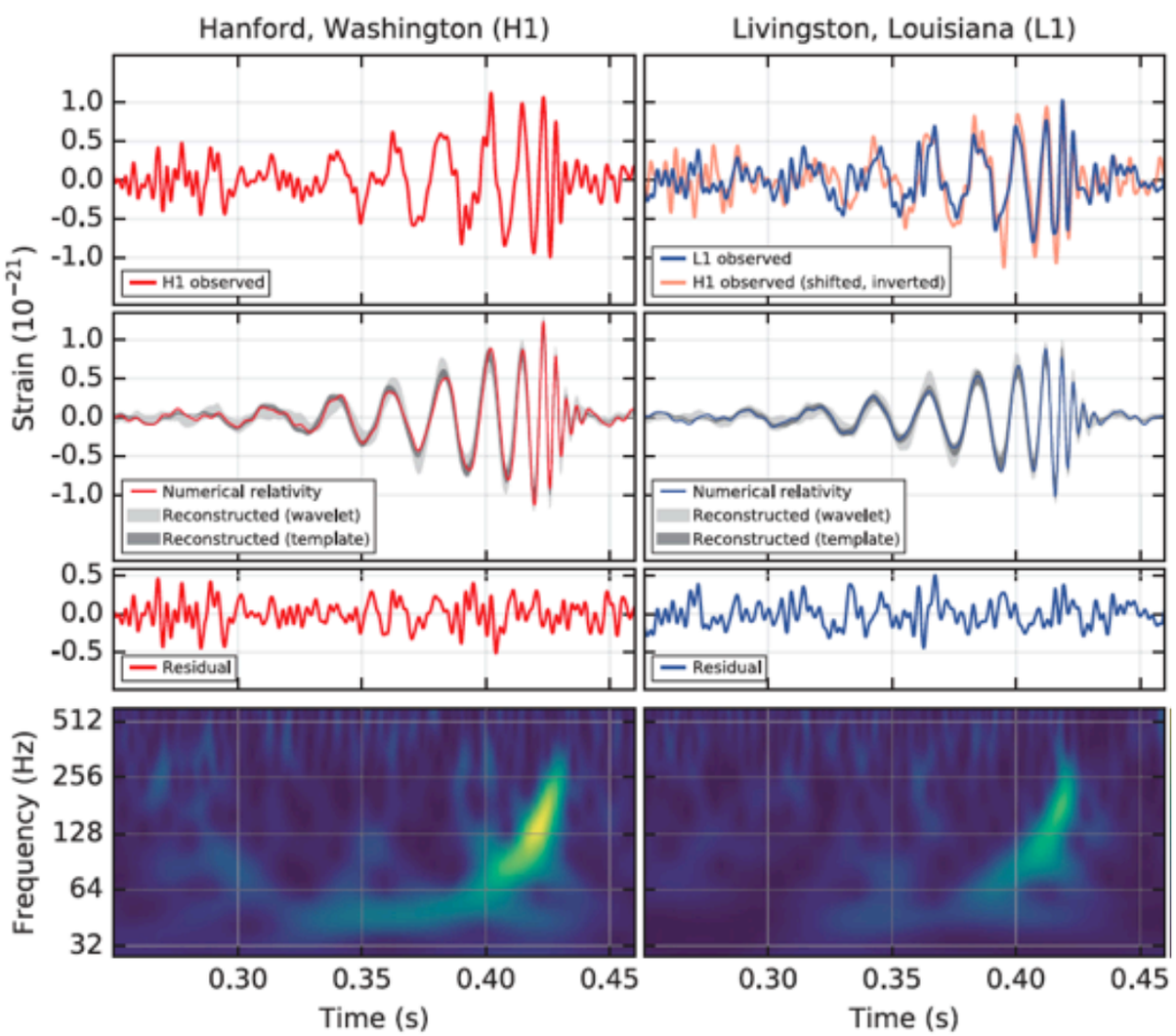
Gravitational-wave Sources



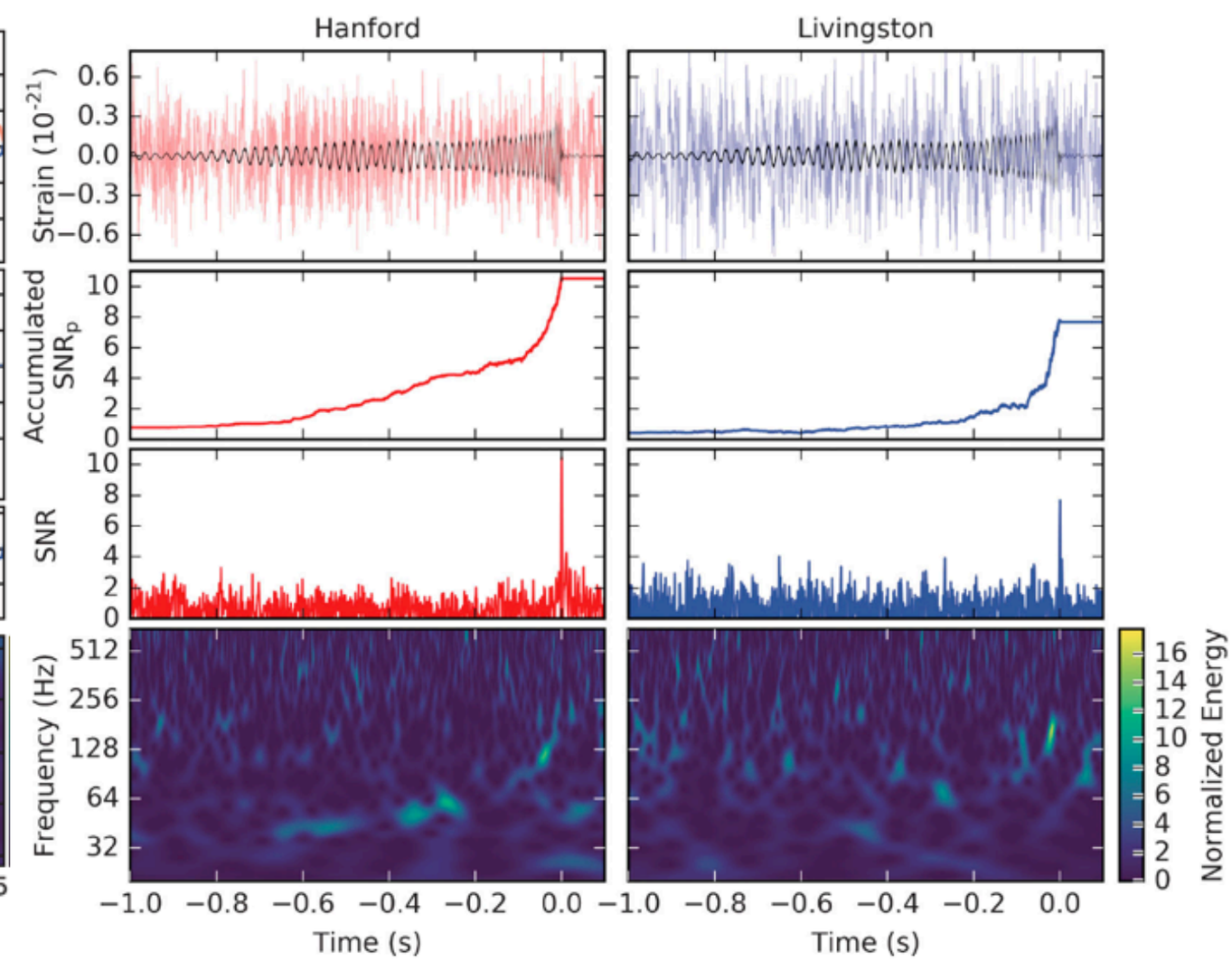


Do we really need a matched filter search?

GW150914



GW151226



Formal Definition of Matched Filter

Cross-correlate data with template, weighted by detector noise:

$$Z(t) = A \int_{\infty}^{\infty} \frac{\tilde{s}(f)\tilde{h}^*(f)e^{2\pi i f t}}{S_n(f)} df$$

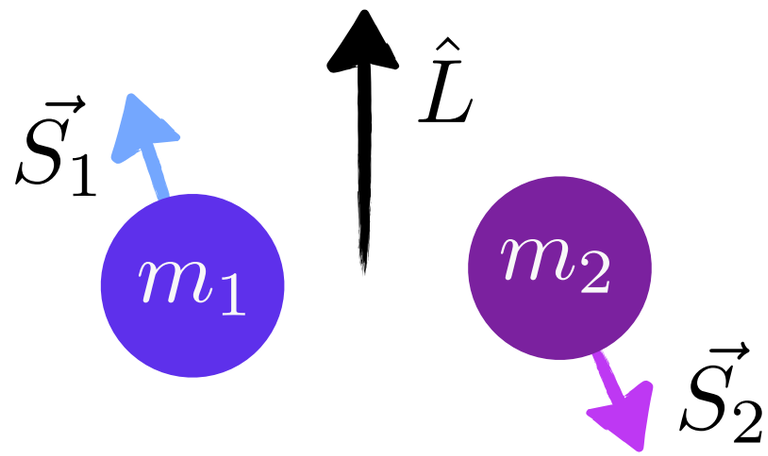
Cross-correlate template with template, weighted by detector noise:

$$\sigma^2 = 2 \int_{0}^{\infty} \frac{\tilde{h}(f)\tilde{h}^*(f)}{S_n(f)} df$$

Normalize output of optimal filter:

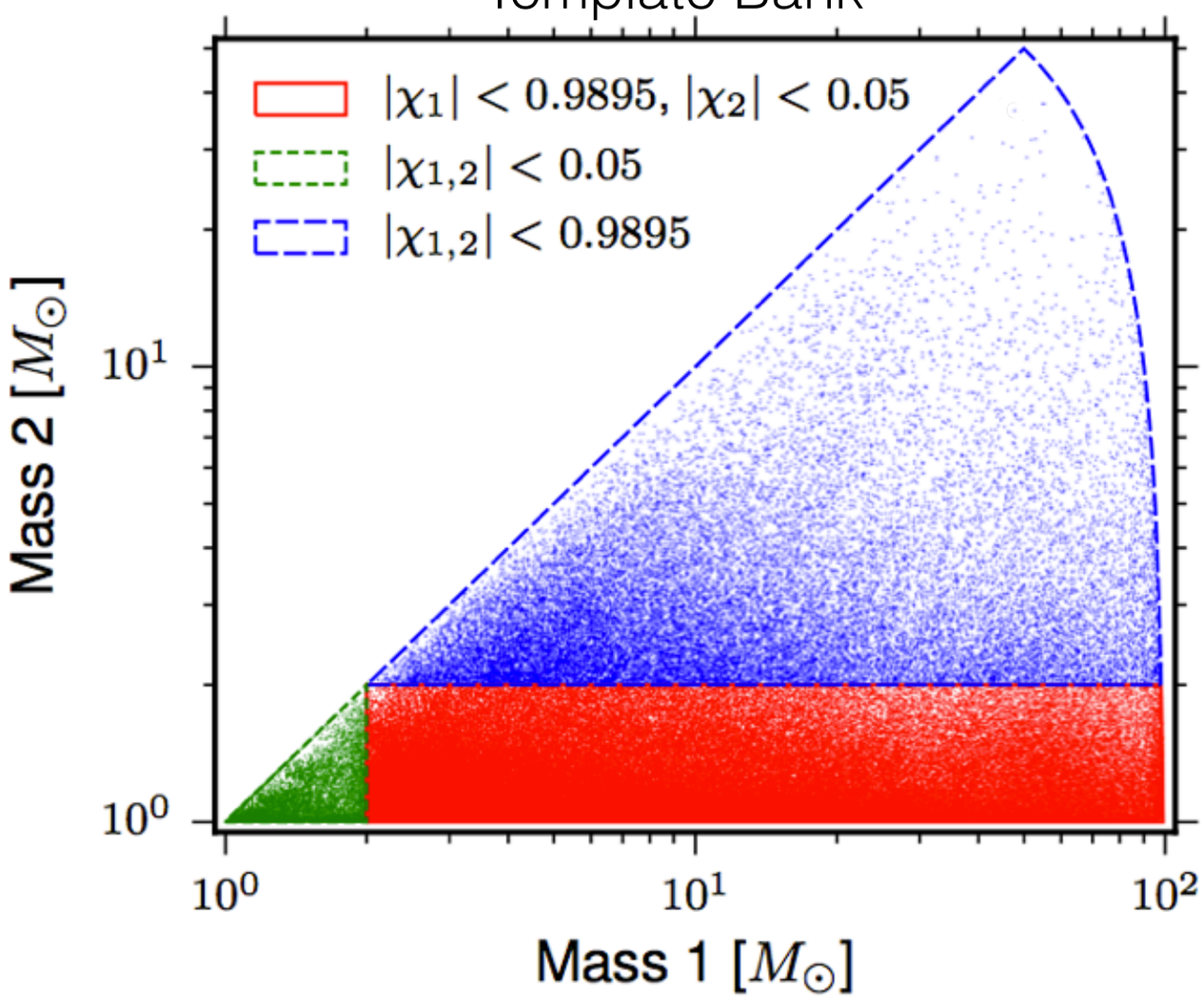
$$\rho(t) = \frac{|Z(t)|}{\sigma}$$

Matched Filter Search

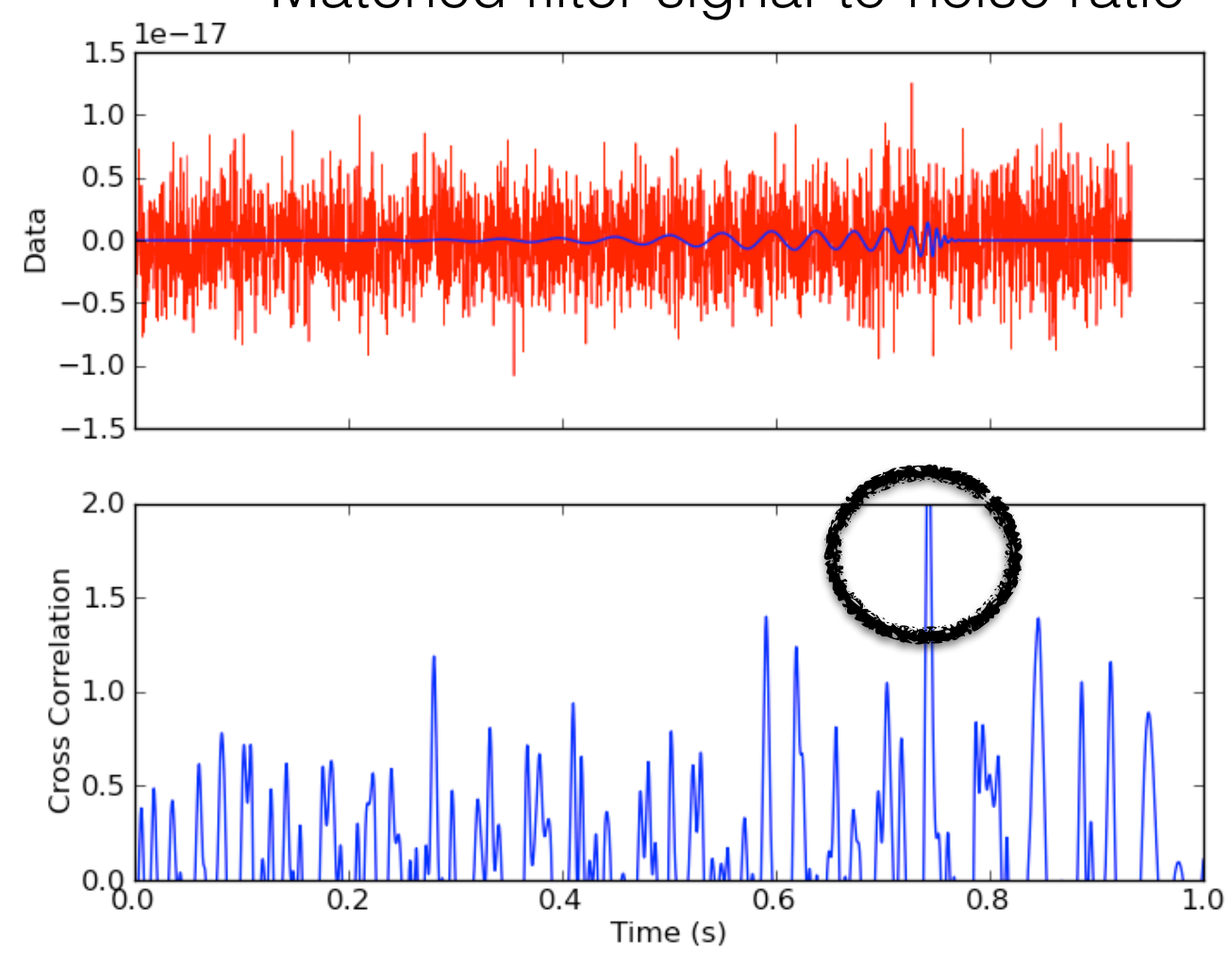


$$\chi_{1,2} \propto \vec{S_{1,2}} \cdot \hat{L}$$

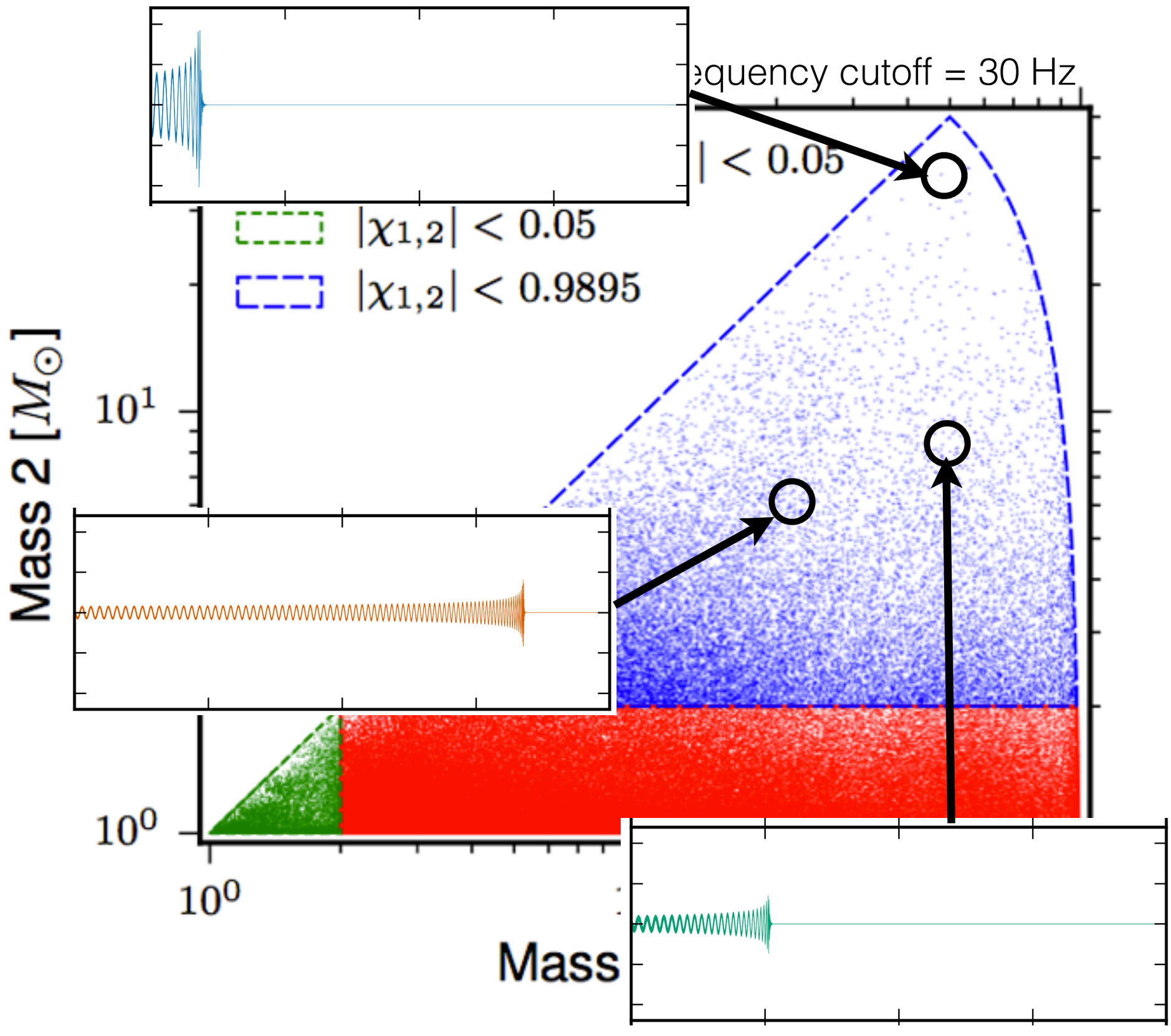
Template Bank



Matched filter signal-to-noise ratio

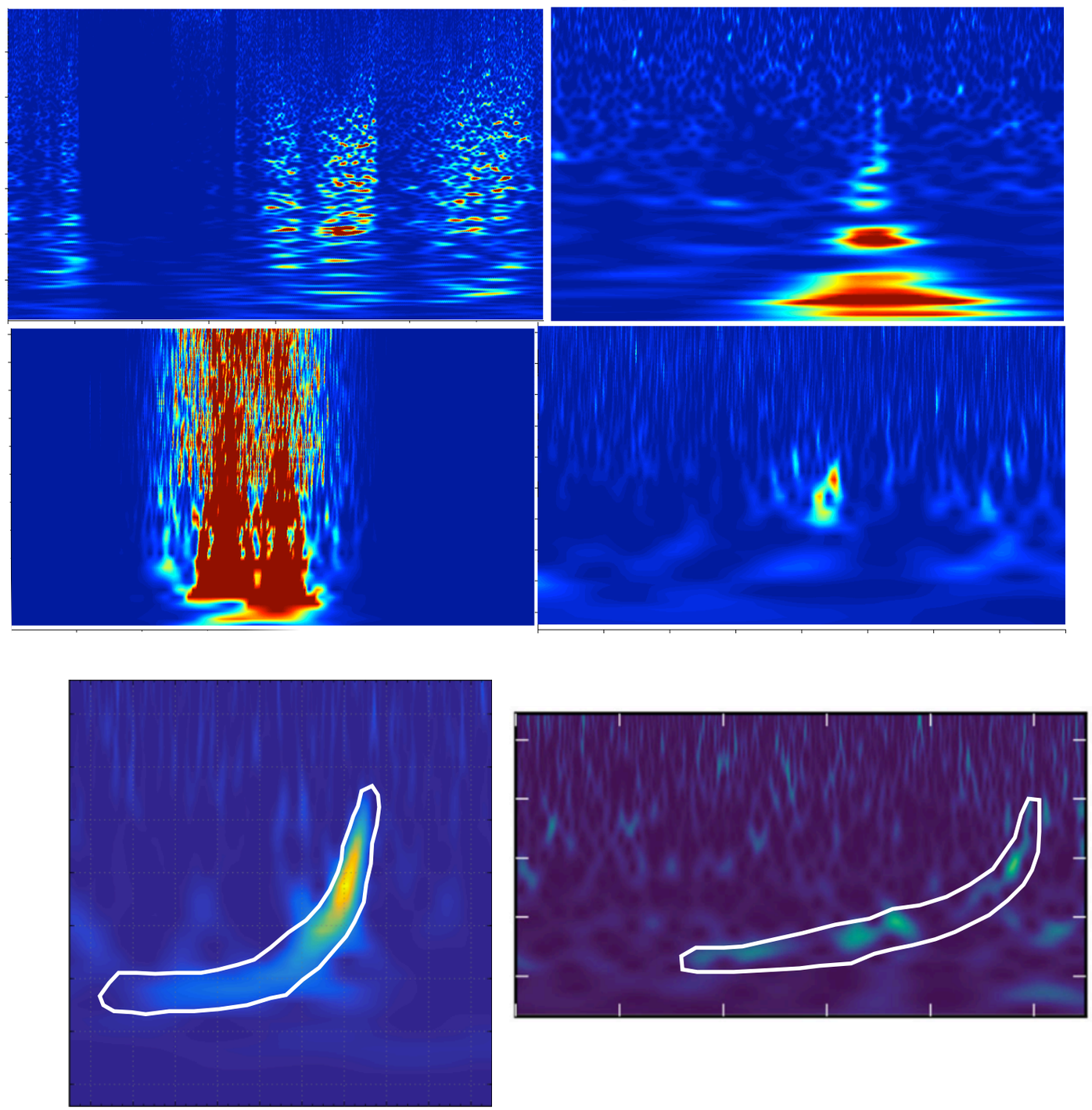


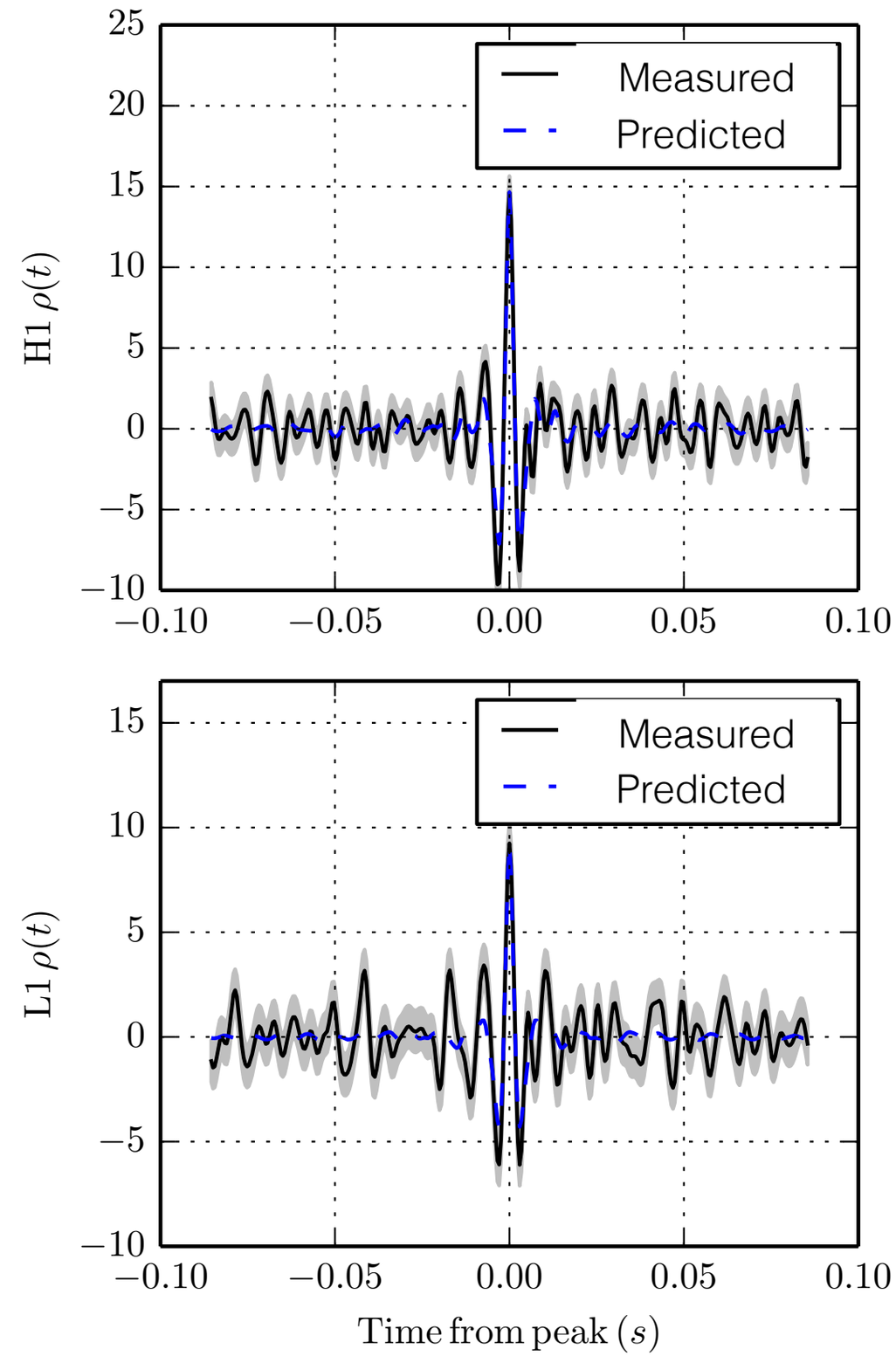
The O1 UberBank



Signal-based Vetoes

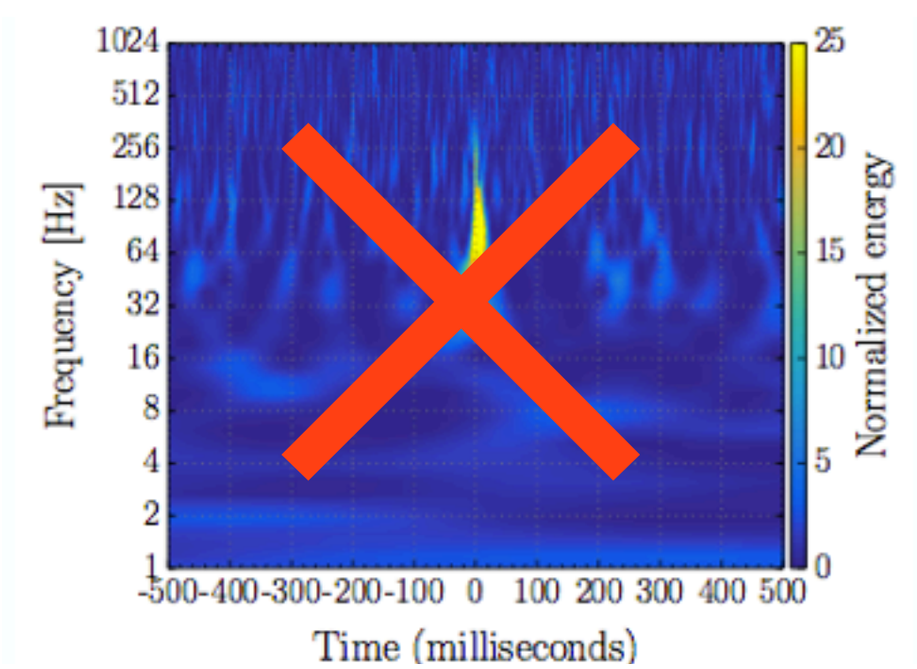
Time-frequency Spectrograms:
Glitches versus Signals

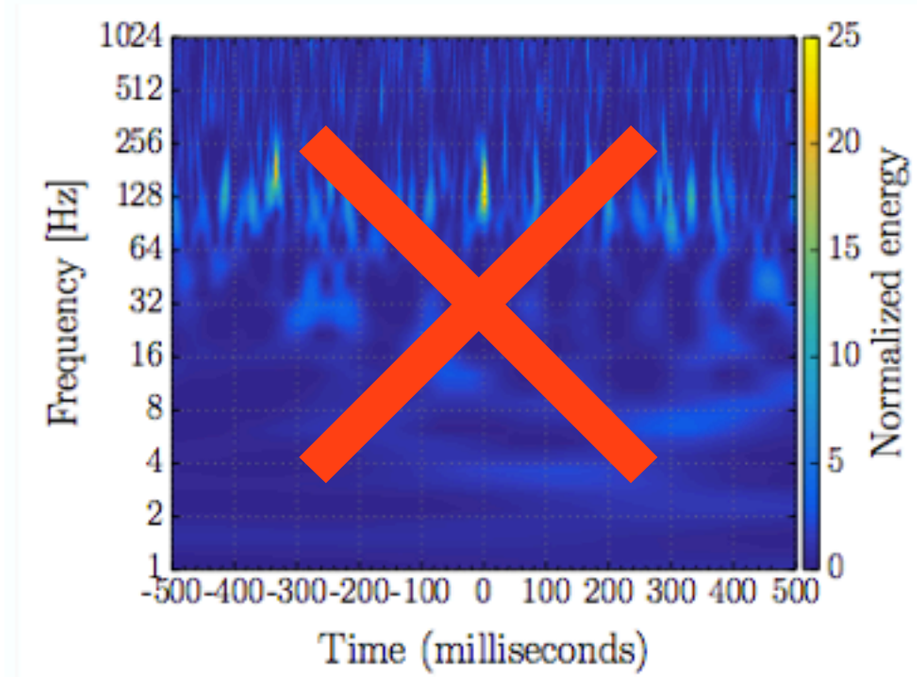


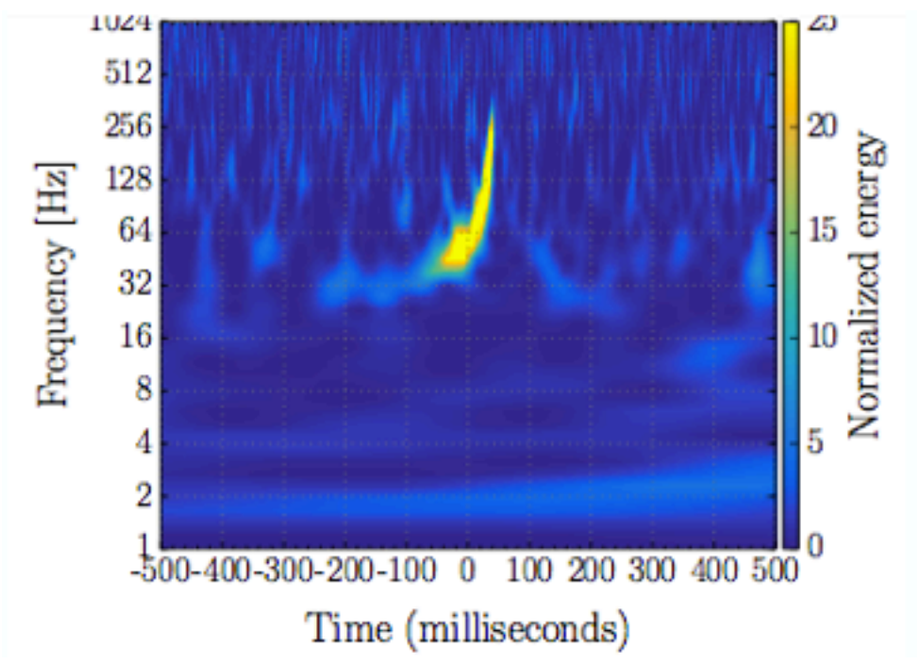


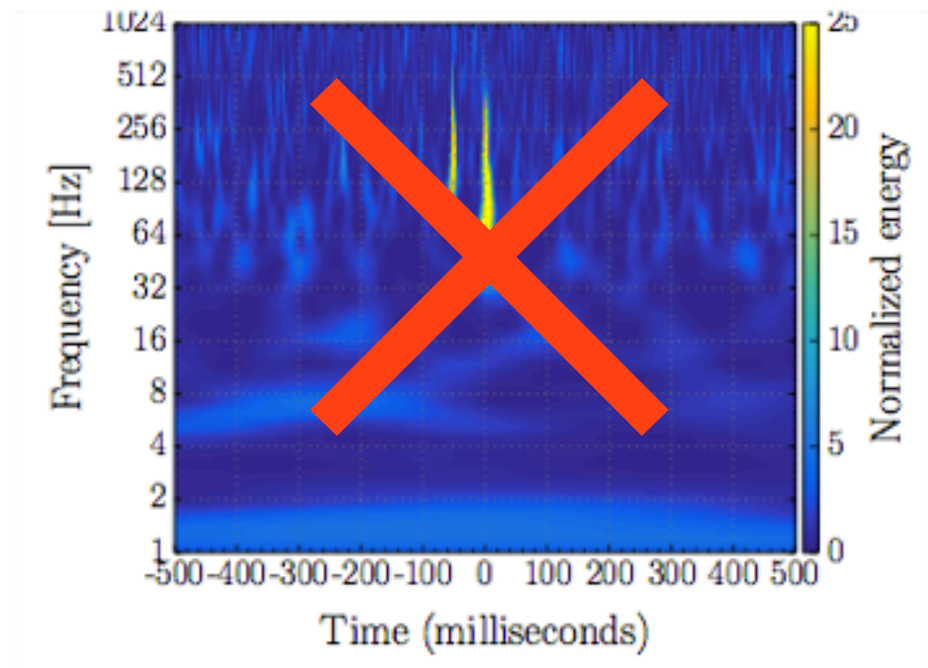
Messick, et al, arXiv 1604.04324

Which glitch is not like the others?

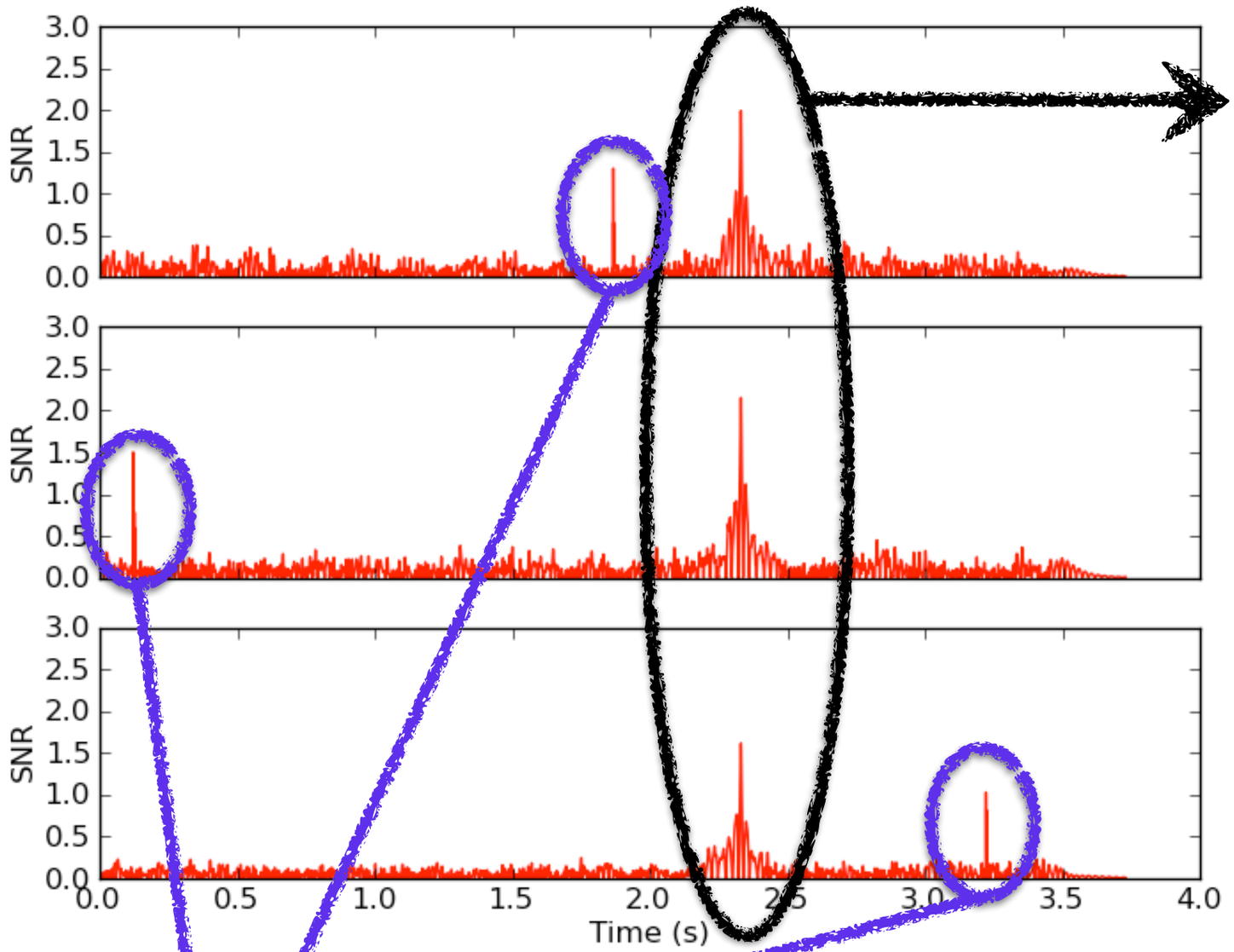








Coincident Triggers



Coincident GW candidate

Background noise model

Likelihood-based ranking statistic

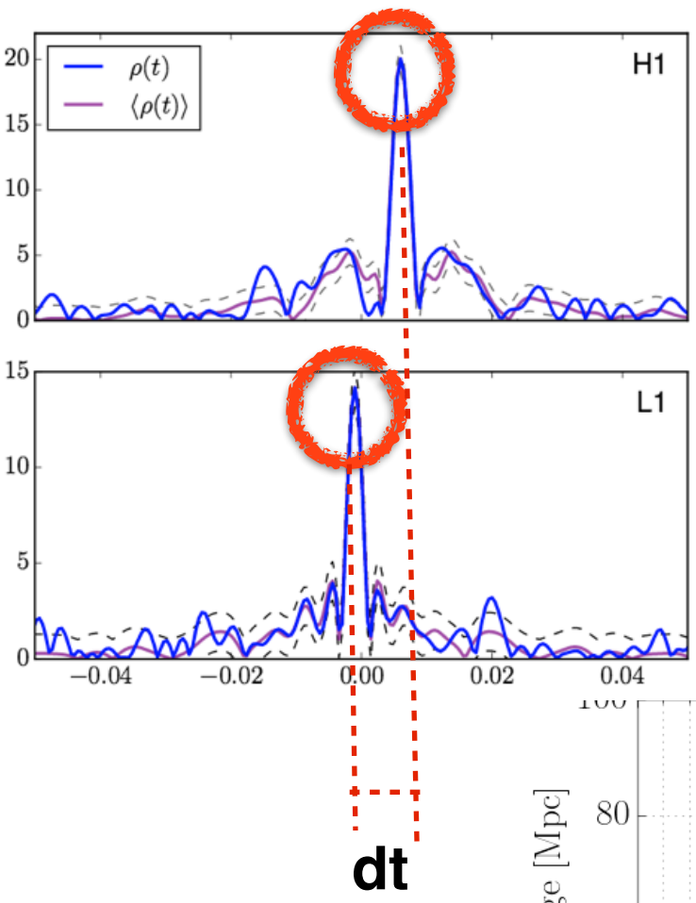
$$\mathcal{L} = \frac{P\left(d_{\mathrm{H}}, d_{\mathrm{L}} | \mathrm{signal}\right)}{P\left(d_{\mathrm{H}} | \mathrm{noise}\right) P\left(d_{\mathrm{L}} | \mathrm{noise}\right)}$$

Numerator: semi-analytic signal model

Denominator: background noise model

Likelihood Parameters

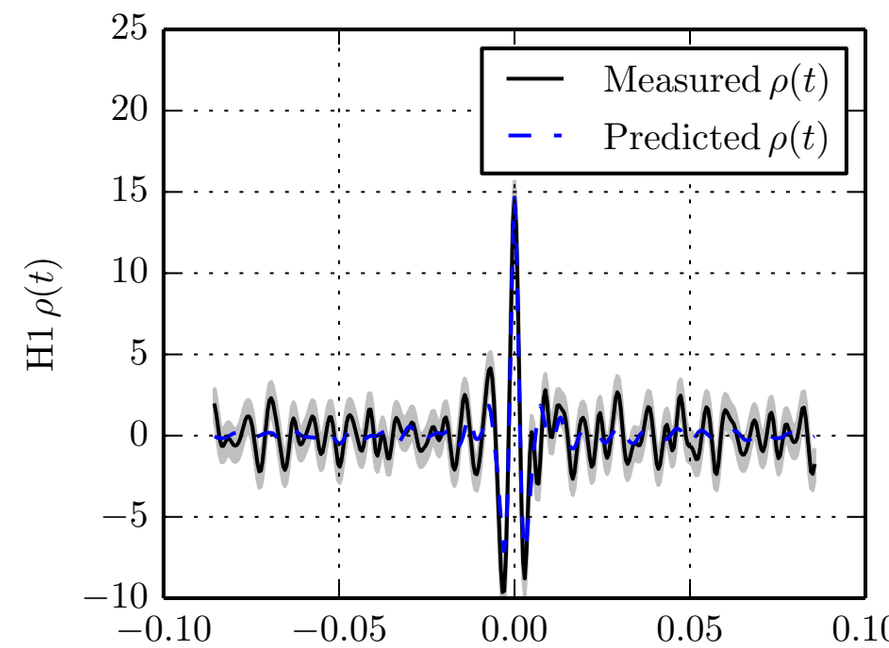
Signal-to-noise ratio



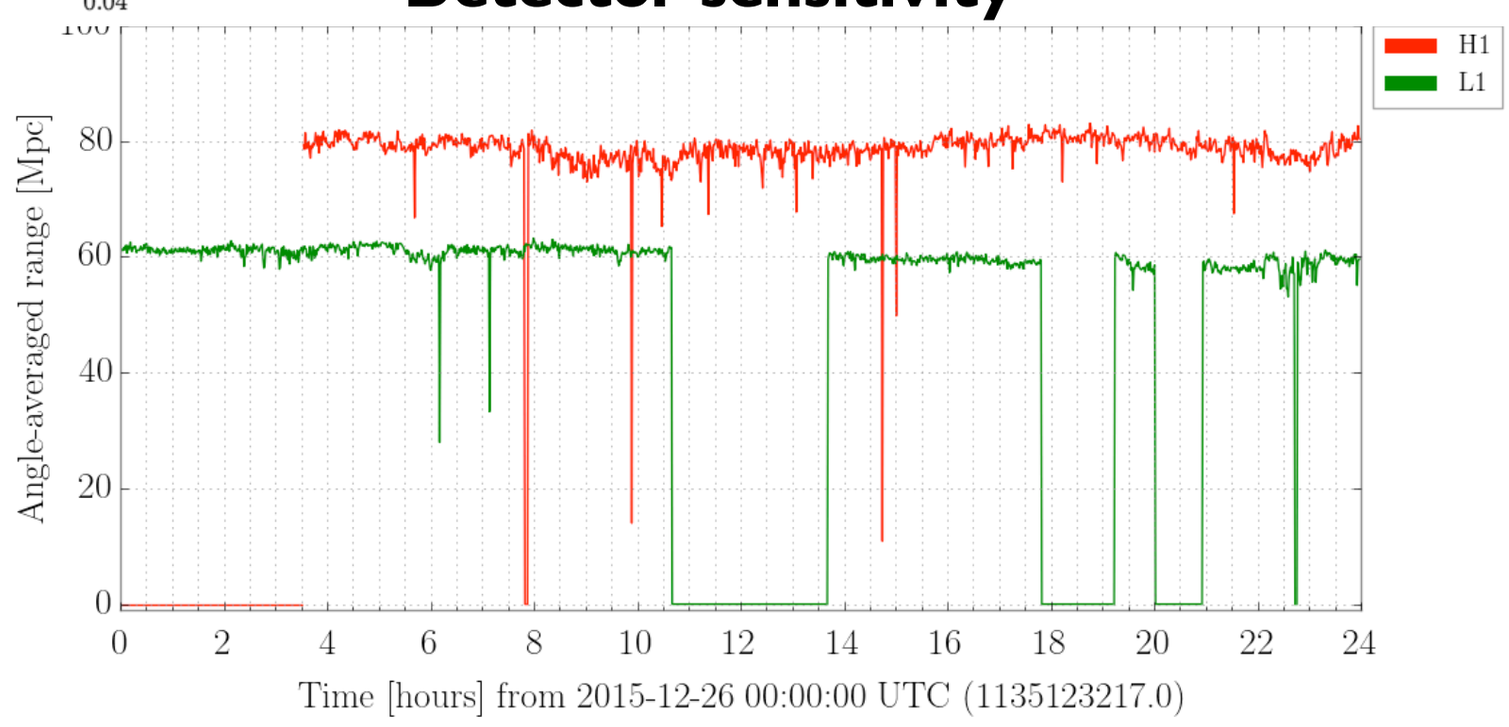
A multivariate ranking statistic

$$\mathcal{L} = \frac{P\left(d_{\mathrm{H}}, d_{\mathrm{L}} | \mathrm{signal}\right)}{P\left(d_{\mathrm{H}} | \mathrm{noise}\right) P\left(d_{\mathrm{L}} | \mathrm{noise}\right)}$$

Signal-based veto

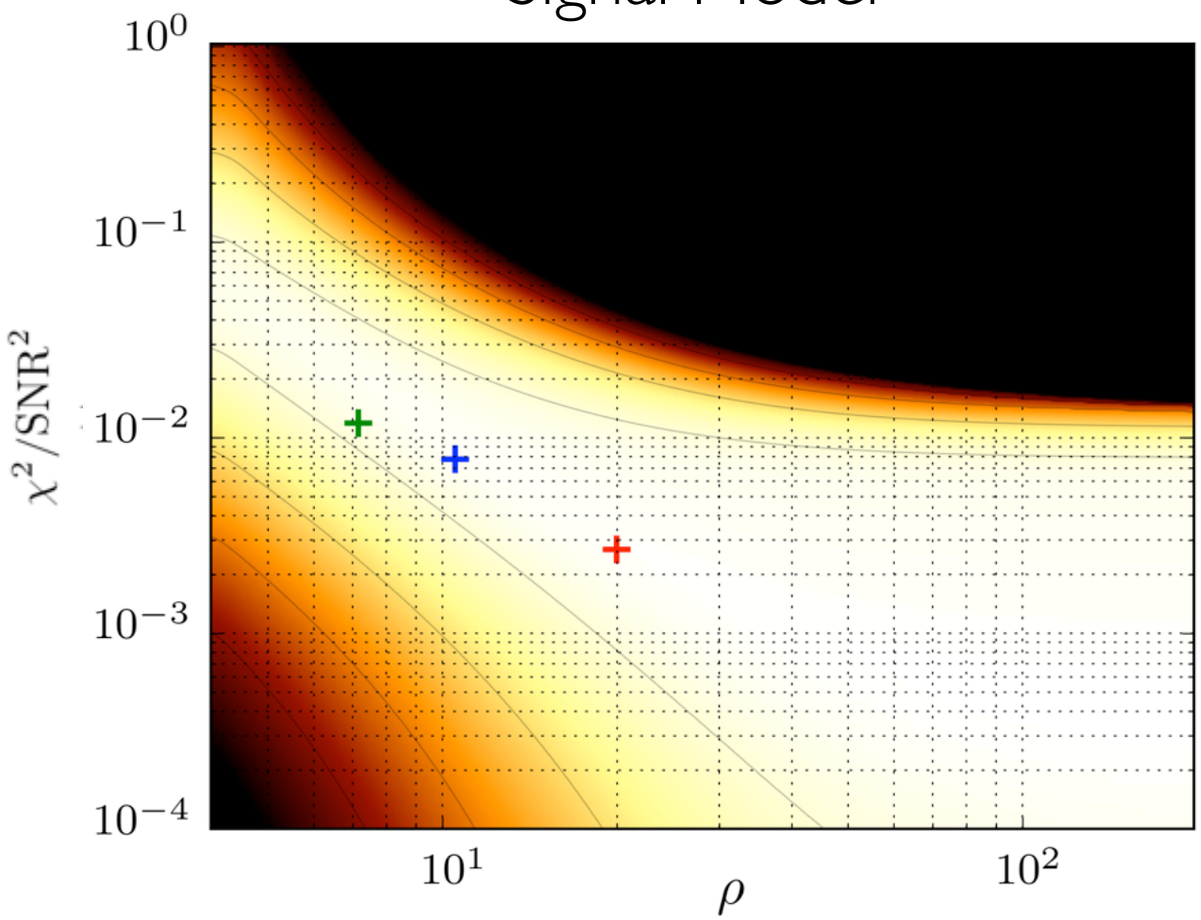


Detector sensitivity

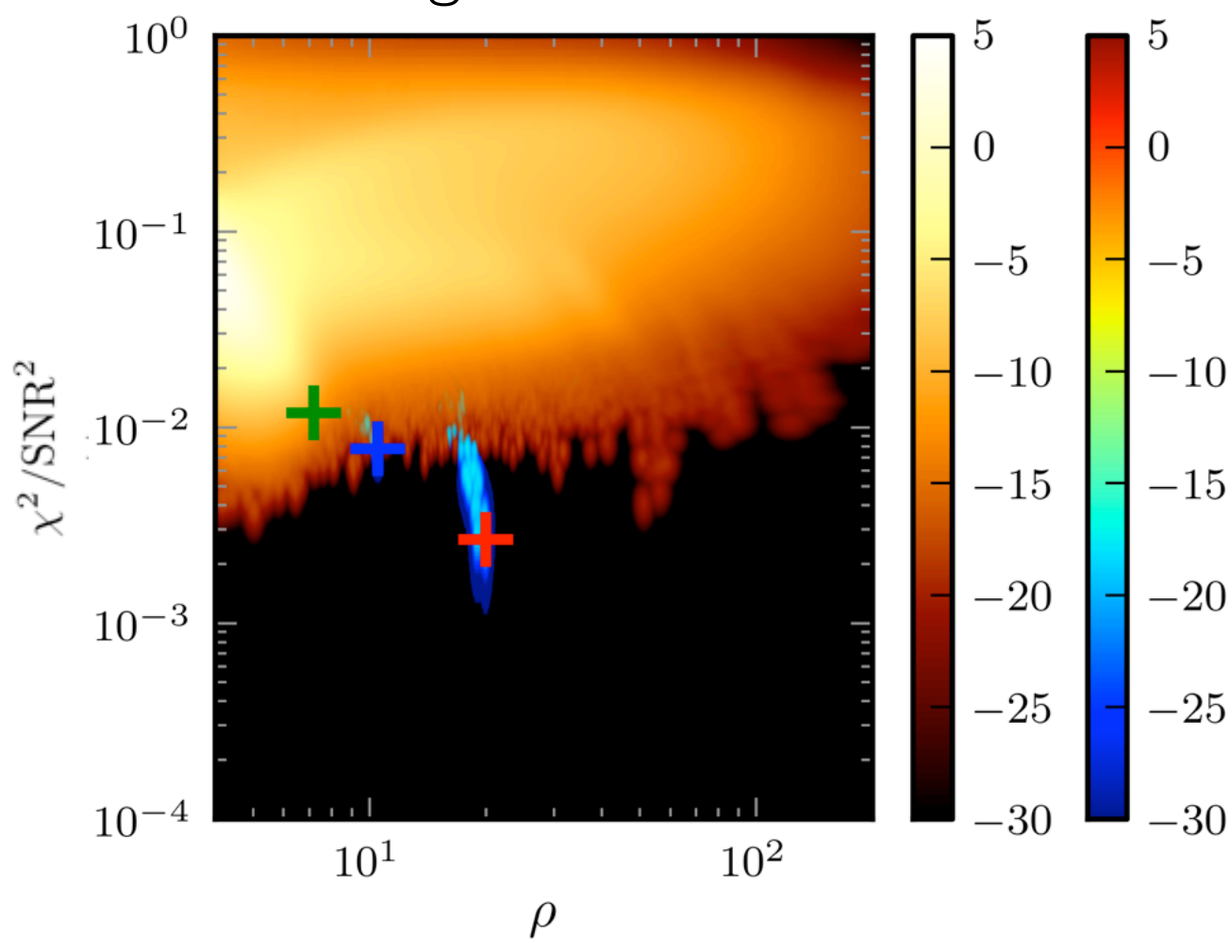


Signal and Background Models

Signal Model



Background Model

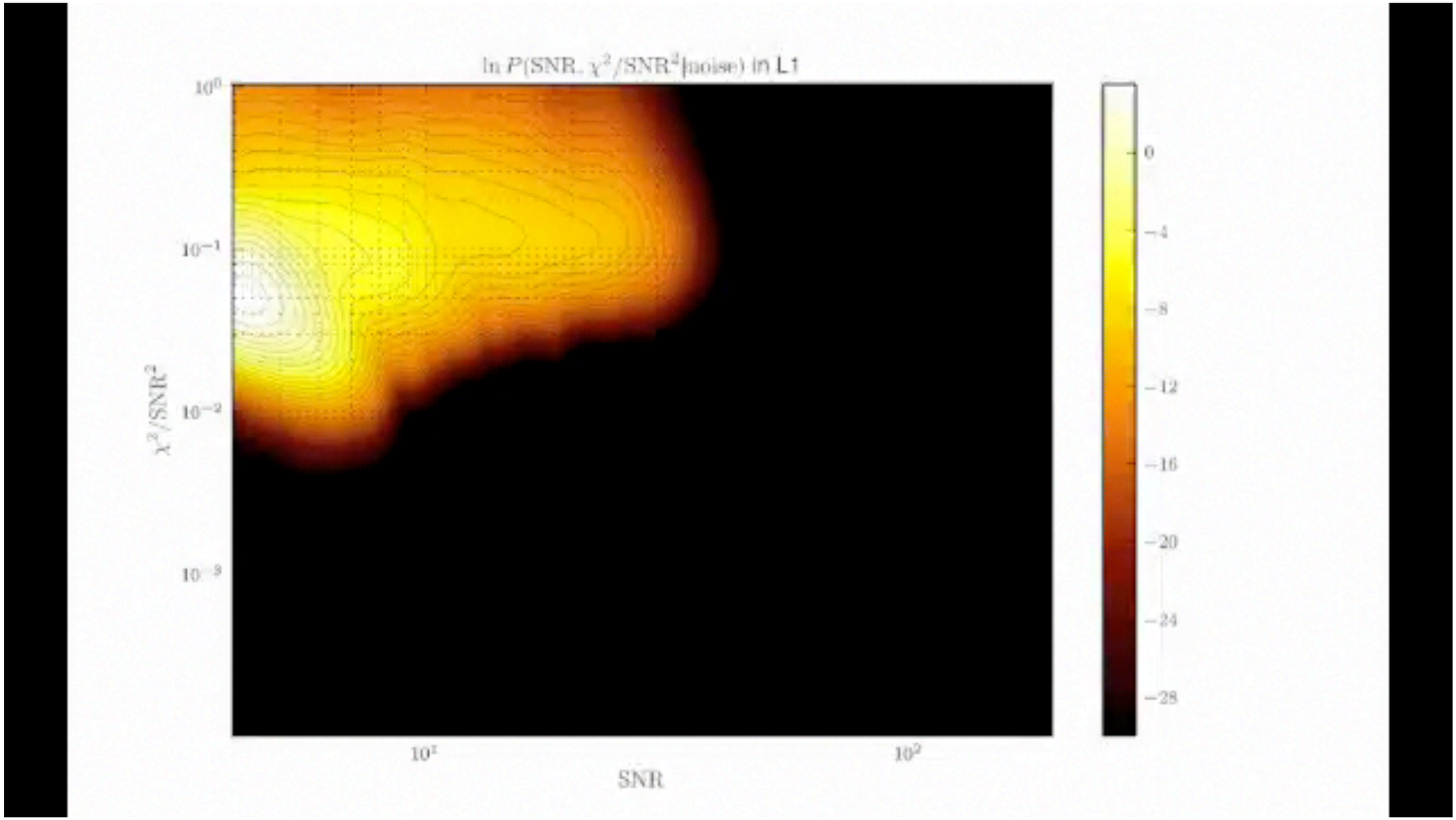


+ - GW150914

+ - GW151226

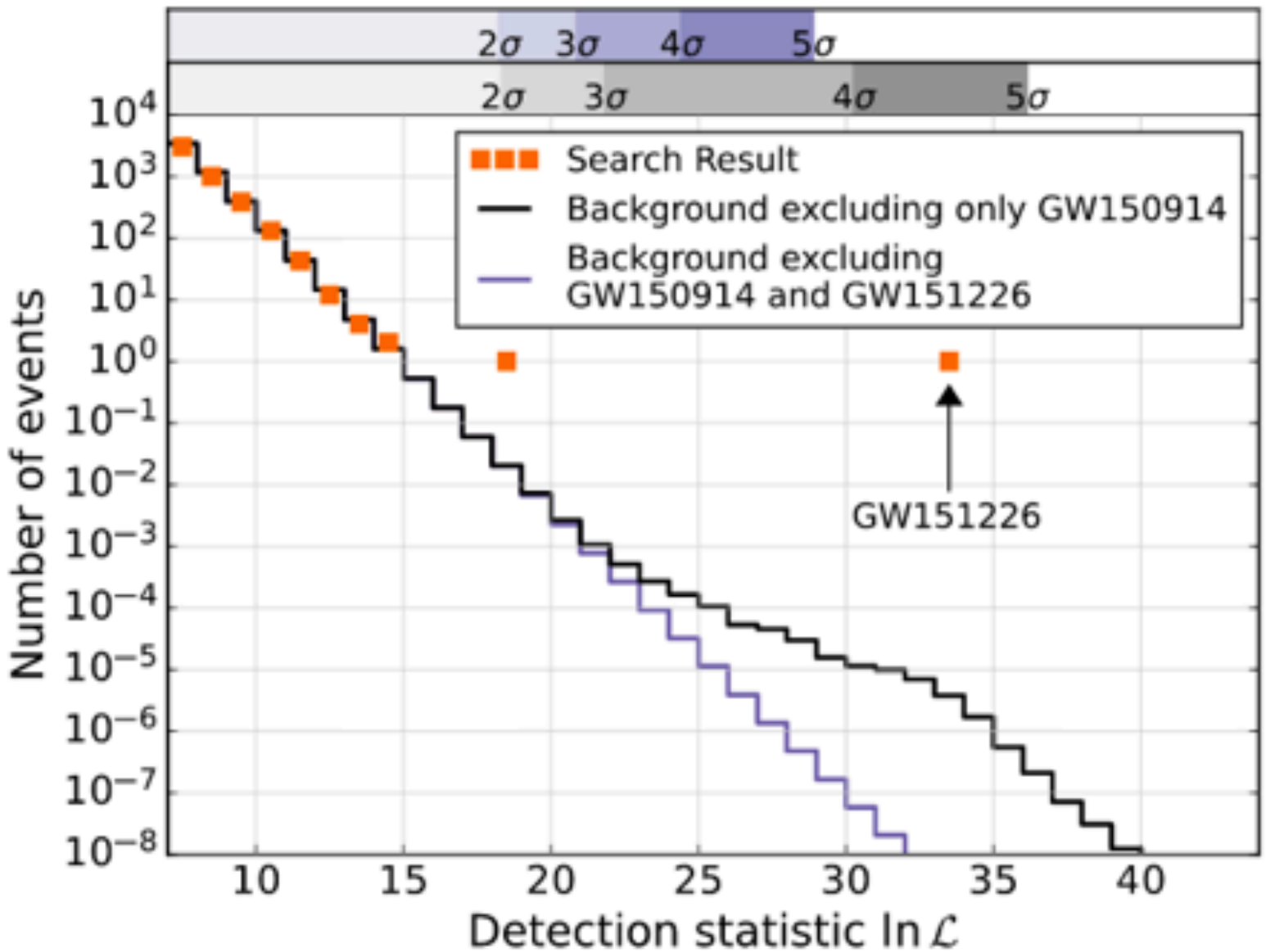
+ - LVT151012

Evolution of Noise Model with Mass



The search background is calculated separately for each mass and chi bin.

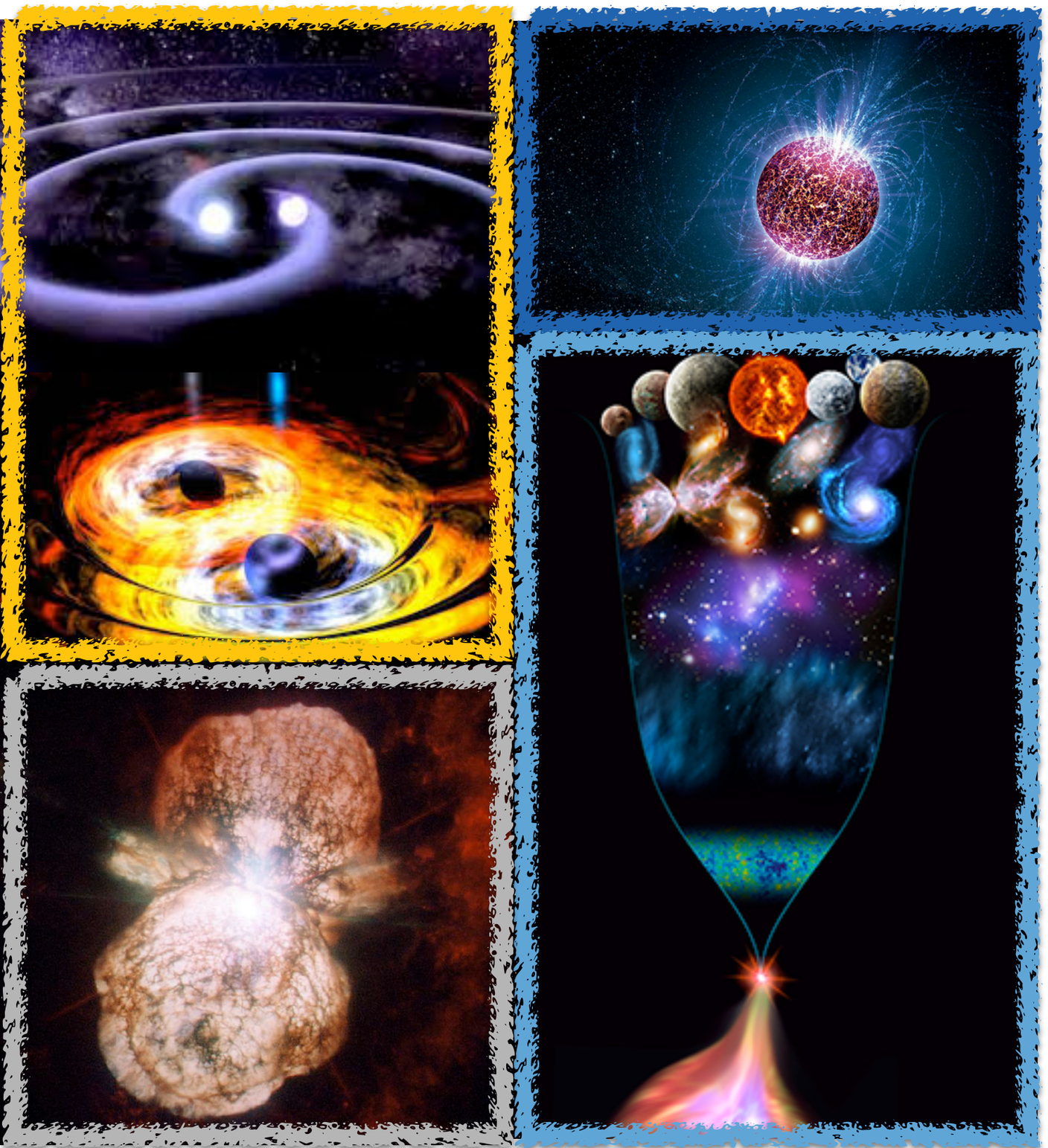
Estimating Significance

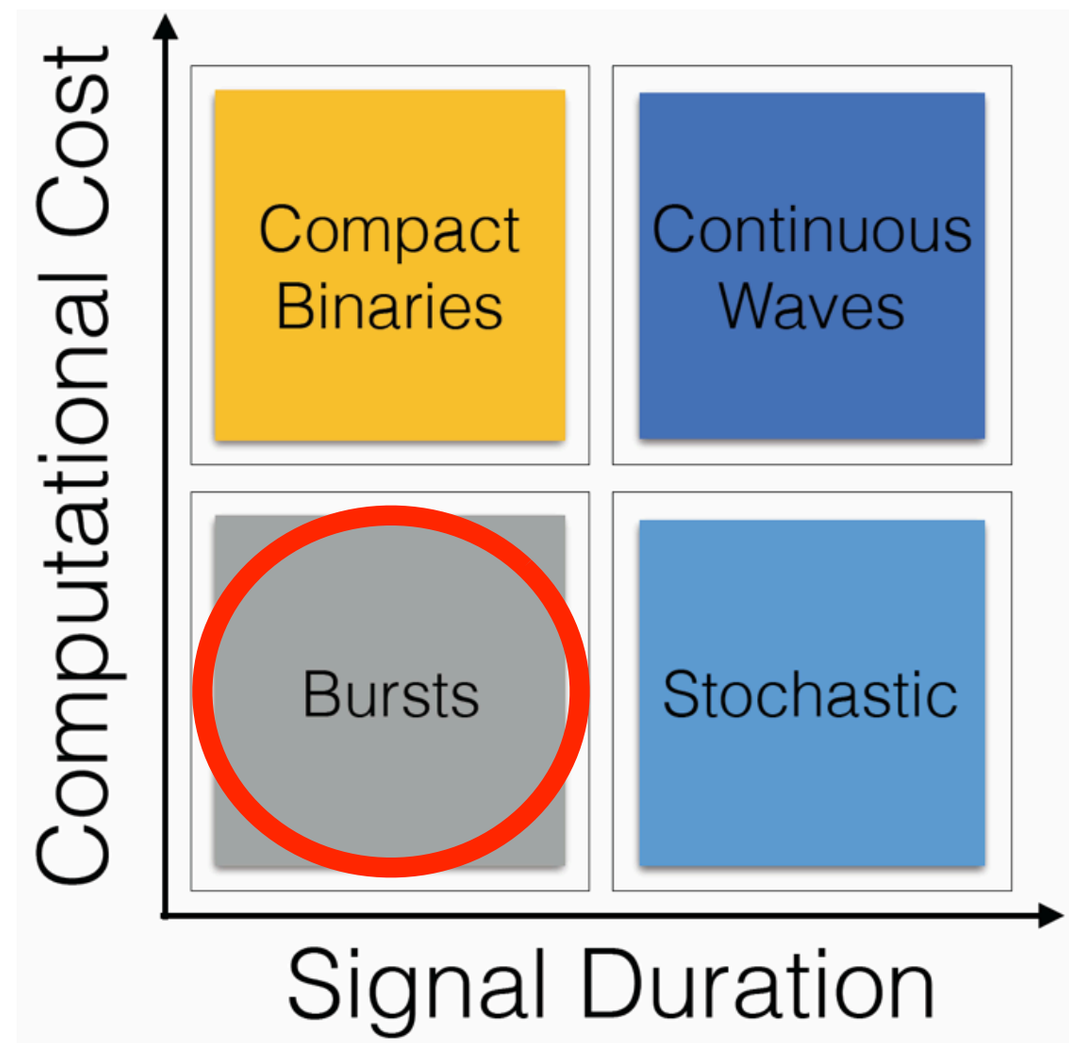


LVC, PRL 116, 241103 (2016)

Only when we leave coincident triggers in our noise model, i.e. gravitational waves, do we form accidental coincidences at the same level of significance or higher.

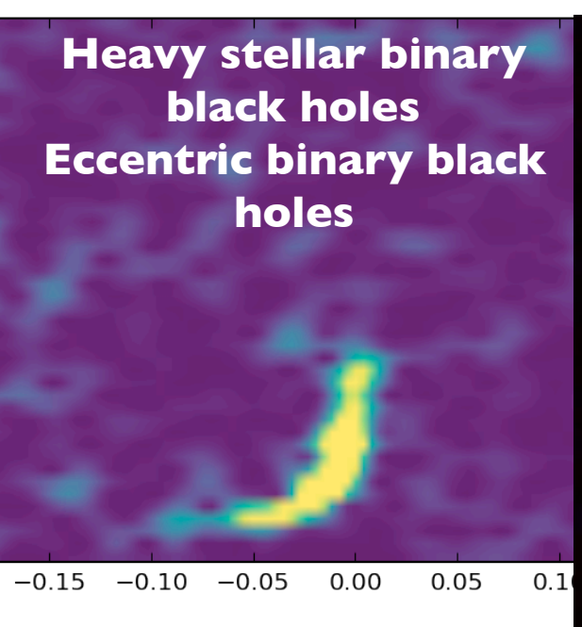
Gravitational-wave Sources

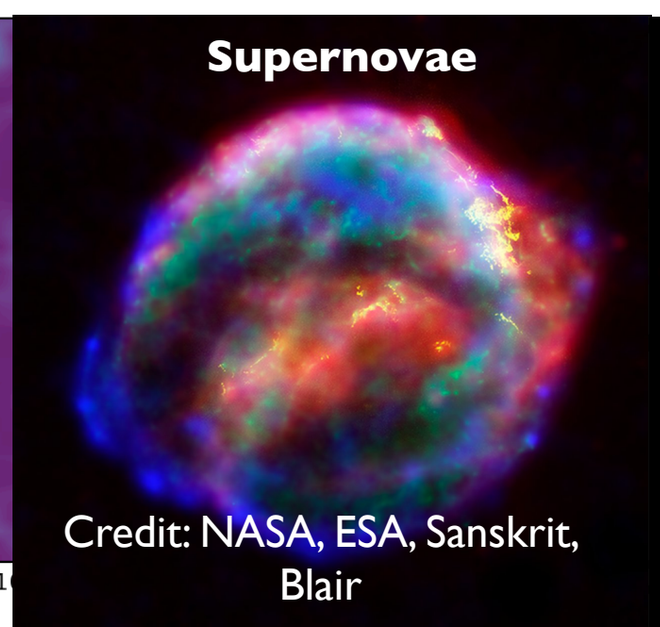


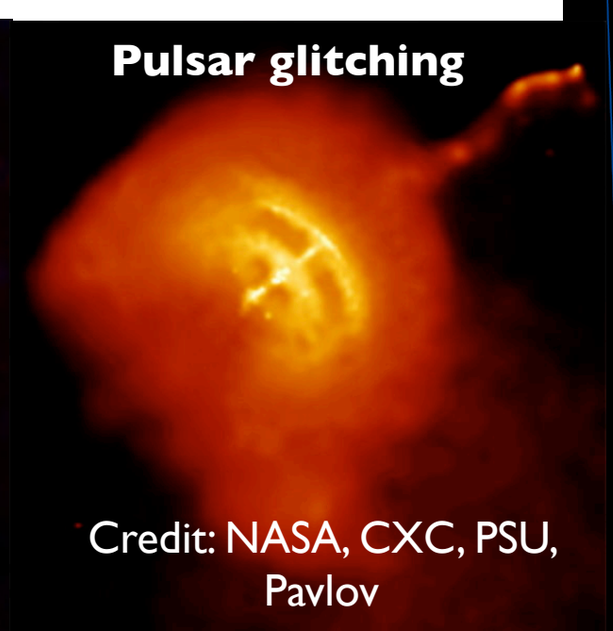


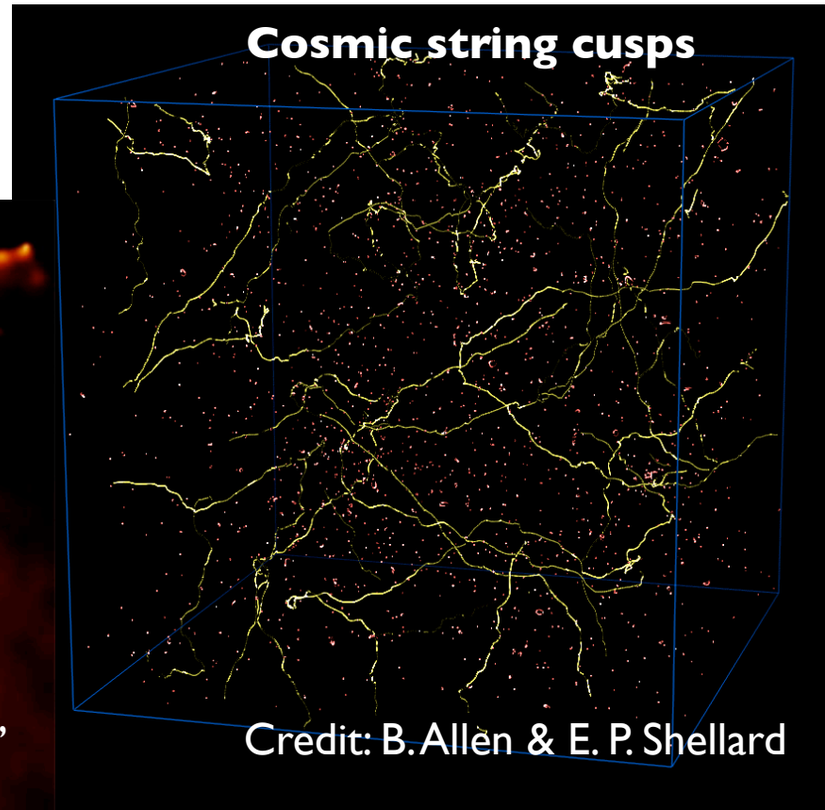
Burst Signals

- GW searches with minimal assumptions
  - Do not assume accurate source models - affected by noise
- Limited ability to measure waveforms, sky positions, polarizations
- Can detect the unexpected



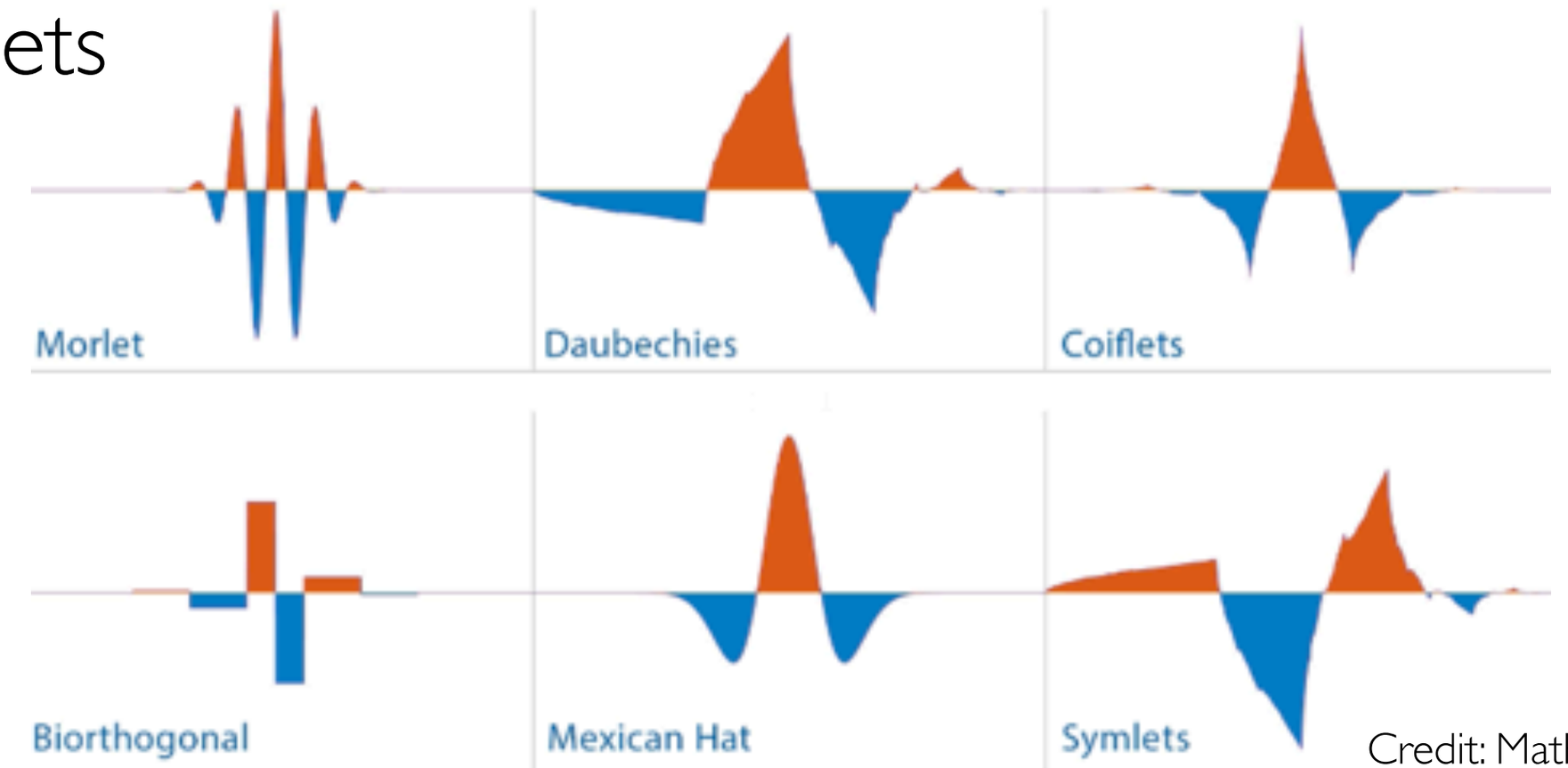






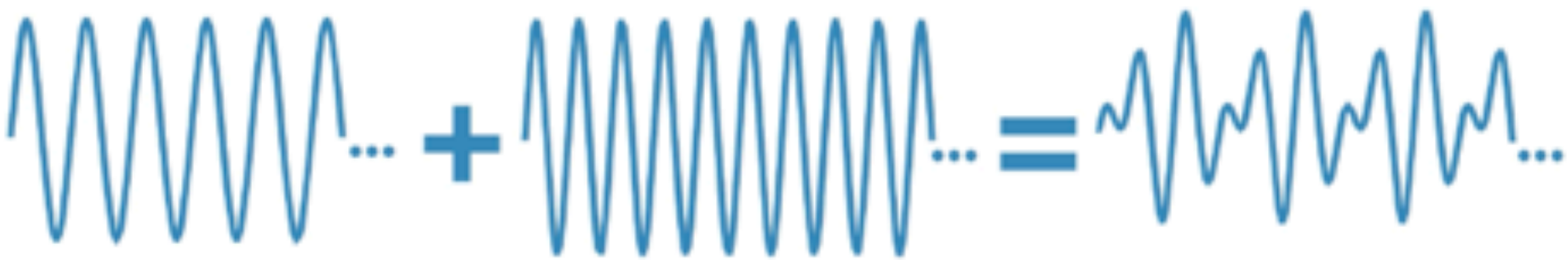
Burst Analysis with Wavelets

- Wavelets are waveforms of limited duration and bandwidth
- GW bursts can be described as superposition of wavelets



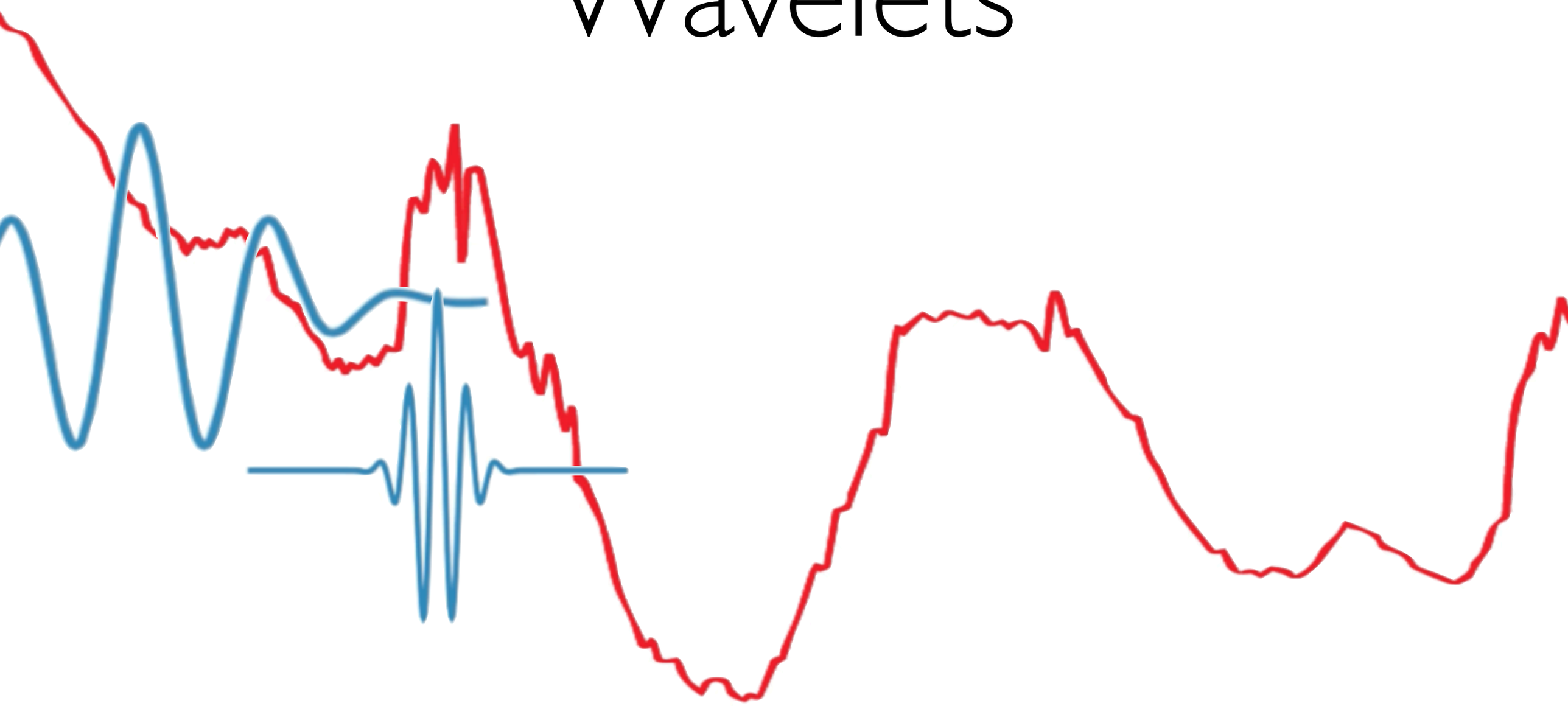
Credit: MathWorks

Burst Analysis with Wavelets



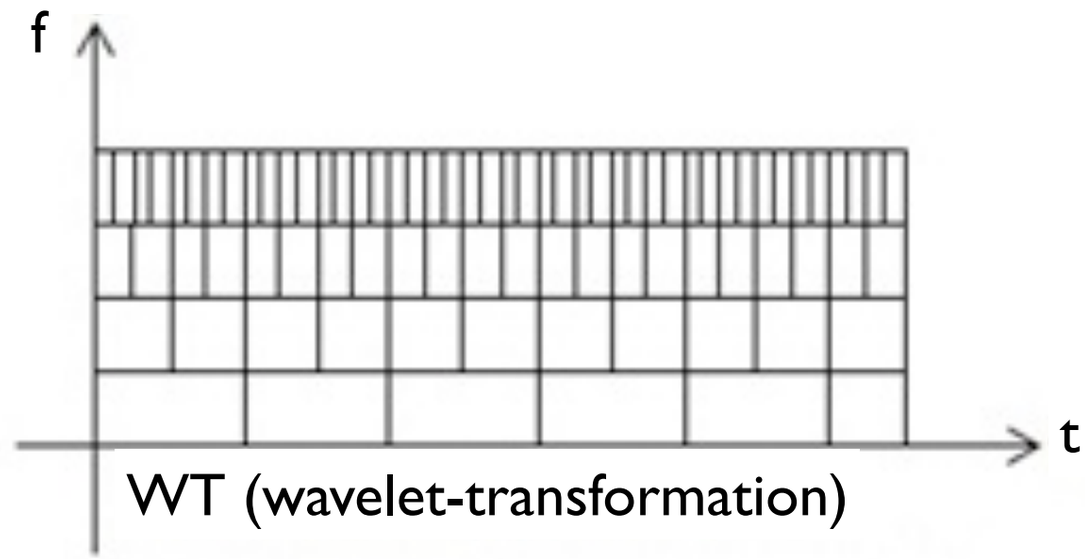
The Fourier Transform does not represent abrupt changes efficiently.
Sum of Sine waves are not localized in time or space and oscillate forever.

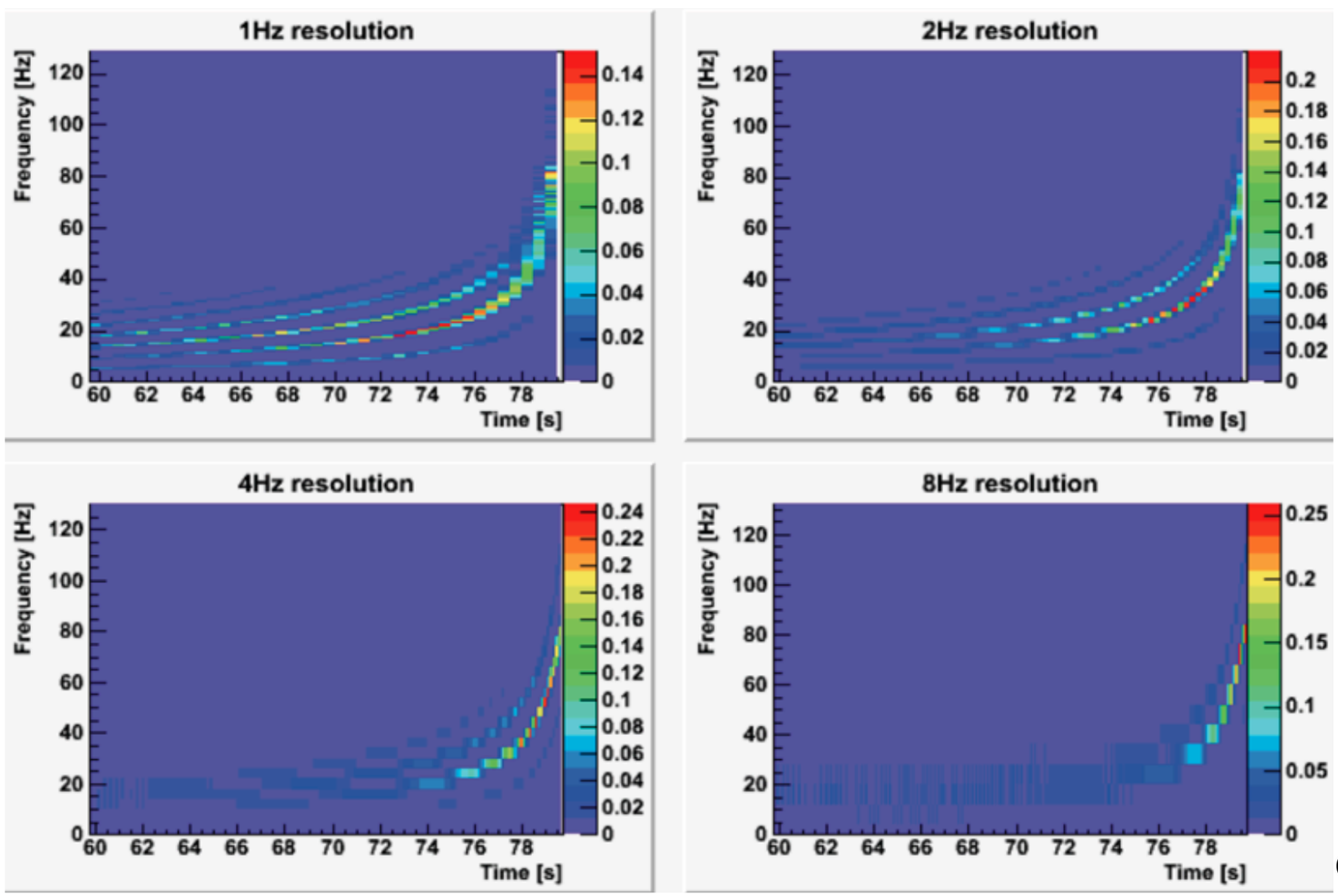
Burst Analyses with Wavelets



For time series that have abrupt changes, we need functions that are well localized in time and frequency.

Time-frequency Pixel Maps

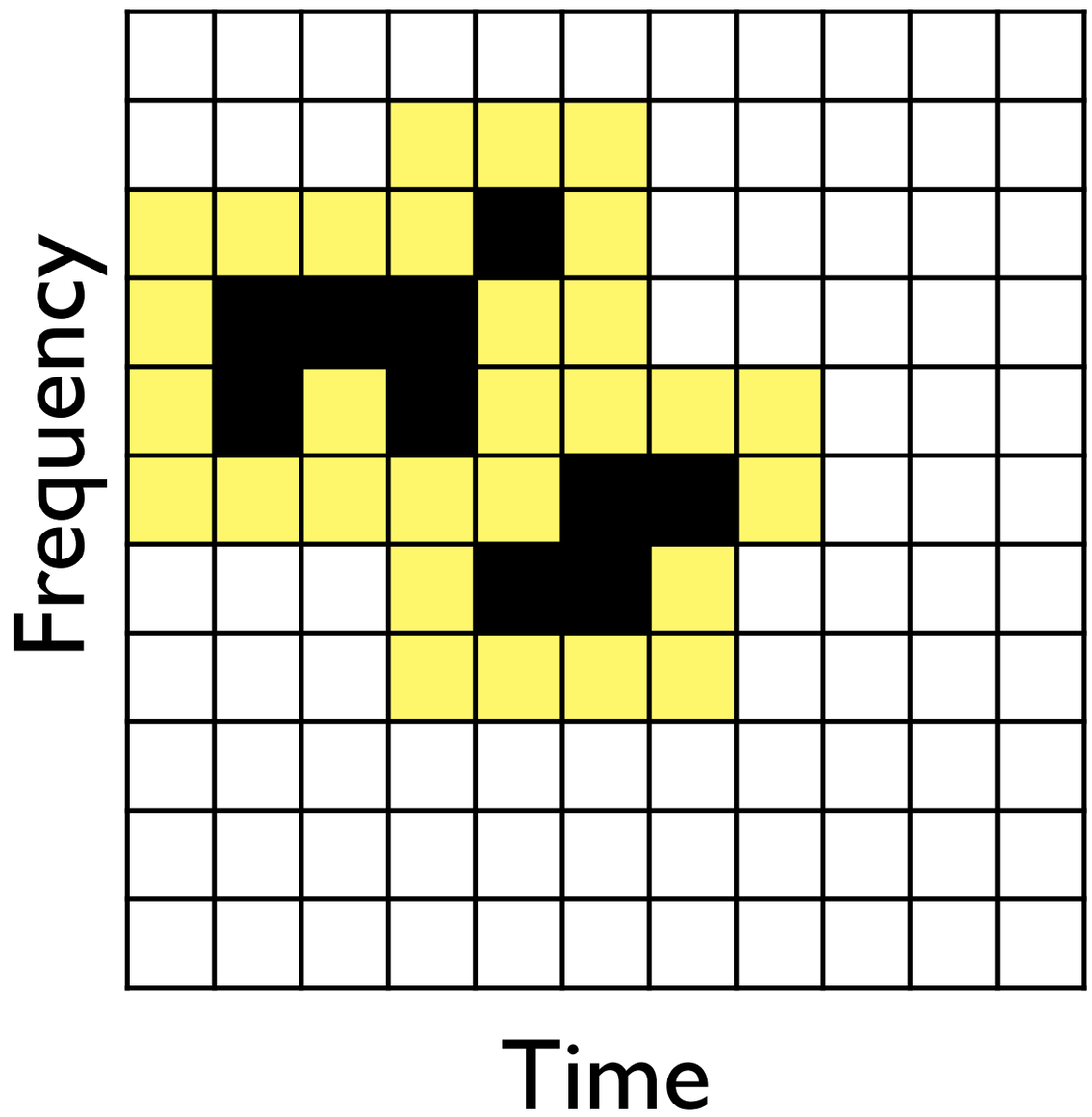




Example: (10, 10) $M_\odot$ binary black hole merger with an eccentric orbit at 4 different time-frequency representations.

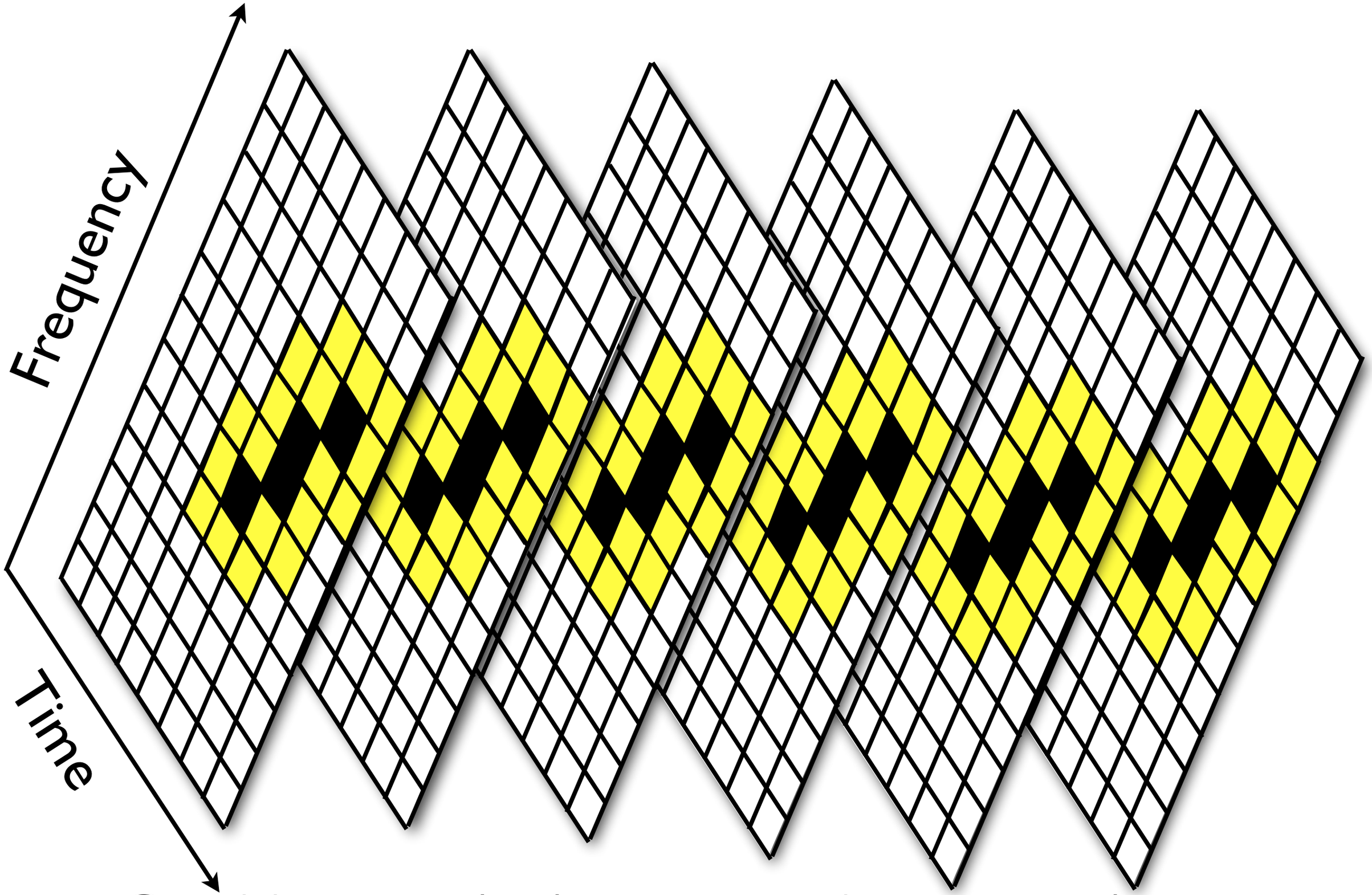
Credit: S. Klimenko

Cluster Selection



Identify time-frequency areas with excess energy above a pre-determined threshold.

Cluster excesses together.



Combine wavelet layers to make a supercluster.
These form burst events.

Coherent Burst Search

- Wavelet amplitudes need to be calculated and signal reconstructed for each time-frequency pixel and sky location
- Joint analysis from all detectors (H1, L1, V1,...)
- Synchronize detectors by searching over entire sky (~200000 points)

Maximum Likelihood Statistic

$$L = c_c E_s$$

similarity of waveforms in detectors

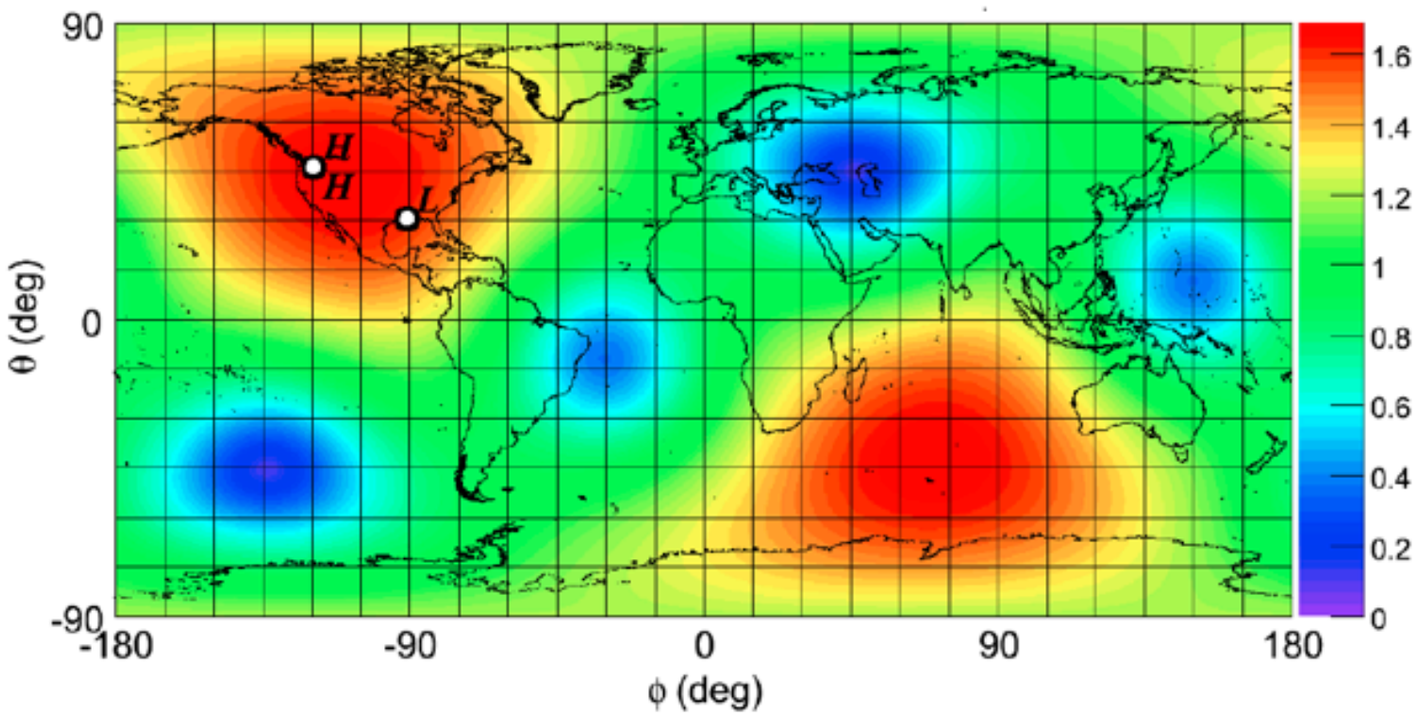
total energy of reconstructed waveforms

$$c_c = \frac{E_c}{E_c + E_n}$$

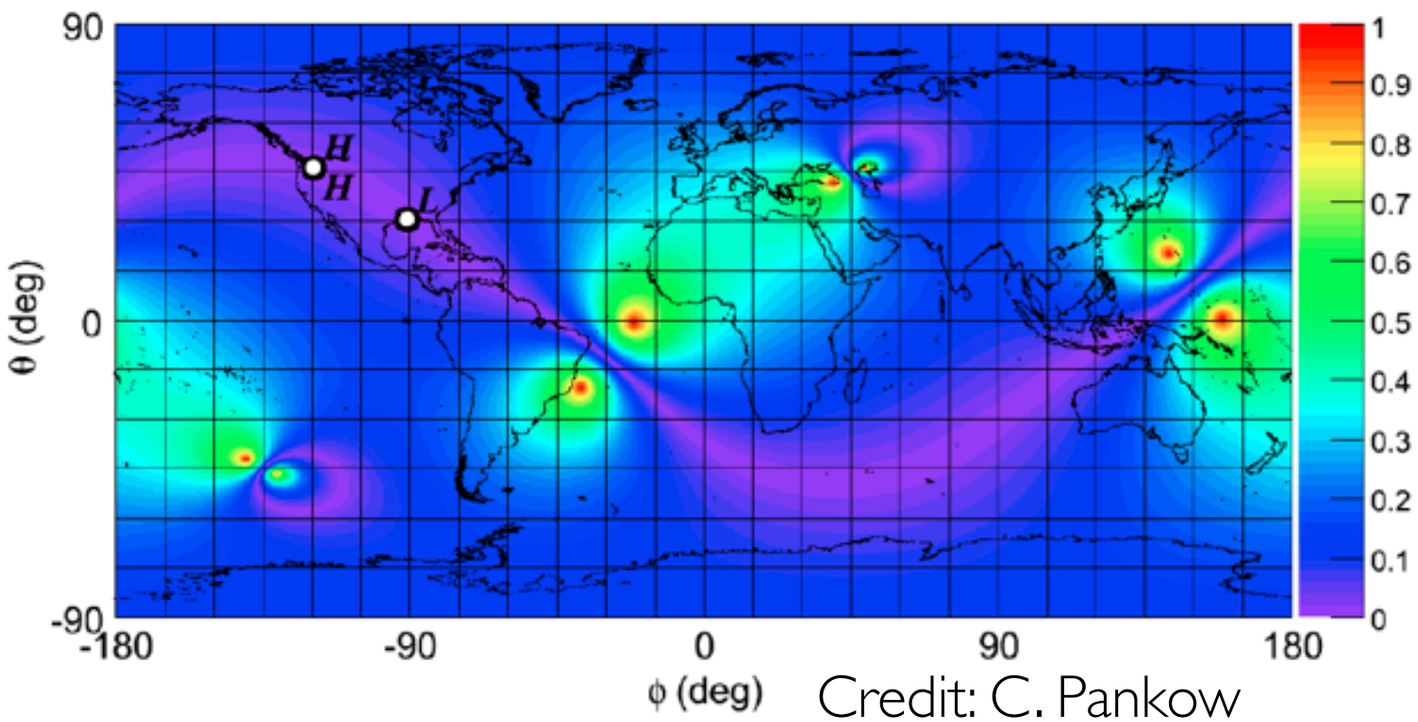
E_c - coherent energy

E_n - energy of residual noise after signal subtracted

Example: 2-Detector Constraint



Magnitude of antenna pattern vectors in for L1-H1 network



Ratio of antenna pattern vectors

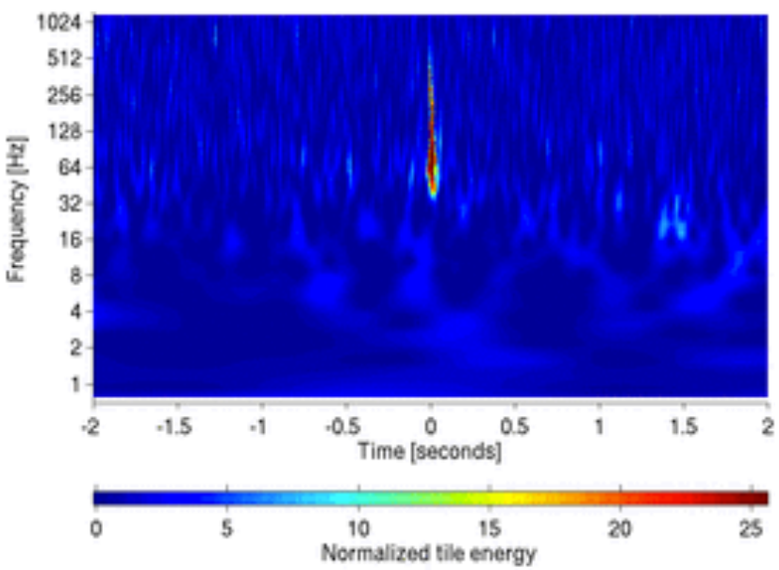
$$|f_\times| << |f_+|$$

Credit: C. Pankow

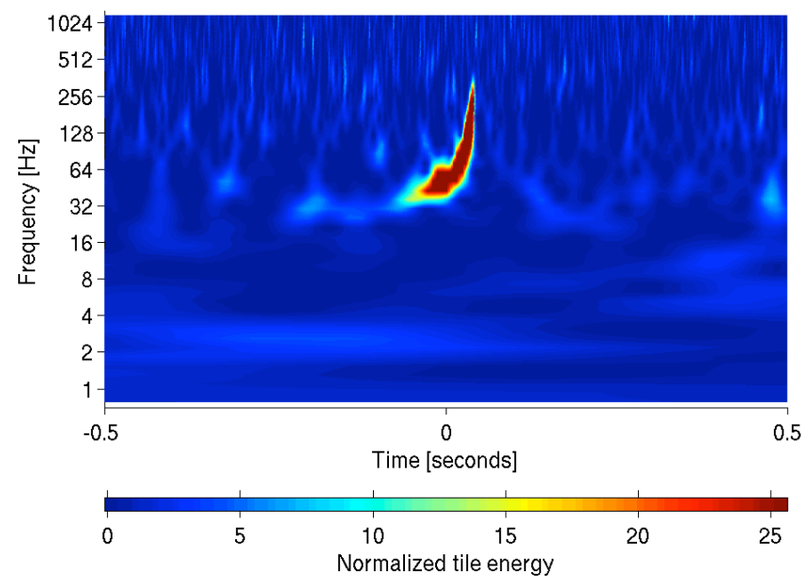
LIGO network nearly aligned; blind to second polarization in most sky locations.

3 Search Classes

C1
time-frequency morphology of known populations of noise



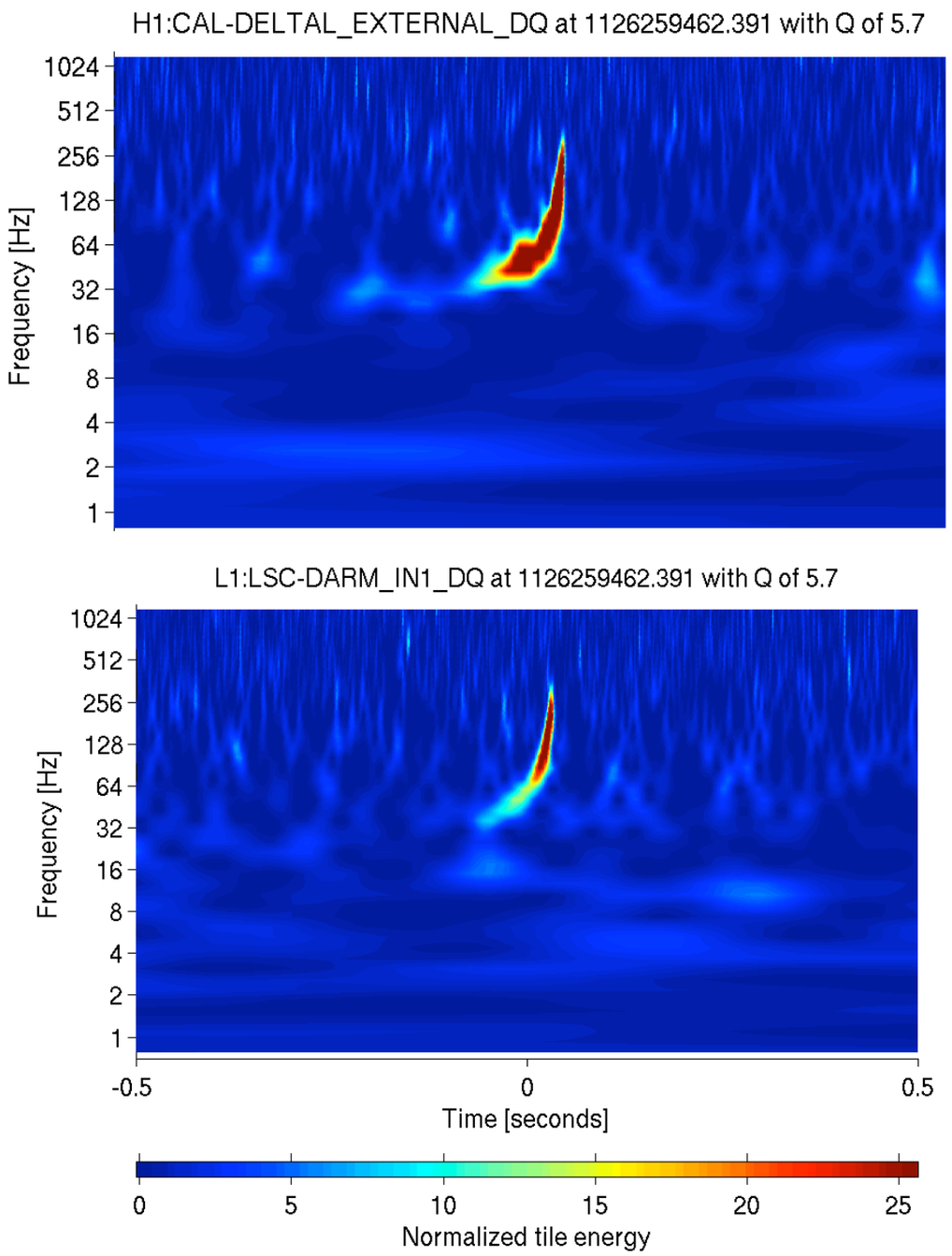
C2
frequency increasing with time

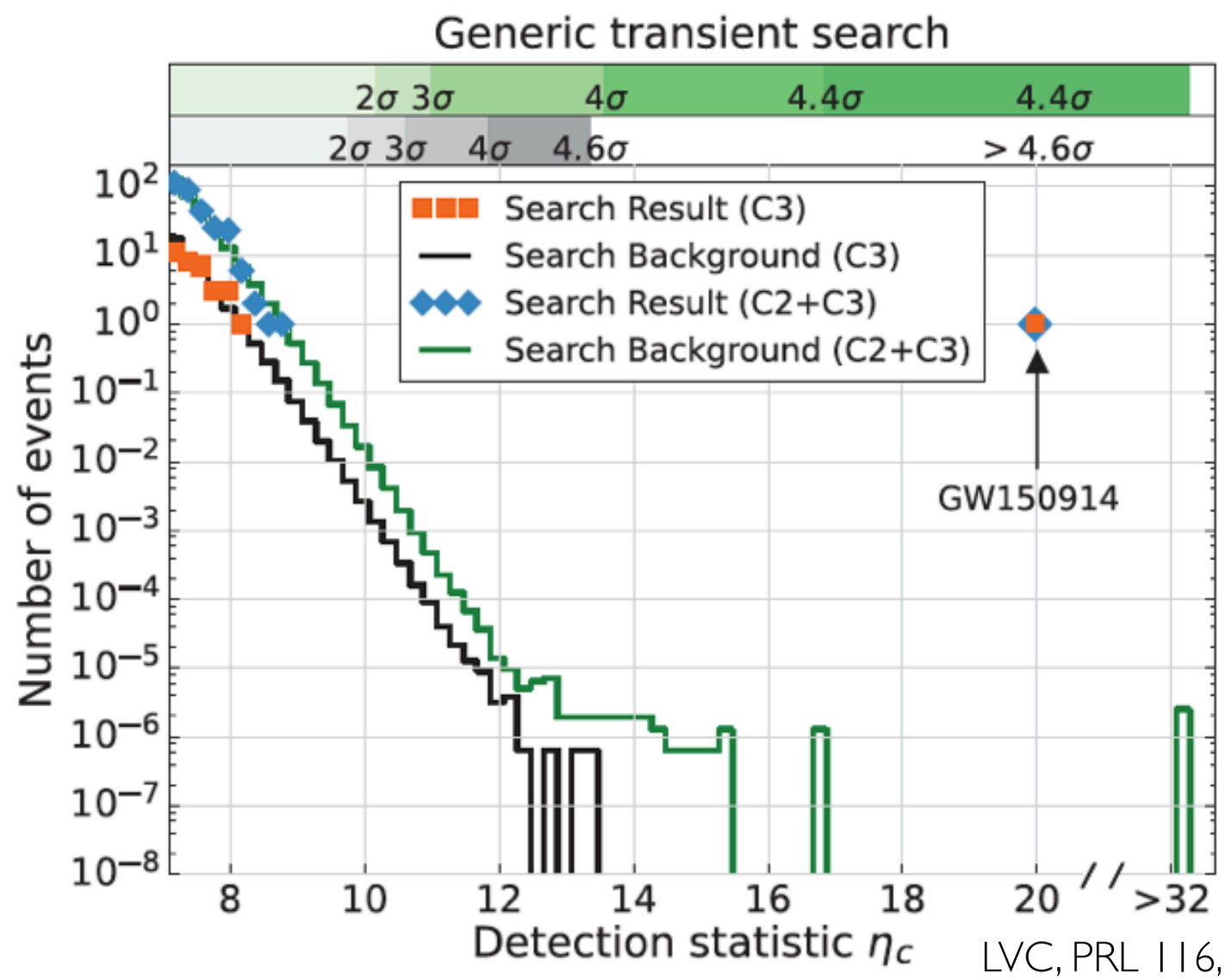


C3
all remaining events

?

A Burst Result

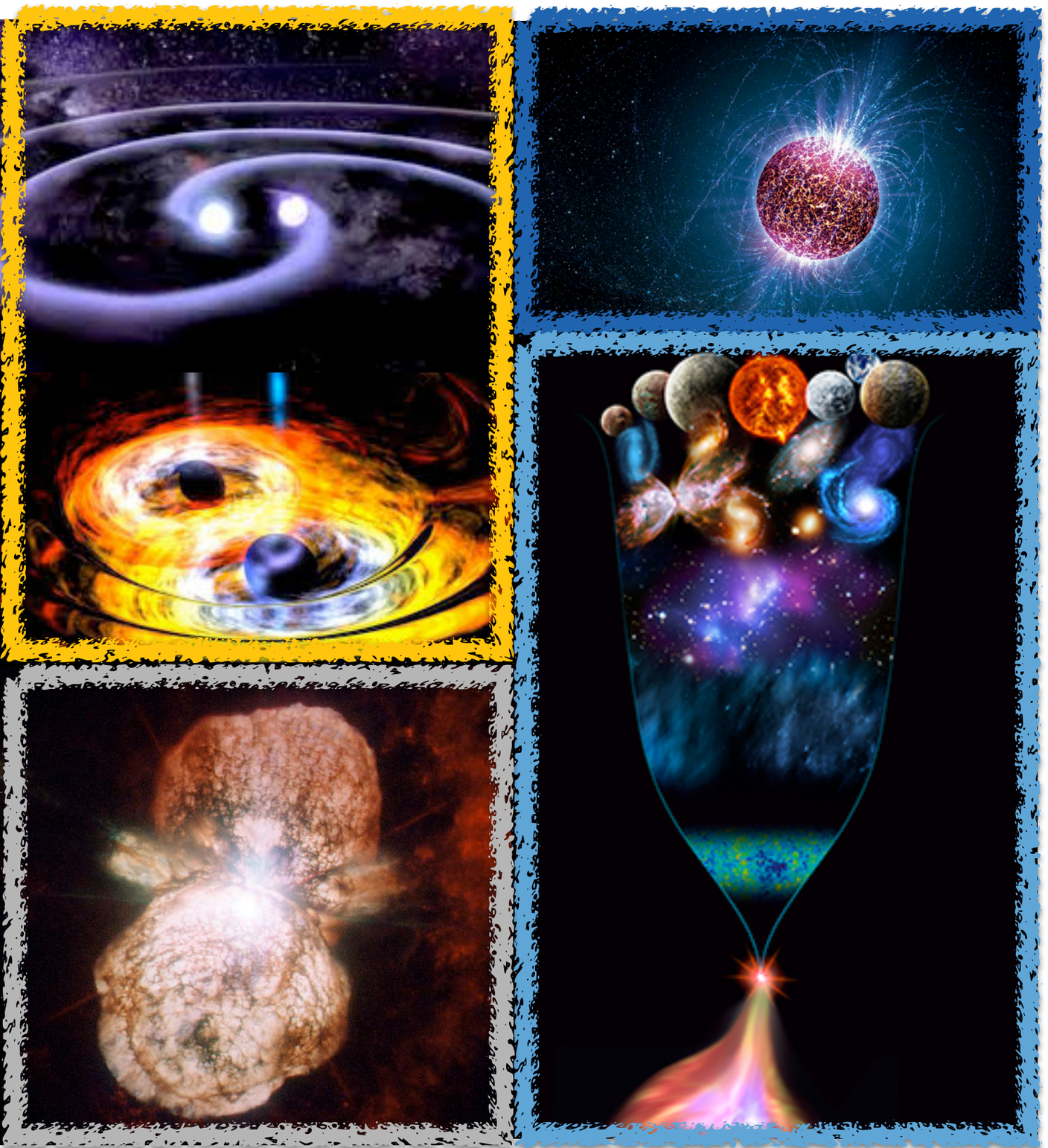


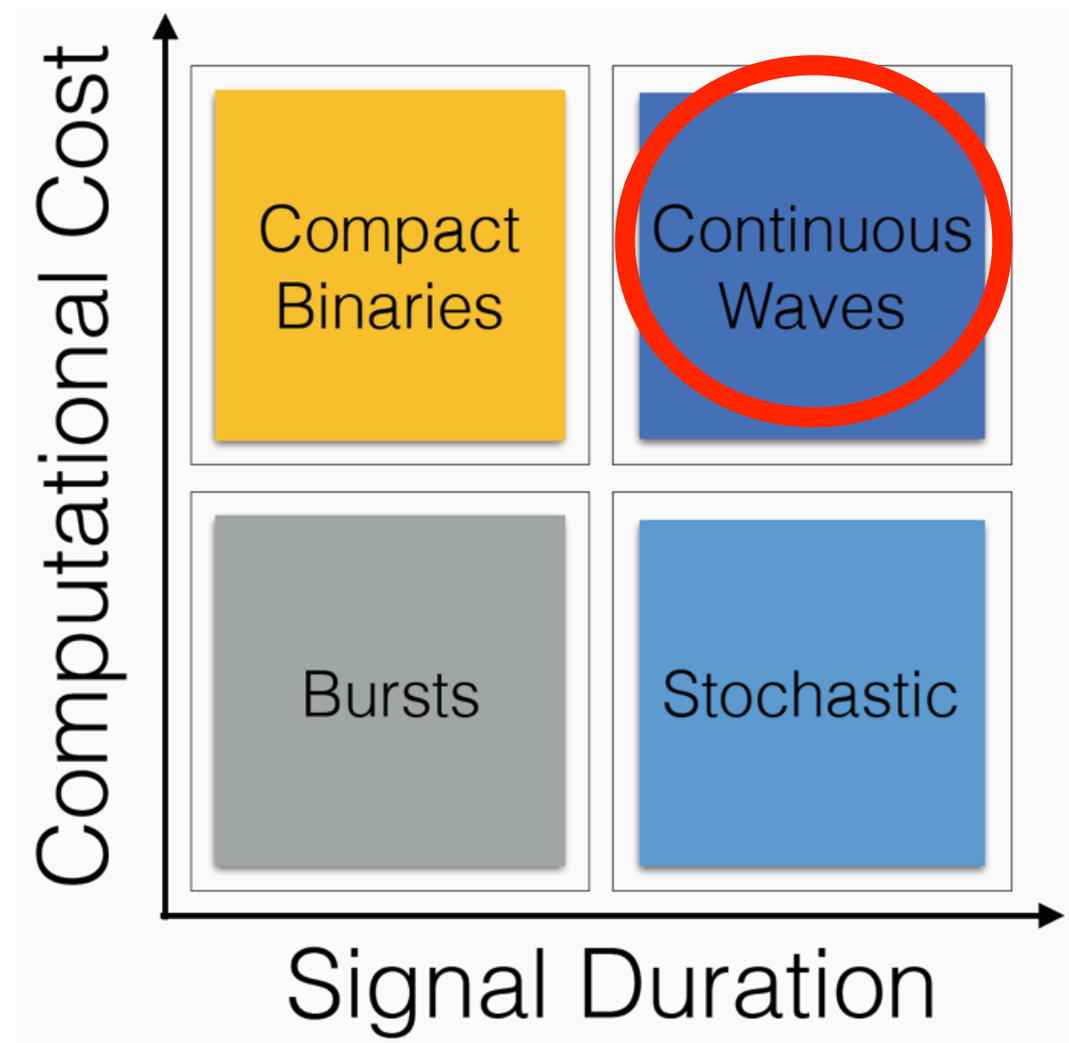


LVC, PRL 116, 061102 (2016)

The first gravitational wave was found by a burst pipeline, within 3 minutes!

Gravitational-wave Sources

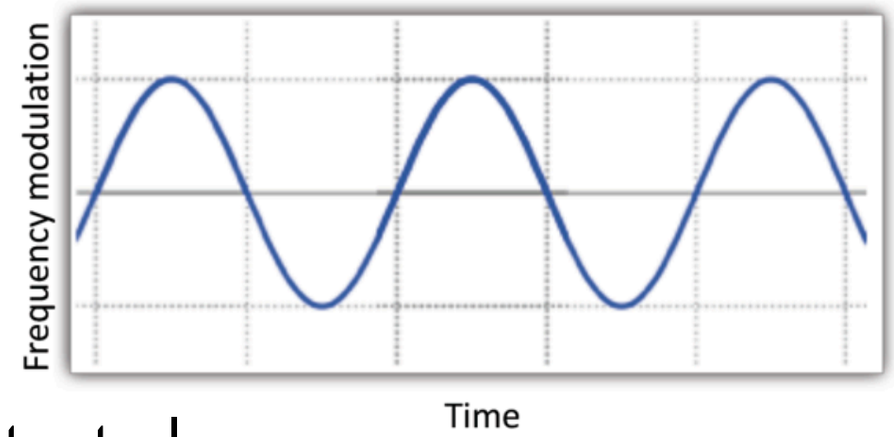




Types of CW Searches

- **Targeted searches** for known pulsars: known position, rotation frequency and spin down
- **Directed searches** in known position: SNRs, galactic center, accreting neutron stars in low-mass x-ray binaries
- **All-sky searches** for isolated, unknown sources

The Continuous Wave from an Isolated NS

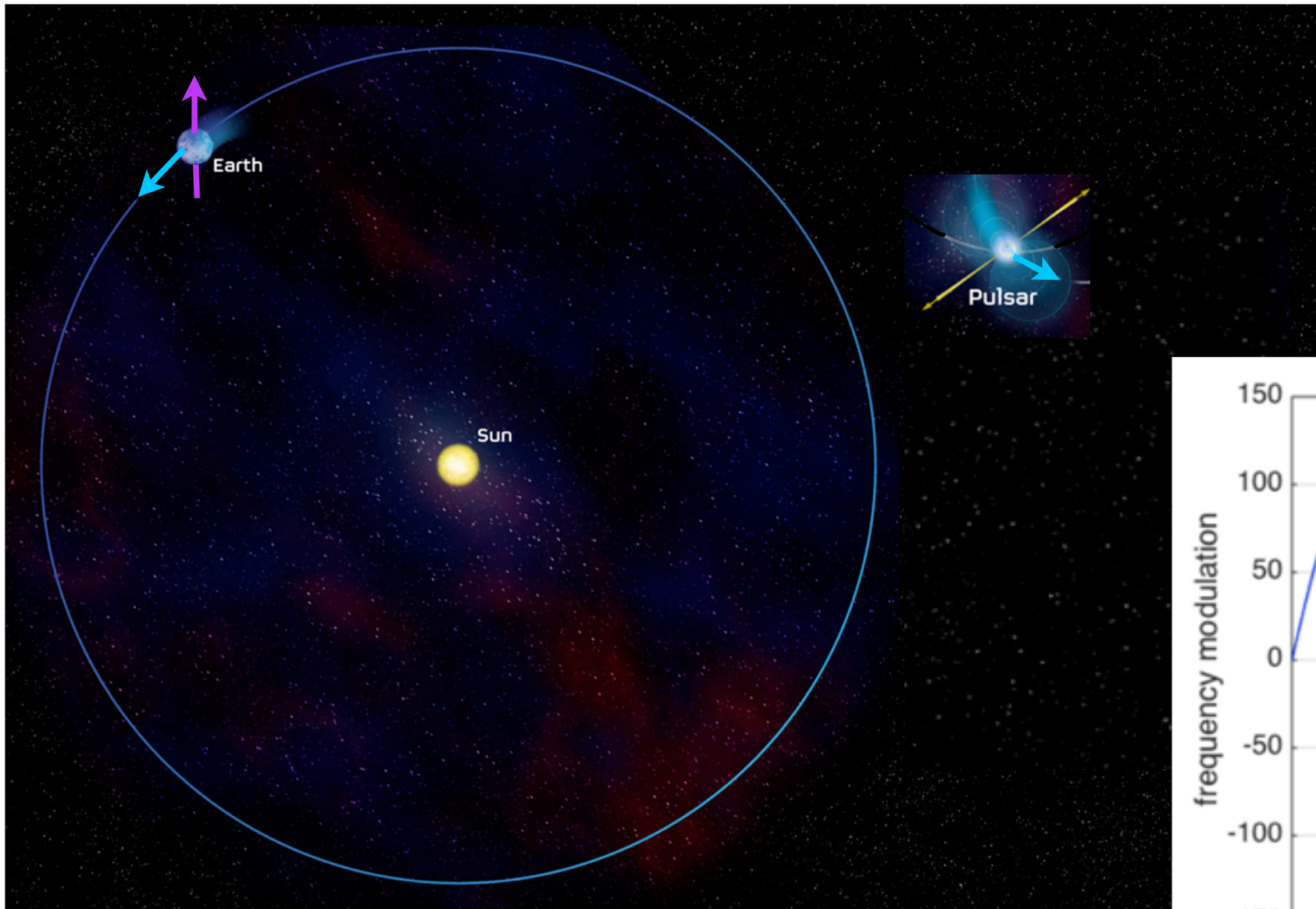


- The source should emit a nearly monochromatic sinusoidal wave
- Limit on observation comes from total available observation time
- But the detector will see a modified signal

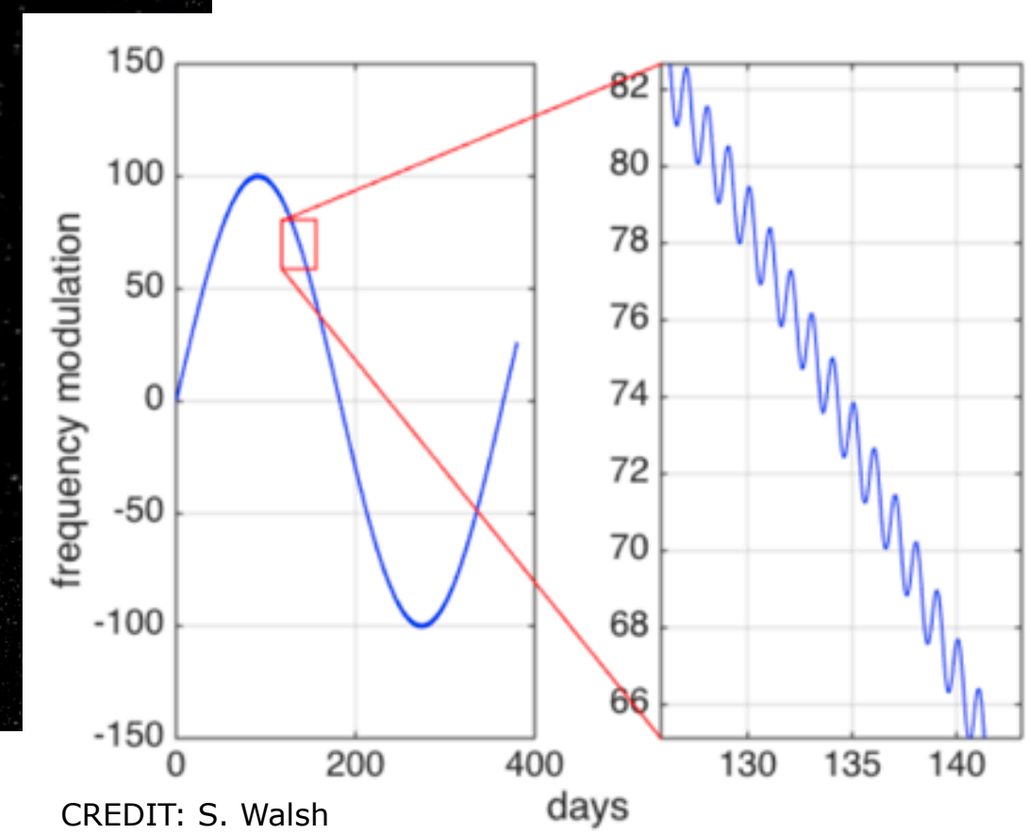
$$\vec{\mathcal{A}} = \{f, \dot{f}, \ddot{f}, \ldots, \alpha, \delta, h_0, \cos\iota, \psi, \phi_0\}$$

  - Four phase evolution parameters
  - Four amplitude parameters

Doppler Effect Due to Earth Rotation and Orbit



CREDIT: Bill Saxton, NRAO/AUI/NSF



CREDIT: S. Walsh

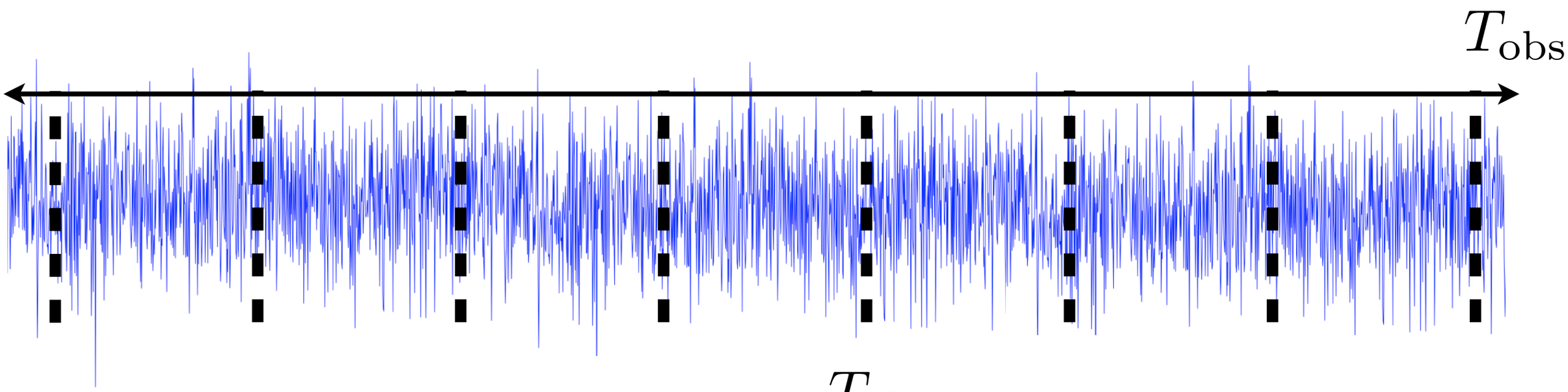
The All-sky Search Goal

$$\mathrm{SNR} \propto \frac{h_0}{\sqrt{S_n}}\sqrt{T}$$

- Search over broad frequency, spin-down range over whole sky over many months of data
- Not really computationally feasible
- Solution: use semi-coherent methods

Semi-coherent Methods

Split data into short segments which are analyzed coherently

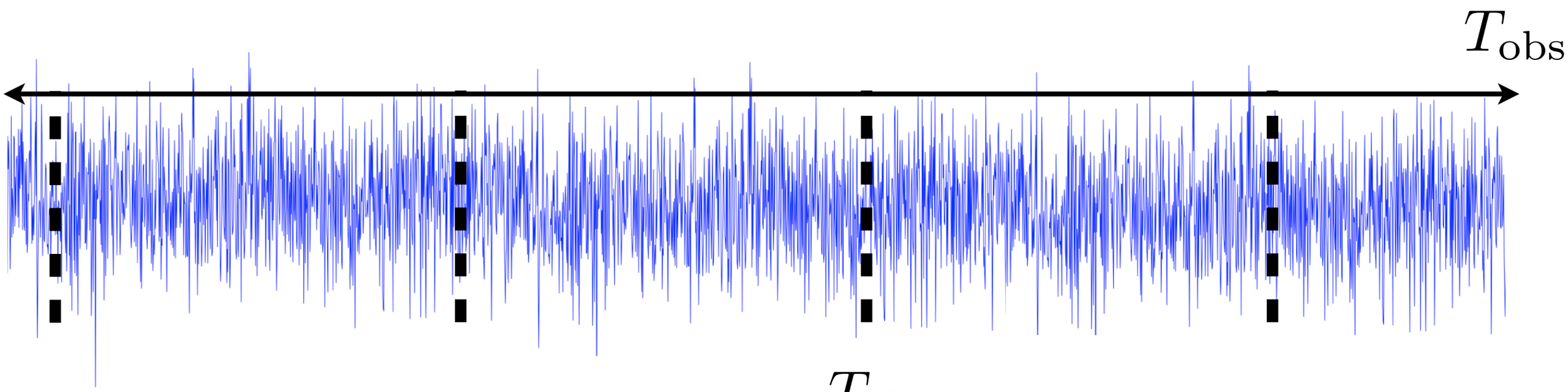


$$\Delta T = \frac{T_{\rm obs}}{N}$$

Short (~1000s): signal stays in single Fourier bin

Semi-coherent Methods

Split data into short segments which are analyzed coherently



$$\Delta T = \frac{T_{\rm obs}}{N}$$

Long (~hrs/days): need to account for signal modulation within segment

F-Statistic

$$\Lambda(x) \equiv \max\mathcal{L}\left(x; \mathcal{A}\right) = \exp^{\mathcal{F}(x)}$$

Maximize with respect to four unknown amplitude parameters:

$$\{h_0, \cos\iota, \psi, \phi_0\}$$

Dimension of parameter space: 8 to 4

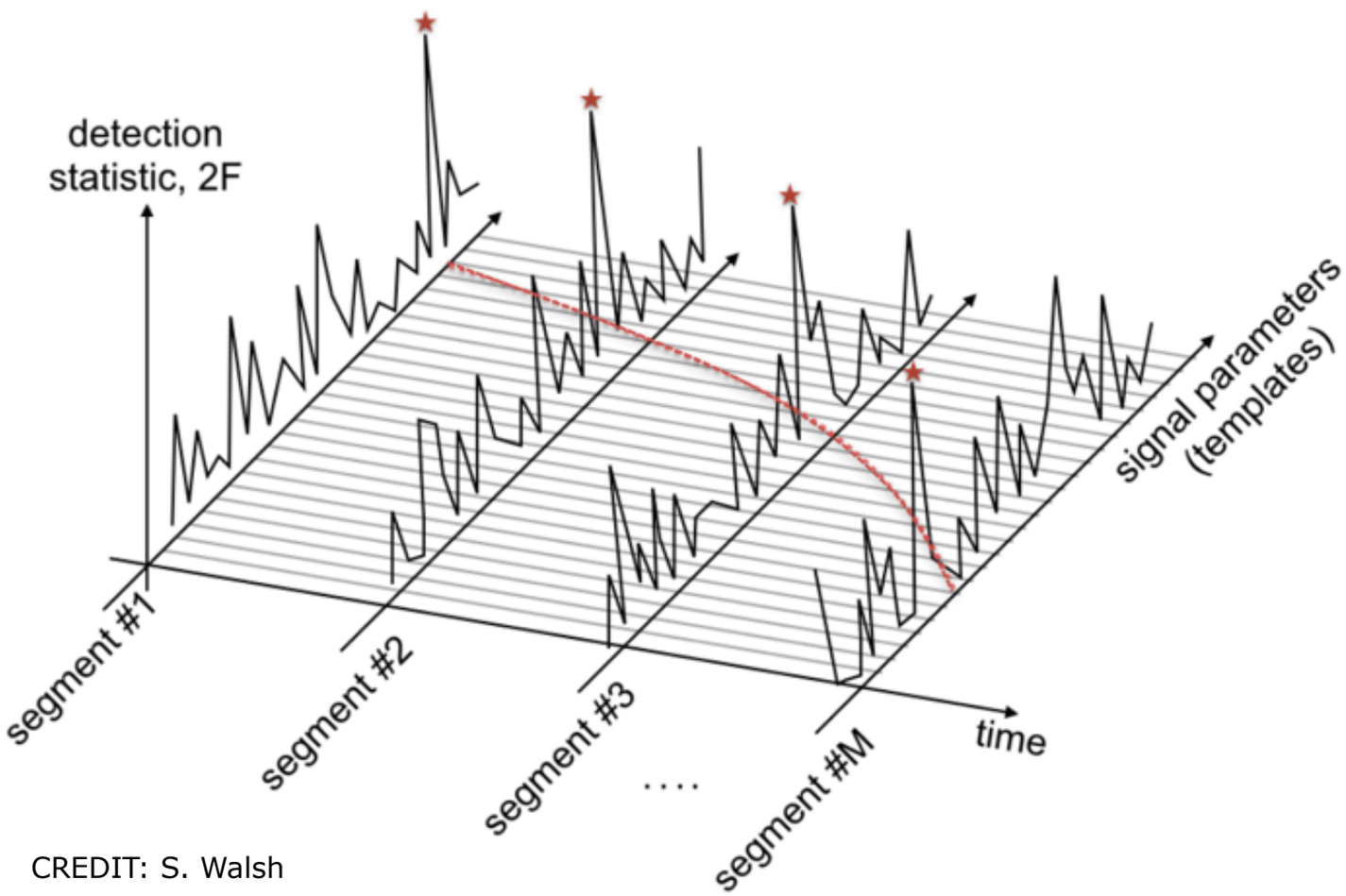
Under the assumption of white detector noise, F-statistic will look like

$$\mathcal{F} \approx \frac{2}{\sigma^2}\left(\frac{|F_a|^2}{\langle a^2\rangle} + \frac{|F_b|^2}{\langle b^2\rangle}\right)$$

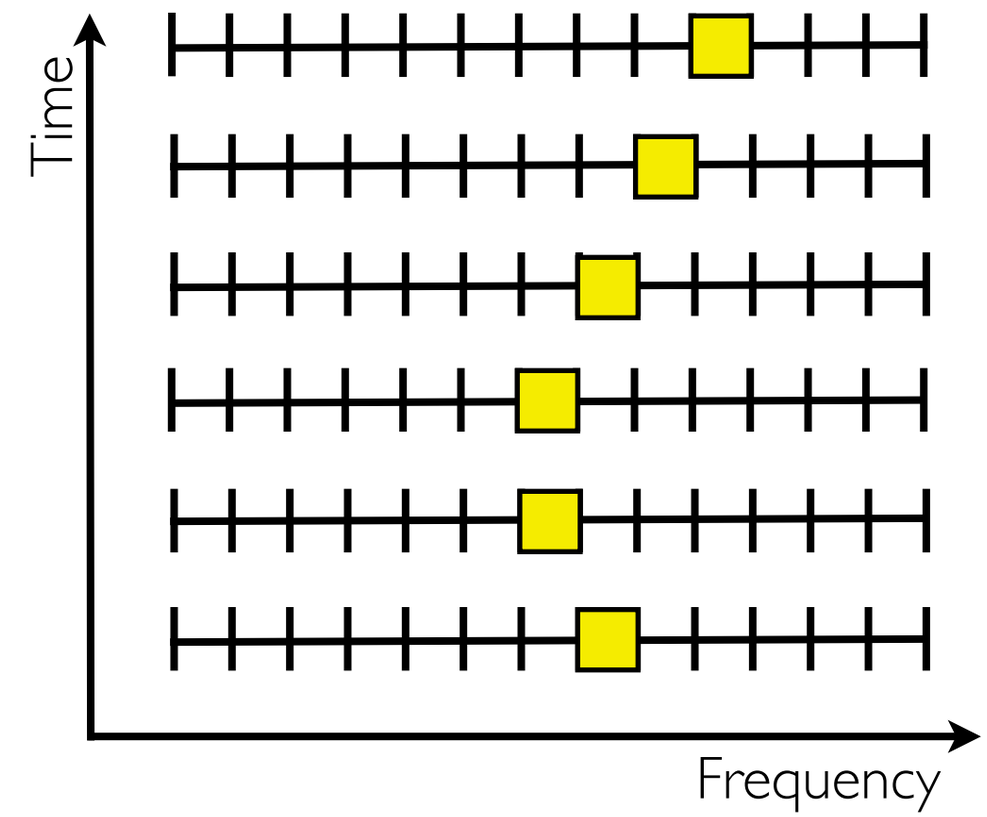
Variance of the data

Functions incorporating the amplitude modulations that depend on location and orientation of detector on the Earth, position of GW source in sky, periodic functions of time

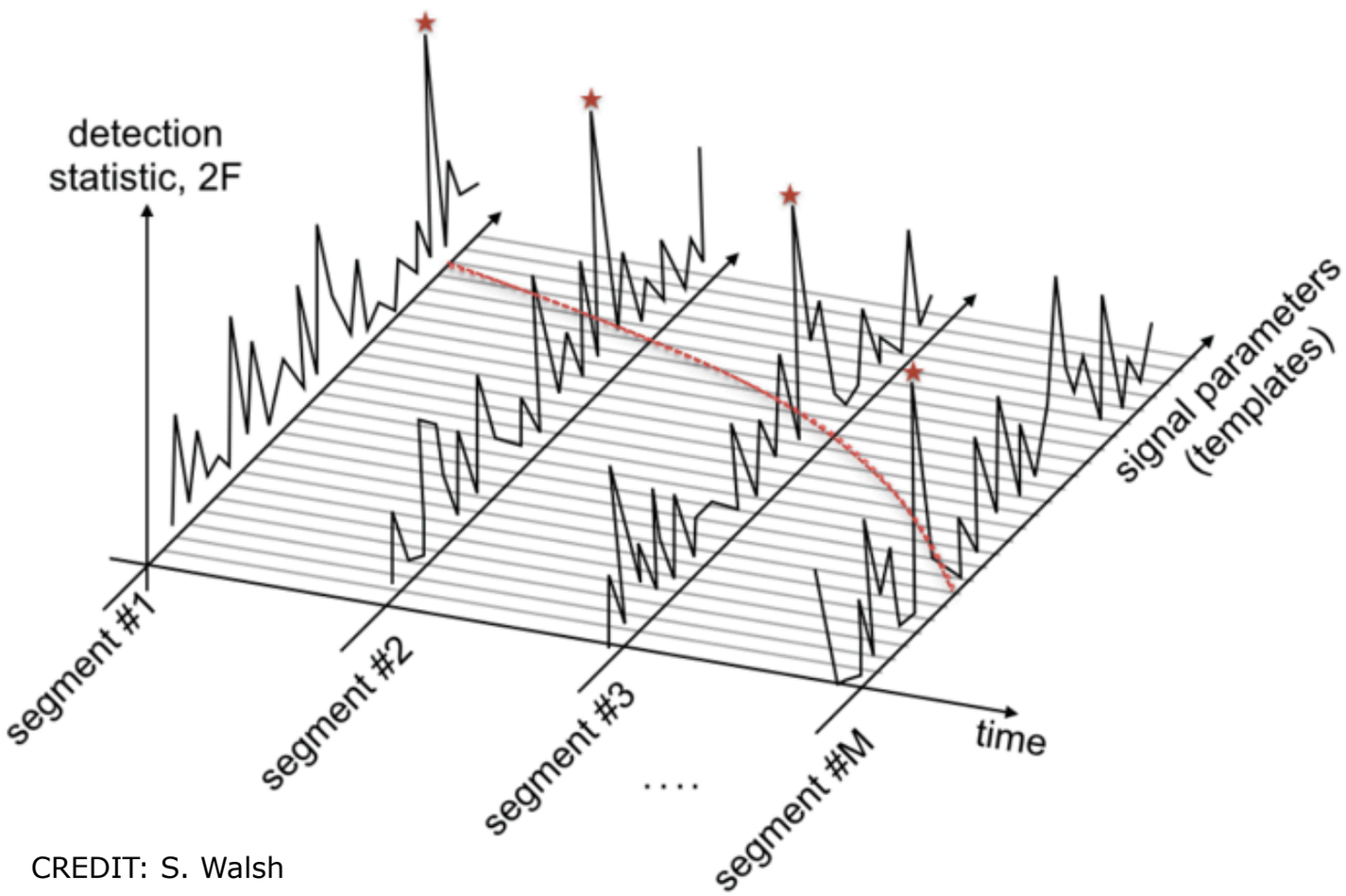
Stack-Slide Method



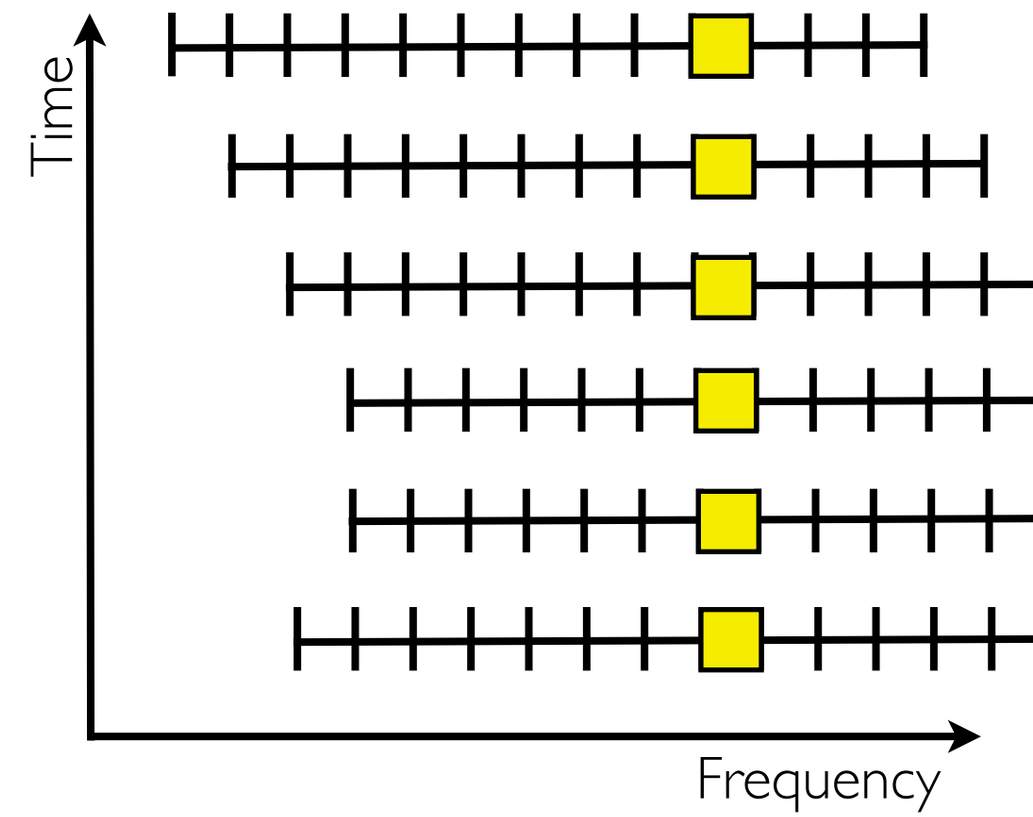
CREDIT: S. Walsh



Stack-Slide Method

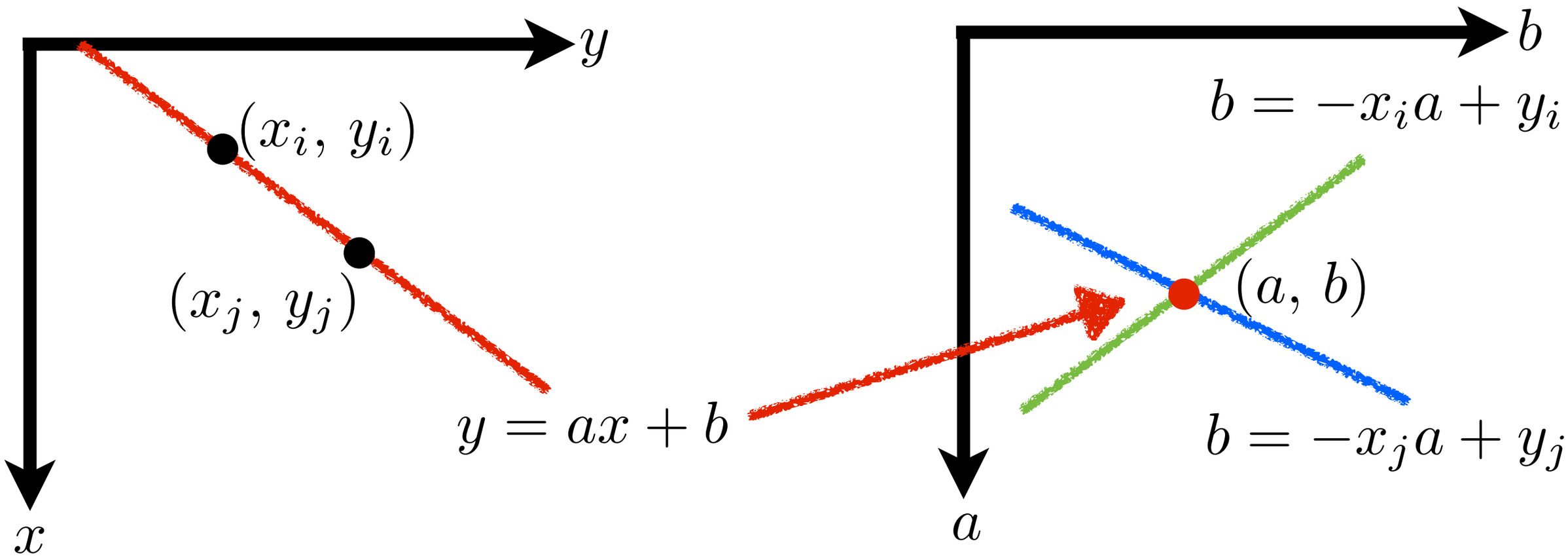


CREDIT: S. Walsh

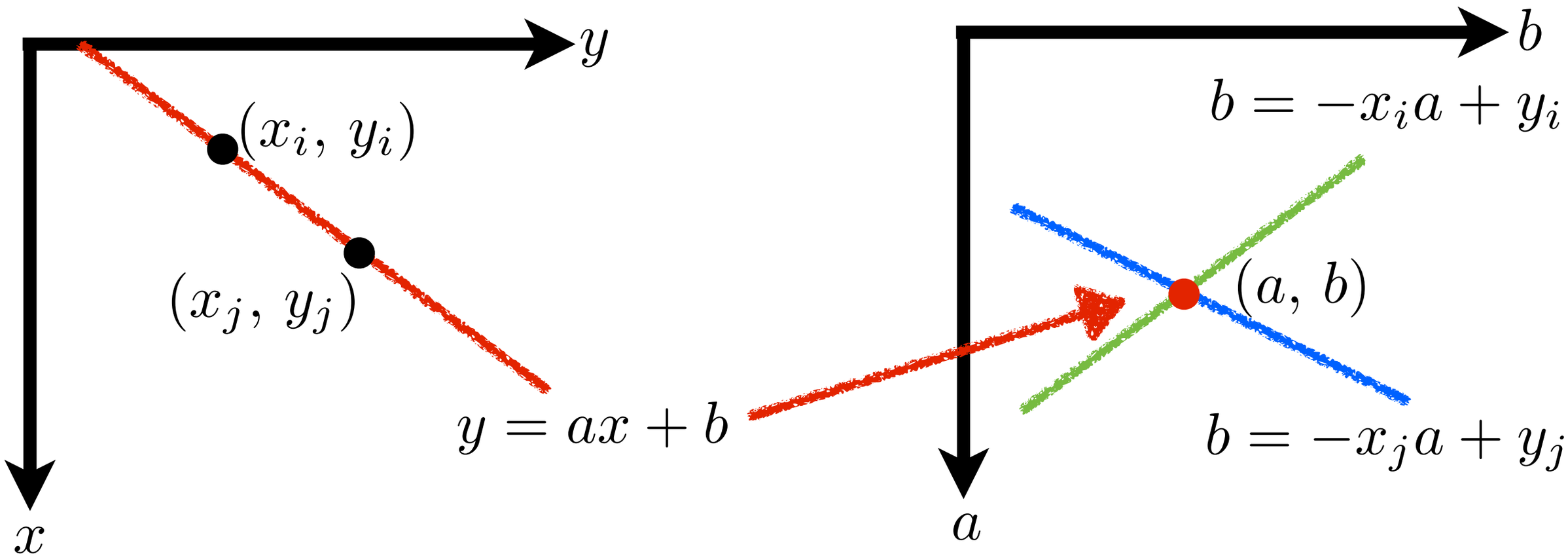


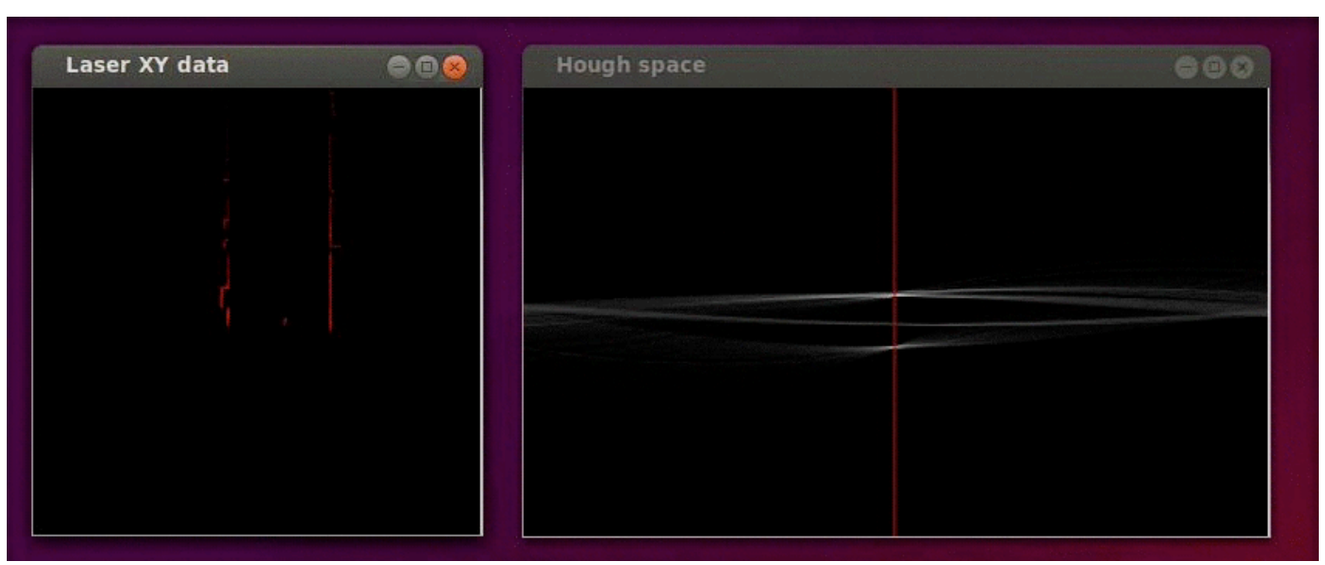
Add power after frequency bins are shifted.

Hough Transform Method



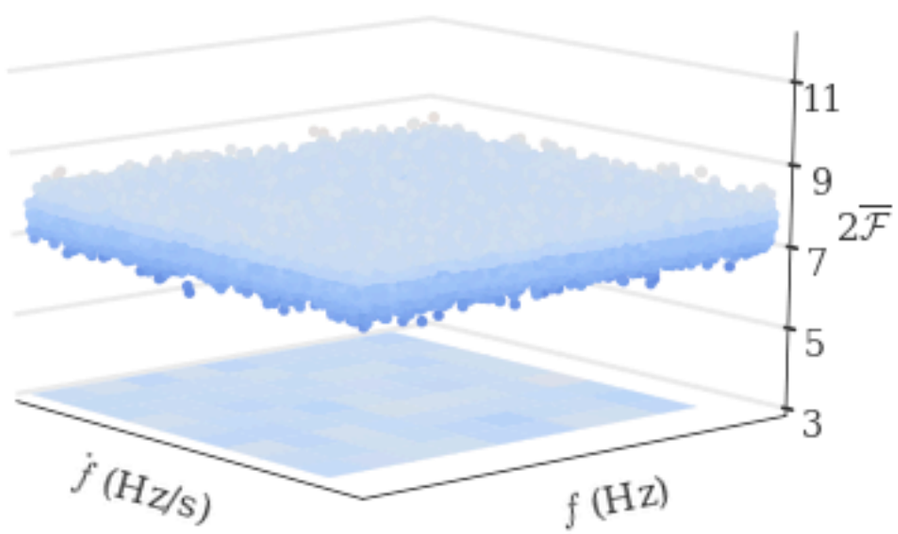
Hough Transform Method

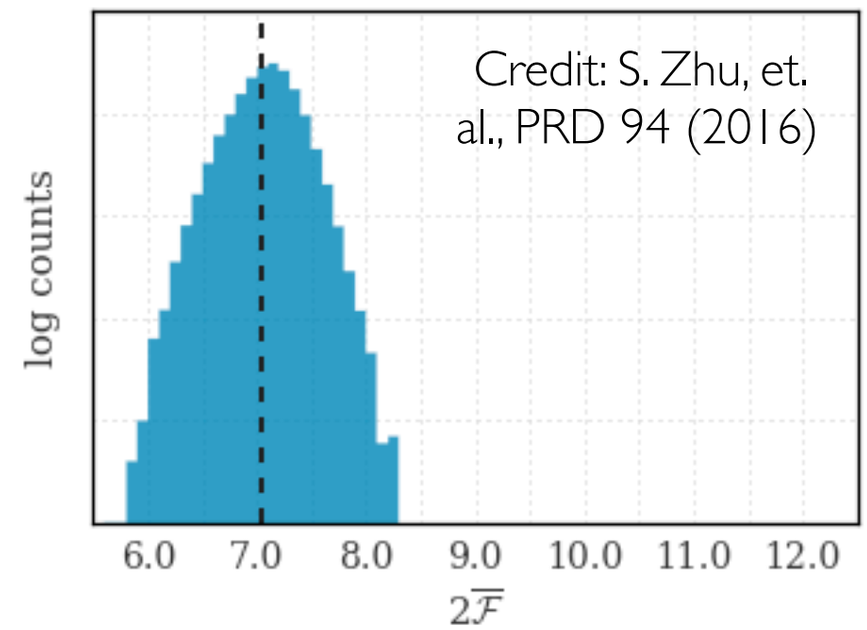


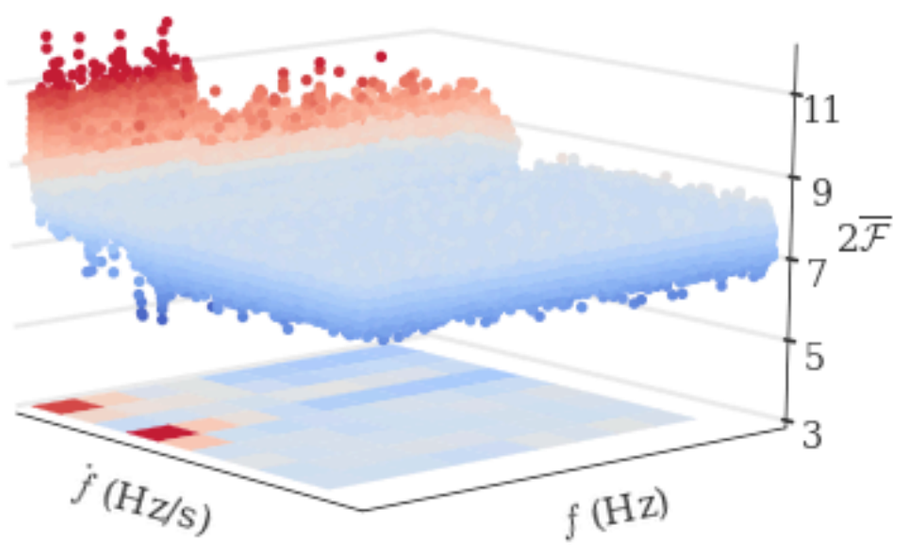


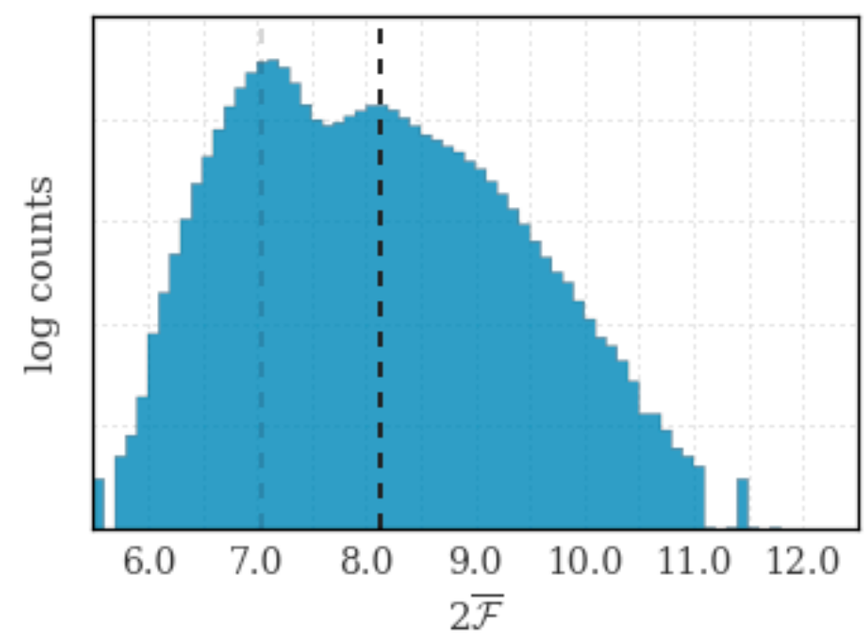
Credit: Cornelissen, et. al.

Identify Interesting Frequency Bands



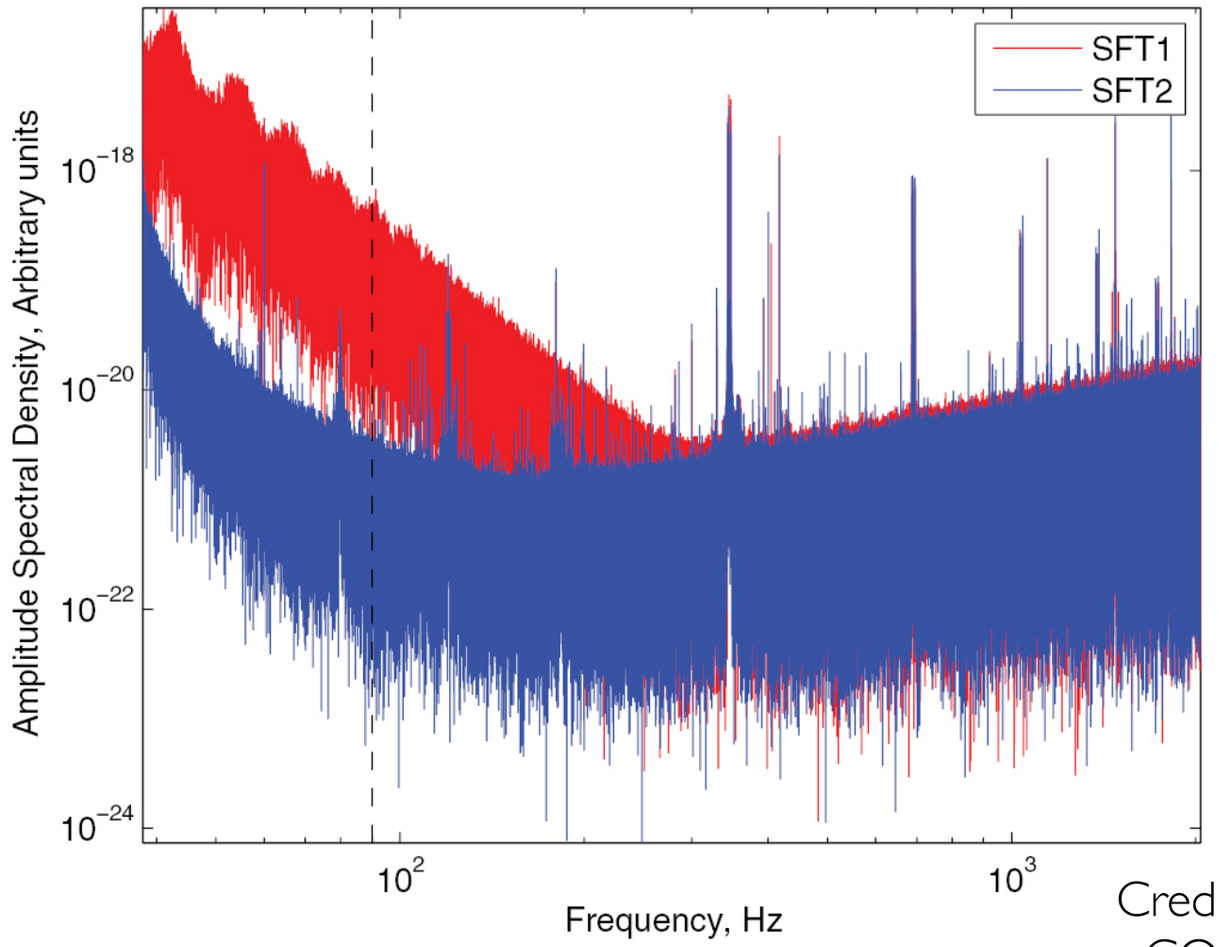






Identify 50 mHz bands disturbed by detector artifacts.
Maximum density and mean 2F value are metrics to identify disturbed bands.

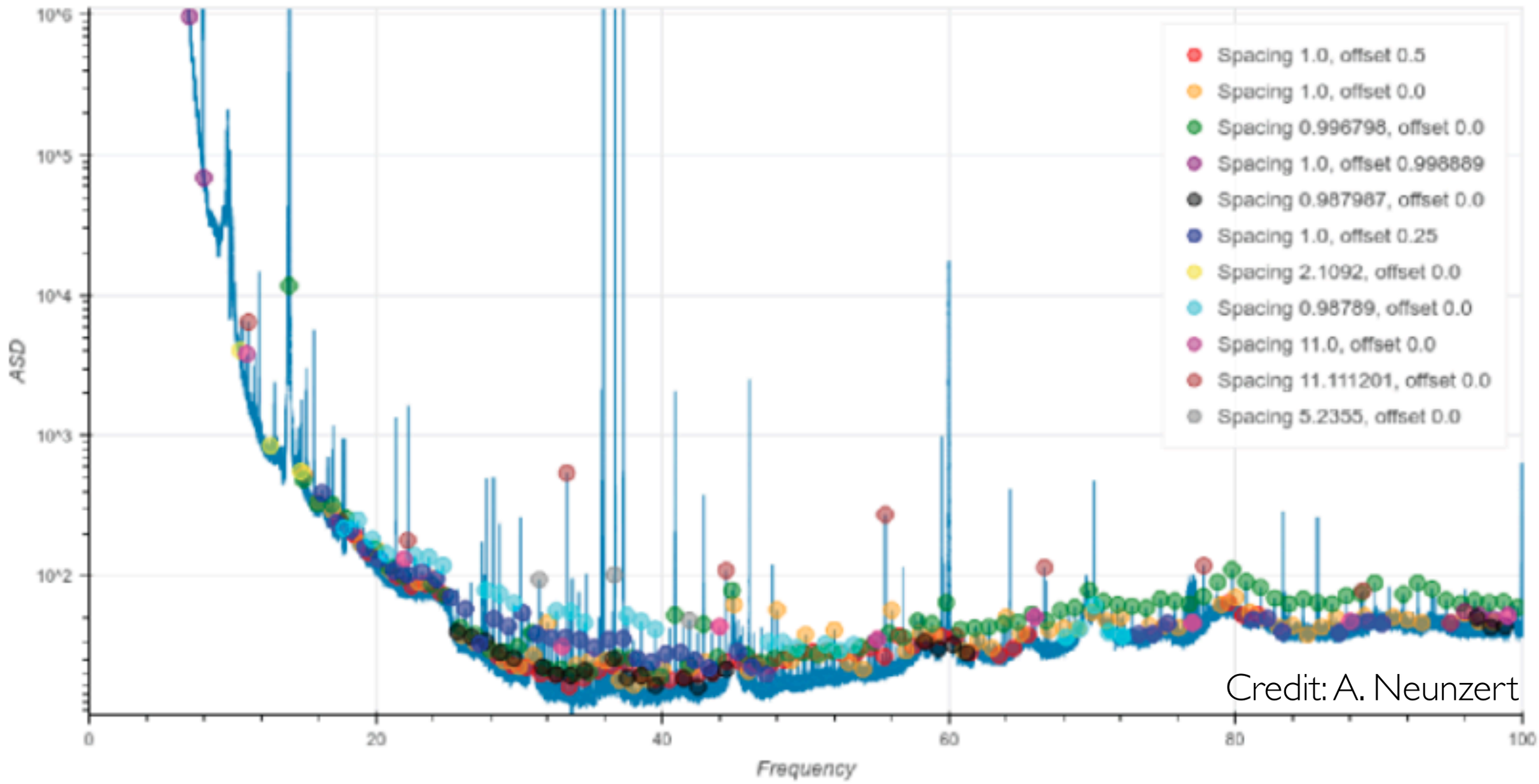
Contamination in CW Searches



Credit: Davies, et al
CQG 34 (2016)

Two power spectra, one with an unusually high low-frequency noise contribution up to 300 Hz.

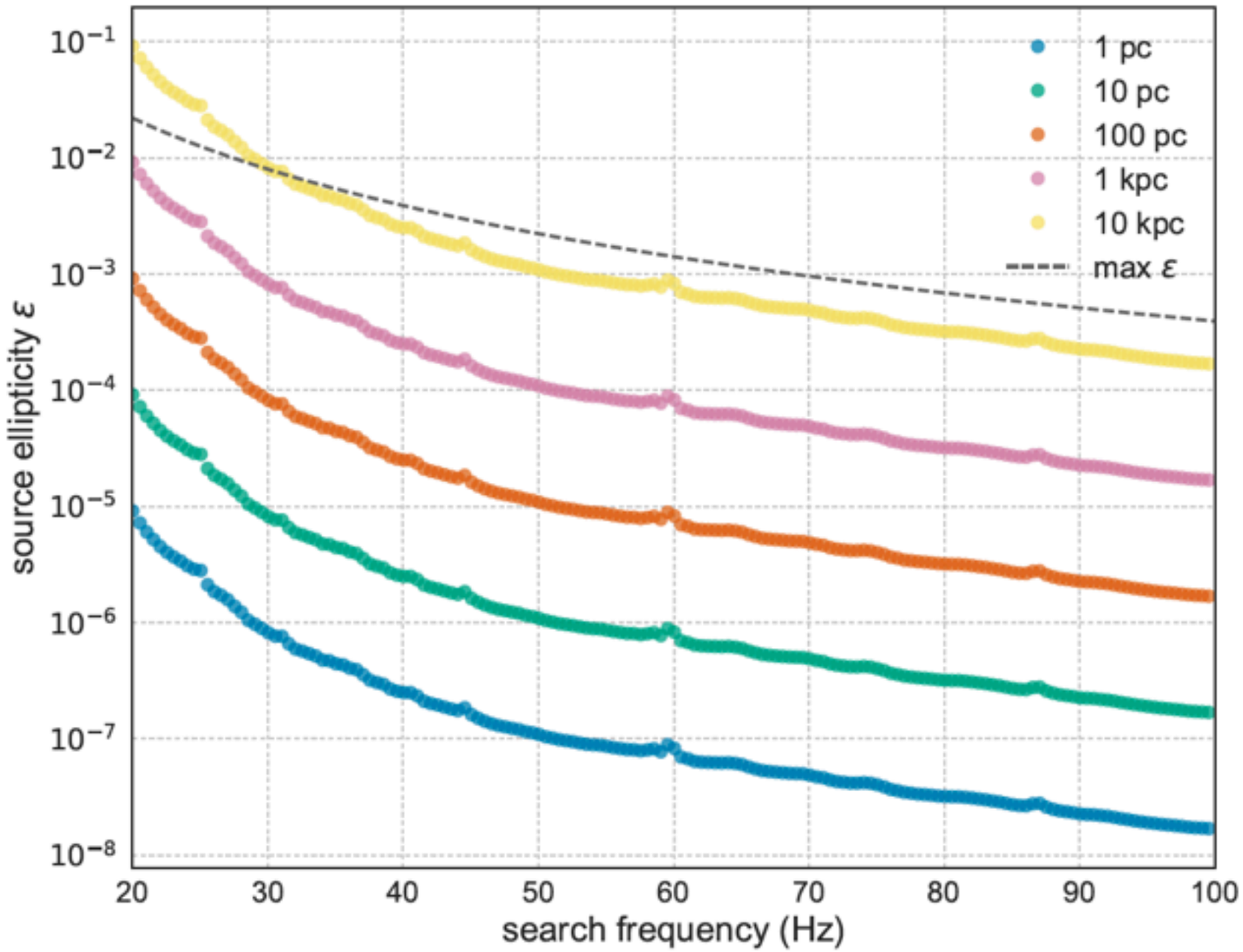
Contamination in CW Searches



Harmonics of single-frequency noise source create comb-like patterns in noise spectrum.

If not flagged, searches can identify them as possible sources.

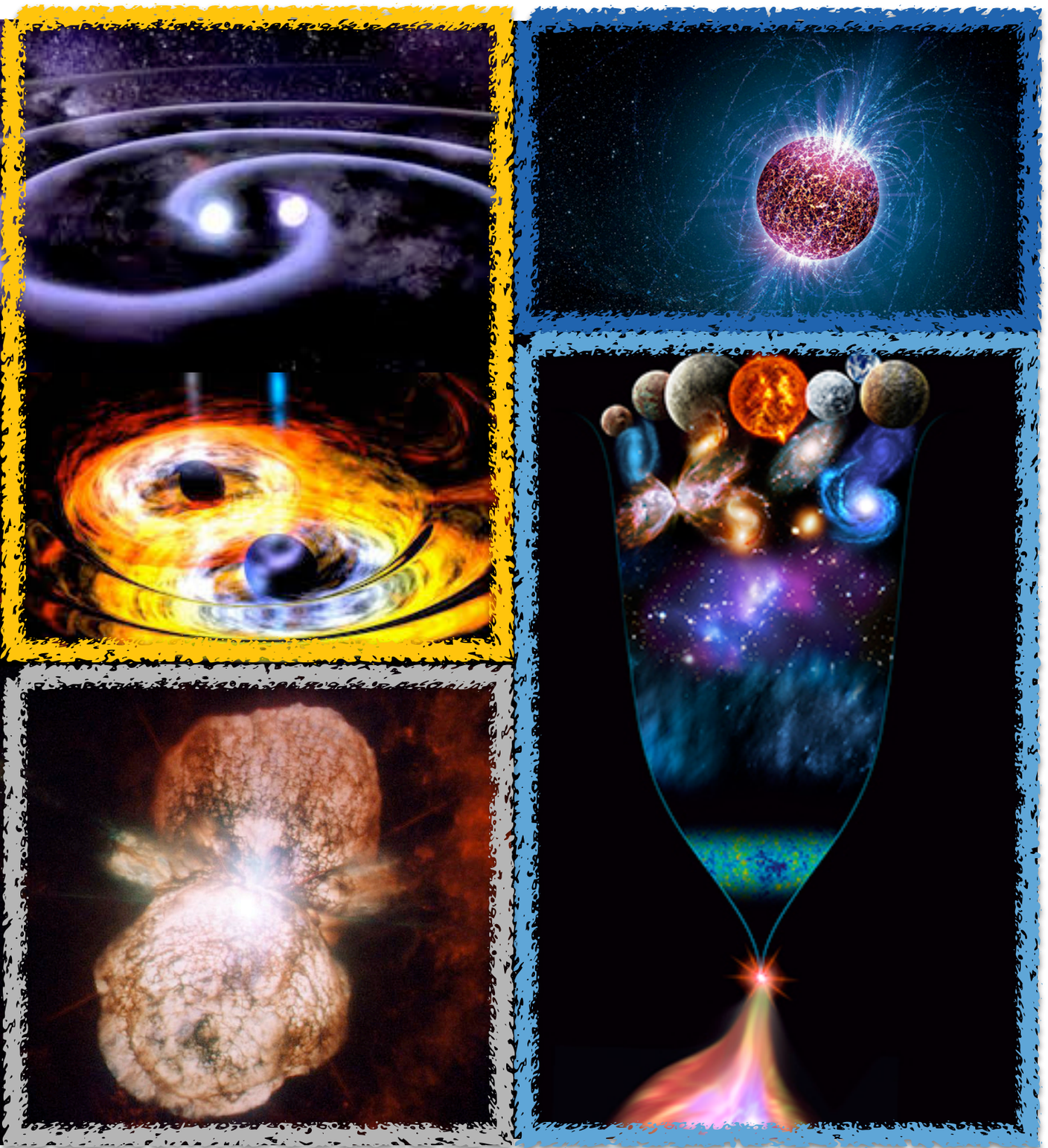
A CW Result

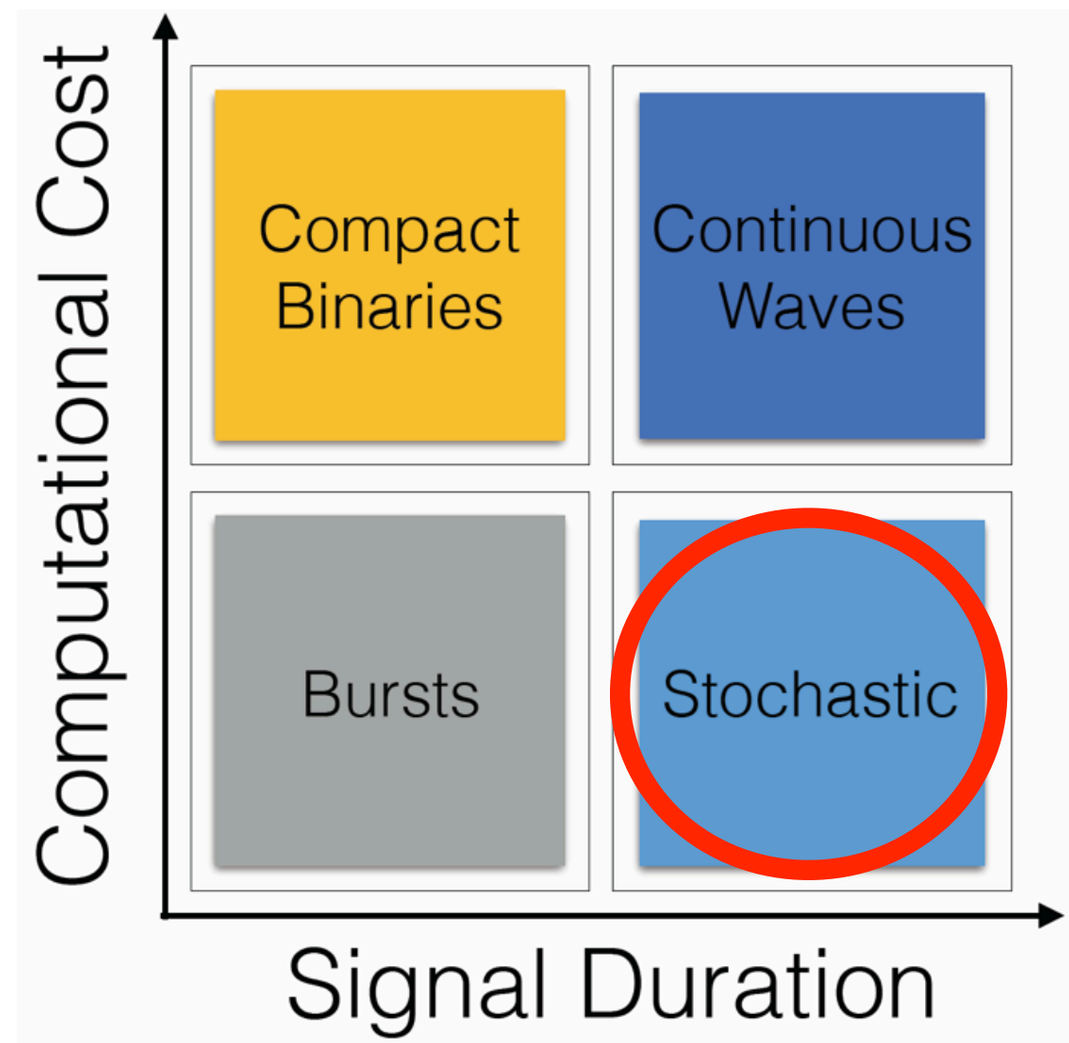


LVC, arXiv: 1707.02669

At 55Hz, the O1 all-sky search can exclude sources with ellipticity above 10^{-5} within 100pc.

Gravitational-wave Sources





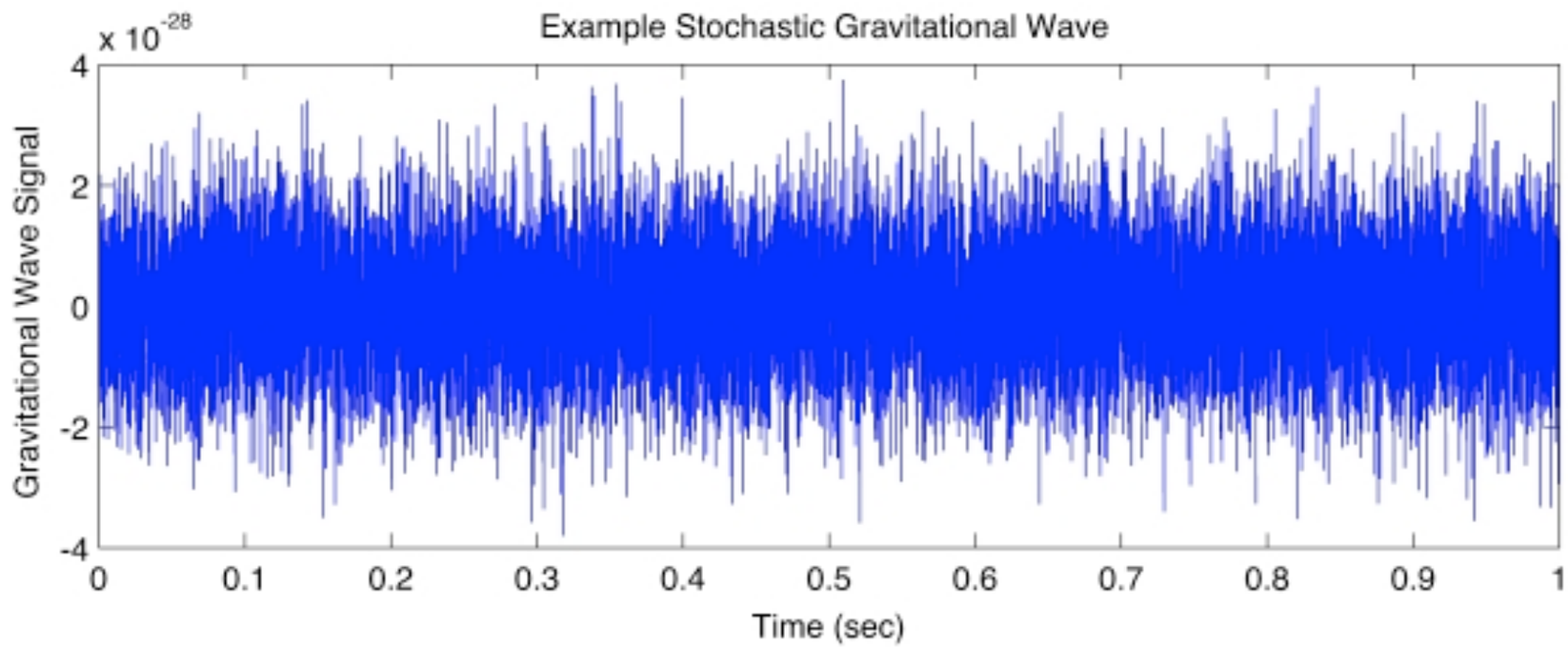
What is a stochastic background?

- Stochastic (random) background of gravitational radiation

- Can arise from superposition of large number of unresolved GW sources
  1. Cosmological origin
  2. Astrophysical origin

- Strength of background measured as gravitational wave energy density ρ_{GW}

Cosmological Gravitational Wave Background



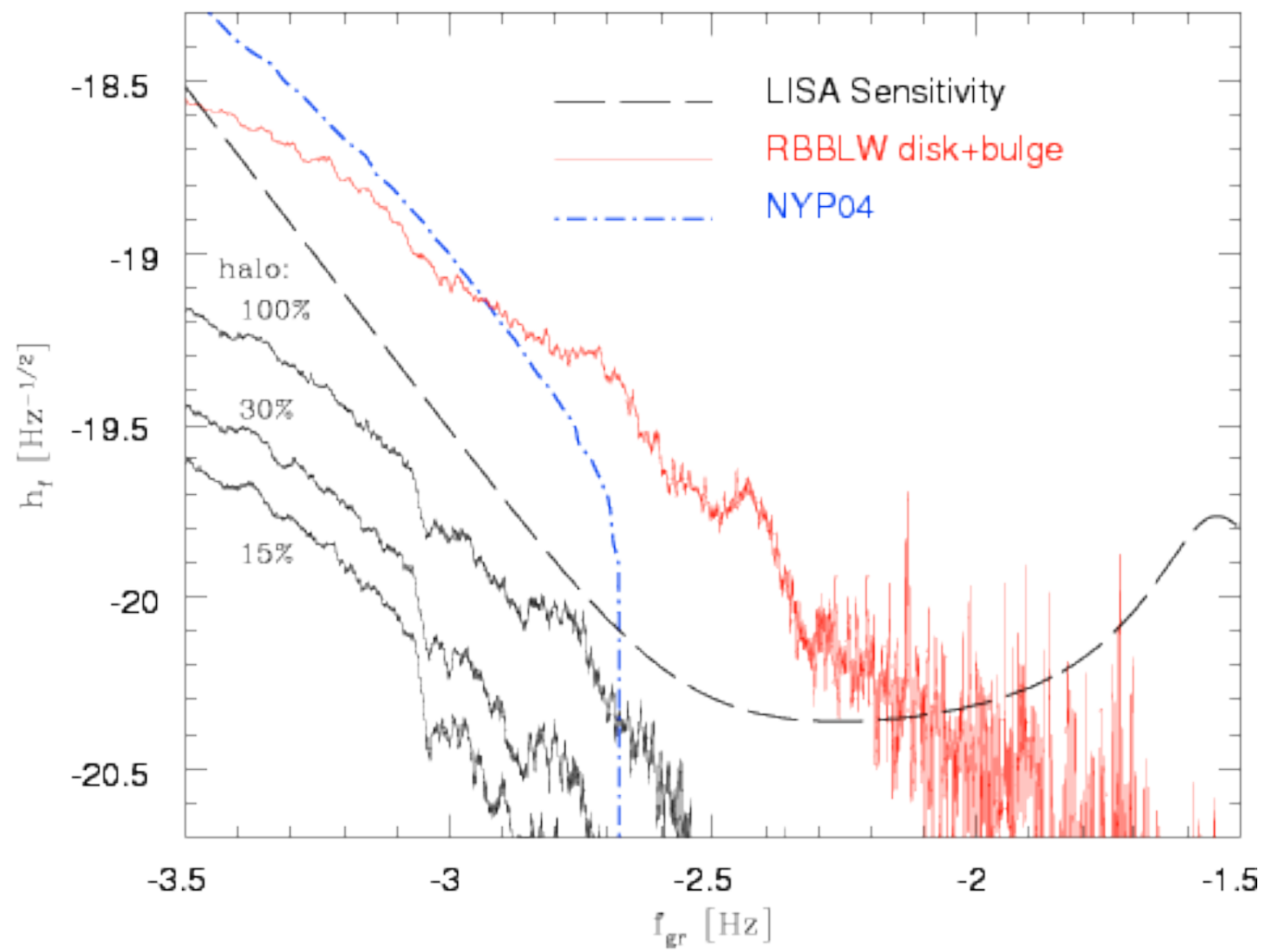
GW spectrum: $\Omega_{\mathrm{GW}}(f) = \frac{f}{\rho_c}\frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}f}$

Critical energy density of universe: $\rho_c = \frac{3c^2 H_0^2}{8\pi G}$

Astrophysical Gravitational Wave Backgrounds

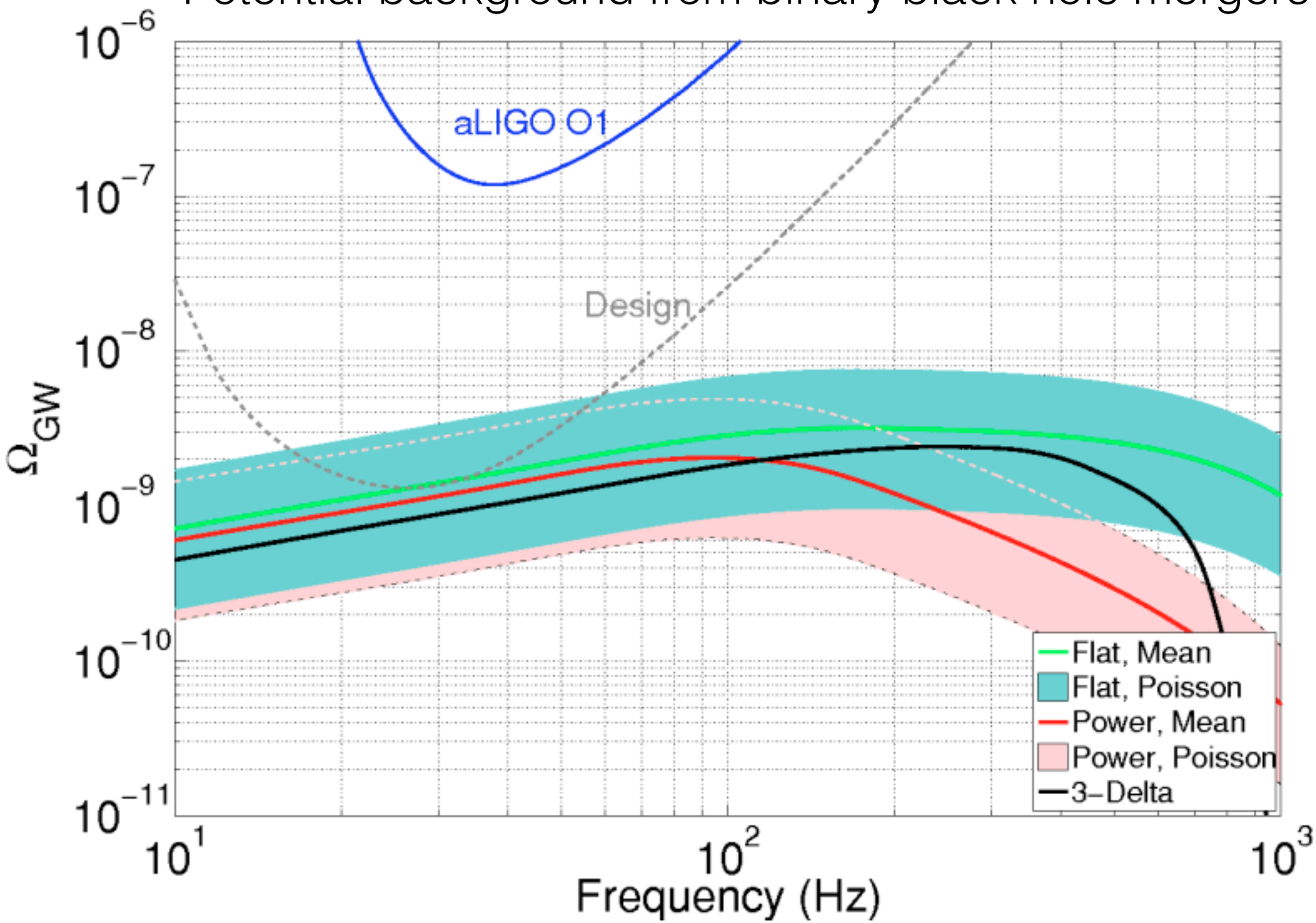
- Produced by an extremely large number of weak, independent, and unresolved gravity-wave sources
  - Binary black holes and/or neutron stars
  - Supernovae
  - White dwarf binaries

For example, the expected contribution from double white dwarfs for LISA



Astrophysical Gravitational Wave Backgrounds

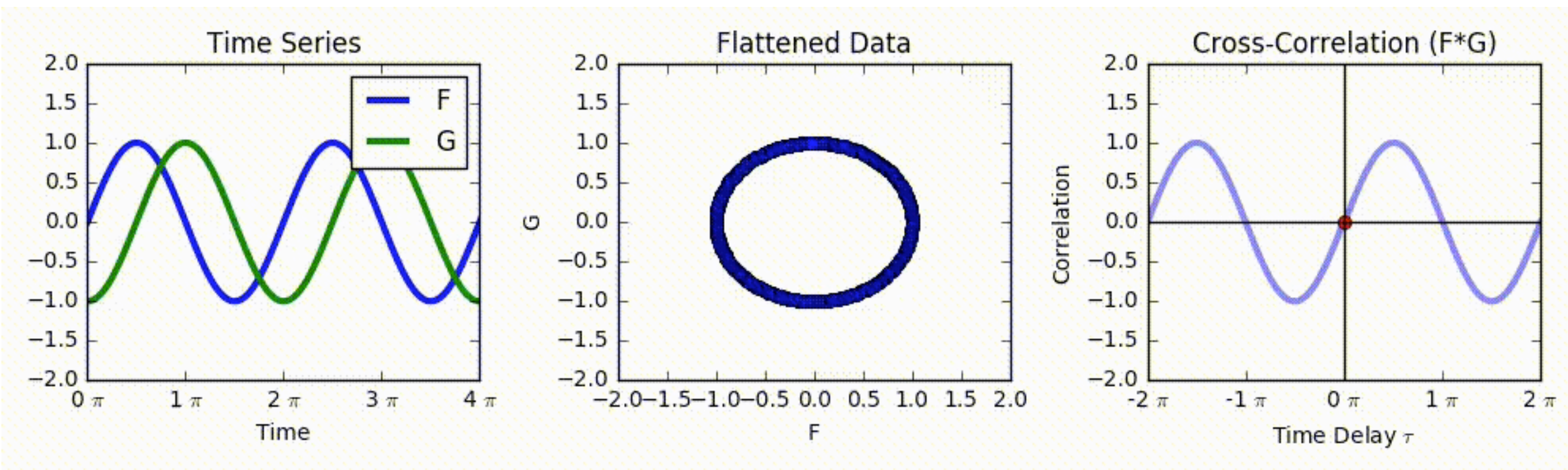
Potential background from binary black hole mergers



Detecting Stochastic Backgrounds

Cross-correlation of two interferometers' data streams multiplied by a filter function:

$$Y = \int_{-\infty}^{+\infty} df \int_{-\infty}^{+\infty} df' \delta_T(f - f') \tilde{s}_1(f)^* \tilde{s}_2(f') \tilde{Q}(f')$$



Detecting Stochastic Backgrounds

The filter function has the form:

$$\tilde{Q}(f) = N \frac{\gamma(f)\Omega_{\mathrm{GW}}(f)H_0^2}{f^3 P_1(f) P_2(f)}$$

overlap reduction function: $\gamma(f)$

power law template for GW spectrum: $\Omega_{\mathrm{GW}}(f) = \Omega_\alpha \left(f/100\,\mathrm{Hz}\right)^\alpha$

present value of Hubble parameter: H_0

noise in detector 1: $P_1(f)$

noise in detector 2: $P_2(f)$

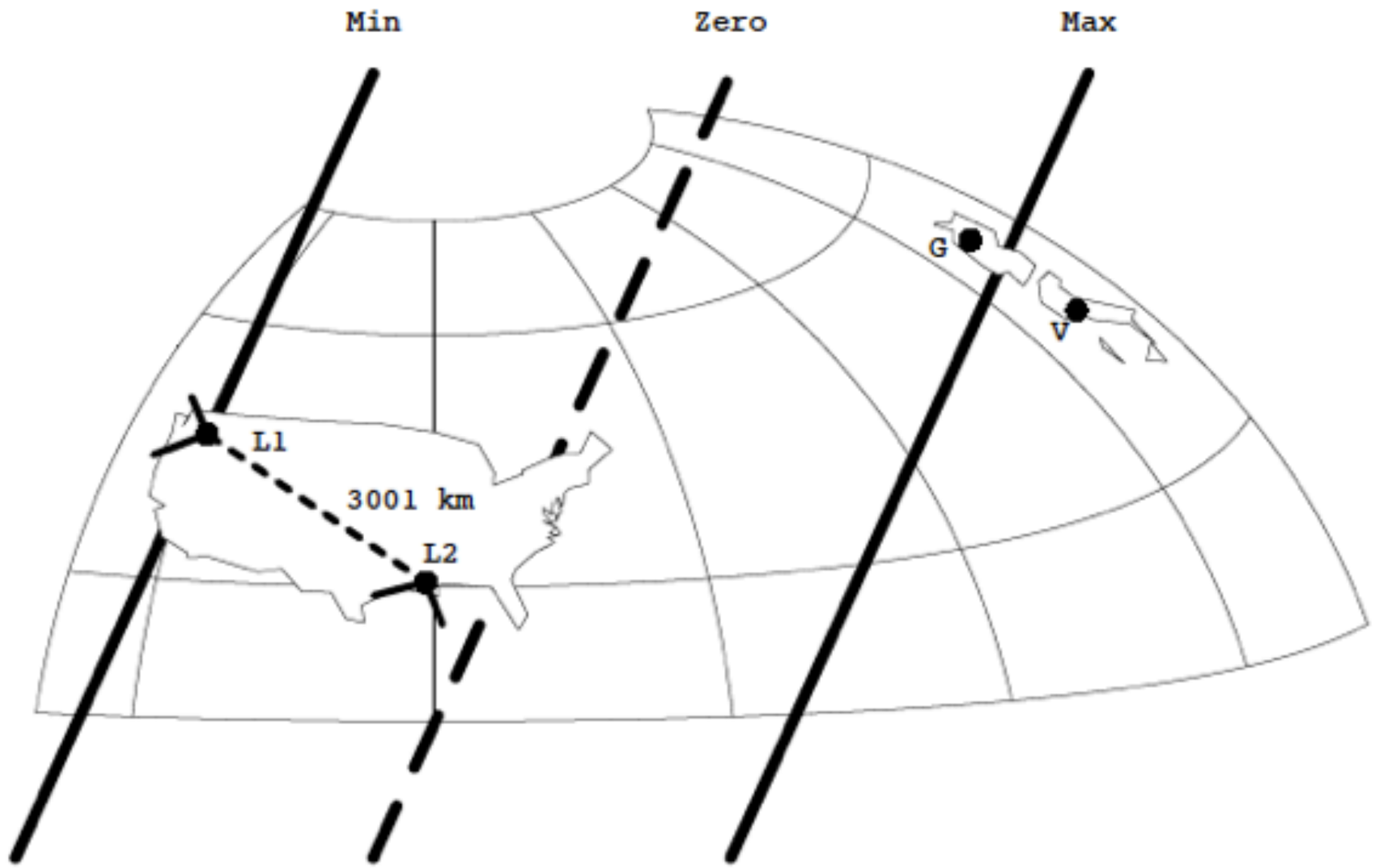
Purpose: Enhance SNR at frequencies where signal is strong and suppress SNR at frequencies where detector noise is large.

Overlap Reduction Function

Signal in two detectors will not be exactly the same because:

i) time delay between detectors

ii) non-alignment of detector



Get involved...

Einstein@Home uses your computer's idle time to search for weak astrophysical signals from spinning neutron stars (often called pulsars) using data from the LIGO gravitational-wave detectors, the Arecibo radio telescope, and the Fermi gamma-ray satellite.

Gravity Spy - citizen science project for identifying and classifying glitches