<u>Announcements</u>

Please turn in Assignment 3 and pick up Assignment 4 You can also email assignments to the TAs: Ka Wa Tsang (<u>kwtsang@nikhef.nl</u>) Pawan Gupta (<u>p.qupta@nikhef.nl</u>)

Did you receive my email last week?

Reminder: Visualization Project due May 21

Gravitational Wave Data Analysis

Lecture 4: Gravitational Waves MSc Course

Output of GW detector

Input of detector has the form $h(t) = D^{ij}h_{ij}(t)$

 D^{ij} is the detector tensor; depends on detector geometry.

Output of detector is related to input by $\tilde{h}_{out} = T(f)\tilde{h}(f)$ T(f) is the transfer function.

However, output of any real detector will have noise, so output will really be given by

$$s_{\text{out}}(t) = h_{\text{out}}(t) + n_{\text{out}}(t)$$
$$s(t) = h(t) + n(t)$$

Detector noise

To compare detector performances, we use the detectors' noise n(t).

If noise is stationary, then the different Fourier components are uncorrelated. Then the ensemble average of the Fourier components of the noise is of the form:

$$\langle \tilde{n}^*(f)\tilde{n}(f')\rangle = \delta(f-f')\frac{1}{2}S_n(f)$$

Ensemble average can be replaced by time average:

$$\left< |\tilde{n}(f)|^2 \right> = \frac{1}{2} S_n(f) T \qquad \Delta f = \frac{1}{T}$$

T is time of experiment.

Noise spectral density

 $S_n(f)$ is the noise spectral density (aka noise spectral sensitivity or noise power spectrum):





Detector Pattern Functions

Polarization tensors where $\hat{\mathbf{u}}, \hat{\mathbf{v}}$ are orthogonal to propagation direction

$$e_{ij}^{+}(\hat{\mathbf{n}}) = \hat{\mathbf{u}}_{i}\hat{\mathbf{u}}_{j} - \hat{\mathbf{v}}_{i}\hat{\mathbf{v}}_{j}$$
$$e_{ij}^{\times}(\hat{\mathbf{n}}) = \hat{\mathbf{u}}_{i}\hat{\mathbf{v}}_{j} + \hat{\mathbf{v}}_{i}\hat{\mathbf{u}}_{j}$$

In the frame where \hat{n} is along the \hat{z} direction, we can choose

$$\hat{u} = \hat{x} \qquad \hat{v} = \hat{y}$$
$$e_{ab}^{+} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}_{ab} \qquad e_{ab}^{\times} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}_{ab}$$

Detector Pattern Functions
GW with given propagation direction
$$\hat{n}$$
 is given by
 $h_{ij}(t, \mathbf{x}) = \sum_{A=+,\times} e_{ij}^A(\hat{n}) \int_{-\infty}^{\infty} df \, \tilde{h}_A(f) e^{-2\pi i f(t-\hat{\mathbf{n}}\cdot\mathbf{x}/c)}$
 $h_{ij}(t) = \sum_{A=+,\times} e_{ij}^A(\hat{n}) \int_{-\infty}^{\infty} df \, \tilde{h}_A(f) e^{-2\pi i f t}$
 $= \sum_{A=+,\times} e_{ij}^A(\hat{n}) h_A(t)$

Fold in the detector tensor:

$$h(t) = D^{ij}h_{ij}(t)$$

$$h(t) = \sum_{A=+,\times} D^{ij}e^{A}_{ij}(\hat{n})h_{A}(t)$$

Detector Pattern Functions

Detector pattern functions depend on the direction of propagation $\hat{n} = (\theta, \phi)$ of the wave.



Detector Pattern Functions

 $h(t) = h_{+}(t)F_{+}(\theta,\phi) + h_{\times}(t)F_{\times}(\theta,\phi)$

×

GWs have two polarizations, usually called + and x.



Detectors are not omnidirectional but exhibit an antenna pattern.

Credit: T. Fricke

Interferometric GW Detector Pattern Functions

$$F_{+}(\theta,\phi;\psi=0) = \frac{1}{2} \left(1 + \cos^{2}\theta\right) \cos 2\phi$$

 $F_{\times}(\theta,\phi;\psi=0) = \cos\theta\sin 2\phi$

Thus GW interferometers have blind directions. For instance, for a GW with plus polarization,



$$\phi = \pi/4$$
 and $F_+ = 0$

This wave produces the same displacement in the y and x arm.

Differential phase shift vanishes!



Finding signals

Naively, we can detect a GW signal only when |h(t)| is larger than |n(t)|

But we will rather be in the situation $|h(t)| \ll |n(t)|$

Fundamental question:

How can we dig out the GW signal from a much larger noise?

Answer: We can detect values of h(t) much smaller than the floor of the noise if we know, at least to some level of accuracy, the form of h(t).



Fundamental question:

How can we dig out the GW signal from a much larger noise?

$$\frac{1}{T} \int_0^T dt \, n(t) h(t) \sim \left(\frac{\tau_0}{T}\right)^{1/2} n_0 h_0$$

Goes to zero for large T!

Thus, it is not necessary to have $h_0 > n_0$.

It is sufficient to have $h_0 > (\tau_0/T)^{1/2} n_0$.







Let's turn the previous procedure into a method we can use...

$$\hat{s} = \int_{-\infty}^{\infty} dt \, s(t) K(t)$$

Consider K(t) as a generic filter function.

Goal: Obtain the highest possible signal-to-noise ratio (SNR).

If we know the form of h(t), what is the filter function K(t) that maximizes the SNR?

Define the signal-to-noise ratio...

$$\langle s(t) \rangle = \langle n(t) \rangle + \langle h(t) \rangle$$

 $\langle n(t) \rangle = 0$ (if noise is stationary)

Signal: expected value if signal is present
$$S = \int_{-\infty}^{\infty} dt \langle s(t) \rangle K(t) = \int_{-\infty}^{\infty} dt h(t) K(t) = \int_{-\infty}^{\infty} df \tilde{h}(f) \tilde{K}^{*}(f)$$

Noise: root-mean-square value if no signal present

$$N^{2} = \left[\left\langle \hat{s}^{2}(t) \right\rangle - \left\langle \hat{s}(t) \right\rangle^{2} \right]_{h=0} = \int_{-\infty}^{\infty} dt \, dt' K(t) K(t') \left\langle n(t)n(t') \right\rangle$$
$$\mathbf{0} = \int_{-\infty}^{\infty} df \left(\frac{1}{2} \right) S_{n}(f) \left| \tilde{K}(f) \right|^{2}$$

Define the signal-to-noise ratio...

$$\frac{S}{N} = \frac{\int_{-\infty}^{\infty} df \tilde{h}(f) \tilde{K}^*(f)}{\left[\int_{-\infty}^{\infty} df \left(\frac{1}{2}\right) S_n(f) \left|\tilde{K}(f)\right|^2\right]^{1/2}}$$

Convenient definition: noise-weighted scalar product between two real functions A(t) and B(t)

$$(A|B) = \operatorname{Re} \int_{-\infty}^{\infty} df \frac{\tilde{A}^*(f)\tilde{B}(f)}{(1/2)S_n(f)}$$

$$= 4 \operatorname{Re} \int_0^\infty df \frac{\tilde{A}^*(f)\tilde{B}(f)}{S_n(f)}$$

Define the signal-to-noise ratio...

Using this scalar product definition, we have:

$$\frac{S}{N} = \frac{(u|h)}{(u|u)^{1/2}} \quad \text{where} \quad \tilde{u}(f) = \frac{1}{2}S_n(f)\tilde{K}(f)$$

We are searching for vector $u/(u|u)^{1/2}$ such that its scalar product with vector *h* is maximum.

They should be parallel (i.e. proportional): $\tilde{K}(f) = \text{const.} \frac{h(f)}{S_n(f)}$

This is the **Wiener filter** (aka matched filter).

Define the signal-to-noise ratio...

Optimal value of signal-to-noise ratio:

$$\left(\frac{S}{N}\right) = (h|h)^{1/2}$$

$$\left(\frac{S}{N}\right)^2 = 4 \int_0^\infty df \frac{|\tilde{h}(f)|^2}{S_n(f)}$$

Completely generic and independent of form of $\tilde{h}(f)$



If a particularly noisy period of data is identified, and if it is correlated with a known instrument problem, then the data may be removed.

Coincidence Window

Light travel time between detectors plus k standard deviations. $|t_1 - t_2| \leq (\Delta t)_{\text{light}} + k \left(\sigma_1^2 + \sigma_2^2\right)^{1/2}$ σ_1^2, σ_2^2 - variances on arrival times.

Coincidence must be within a few tens of milliseconds.



Gravitational-wave Sources



Computational Cost Compact Continuous **Binaries** Waves **Bursts** Stochastic

Signal Duration

Do we really need a matched filter search?

GWI509I4

GWI5I226



Formal Definition of Matched Filter

Cross-correlate data with template, weighted by detector noise:

$$Z(t) = A \int_{\infty}^{\infty} \frac{\tilde{s}(f)\tilde{h}^*(f)e^{2\pi i ft}}{S_n(f)} df$$

Cross-correlate template with template, weighted by detector noise:

$$\sigma^2 = 2 \int_0^\infty \frac{\tilde{h}(f)\tilde{h}^*(f)}{S_n(f)} df$$

Normalize output of optimal filter:

$$\rho(t) = \frac{|Z(t)|}{\sigma}$$





Signal-based Vetoes

25Time-frequency Spectrograms: Measured Glitches versus Signals 20Predicted 15H1 $\rho(t)$ 10 III WAATGAR 5 0 -5-10-0.050.00 0.05 0.10 -0.1015Measured Predicted 10 L1 $\rho(t)$ 5 -5-10-0.10-0.050.00 0.05 0.10 Time from peak (s)

Messick, et al, arXiv 1604.04324

Which glitch is not like the others?







Likelihood Parameters



Signal and Background Models



+ - GW150914 + - GW151226 + - LVT151012

Evolution of Noise Model with Mass



The search background is calculated separately for each mass and chi bin.
Estimating Significance



Only when we leave coincident triggers in our noise model, i.e. gravitational waves, do we form accidental coincidences at the same level of significance or higher.

Gravitational-wave Sources





Signal Duration

Burst Signals

- GW searches with minimal assumptions
 - Do not assume accurate source models affected by noise
- Limited ability to measure waveforms, sky positions, polarizations
- Can detect the unexpected







Burst Analysis with Wavelets

- Wavelets are waveforms of limited duration and bandwidth
- GW bursts can be described as superposition of wavelets Coiflets Daubechies Morlet Biorthogonal Mexican Hat Symlets Credit: MathWorks

Burst Analysis with Wavelets

$\mathsf{MMM}^{\mathsf{H}} = \mathsf{MMMM}^{\mathsf{H}} = \mathsf{MMMM}^{\mathsf{H}}$

The Fourier Transform does not represent abrupt changes efficiently. Sum of Sine waves are not localized in time or space and oscillate forever.

Burst Analyses with Wavelets

For time series that have abrupt changes, we need functions that are well localized in time and frequency.

Time-frequency Pixel Maps





Example: (10, 10) M_☉ binary black hole merger with an eccentric orbit at 4 different timefrequency representations.

Credit: S. Klimenko

Cluster Selection



Identify time-frequency areas with excess energy above a predetermined threshold.

Cluster excesses together.

Time



Coherent Burst Search

- Wavelet amplitudes need to be calculated and signal reconstructed for each timefrequency pixel and sky location
- Joint analysis from all detectors (HI, LI, VI,...)
- Synchronize detectors by searching over entire sky (~20000 points)

Maximum Likelihood Statistic

similarity of waveforms in detectors total energy of reconstructed waveforms

$$c_c = \frac{E_c}{E_c + E_n}$$

 $L = c_c$

 E_c - coherent energy

E_n - energy of residual noise after signal subtracted

Example: 2-Detector Constraint



Magnitude of antenna pattern vectors in for LI-HI network

Ratio of antenna pattern vectors

 $|f_{\mathsf{X}}| << |f_{\mathsf{+}}|$

LIGO network nearly aligned; blind to second polarization in most sky locations.

3 Search Classes



A Burst Result

H1:CAL-DELTAL_EXTERNAL_DQ at 1126259462.391 with Q of 5.7



L1:LSC-DARM_IN1_DQ at 1126259462.391 with Q of 5.7





The first gravitational wave was found by a burst pipeline, within 3 minutes!

Gravitational-wave Sources





Signal Duration

Types of CW Searches

- **Targeted searches** for known pulsars: known position, rotation frequency and spin down
- **Directed searches** in known position: SNRs, galactic center, accreting neutron stars in low-mass x-ray binaries
- All-sky searches for isolated, unknown sources

The Continuous Wave from an Isolated NS

• The source should emit a nearly monochromatic sinusoidal wave



- Limit on observation comes from total available observation time
- But the detector will see a modified signal

$$\vec{\mathcal{A}} = \{f, \dot{f}, \ddot{f}, \dots, \alpha, \delta, h_0, \cos \iota, \psi, \phi_0\}$$

- Four phase evolution parameters
- Four amplitude parameters

Doppler Effect Due to Earth Rotation and Orbit



The All-sky Search Goal

$$\mathrm{SNR} \propto \frac{h_0}{\sqrt{S_n}} \sqrt{T}$$

- Search over broad frequency, spin-down range over whole sky over many months of data
- Not really computationally feasible
- Solution: use semi-coherent methods

Semi-coherent Methods

Split data into short segments which are analyzed coherently



Short (~1000s): signal stays in single Fourier bin

Semi-coherent Methods

Split data into short segments which are analyzed coherently



Long (~hrs/days): need to account for signal modulation within segment

F-Statistic

$$\Lambda(x) \equiv \max \mathcal{L}\left(x; \mathcal{A}\right) = \exp^{\mathcal{F}(x)}$$

Maximize with respect to four unknown amplitude parameters: (1 - 1)

 $\{h_0, \cos\iota, \psi, \phi_0\}$

Dimension of parameter space: 8 to 4

Under the assumption of white detector noise, F-statistic will look like

Variance of the data

Functions incorporating the amplitude modulations that depend on location and orientation of detector on the Earth, position of GW source in sky, periodic functions of time

Stack-Slide Method



Stack-Slide Method



Add power after frequency bins are shifted.

Hough Transform Method



Hough Transform Method





Credit: Cornelissen, et. al.

Identify Interesting Frequency Bands



Identify 50 mHz bands disturbed by detector artifacts. Maximum density and mean 2F value are metrics to identify disturbed bands.

Contamination in CW Searches



Two power spectra, one with an unusually high low-frequency noise contribution up to 300 Hz.

Contamination in CW Searches



Harmonics of single-frequency noise source create comb-like patterns in noise spectrum.

If not flagged, searches can identify them as possible sources.

A CW Result



At 55Hz, the O1 all-sky search can exclude sources with ellipticity above 10⁻⁵ within 100pc.

Gravitational-wave Sources





What is a stochastic background?

- Stochastic (random) background of gravitational radiation
- Can arise from superposition of large number of unresolved GW sources
 - 1. Cosmological origin
 - 2. Astrophysical origin
- Strength of background measured as gravitational wave energy density $\rho_{\rm GW}$

Cosmological Gravitational Wave Background



GW spectrum:
$$\Omega_{\rm GW}(f) = \frac{f}{\rho_c} \frac{\mathrm{d}\rho_{\rm GW}}{\mathrm{d}f}$$

Critical energy density of universe: $\rho_c = \frac{3c^2H_0^2}{8\pi G}$

Astrophysical Gravitational Wave Backgrounds

- Produced by an extremely large number of weak, independent, and unresolved gravity-wave sources
 - Binary black holes and/or neutron stars
 - Supernovae
 - White dwarf binaries

For example, the expected contribution from double white dwarfs for LISA



Astrophysical Gravitational Wave Backgrounds



Detecting Stochastic Backgrounds

Cross-correlation of two interferometers' data streams multiplied by a filter function:

$$Y = \int_{-\infty}^{+\infty} df \int_{-\infty}^{+\infty} df' \delta_T (f - f') \tilde{s}_1(f)^* \tilde{s}_2(f') \tilde{Q}(f')$$


Detecting Stochastic Backgrounds

The filter function has the form:

$$\tilde{Q}(f) = N \frac{\gamma(f)\Omega_{\rm GW}(f)H_0^2}{f^3 P_1(f)P_2(f)}$$

overlap reduction function: $\gamma(f)$

power law template for GW spectrum: $\Omega_{\rm GW}(f) = \Omega_{\alpha} \left(f/100 \, {\rm Hz} \right)^{\alpha}$ present value of Hubble parameter: H_0

noise in detector 1: $P_1(f)$

noise in detector 2: $P_2(f)$

Purpose: Enhance SNR at frequencies where signal is strong and suppress SNR at frequencies where detector noise is large.

Overlap Reduction Function

Signal in two detectors will not be exactly the same because: i) time delay between detectors ii) non-alignment of detector



Get involved...

Einstein@Home uses your computer's idle time to search for weak astrophysical signals from spinning neutron stars (often called pulsars) using data from the LIGO gravitational-wave detectors, the Arecibo radio telescope, and the Fermi gamma-ray satellite.

Gravity Spy - citizen science project for identifying and classifying glitches

