Physics with GWs
Lecture 9: Gravitational Waves MSc Course
• Physics
  • Tests of General Relativity
  • Cold, high density nuclear matter
  • Galaxy dynamics
  • Cosmology
Strong field tests of General Relativity

Strong field tests of General Relativity

Remove most probable GR waveform from data.

Calibrated against waveforms from direct numerical integration of Einstein equations.

Analysis reveals that GW150914 residual favors instrumental noise over the presence of coherent signal or glitches.
Mass and spin parameters predicted from binary inspiral

versus

Mass and spin inferred from post-inspiral signal

Numerical relativity provides fitting formulas for relations between the binary’s components and final masses and spins.

Analysis reveals that GW150914 inspiral and post-inspiral have significant region of overlap.
Parameterized deviations from GR

Look for possible departures from GR, parameterized by set of testing coefficients.

Orbital phase during inspiral is function of ever increasing orbital speed:

$$\Phi(v) = \left(\frac{v}{c}\right)^{-5} \left[ \varphi_{0\text{PN}} + \varphi_{0.5\text{PN}} \left(\frac{v}{c}\right) + \varphi_{1\text{PN}} \left(\frac{v}{c}\right)^2 + \ldots + \varphi_{2.5\text{PN}}(v) \log \left(\frac{v}{c}\right) \left(\frac{v}{c}\right)^5 + \ldots + \varphi_{3.5\text{PN}} \left(\frac{v}{c}\right)^7 \right]$$

In GR, these have known functions.

Orbital phase between inspiral and merger-ringdown parameterized by $\beta_j$.

Orbital phase of merger-ringdown parameterized by $\alpha_j$. 
Parameterized deviations from GR

Look for possible departures from GR, parameterized by set of testing coefficients.

GW150914 provided probe of late inspiral and merger.
GW151226 provided opportunity to probe PN inspiral with many more waveform cycles.

Parameterized deviations from GR

Look for possible departures from GR, parameterized by set of testing coefficients.
Parameterized deviations from GR

Look for possible departures from GR, parameterized by set of testing coefficients.

Posterior distributions for deviations can be combined to yield stronger constraints.

No evidence for disagreement with predictions of GR. Accuracies will improve with $\sqrt{N}$.
In GR, GWs are nondispersive.

But modifications to the dispersion relation can arise in theories that include violations of local Lorentz invariance.

Thus, modified propagation of GWs can be mapped to Lorentz violation.
Constraints on Lorentz violations

Consider modified dispersion relation of the form:

\[ E^2 = p^2 c^2 + Ap^\alpha c^\alpha \]

\( A \) - amplitude of dispersion. GR predicts \( A=0 \).

Several modified theories of gravity predict specific values of \( \alpha \geq 0 \).

Dispersion occurs during propagation of GW toward Earth. GW170104 provides the best constraint since it was the furthest signal so far. Redshift \( \sim 0.2 \).
Constraints on Lorentz violations

Combined posterior of GW150914, GW151226, and GW170104

![Graph showing constraints on Lorentz violations](image)

\[ |A| \sim [\text{peV}^2 - \alpha] \]

\( (\alpha = 2.5) \)

multifractal spacetime

LVC, PRL 118, 221101 (2017).
Constraints on Lorentz violations

Combined posterior of GW150914, GW151226, and GW170104

\( |A| \) [\text{peV}^2 - \alpha]

\( \alpha = 3 \)

doubly special relativity

LVC, PRL 118, 221101 (2017).
Constraints on Lorentz violations

Combined posterior of GW150914, GW151226, and GW170104

extra-dimensional theories

LVC, PRL 118, 221101 (2017).
Constraints on Lorentz violations

Combined posterior of GW150914, GW151226, and GW170104

\(|A| [\text{peV}^2 - \alpha]\)

\(A > 0\)

\(A < 0\)

\((\alpha = 2)\) degenerate with arrival time of signal
Constraints on Lorentz violations

Combined posterior of GW150914, GW151226, and GW170104

\[(\alpha = 0, A > 0) \text{ massive-graviton theories}\]
Massive graviton

\( (\alpha = 0, A > 0) \) - reparameterized to derive lower bound on graviton Compton wavelength

Finite Compton wavelength \( \Rightarrow \) nonzero mass

\[ \lambda_g > 1.6 \times 10^{13} \text{ km} \]

\[ m_g \leq 7.7 \times 10^{-23} \text{ eV} / c^2 \]

LVC, PRL 118, 221101 (2017).
Gravitational-wave Polarizations

General relativity predicts only two tensor GW polarizations.

Alternate theories allow for up to four additional vector and scalar modes.

In principle, full generic metric theories predict any combination of tensor, vector or scalar polarizations.
Two LIGO detectors are almost aligned so they can’t really give us information on other polarizations.

With Virgo, we get a little more information.

Consider models where polarization states are pure tensor, pure vector, or pure scalar.

Bayes’ factors for GW170814 (triple BBH):

\[
\frac{P(\theta|\text{tensor})}{P(\theta|\text{vector})} = 200, \quad \frac{P(\theta|\text{tensor})}{P(\theta|\text{scalar})} = 1000
\]

Network of at least six detectors is required to determine the polarization content of GW transient.

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Neutron Stars

Unique natural laboratories for studying behavior of cold, high-density nuclear matter.

Behavior is governed by equation of state (EoS), relationship between pressure and density:

- determines relation between NS mass and radius
- determines stellar moment of inertial
- determines tidal deformability

Thus measurement of NS masses, radii, moments of inertia and tidal effects provide information about EoS.
Effect on GW signal

Credit: K. Hotokezaka
Effect on GW signal

Inspiral

Most prominent effect: tidal deformation of each star’s gravitational field on its companion accelerates decay of inspiral.

Often this is the easiest effect to measure.
Effect on GW signal

Most prominent effect: deformation of each NS due to its own spin has effect on orbital evolution for NSs with large spin.

Effect is degenerate with mass ratio and spins so is difficult to measure.
Effect on GW signal

**Merger-ringdown**

Dominant effect: EoS affects GW behavior during merger and post-merger, as well as its outcome in general.

Can provide lots of physics put signal is buried in photon shot noise at relevant frequencies.
Tidal Deformability

Deformability of each star: \( \Lambda_{1,2} = \frac{2}{3} k_2 \left( \frac{R_{1,2} c^2}{G m_{1,2}} \right)^5 \)

- \( R \) - radius
- \( m \) - mass
- \( k_2 \) - tidal Love number that depends on mass and EoS

Tidal effects imprinted in gravitational-wave signal through binary tidal deformability:

\[
\tilde{\Lambda} = \frac{16}{13} \frac{(12q + 1) \Lambda_1 + (12 + q) q^4 \Lambda_2}{(1 + q)^5} \\
q = \frac{m_2}{m_1} \leq 1
\]
Tidal Deformability and EoSs

Tidal deformability as a function of mass for a set of polytropes.

De, et al. arXiv: 1804.08583v1
GW170817 Parameter Estimation

We know the location! Fix location in the sky.

Assume two NSs with properties that are described by the same EoS.

Small spin prior in agreement with galactic binary NS spin measurements.

Choice of mass prior can have an impact on the measurement of the tidal deformability.
GW170817 Parameter Estimation

\[ m_{1,2} \sim U[1, 2] M_\odot \]
GW170817 Parameter Estimation

$m_{1,2} \sim N(\mu = 1.33, \sigma = 0.09)M_\odot$

Fit to masses of NSs in double NS systems
Fit to masses of recycled and slow pulsars in the Galaxy

\[ m_1 \sim N(\mu = 1.54, \sigma = 0.23) M_\odot \]
\[ m_2 \sim N(\mu = 1.49, \sigma = 0.19) M_\odot \]
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• Astrophysical Rates
GWs carry away energy, angular momentum and linear momentum. Remnant black hole inherits a recoil velocity or “kick”.

From black hole perturbation theory:

\[ V_F \sim 10 - 100 \text{ km/s} \]
\[ V_{\text{max}} \lesssim 500 \text{ km/s} \]

Compare with escape velocities from:

Globular clusters: \(~30 \text{ km/s}\)
Galaxies: \(~1000 \text{ km/s}\)
Binary black holes and kicks

Existence of kick provides preferred direction within orbital plane: that of the kick.

Use parameter estimation to estimate magnitude of kick and direction.

Final kick
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Cosmology
Measuring the Hubble Constant

**Hubble constant:**
1. Represents local expansion of Universe
2. Sets overall scale of Universe
3. Fundamental importance to cosmology

\[ v_H = H_0 d \]

Distance from GW signal:

\[ d = 44 \text{ Mpc} \]

Recessional velocity from galaxy’s velocity:

\[ v_H = 3000 \text{ km/s} \]
Measuring the Hubble Constant

\[ H_0 = 70.0^{+12.0}_{-8.0} \text{ km s}^{-1} \text{ Mpc}^{-1} \]

Consistent with SHoES (nearby) and Planck (very distant) estimates. Heralds the age of multimessenger cosmology.