# Announcements

Visualization Project information uploaded to website. Due May 25th.

No class this Friday. The next class will be next Tuesday, May 8th. Syllabus will be updated shortly and email sent.

# Astrophysical Sources of Gravitational Waves

Lecture 5: Gravitational Waves MSc Course

# Review: Generation of Gravitational Waves

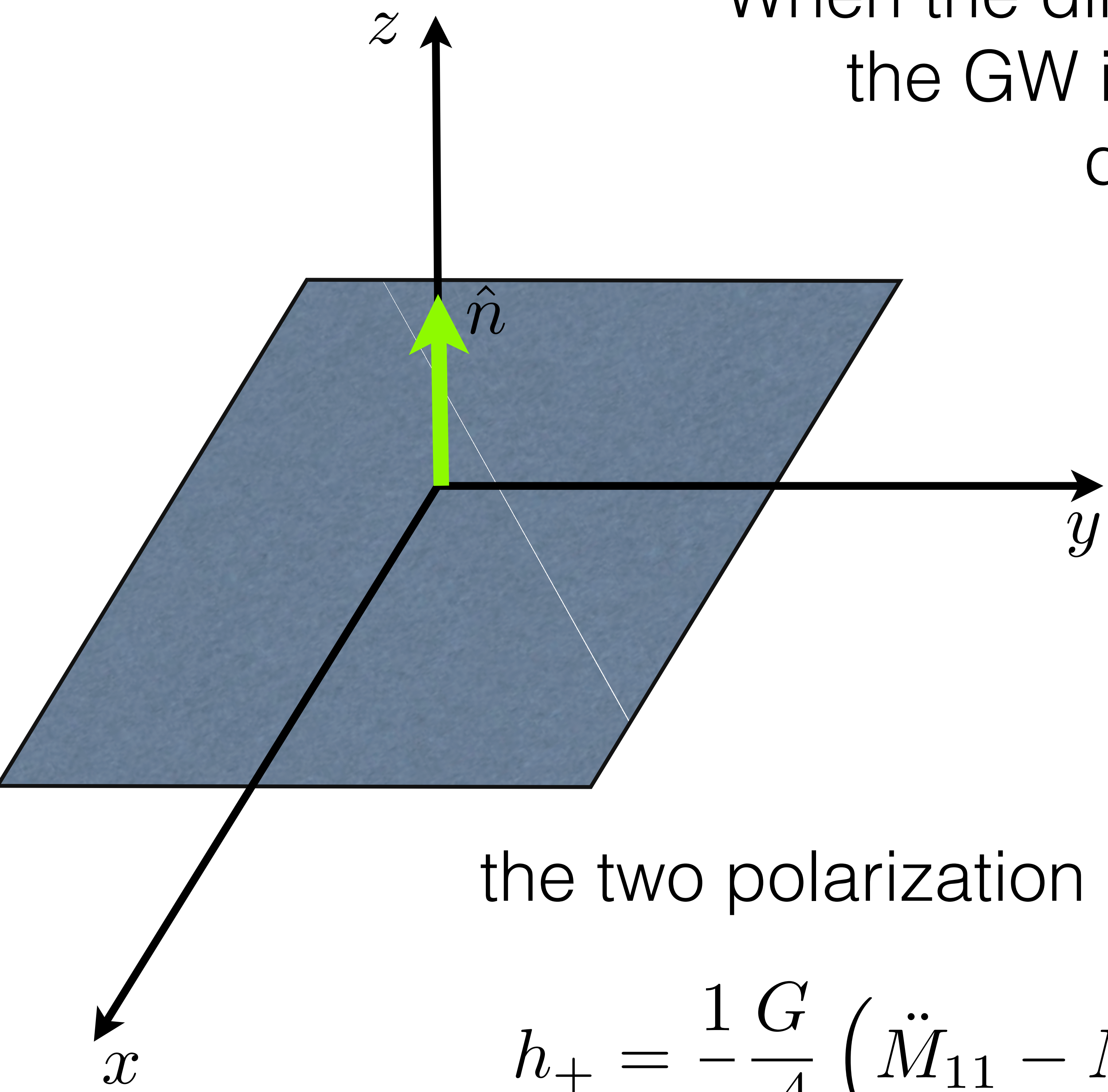
To leading order in *v/c*, we can eliminate the multipole moments in favor of the mass moments to get a solution of the form:

$$\left[h_{ij}^{\mathrm{TT}}(t,\vec{x})\right]_{\mathrm{quad}} = \frac{1}{r}\frac{2G}{c^4}\Lambda_{ij,kl}(\hat{n})\ddot{M}^{kl}\,(t-r/c)$$

where we have used: $S^{ij} = \frac{1}{2}\ddot{M}^{ij}$

Mass quadrupole radiation!

# Case I: Propagation in ẑ




When the direction of propagation $\hat{n}$ of the GW is equal to $\hat{z}$, $P_{ij}$ is the diagonal matrix:

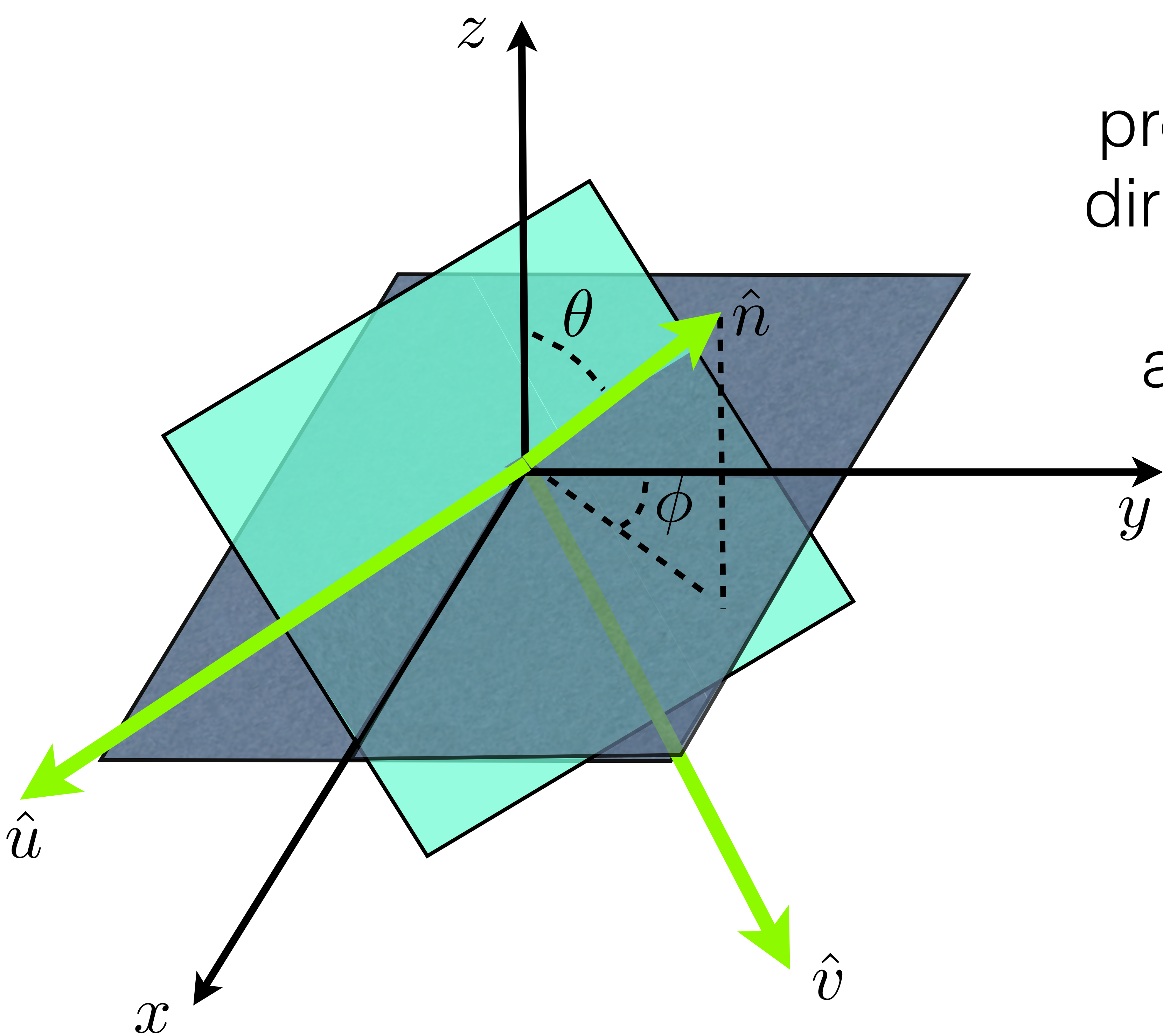
$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

i.e., a projector on the $(x, y)$ plane,

the two polarization amplitudes have the form

$$h_+ = \frac{1}{r}\frac{G}{c^4}\left(\ddot{M}_{11} - \ddot{M}_{22}\right) \qquad h_\times = \frac{2}{r}\frac{G}{c^4}\ddot{M}_{12}$$

# Case II: Propagation in $\hat{n}$




When the wave propagates in a **generic** direction $\hat{n}$, we introduce two unit vectors $\hat{u}$ and $\hat{v}$, orthogonal to $\hat{n}$

The vector $\hat{u}$ is in the $(\hat{x}, \hat{y})$ plane while $\hat{v}$ points downward with respect to the $(\hat{x}, \hat{y})$ plane.

# Case II: Propagation in $\hat{n}$

For a generic propagation direction, the two polarization amplitudes have the form:

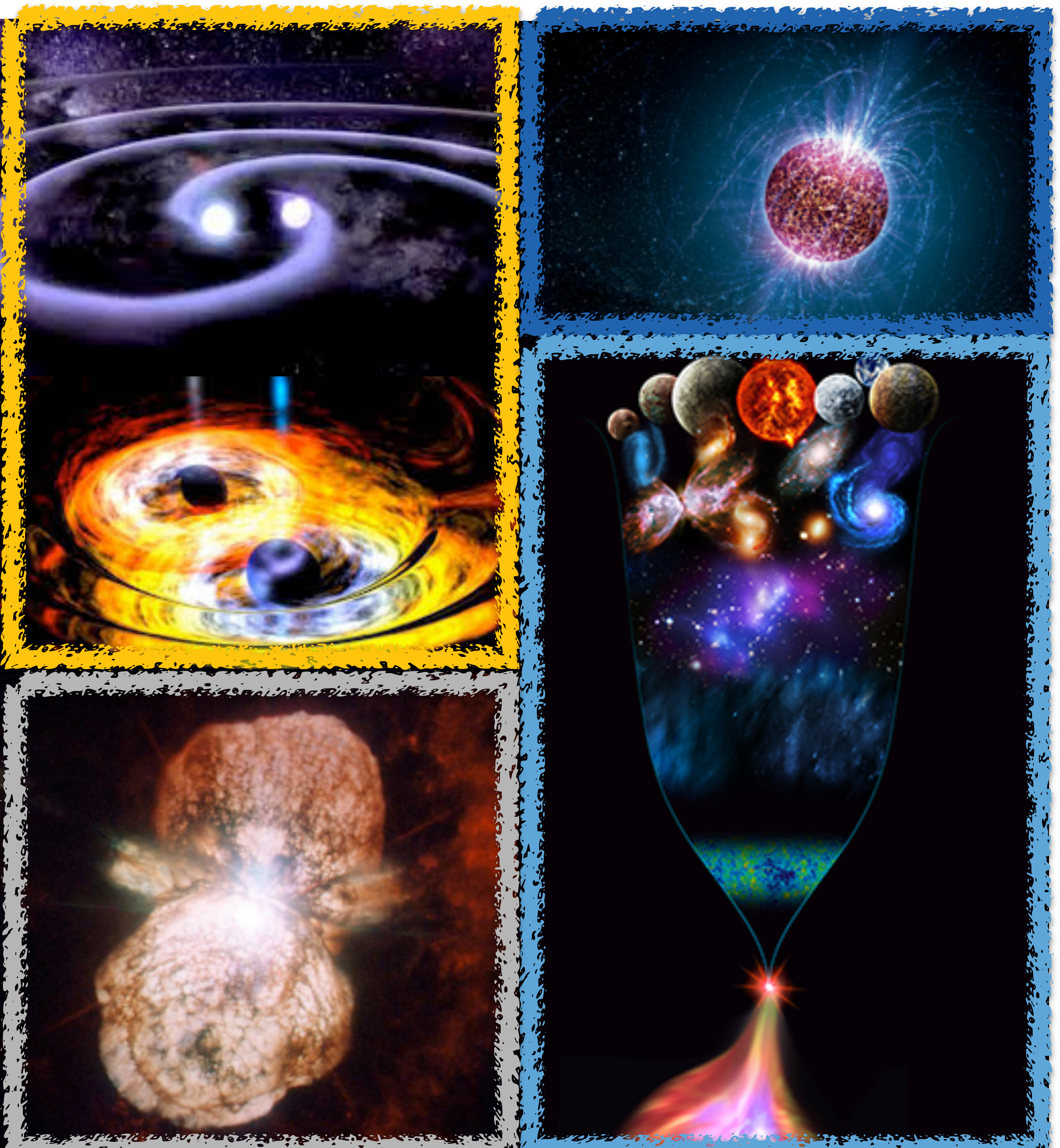
$$\begin{aligned} h_+(t;\theta,\phi) = \frac{1}{r}\frac{G}{c^4}[&\ddot{M}_{11}\left(\cos^2\phi - \sin^2\phi\cos^2\theta\right) \\ &+ \ddot{M}_{22}\left(\sin^2\phi - \cos^2\phi\cos^2\theta\right) \\ &- \ddot{M}_{33}\sin^2\theta \\ &- \ddot{M}_{12}\sin 2\phi\left(1+\cos^2\theta\right) \\ &+ \ddot{M}_{13}\sin\phi\sin 2\theta \\ &+ \ddot{M}_{23}\cos\phi\sin 2\theta] \end{aligned}$$

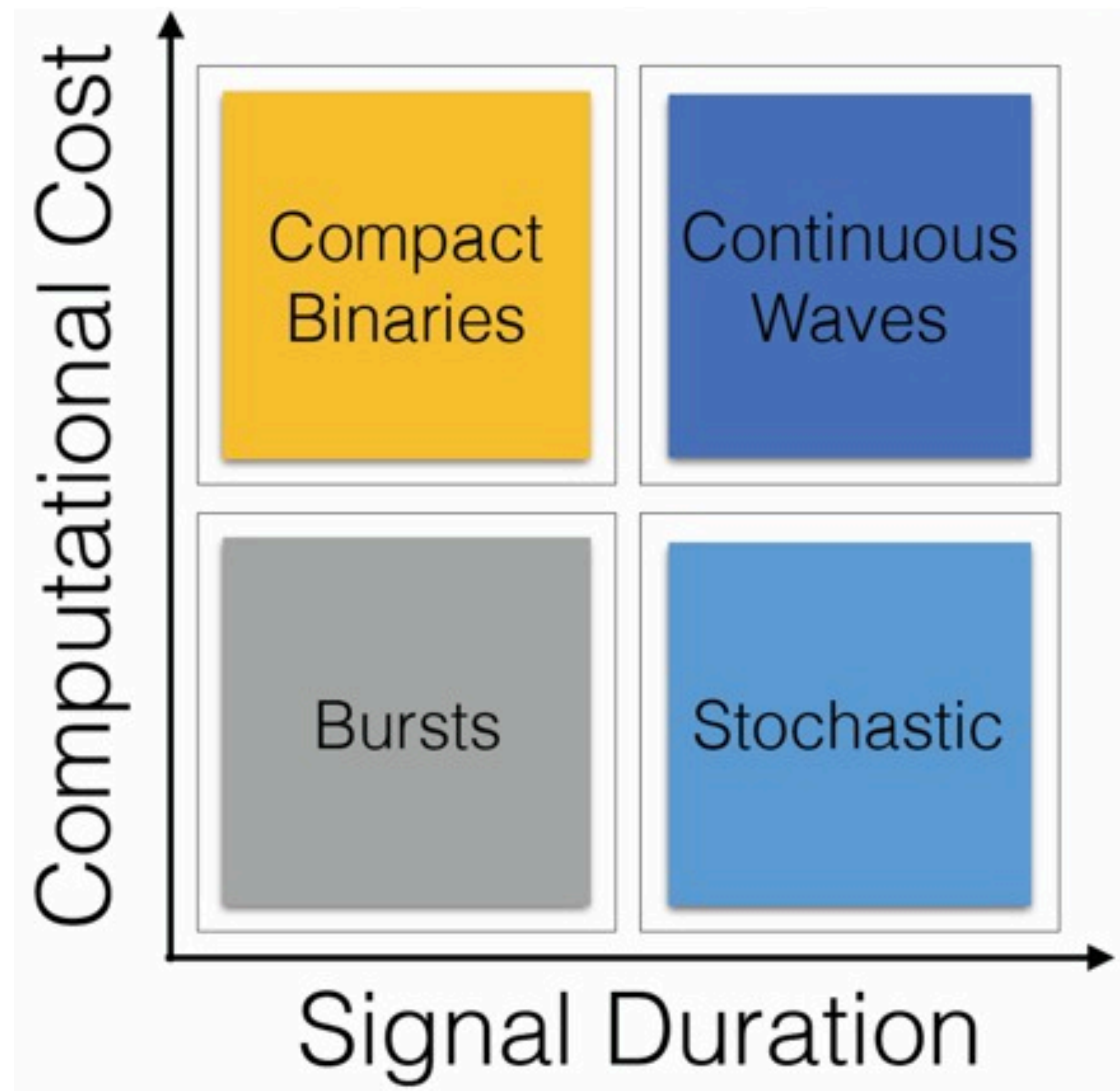
$$\begin{aligned} h_\times(t;\theta,\phi) = \frac{1}{r}\frac{G}{c^4}[&(\ddot{M}_{11} - \ddot{M}_{22})\sin 2\phi\cos\theta \\ &+ 2\ddot{M}_{12}\cos 2\phi\cos\theta \\ &- 2\ddot{M}_{13}\cos\phi\sin\theta \\ &+ 2\ddot{M}_{23}\sin\phi\sin\theta] \end{aligned}$$

# LIGO/Virgo Astrophysical Sources

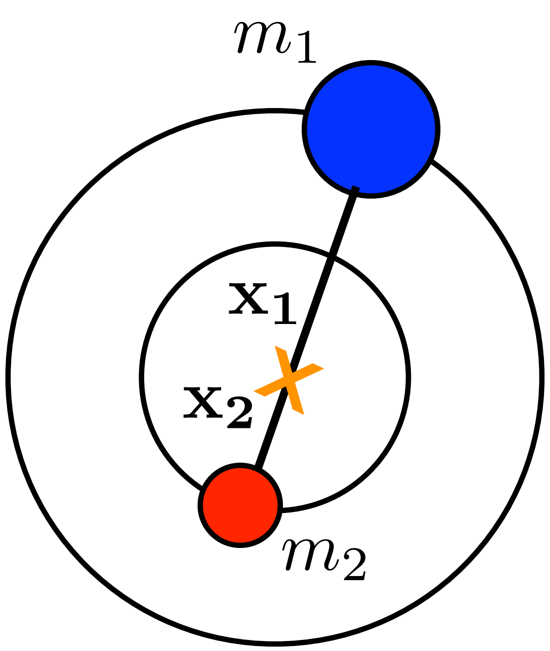
- Coalescing Binaries
  - Binary Neutron Stars
  - Binary Black Holes
- Continuous Waves
- Transient Bursts
- Stochastic Background

# LIGO/Virgo Astrophysical Sources





# Example I: Quadrupole radiation from a mass in circular orbit




The usual center-of-mass coordinate is:

$$\mathbf{x}_{\mathrm{CM}} = \frac{m_1\mathbf{x_1} + m_2\mathbf{x_2}}{m_1 + m_2}$$

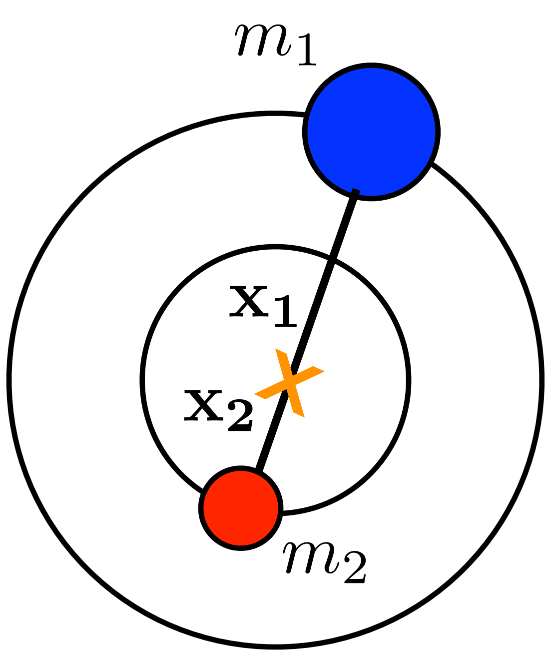
$\mathbf{x}_0 = \mathbf{x}_1 - \mathbf{x}_2$ is the relative coordinate of an isolated two-body system in the center-of-mass frame.

If we chose the origin of the coordinate system at $\mathbf{x}_{\mathrm{CM}} = 0$,

then the second mass moment is: $M^{ij}(t) = \mu x_0^i(t) x_0^j(t)$

where $\mu = \dfrac{m_1 m_2}{m_1 + m_2}$ is the reduced mass.

# Example I: Quadrupole radiation from a mass in circular orbit




$$\mathbf{x}_0 = \mathbf{x}_1 - \mathbf{x}_2$$

Choose $(x, y, z)$ frame so orbit is in $(x, y)$ plane.

Orbit is given by:

$$x_0(t) = R\cos(\omega_s t + \pi/2)$$

$$y_0(t) = R\sin(\omega_s t + \pi/2)$$

$$z_0(t) = 0$$

The only non-vanishing second mass moment components are:

$$M_{11} = \mu R^2 \frac{1 - \cos 2\omega_s t}{2}$$

$$M_{22} = \mu R^2 \frac{1 + \cos 2\omega_s t}{2}$$

$$M_{12} = -\frac{1}{2}\mu R^2 \sin 2\omega_s t$$

Compute $\ddot{M}_{ij}$ . Plug into generic expressions for polarization amplitudes to get:

$$h_+(t; \theta, \phi) = \frac{1}{r}\frac{4G\mu\omega_s^2 R^2}{c^4}\left(\frac{1 + \cos^2\theta}{2}\right)\cos(2\omega_s t_{\rm ret} + 2\phi)$$

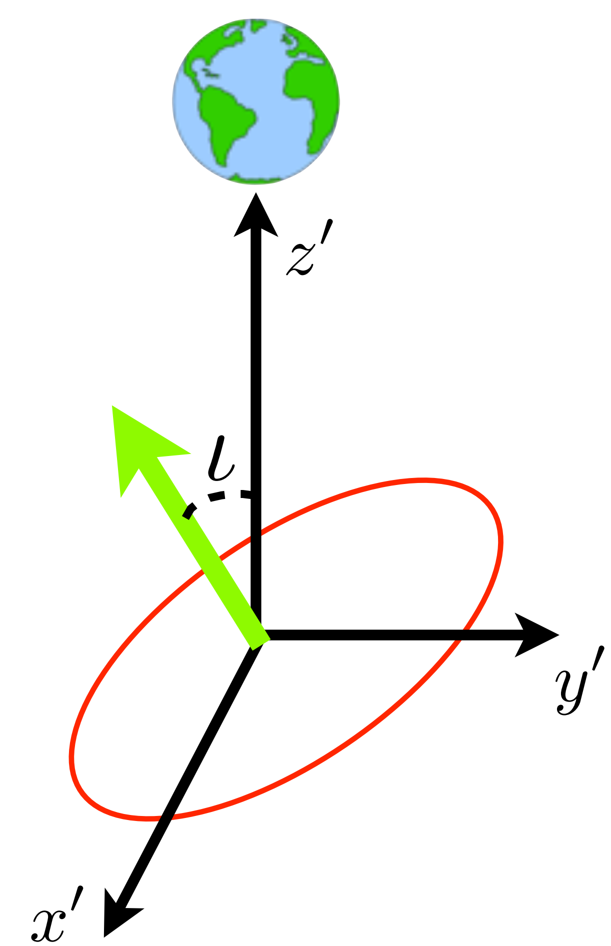
$$h_\times(t; \theta, \phi) = \frac{1}{r}\frac{4G\mu\omega_s^2 R^2}{c^4}\cos\theta\sin(2\omega_s t_{\rm ret} + 2\phi)$$

# Example I: Quadrupole radiation from a mass in circular orbit

$$h_+(t;\theta,\phi) = \frac{1}{r}\frac{4G\mu\omega_s^2 R^2}{c^4}\left(\frac{1+\cos^2\theta}{2}\right)\cos(2\omega_s t_{\mathrm{ret}} + 2\phi)$$

$$h_\times(t;\theta,\phi) = \frac{1}{r}\frac{4G\mu\omega_s^2 R^2}{c^4}\cos\theta\sin(2\omega_s t_{\mathrm{ret}} + 2\phi)$$

Quadrupole radiation is at **twice** the frequency $\omega_s$ of the source: $\omega_{\mathrm{gw}} = 2\omega_s$




A rotation of the source by $\Delta\phi$ is the same as a time translation so that

$$\omega_s \Delta t = \Delta\phi$$

The angle $\theta$ is equal to the angle $\iota$ between the normal to the orbit and the line-of-site.

# Example I: Quadrupole radiation from a mass in circular orbit

Use Kepler's law, the chirp mass, and the GW frequency to rewrite the solutions.

$$\omega_s^2 = \frac{GM}{R^3} \qquad M_c = \mu^{3/5} M^{2/5} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} \qquad \omega_{\rm gw} = 2\omega_s$$

$$\omega_{\rm gw} = 2\pi f_{\rm gw}$$

$$h_+(t) = \frac{4}{r}\left(\frac{GM_c}{c^2}\right)^{5/3}\left(\frac{\pi f_{\rm gw}}{c}\right)^{2/3}\frac{1+\cos^2\theta}{2}\cos(2\pi f_{\rm gw} t_{\rm ret} + 2\phi)$$

$$h_\times(t) = \frac{4}{r}\left(\frac{GM_c}{c^2}\right)^{5/3}\left(\frac{\pi f_{\rm gw}}{c}\right)^{2/3}\cos\theta\sin(2\pi f_{\rm gw} t_{\rm ret} + 2\phi)$$

The amplitudes of the GWs emitted depend on the masses $m_1$ and $m_2$ only through the combination $M_c$.

# Example I: Quadrupole radiation from a mass in circular orbit

Angular distribution of the radiated power in quadrupole approximation:

$$\boxed{\left(\frac{dP}{d\Omega}\right)_{\text{quad}} = \frac{r^2 c^3}{16\pi G}\left\langle \dot{h}_+^2 + \dot{h}_\times^2 \right\rangle}$$

For our binary system example:

$$\left(\frac{dP}{d\Omega}\right)_{\text{quad}} = \frac{2G\mu^2 R^4 \omega_s^6}{\pi c^5} g(\theta)$$

$$g(\theta) = \left(\frac{1+\cos^2\theta}{2}\right)^2 + \cos^2\theta$$

Total power radiated in quadrupole approximation

$$\boxed{P_{\text{quad}} = \left(\frac{dE_{\text{gw}}}{d\Omega}\right)_{\text{quad}} = \frac{r^2 c^3}{16\pi G}\int_{\mathcal{S}} d\Omega \left\langle \dot{h}_+^2 + \dot{h}_\times^2 \right\rangle}$$

For our binary system example:

$$P_{\text{quad}} = \frac{32}{5}\frac{G\mu^2}{c^5} R^4 \omega_s^6$$

# Example I: Quadrupole radiation from a mass in circular orbit

In terms of the chirp mass $M_c$ , the total radiated power in the binary system is

$$P = \frac{32}{5}\frac{c^5}{G}\left(\frac{GM_c\omega_{\rm gw}}{2c^3}\right)^{10/3}$$

# Example I: Quadrupole radiation from a mass in circular orbit

**The emission of GWs costs energy.** Previous equations are only valid if sources are on fixed, circular Keplerian orbit.

$$E_{\rm orbit} = E_{\rm kin} + E_{\rm pot} = -\frac{1}{2}\frac{Gm_1m_2}{R}$$

Kepler's law

$$\omega_s^2 = \frac{GM}{R^3}$$

$$P = \frac{32}{5}\frac{c^5}{G}\left(\frac{GM_c\omega_{\rm gw}}{2c^3}\right)^{10/3}$$

To compensate for loss of energy to GWs, R must decrease in time.

If R decreases, $\omega_s$ increases.

Then power radiated in GWs increases which means R must decrease even more.

Runaway process $\Rightarrow$ binary system must coalesce.

# Example I: Quadrupole radiation from a mass in circular orbit

Changes needed to:

$$h_{+}\left(t;\theta,\phi\right)=\frac{1}{r}\frac{4G\mu\omega_s^2R^2}{c^4}\left(\frac{1+\cos^2\theta}{2}\right)\cos(2\omega_s t_{\mathrm{ret}}+2\phi)$$

$$h_{\times}\left(t;\theta,\phi\right)=\frac{1}{r}\frac{4G\mu\omega_s^2R^2}{c^4}\cos\theta\sin(2\omega_s t_{\mathrm{ret}}+2\phi)$$

In arguments of the trigonometric functions: $\omega_{\mathrm{gw}}t\rightarrow\Phi(t)$

In factors in front of trigonometric functions: $\omega_{\mathrm{gw}}\rightarrow\omega_{\mathrm{gw}}(t)$

May have contributions from derivatives of $R(t)$ and $\omega_{\mathrm{gw}}(t)$.

$\dot{R}(t)$ is negligible as long as $f_{\mathrm{gw}}\ll 13\mathrm{kHz}\,(1.2M_{\odot}/M_c)$

# Example I: Quadrupole radiation from a mass in circular orbit

Time to coalescence $\tau$ measured by the observer:

$$\tau \equiv t_{\rm coal} - t \qquad -\infty < t < t_{\rm coal}$$

Evolution of GW frequency:

$$f_{\rm gw}(\tau) = \frac{1}{\pi}\left(\frac{5}{256\tau}\right)^{3/8}\left(\frac{GM_c}{c^3}\right)^{-5/8}$$

Evolution of arguments of trigonometric functions:

$$\Phi(\tau) = -2\left(\frac{5GM_c}{c^3}\right)^{-5/8}\tau^{5/8} + \Phi_0 \qquad \Phi_0 = \Phi(\tau = 0)$$

Then the GW amplitudes are

$$\boxed{h_+(t) = \frac{1}{r}\left(\frac{GM_c}{c^2}\right)^{5/4}\left(\frac{5}{c\tau}\right)^{1/4}\frac{1+\cos^2\iota}{2}\cos\left[\Phi(\tau)\right]}$$

$$\boxed{h_\times(t) = \frac{1}{r}\left(\frac{GM_c}{c^2}\right)^{5/4}\left(\frac{5}{c\tau}\right)^{1/4}\cos\iota\sin\left[\Phi(\tau)\right]}$$

# Example I: Quadrupole radiation from a mass in circular orbit

In Schwarzschild geometry, there is a minimum value of the radial distance beyond which stable circular orbits are no longer allowed, i.e. the Innermost Stable Circular Orbit (ISCO):

$$r_{\rm ISCO} = \frac{6GM}{c^2}$$

For binaries of BH or NS, a phase of slow adiabiatic inspiral, going through quasi-circular orbit and driven by emission of GWs can only take place at distances $r \gtrsim r_{\rm ISCO}$

$$f_{\rm max} = (f_s)_{\rm ISCO} = \frac{1}{12\sqrt{6}\pi}\frac{c^3}{GM}$$

# Example II: Quadrupole radiation from rotating rigid body

A rigid body is characterized by its inertia tensor:

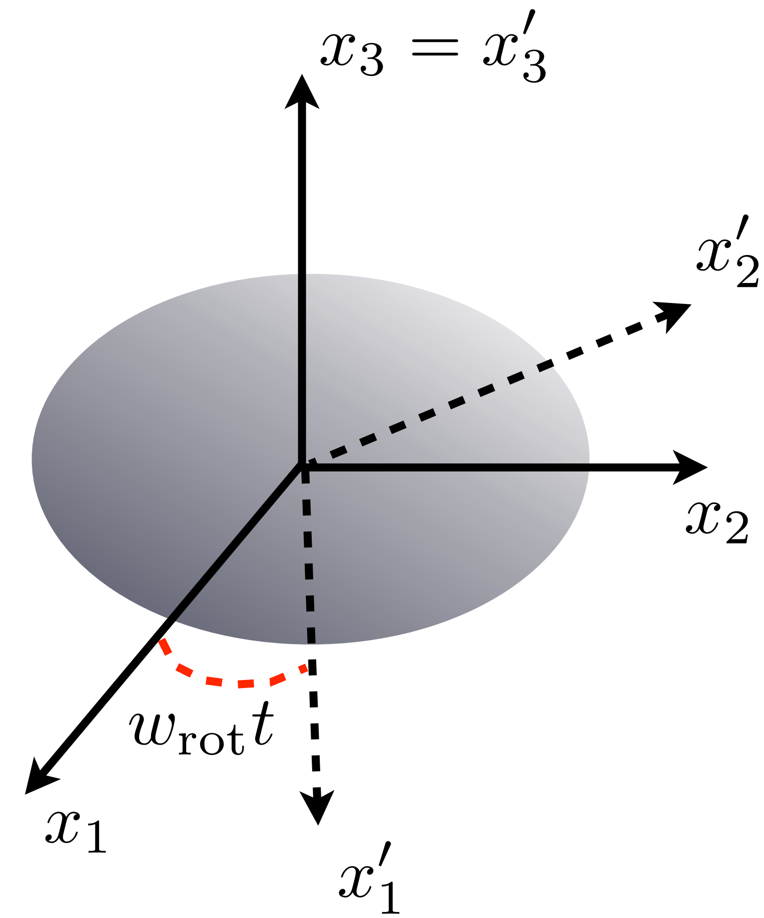
$$I^{ij} = \int d^3x\, \rho(\mathbf{x}) \left( r^2 \delta^{ij} - x^i x^j \right)$$

There is a frame where the inertia tensor is diagonal. The principal moments of inertia are

$$I_1 = \int d^3x'\, \rho(\mathbf{x}') \left( x_2'^2 + x_3'^2 \right)$$

$$I_2 = \int d^3x'\, \rho(\mathbf{x}') \left( x_1'^2 + x_3'^2 \right)$$

$$I_3 = \int d^3x'\, \rho(\mathbf{x}') \left( x_1'^2 + x_2'^2 \right)$$

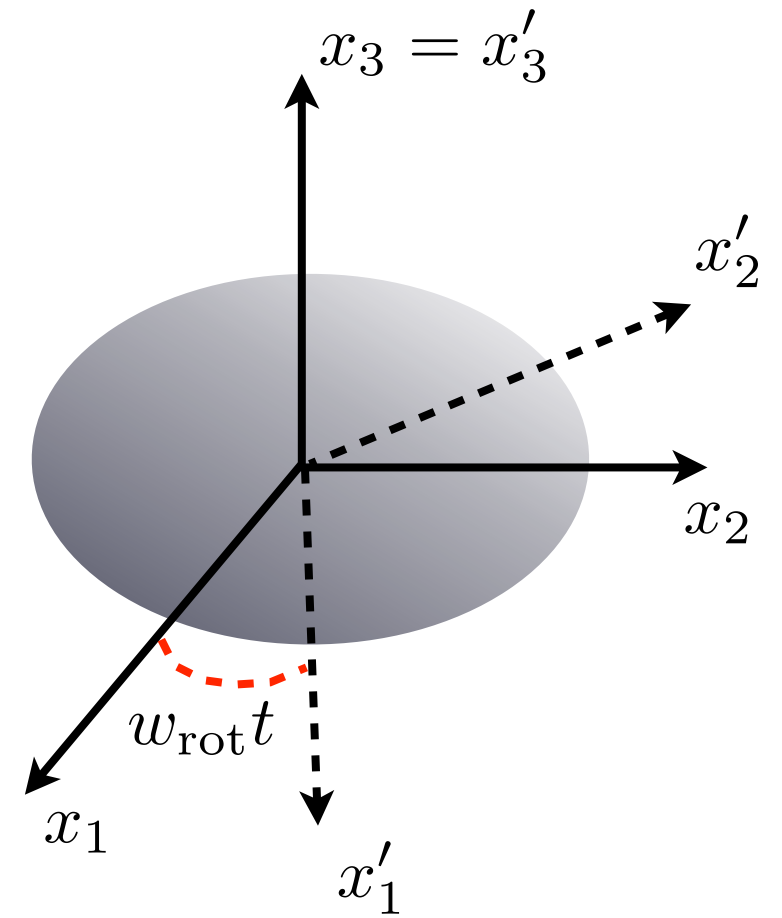



Consider a simple situation in which an ellipsoidal body rotates rigidly about one of its principle axes.

# Example II: Quadrupole radiation from rotating rigid body

$(x'_1, x'_2, x'_3)$ - attached to body and rotate with it

$(x_1, x_2, x_3)$ - fixed reference frame




The two frames are related by time-dependent rotation matrix:

$$x'_i = \mathcal{R}_{ij} x_j$$

$$\mathcal{R}_{ij} = \begin{bmatrix} \cos\omega_{\mathrm{rot}}t & \sin\omega_{\mathrm{rot}}t & 0 \\ -\sin\omega_{\mathrm{rot}}t & \cos\omega_{\mathrm{rot}}t & 0 \\ 0 & 0 & 1 \end{bmatrix}_{ij}$$

The time-dependent inertia tensor is then given as $I = \mathcal{R}^T I' \mathcal{R}$

$$I_{11} = 1 + \frac{I_1 - I_2}{2}\cos 2\omega_{\mathrm{rot}}t \qquad I_{22} = 1 - \frac{I_1 - I_2}{2}\cos 2\omega_{\mathrm{rot}}t$$

$$I_{12} = \frac{I_1 - I_2}{2}\sin 2\omega_{\mathrm{rot}}t \qquad I_{33} = I_3 \qquad I_{13} = I_{23} = 0$$

# Example II: Quadrupole radiation from rotating rigid body

Compare the inertia tensor with the second mass moment:

$$I^{ij} = \int d^3x\, \rho(\mathbf{x}) \left(r^2 \delta^{ij} - x^i x^j\right) \qquad\qquad M^{ij} = \int d^3x\, \rho(\mathbf{x}) x^i x^j$$

They differ by a minus sign and a trace term.

$$M^{ij} = -I^{ij} + \mathrm{Tr}(I)\delta^{ij}$$

But the trace is a constant :

$$\mathrm{Tr}(I) = \mathrm{Tr}(\mathcal{R}^T I' \mathcal{R}) = \mathrm{Tr}(\mathcal{R}\mathcal{R}^T I') = \mathrm{Tr}(I') = I_1 + I_2 + I_3$$

# Example II: Quadrupole radiation from rotating rigid body

So when taking the second time derivative of $M^{ij}$, the trace terms vanish.

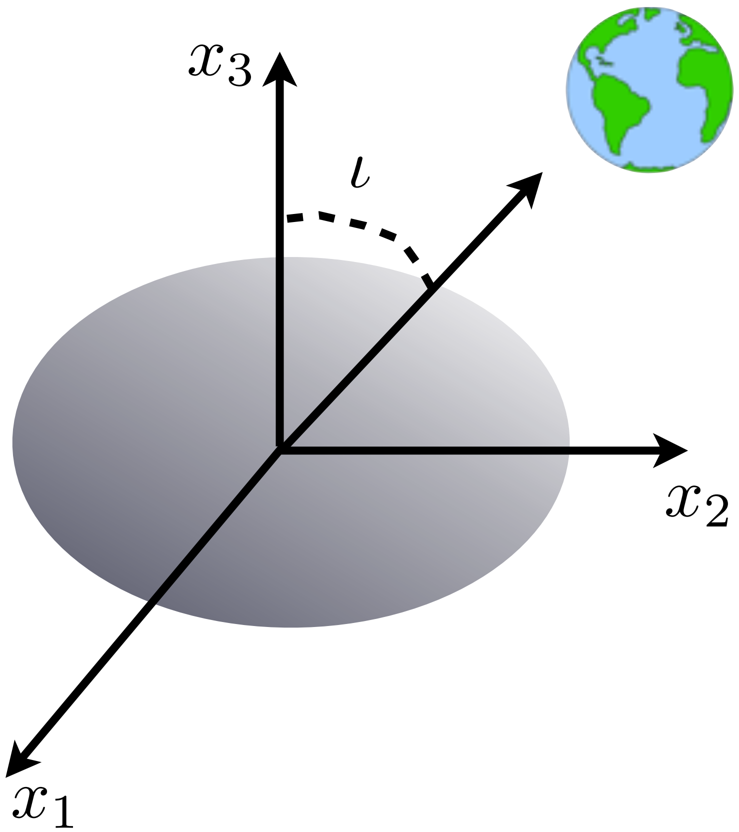
$$M_{11} = -\frac{I_1 - I_2}{2} \cos 2\omega_{\mathrm{rot}} t + \mathrm{constant}$$
$$M_{12} = -\frac{I_1 - I_2}{2} \sin 2\omega_{\mathrm{rot}} t + \mathrm{constant}$$
$$M_{22} = +\frac{I_1 - I_2}{2} \cos 2\omega_{\mathrm{rot}} t + \mathrm{constant}$$
$$M_{13} = M_{23} = M_{33} = \mathrm{constant}$$

Note, there is a time-varying second mass moment only if $I_1 \neq I_2$ .

$M_{ij}$ is a periodic function so we have production of gravitational waves with frequency:

$$\omega_{\mathrm{gw}} = 2\omega_{\mathrm{rot}}$$

# Example II: Quadrupole radiation from rotating rigid body




Use equations for generic propagation.
Set $\theta = \iota$ and $\phi = 0$.

$$\boxed{h_+ = \frac{1}{r}\frac{4G\omega_{\mathrm{rot}}^2}{c^4}\left(I_1 - I_2\right)\frac{1+\cos^2\iota}{2}\cos\left(2\omega_{\mathrm{rot}}t\right)}$$

$$\boxed{h_\times = \frac{1}{r}\frac{4G\omega_{\mathrm{rot}}^2}{c^4}\left(I_1 - I_2\right)\cos\iota\sin\left(2\omega_{\mathrm{rot}}t\right)}$$

Define ellipticity by: $\epsilon \equiv \dfrac{I_1 - I_2}{I_3}$

$$h_+ = h_0\frac{1+\cos^2\iota}{2}\cos\left(2\pi f_{\mathrm{gw}}t\right)$$

$$h_\times = h_0\cos\iota\sin\left(2\pi f_{\mathrm{gw}}t\right)$$

$$h_0 = \frac{4\pi^2 G}{c^4}\frac{I_3 f_{\mathrm{gw}}^2}{r}\epsilon$$

Neutron stars that rotate more rapidly produce a stronger GW signal.

# Example II: Quadrupole radiation from rotating rigid body

Angular distribution of the radiated power in quadrupole approximation:

$$P_{\rm quad} = \left(\frac{dE_{\rm gw}}{d\Omega}\right)_{\rm quad} = \frac{r^2c^3}{16\pi G}\int_{\mathcal{S}} d\Omega \left\langle \dot{h}_+^2 + \dot{h}_\times^2 \right\rangle$$

For our NS example: $P = \frac{32G}{5c^5}\epsilon^2 I_3^2 \omega_{\rm rot}^6$

Then we can say that the rotational energy of the star decreases because of GW emission as

$$\frac{dE_{\rm rot}}{dt} = -\frac{32G}{5c^5}\epsilon^2 I_3^2 \omega_{\rm rot}^6$$

Rotational energy of star rotating around its principal axis is

$$E_{\rm rot} = (1/2) I_3 \omega_{\rm rot}^2$$

Then rotational frequency of neutron star should decrease as

$$\dot{\omega}_{\rm rot} = -\frac{32G}{5c^5}\epsilon^2 I_3 \omega_{\rm rot}^5$$

# Example II: Quadrupole radiation from rotating rigid body

$$\dot{\omega}_{\mathrm{rot}} \sim -\omega_{\mathrm{rot}}^{n}$$

**Table 1**

Braking index measurements for six pulsars. Also given are the pulsar period, period derivative, period second derivative and characteristic age.

| Pulsar names J2000; B1950 | $P$ (ms) | $\dot{P}$ ($10^{-15}$) | $\ddot{P}$ ($s^{-1}$) | $\tau_c$ (yr) | braking index $n$ |
|---|---|---|---|---|---|
| Crab* | | | | | |
| J0534+2200; B0531+21 | 33.085 | 423 | $-3.61 \times 10^{-24}$ | 1240 | 2.51(1)[a] |
| J0540−6919; B0540−69* | 50.499 | 479 | $-1.6 \times 10^{-24}$ | 1670 | 2.140(9)[b] |
| Vela | | | | | |
| J0835−4510; B0833−45 | 89.328 | 125 | | 11 300 | 1.4(2)[c] |
| J1119−6127* | 407.746 | 4022 | $-8.8 \times 10^{-24}$ | 1160 | 2.91(5)[d] |
| J1513−5908; B1509−58* | 150.658 | 1540 | $-1.312 \times 10^{-23}$ | 1550 | 2.839(3)[e] |
| J1846−0258* | 325.684 | 7083 | | 728 | 2.65(1)[f] |

[a] Demiański & Prószyński (1983), Lyne, Pritchard & Smith (1988, 1993).

[b] Livingstone et al. (2005).

[c] Lyne et al. (1996), Dodson, McCulloch & Lewis (2002).

[d] Camilo et al. (2000).

[e] Manchester, Durdin & Newton (1985), Kaspi et al. (1994).

[f] Livingstone et al. (2006).

N. Vranesevic, D.B. Melrose, MNRAS 410, 4 (2011)

$n$ is the braking index.

Experimentally, $n$ ranges between 2 and 3, rather than $n =$ 5 so GW emission is not main energy loss mechanism for rotating pulsars.

Other EM mechanisms dominate.

# LIGO/Virgo Astrophysical Sources

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  - Binary Neutron Stars
  - Binary Black Holes
- Continuous Waves
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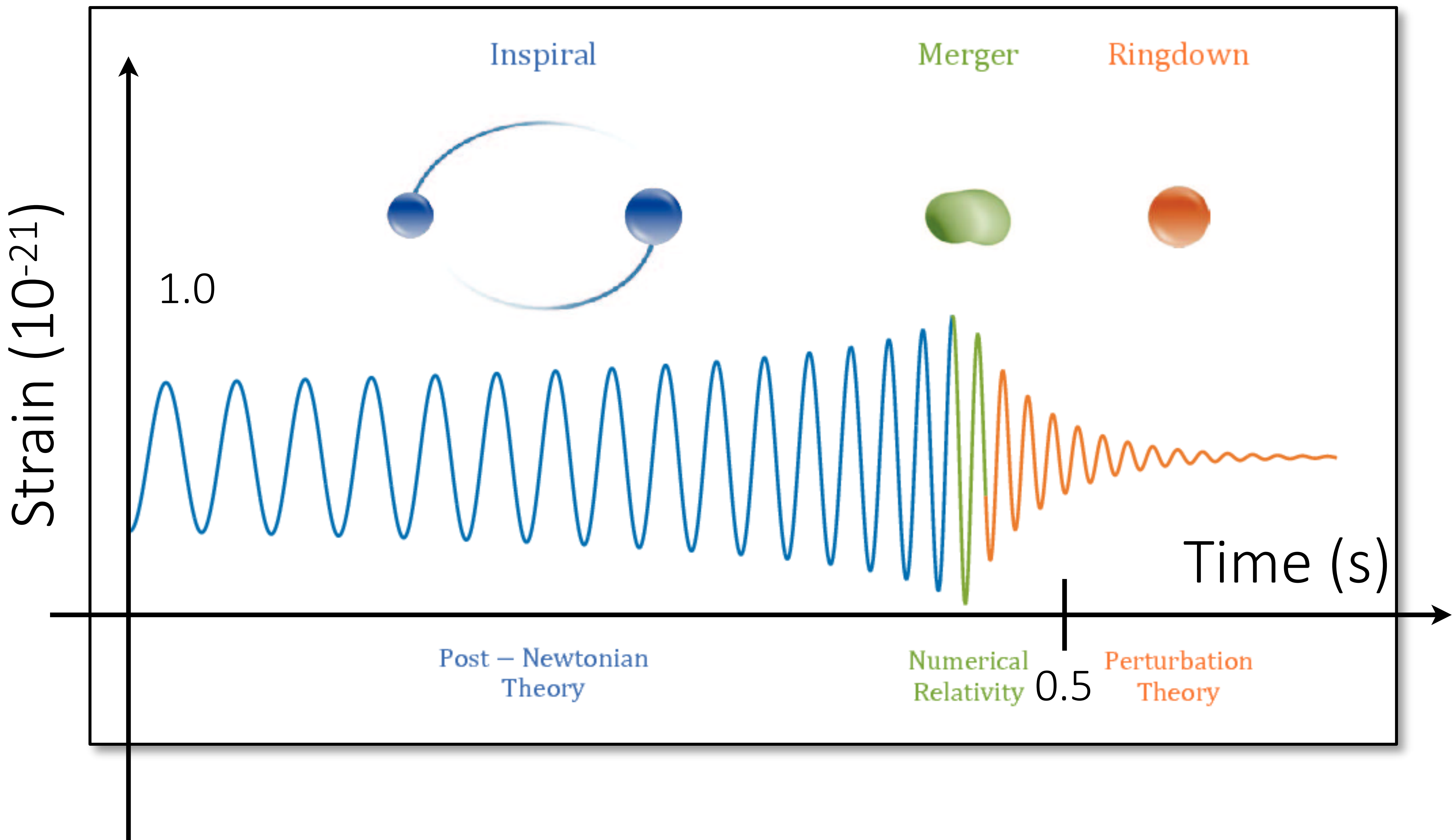
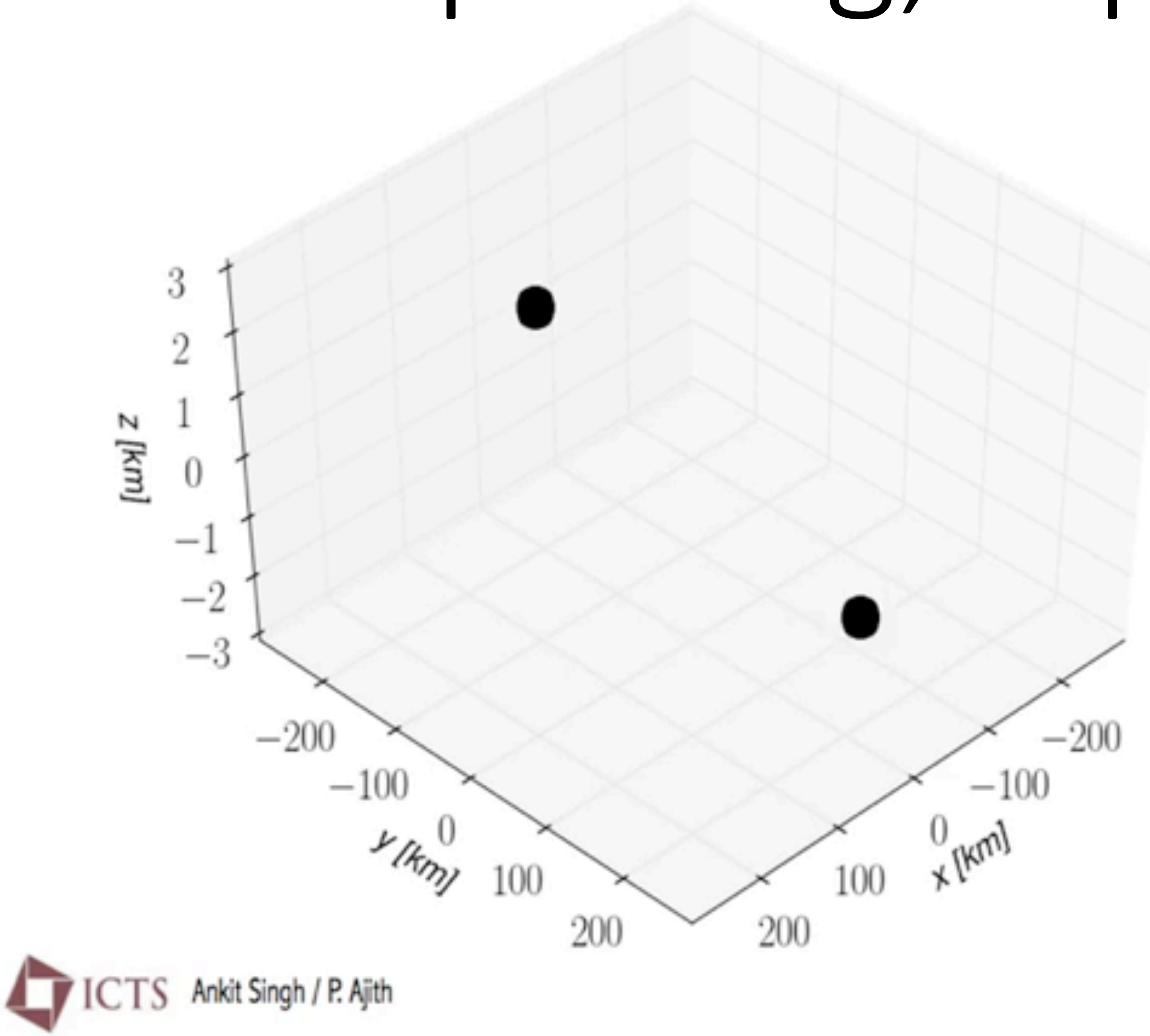
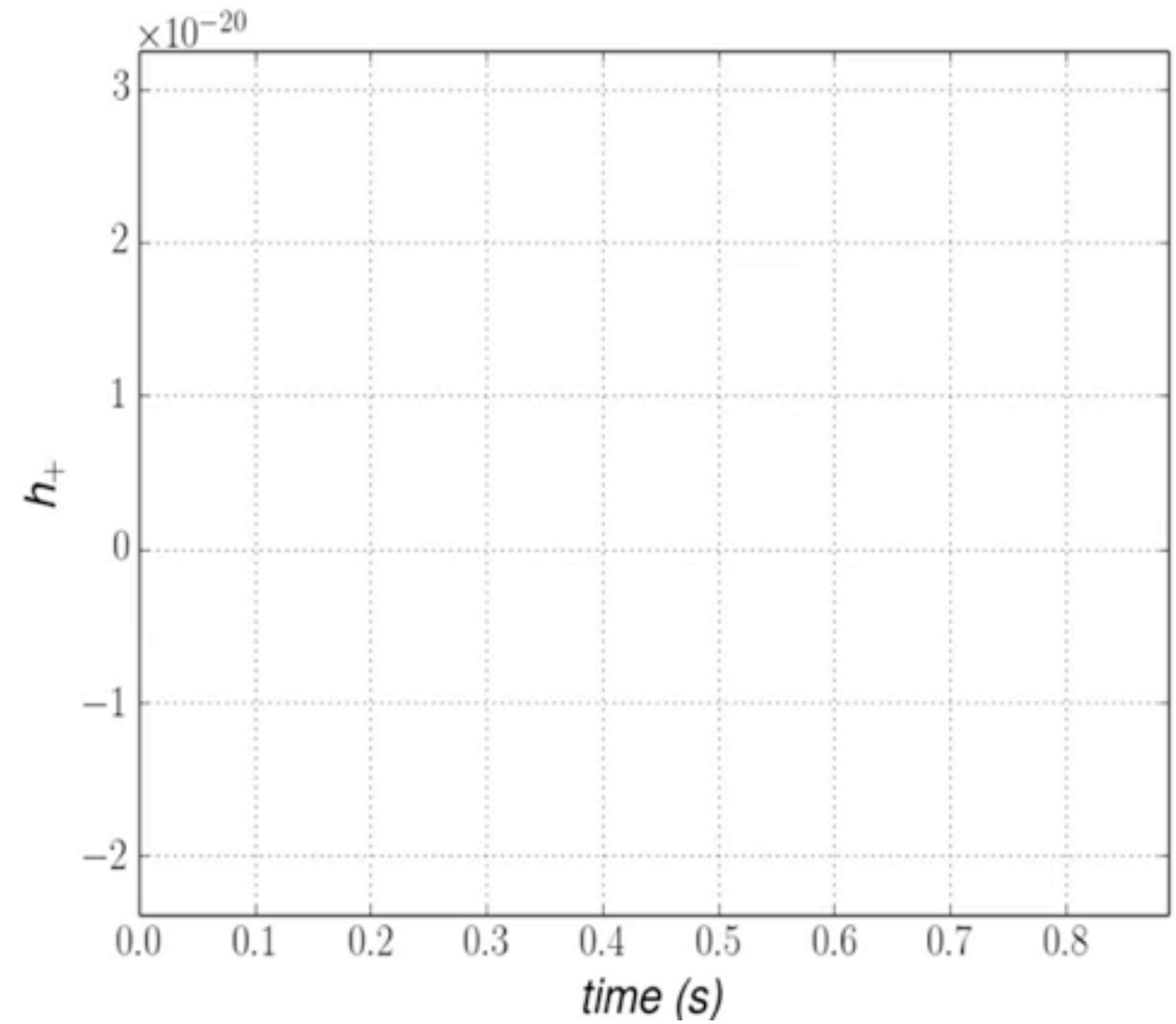
# Coalescing Binaries



# Coalescing Binaries

## Non-spinning, equal mass black holes

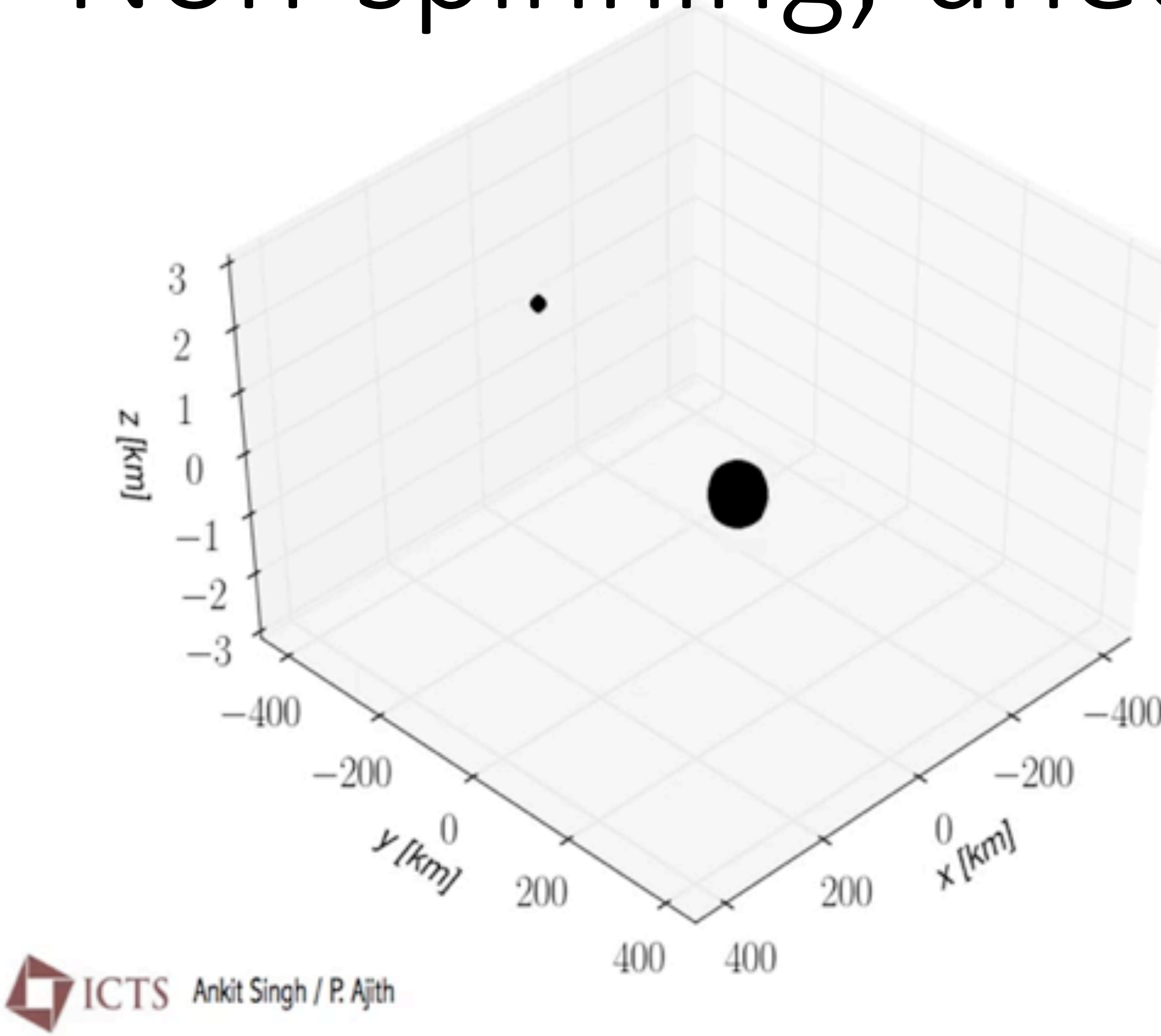


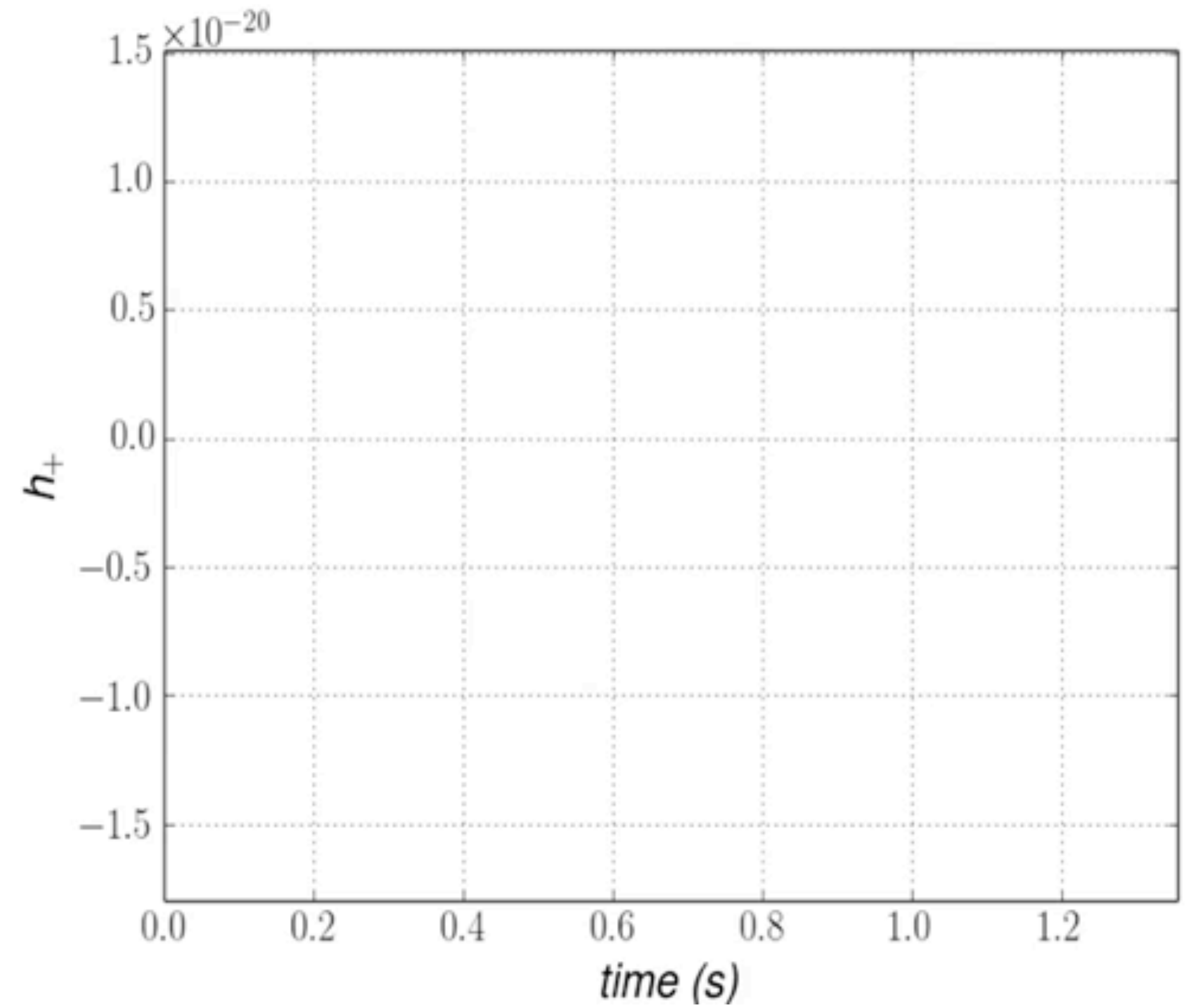




(m1, m2) = (10, 10) $M_\odot$

# Coalescing Binaries

## Non-spinning, unequal mass black holes




(m1, m2) = (4, 16) $M_\odot$

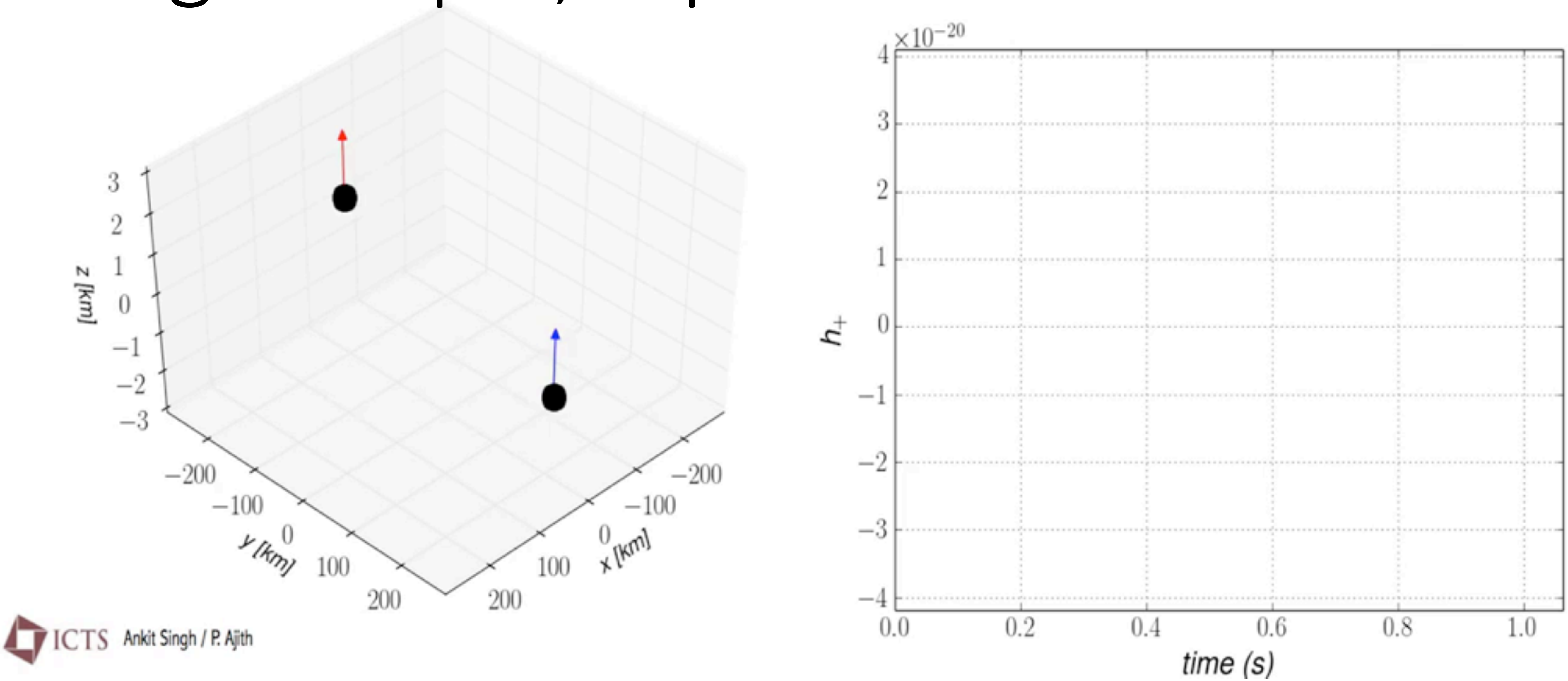
The more massive BH is closer to the center of mass.

The energy radiated is lower than an equal-mass binary.

The binary takes longer to inspiral.

# Coalescing Binaries

## Aligned spin, equal mass black holes

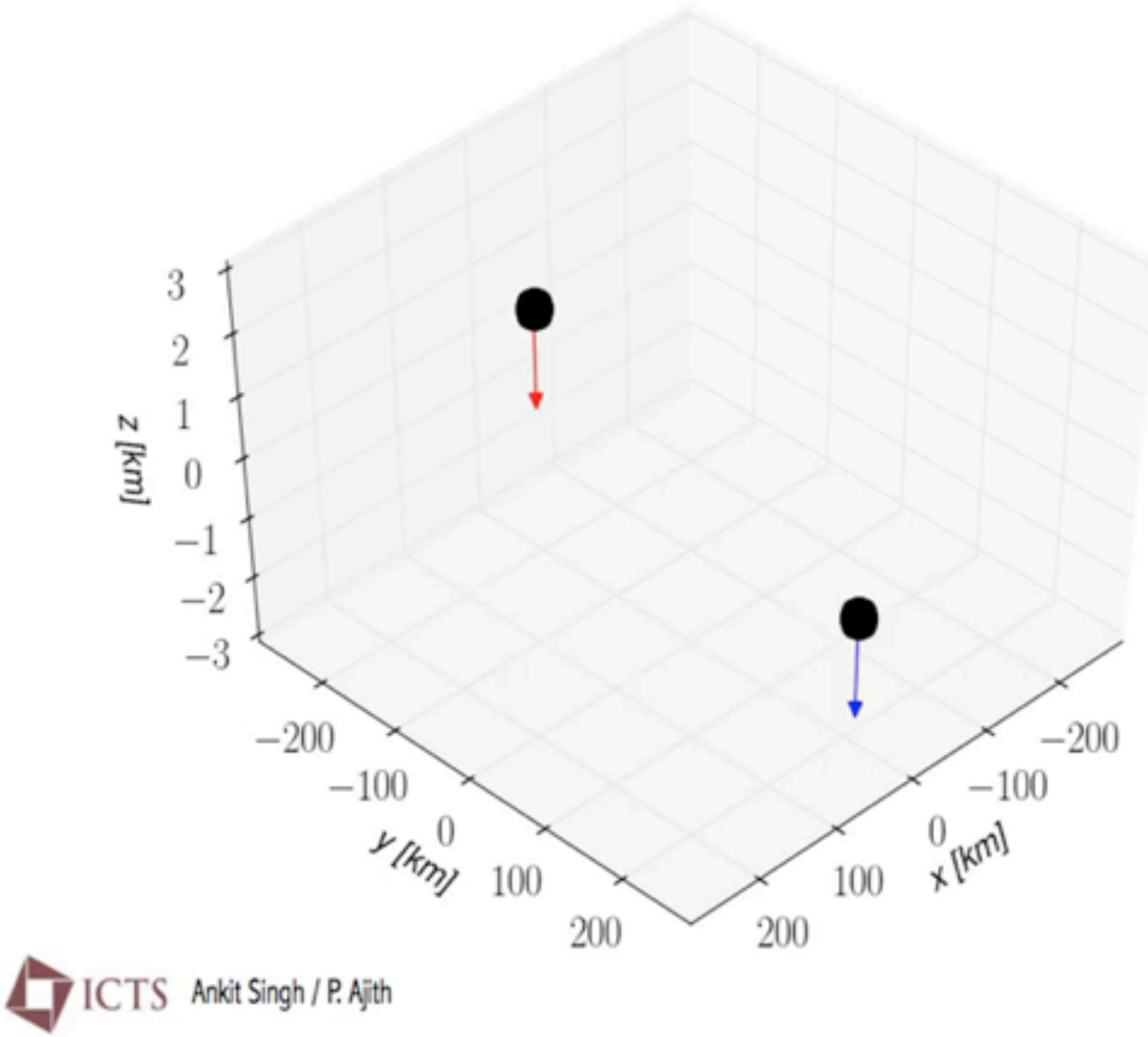


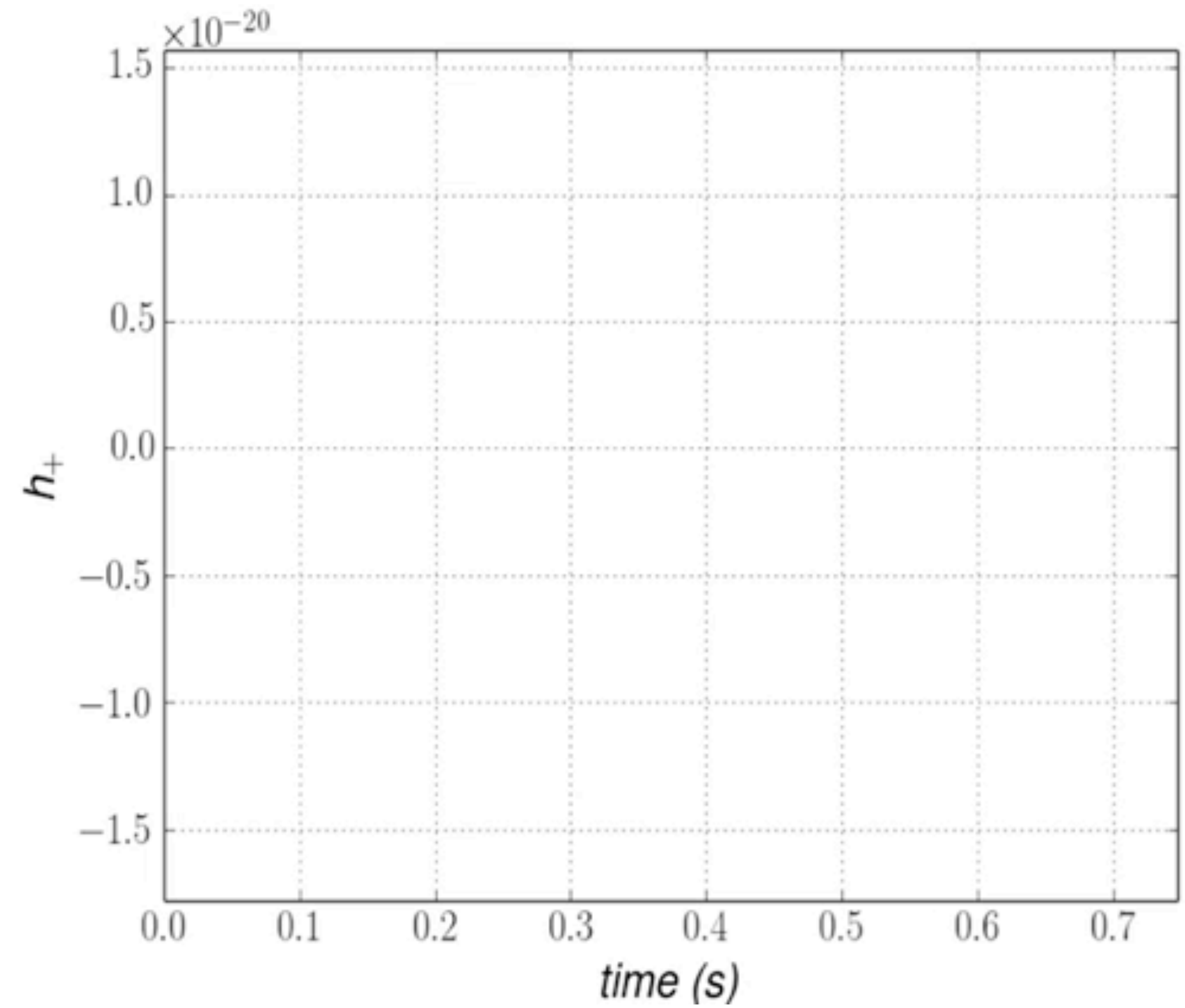

Spin vectors are aligned with orbital angular momentum.

**Orbital hang-up effect**: aligned-spin black holes can inspiral to much closer separations, resulting in longer and stronger GW signals, compared to non-spinning binary.

# Coalescing Binaries

## Anti-aligned spin, equal mass black holes

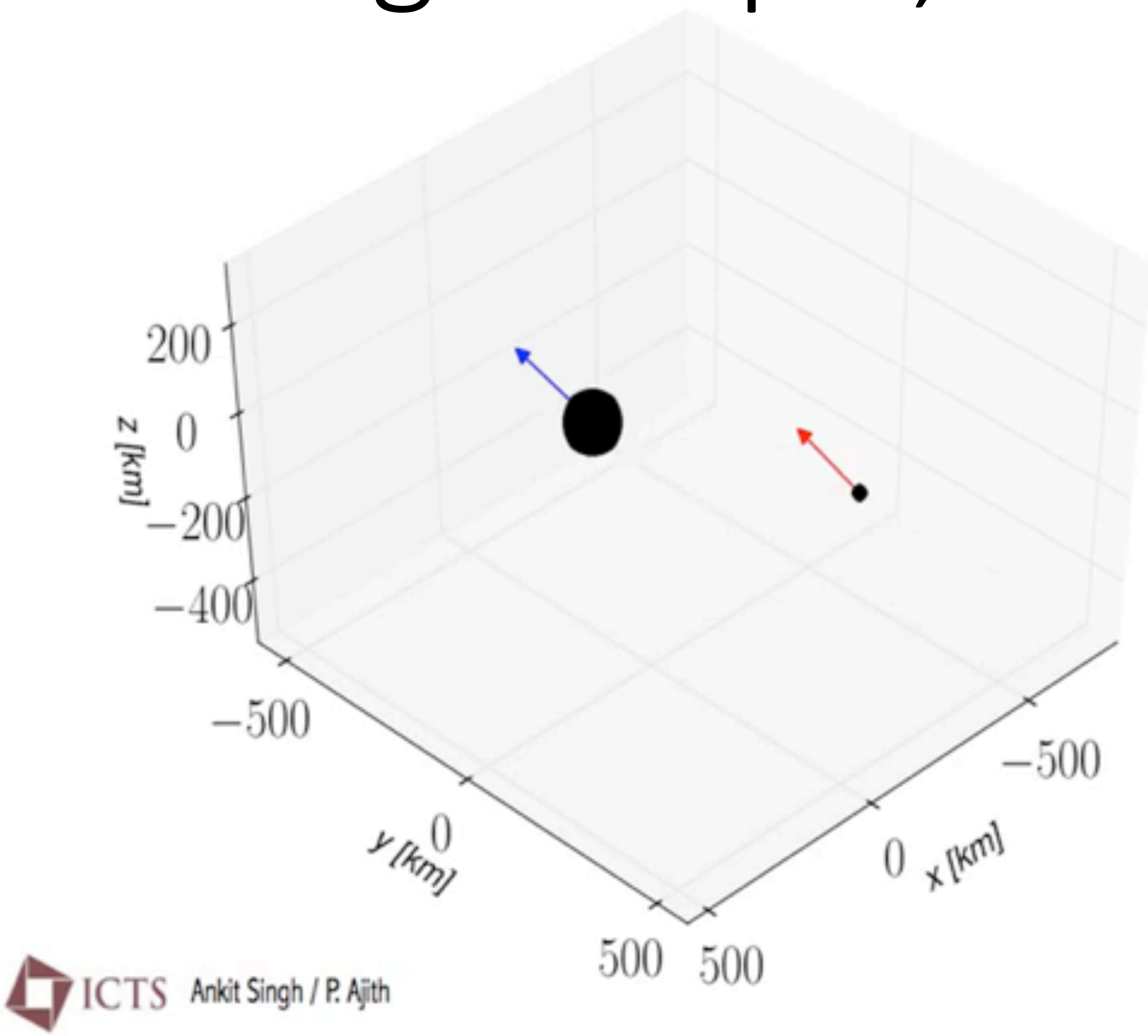


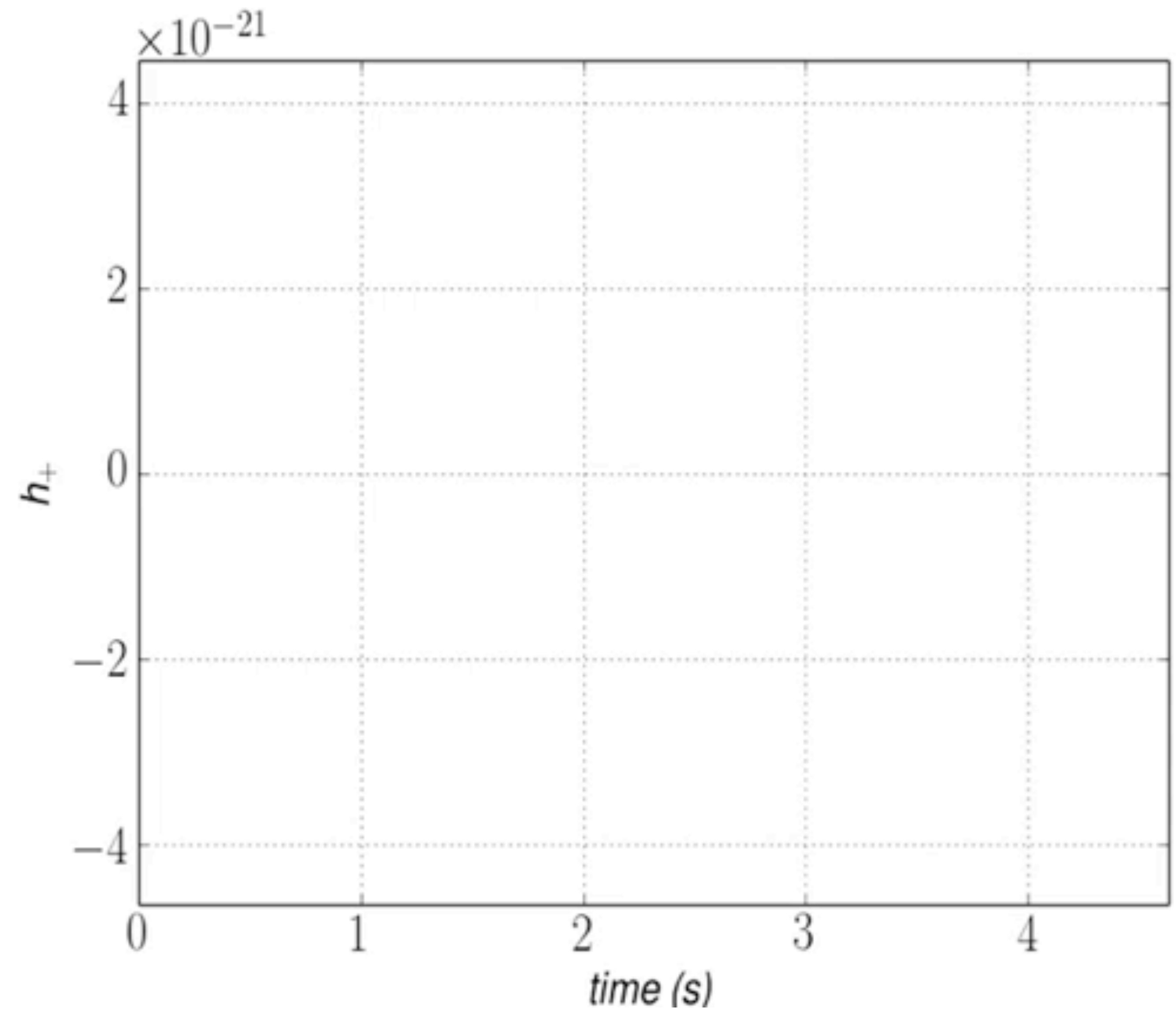




Spin vectors are aligned opposite to orbital angular momentum.
Anti-aligned-spin black holes have shorter and weaker GW signals, compared to non-spinning binary.

# Coalescing Binaries

## Misaligned spin, unequal mass black holes



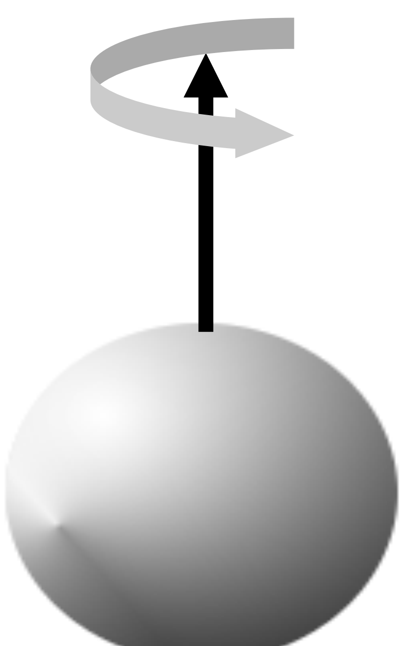



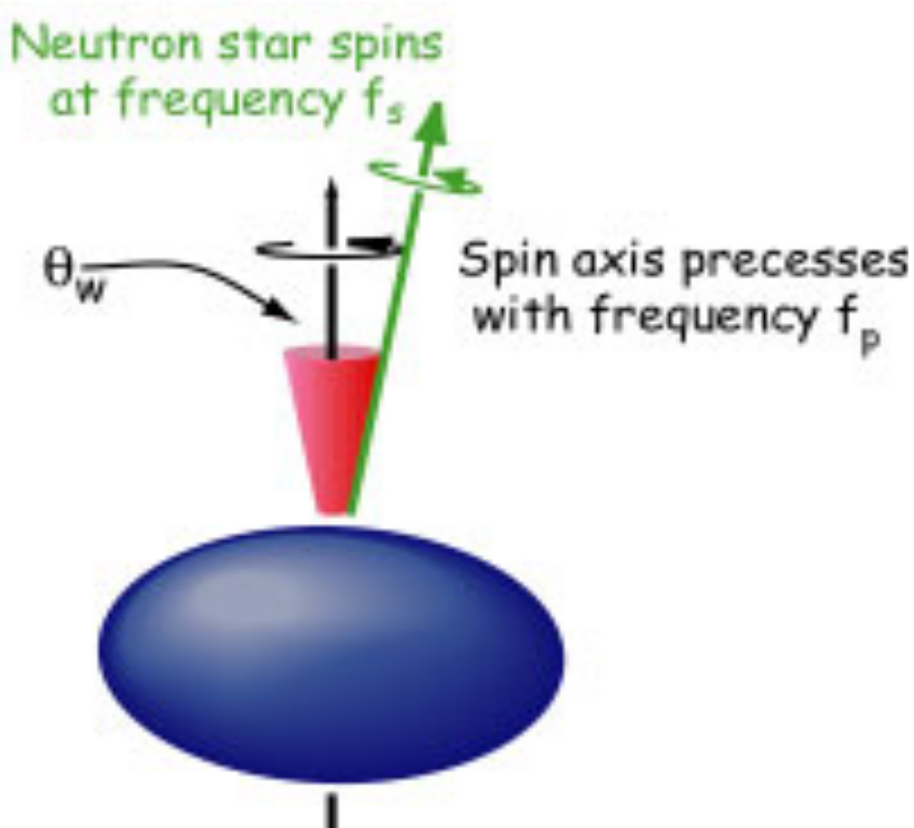

Spin vectors are misaligned with orbital angular momentum.
There are **spin-orbit and spin-spin interactions** between spins and orbital angular momentum that cause spins to precess.
Results in complicated modulations in amplitude and phase of GW signals.

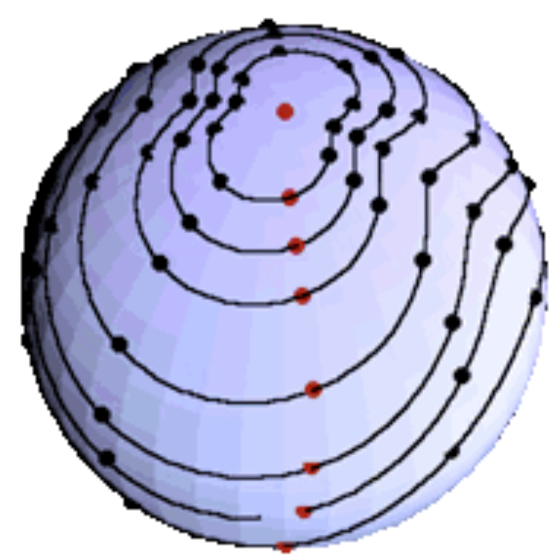
# Continuous Waves

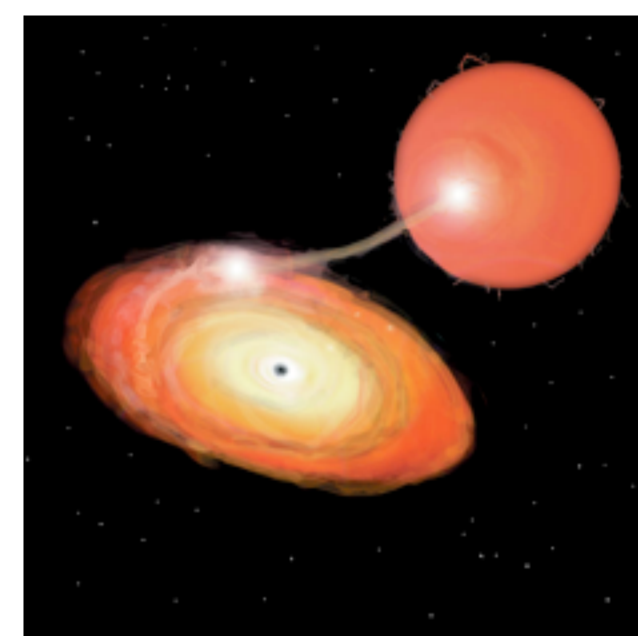
Non-axisymmetric rotating neutron stars;
asymmetry could arise from:

- equatorial ellipticity (mm-high mountain) $f_{\mathrm{GW}} = 2 f_{\mathrm{rot}}$
- free precession around rotation axis $f_{\mathrm{GW}} \sim f_{\mathrm{rot}} + f_{\mathrm{prec}}$
- excitation of long-lasting oscillations $f_{\mathrm{GW}} \sim 4/3 f_{\mathrm{rot}}$
- deformation due to matter accretion $f_{\mathrm{GW}} = 2 f_{\mathrm{rot}}$







# Continuous Waves

Continuous signal with $h \propto \epsilon$ $\quad$ $\mathrm{SNR} \propto \frac{h}{\sqrt{S_n}}\sqrt{T}$

Equatorial ellipticity $\epsilon = \frac{I_{\mathrm{XX}} - I_{\mathrm{YY}}}{I_{\mathrm{ZZ}}}$

Maximum Deformations

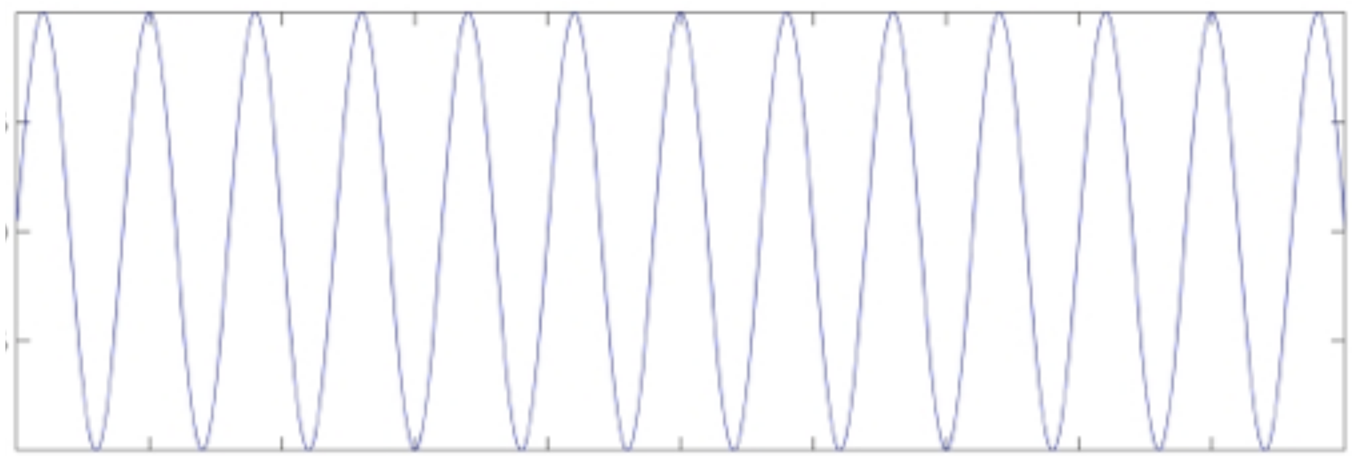
$\epsilon < 10^{-5}$ Normal Neutron Star

$\epsilon < 10^{-3}$ Hybrid Neutron Star

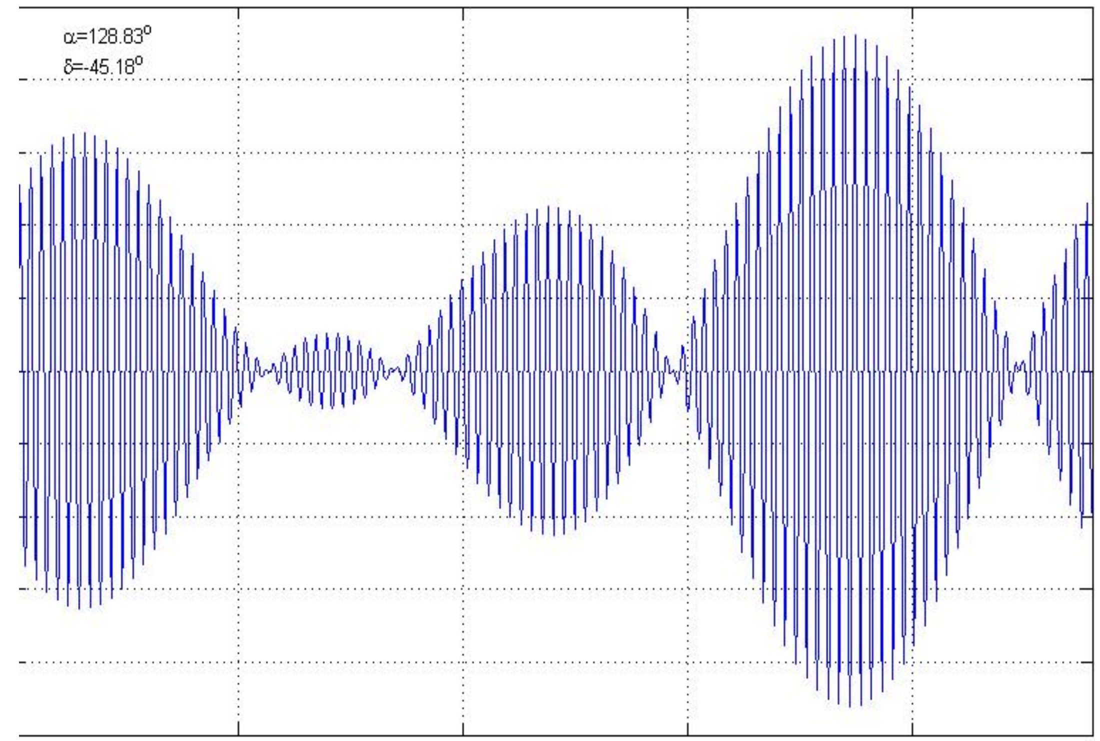
$\epsilon < 10^{-1}$ Extreme Quark Star

# Continuous Waves

At the source



At the detector

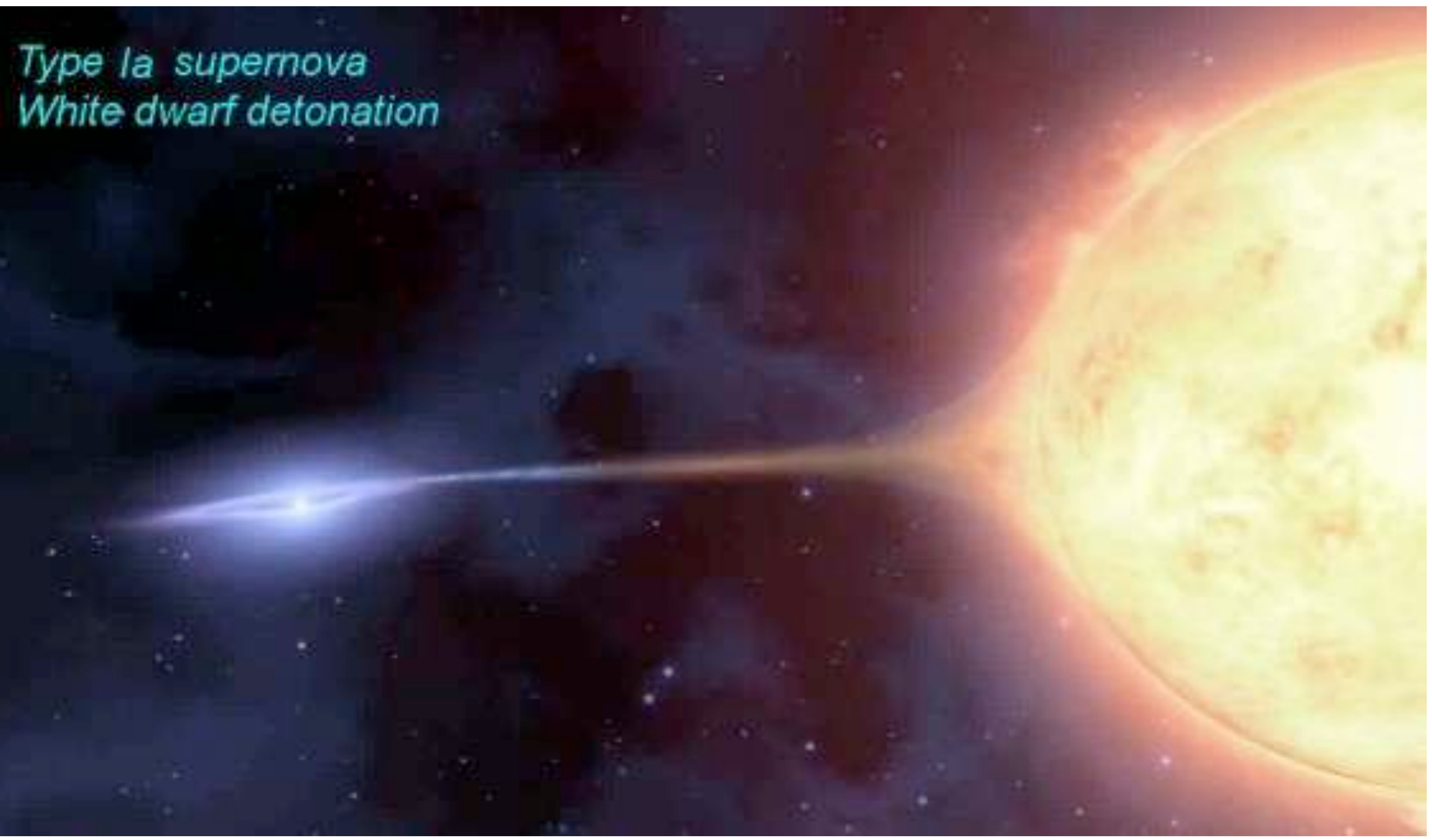


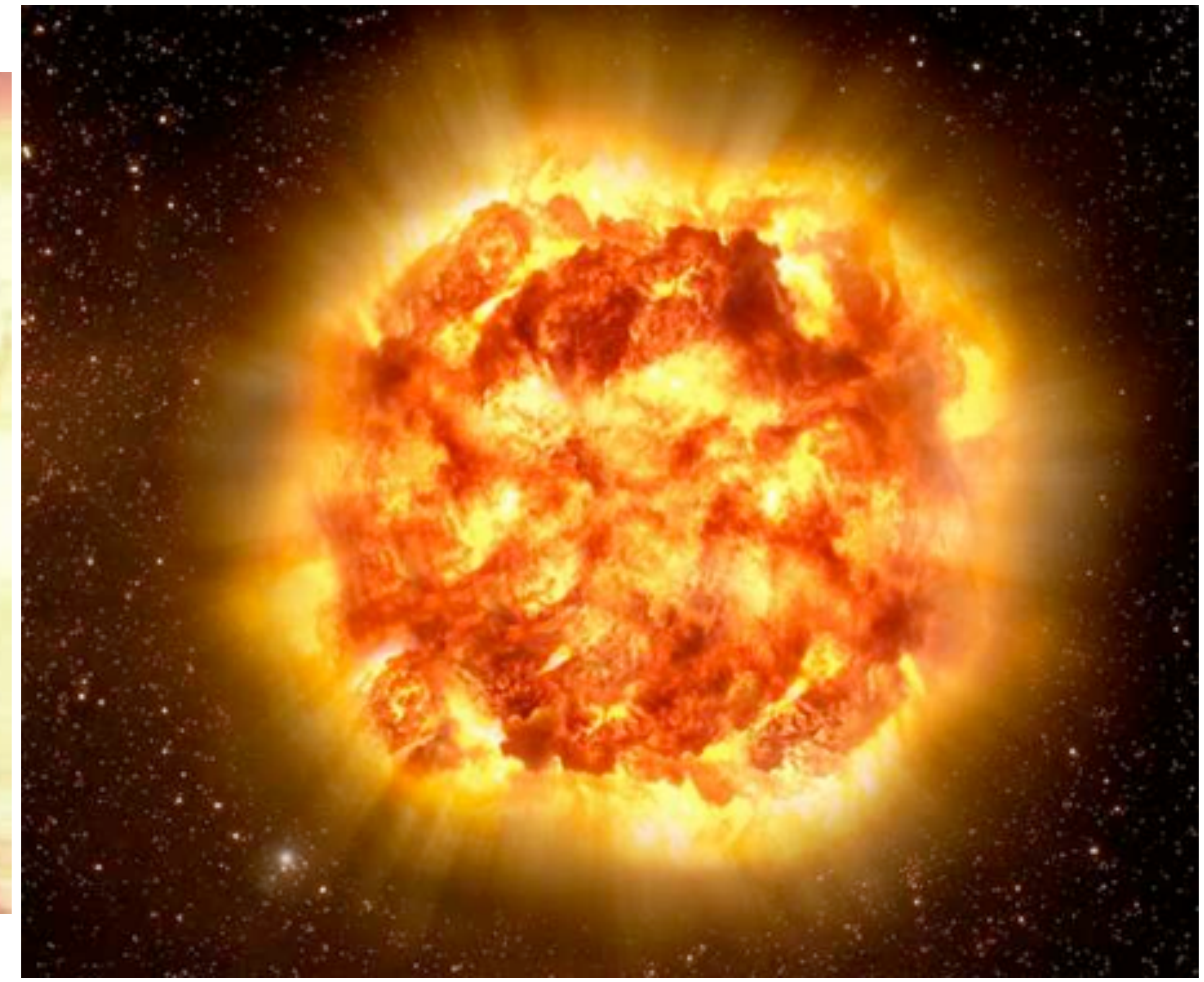

Nearly monochromatic, continuous signal but could have:

- relative velocity between source/detector (Doppler Effect)
- amplitude modulation due to antenna sensitivity of detector
- frequency and phase evolution

# Burst Sources

Supernovae

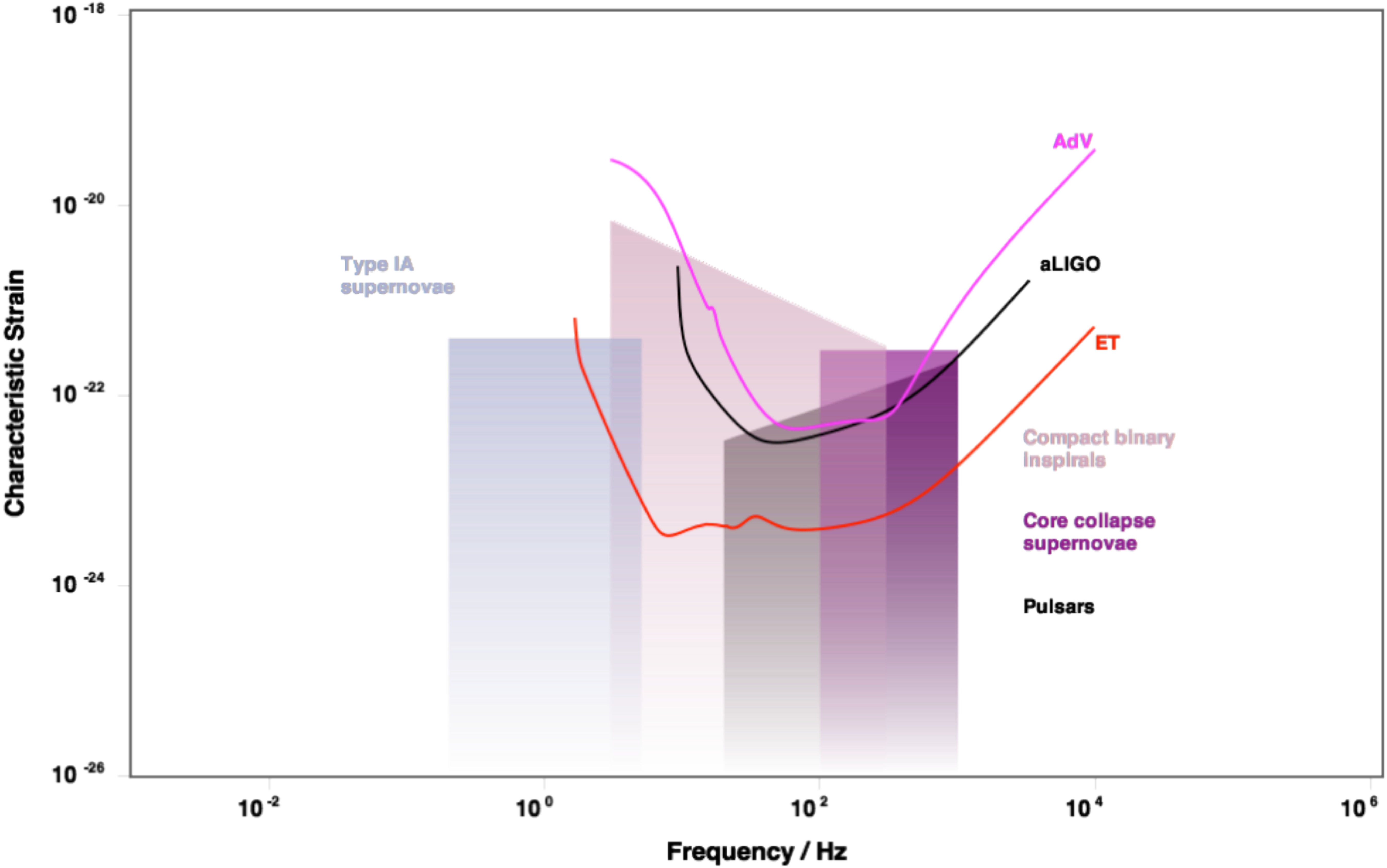


Type Ia supernovae when white dwarfs in binary detonate.

Core collapse supernovae when massive stars die.

# Burst Sources



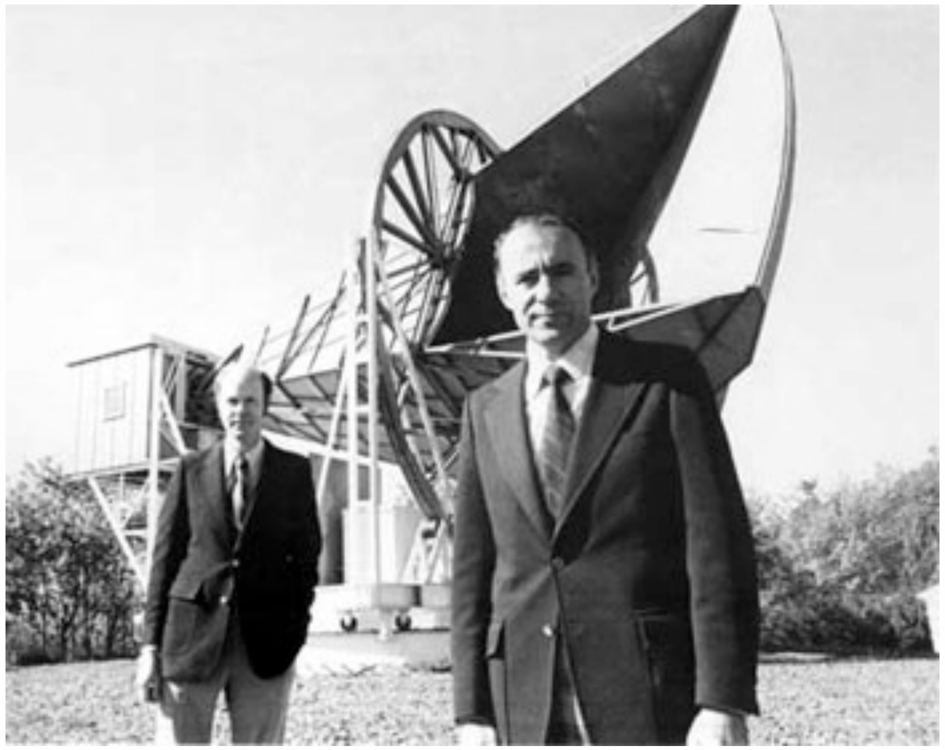
# Stochastic Background

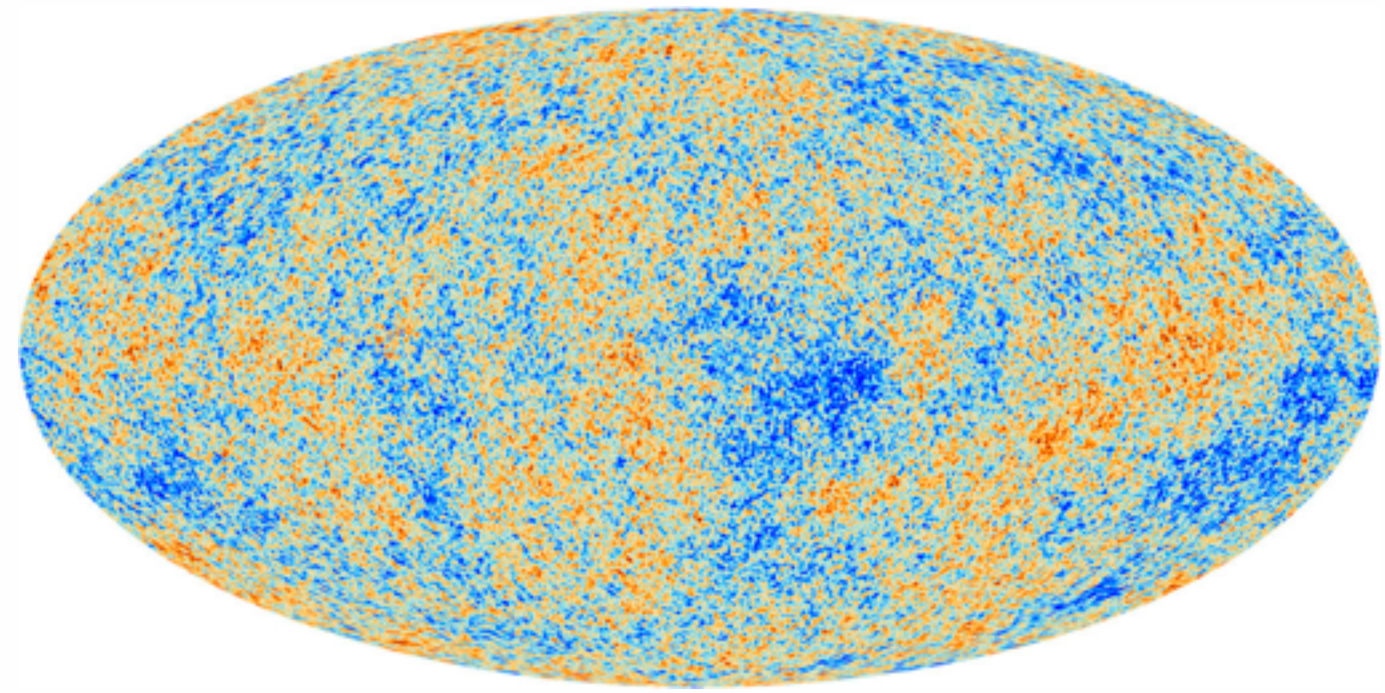
- Stochastic (random) background of gravitational radiation

- Can arise from superposition of large number of unresolved GW sources
  1. Cosmological origin
  2. Astrophysical origin

- Strength of background measured as gravitational wave energy density $\rho_{\mathrm{GW}}$

# Cosmic Microwave Background



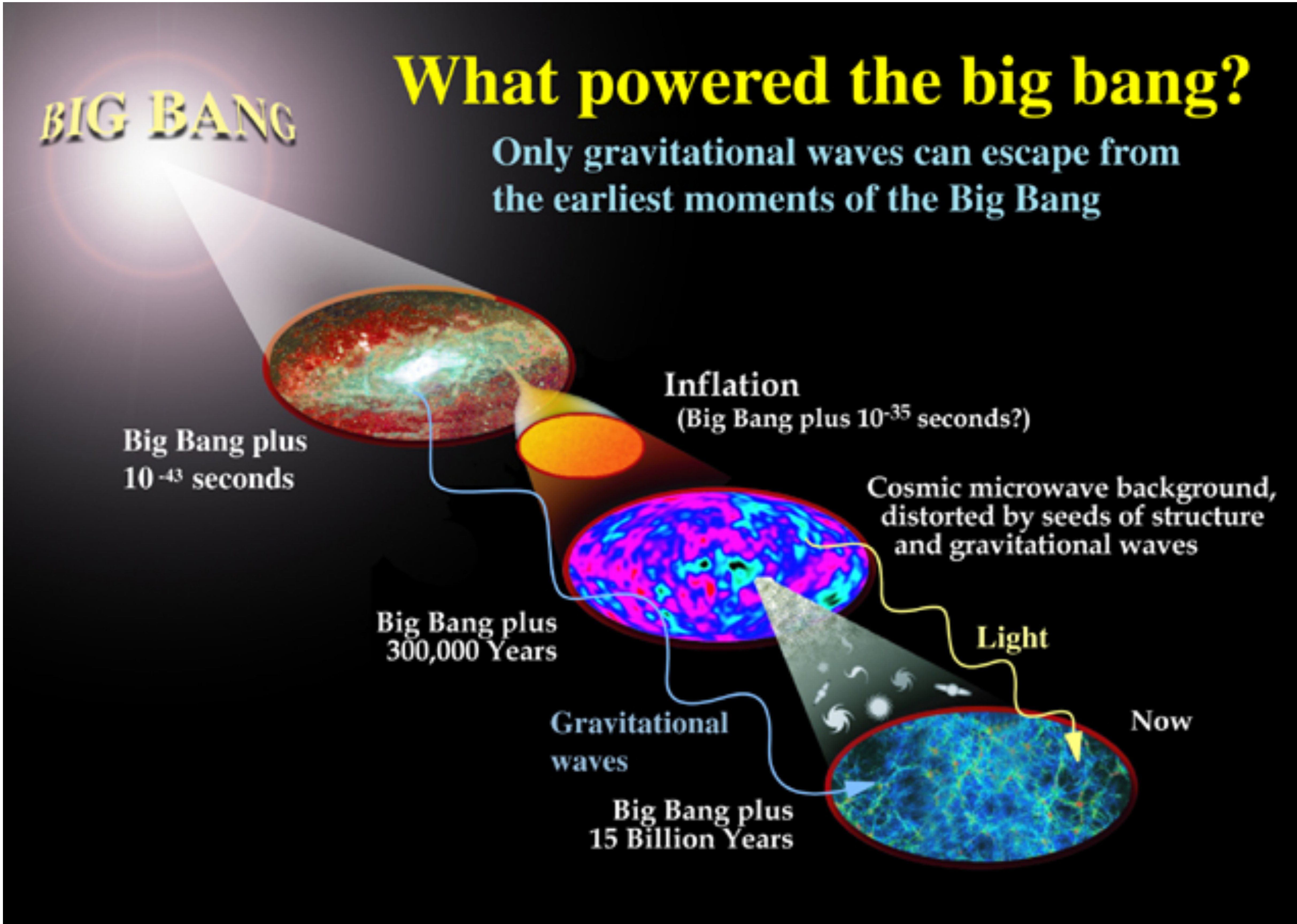


- 1965 - Penzias and Wilson accidently discovered Cosmic Microwave Background (CMB), leftover radiation from 380,000 years Big Bang

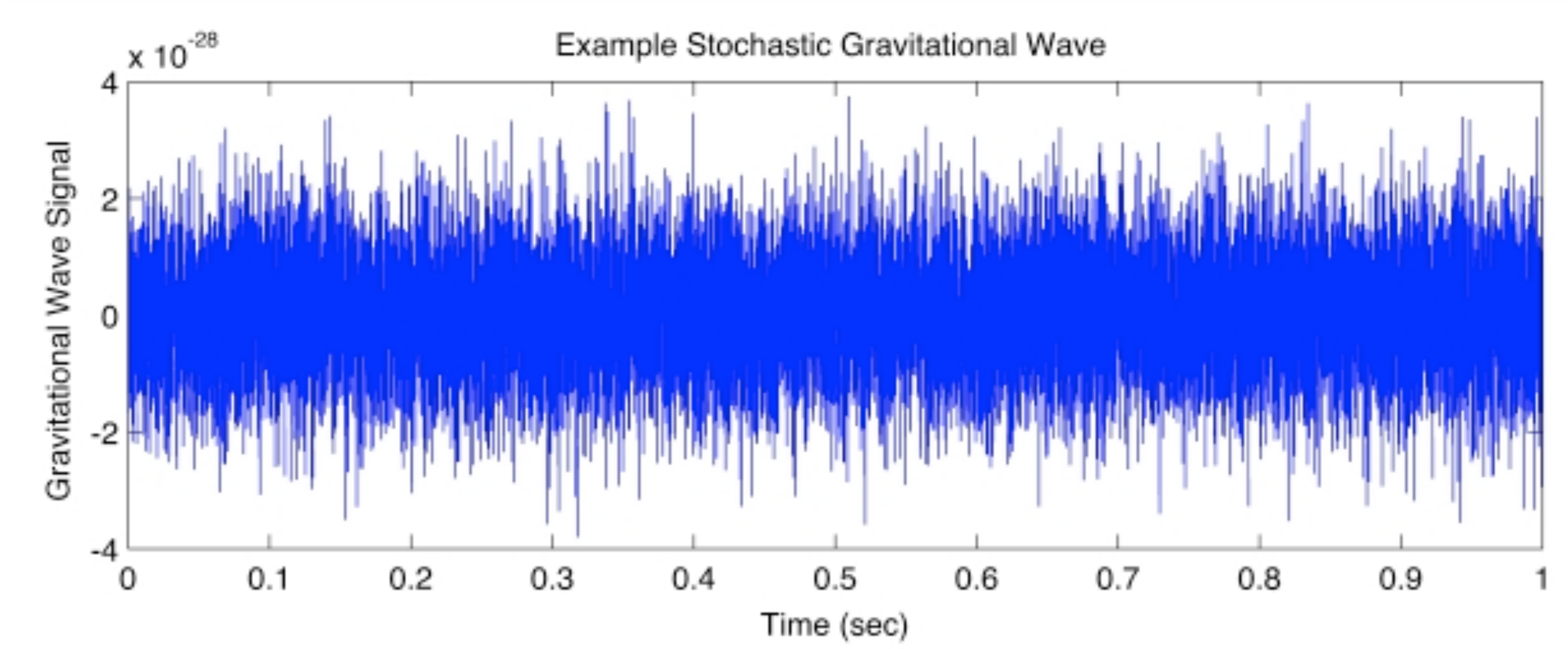
- 1978 - awarded Nobel prize

- CMB as seen by Planck, an ESA observatory
- Wavelengths of photons are greatly redshifted (1mm)
- Effective temperature ~ 2.7K
- Can be detected by far-infrared and radio telescopes

# Cosmological Gravitational Wave Background



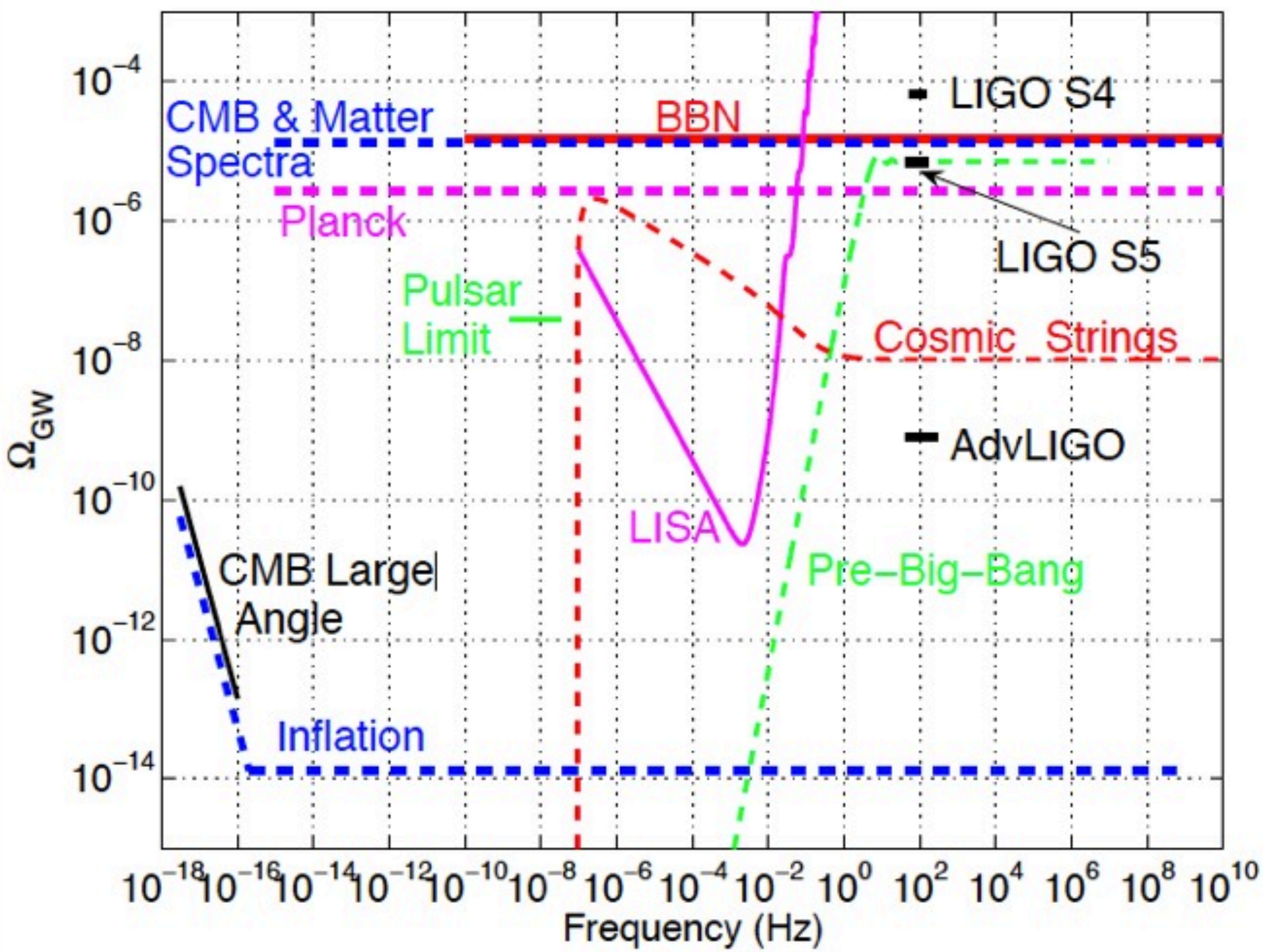
# Cosmological Gravitational Wave Background




GW spectrum: $\Omega_{\mathrm{GW}}(f) = \frac{f}{\rho_c} \frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}f}$

Critical energy density of universe: $\rho_c = \frac{3c^2 H_0^2}{8\pi G}$

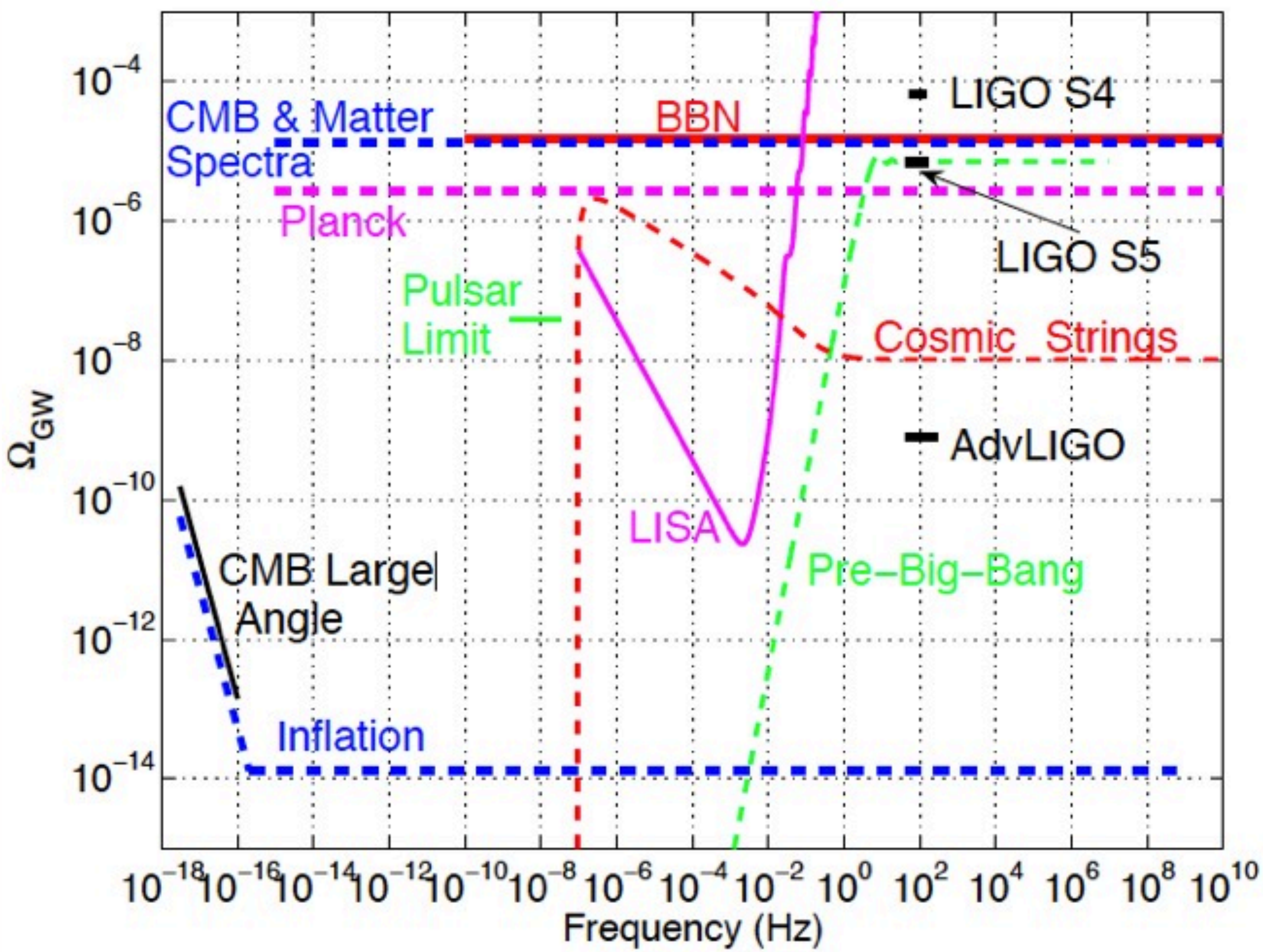
# Cosmological Gravitational Wave Background




**Big-Bang-Nucleosynthesis:** abundances of light nuclei produced

**Cosmic Microwave Background Measurements:** structure of CMB and matter power spectra

# Cosmological Gravitational Wave Background

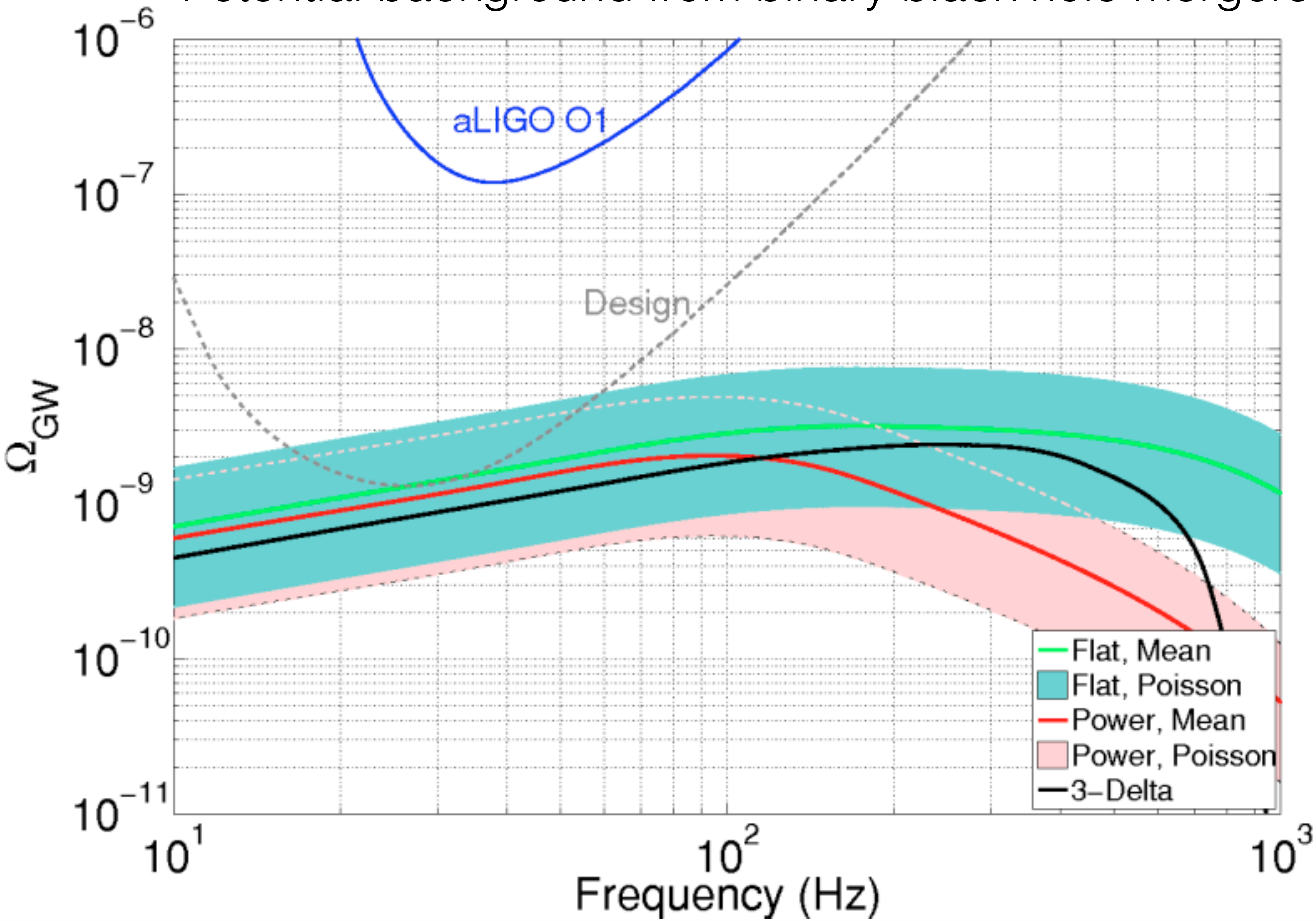



**Inflation:** measuring GWs can test for "stiffness" in early universe

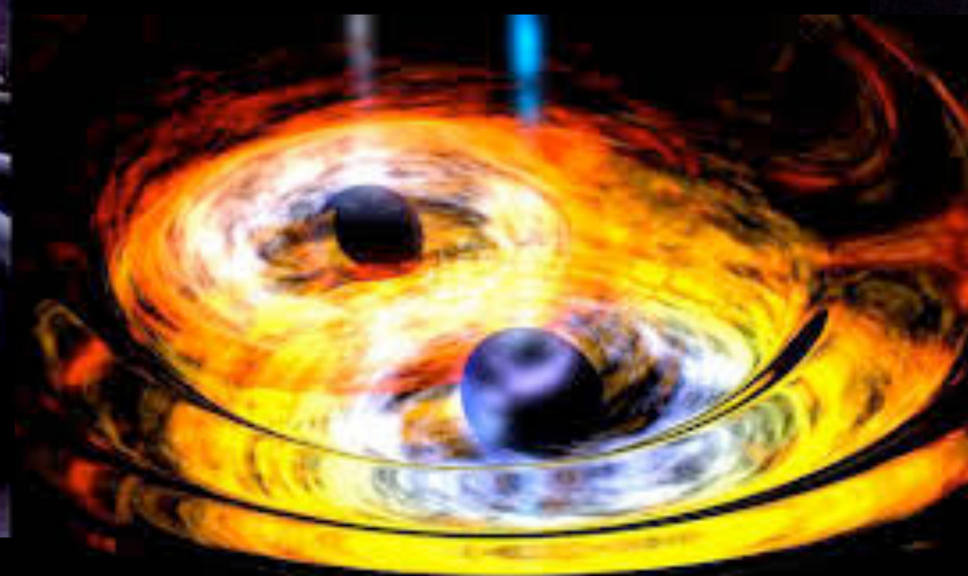
**Models of Cosmic Strings:** topological defects in early universe

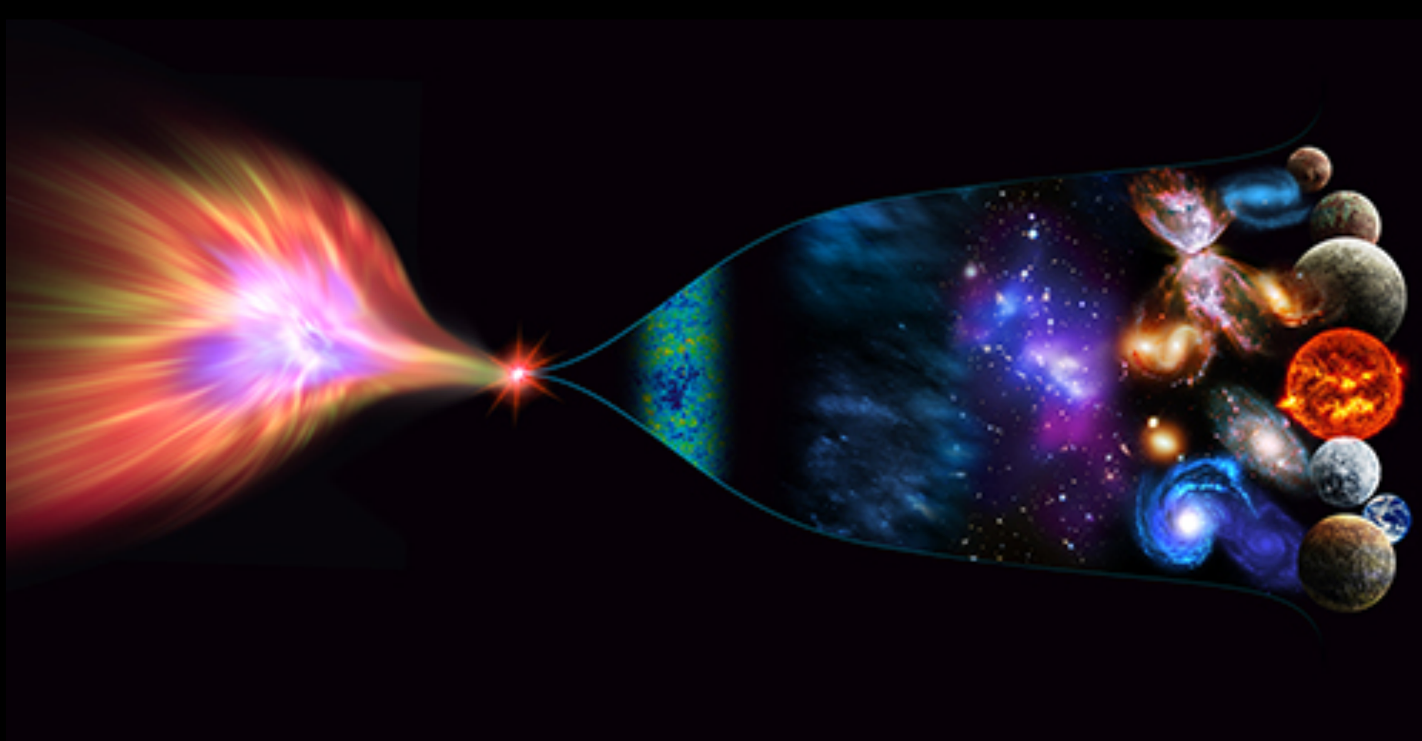
# Astrophysical Gravitational Wave Backgrounds

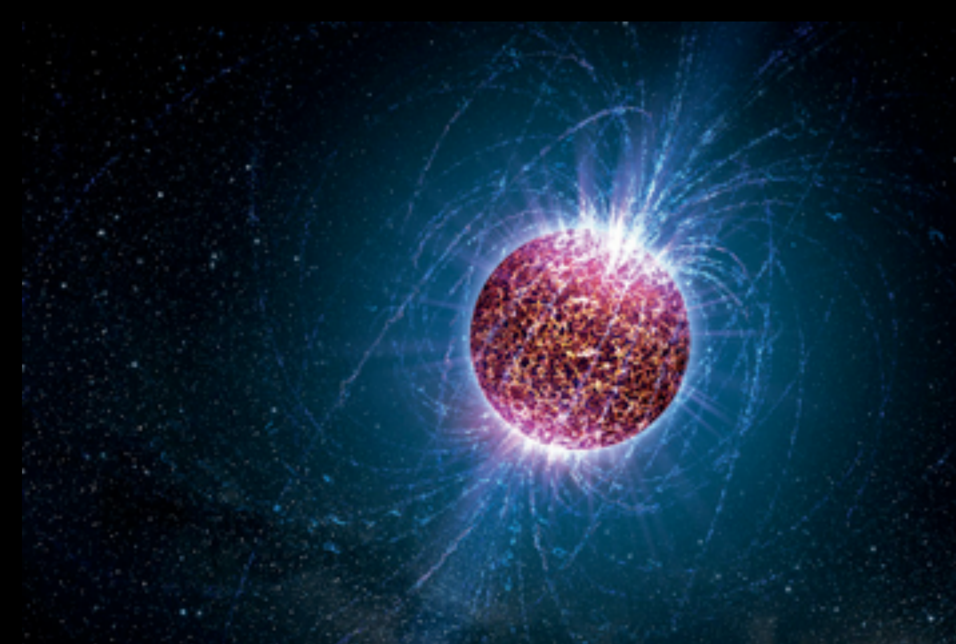
Potential background from binary black hole mergers



# Frequencies of signals as audio

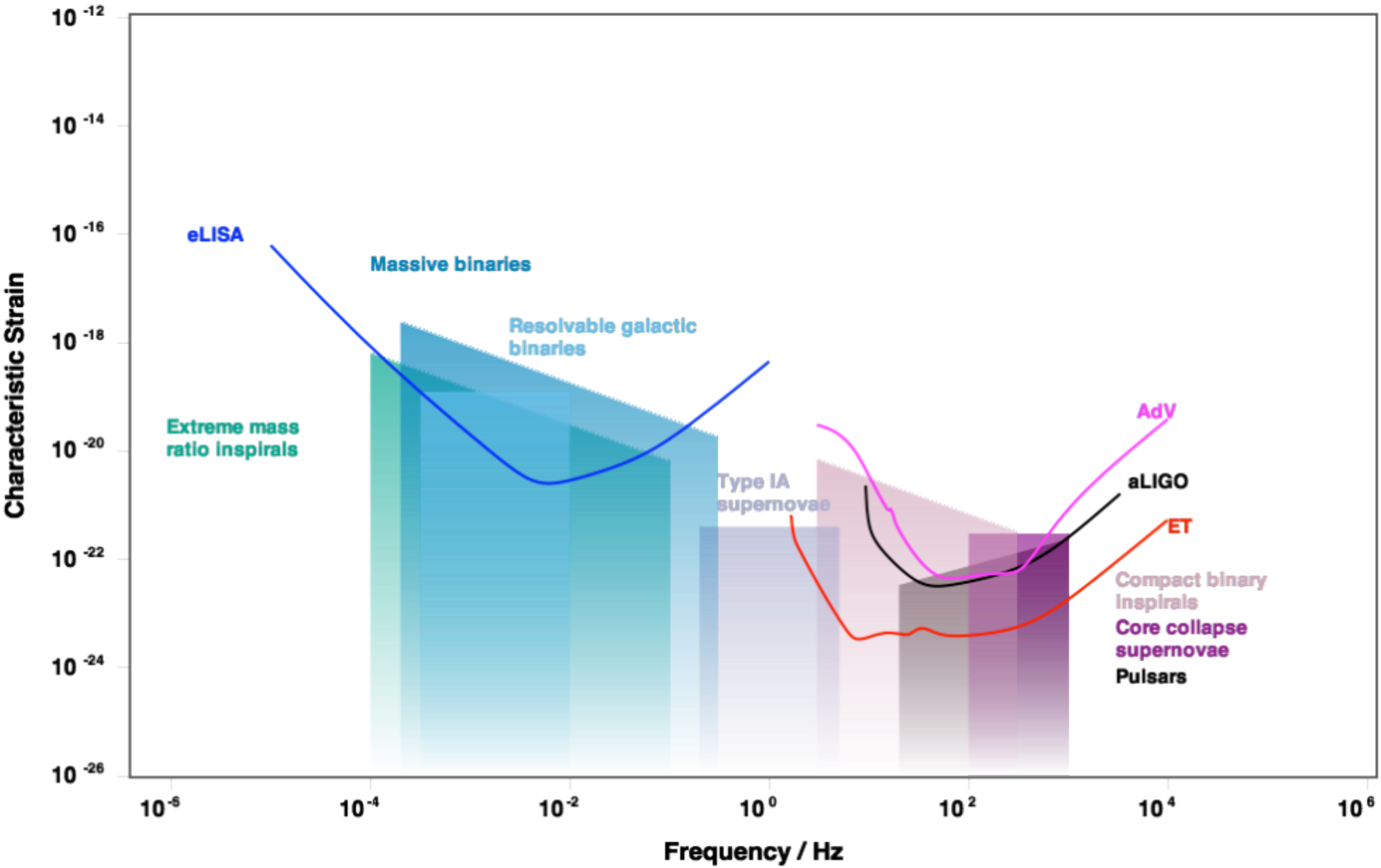






# LISA Sources

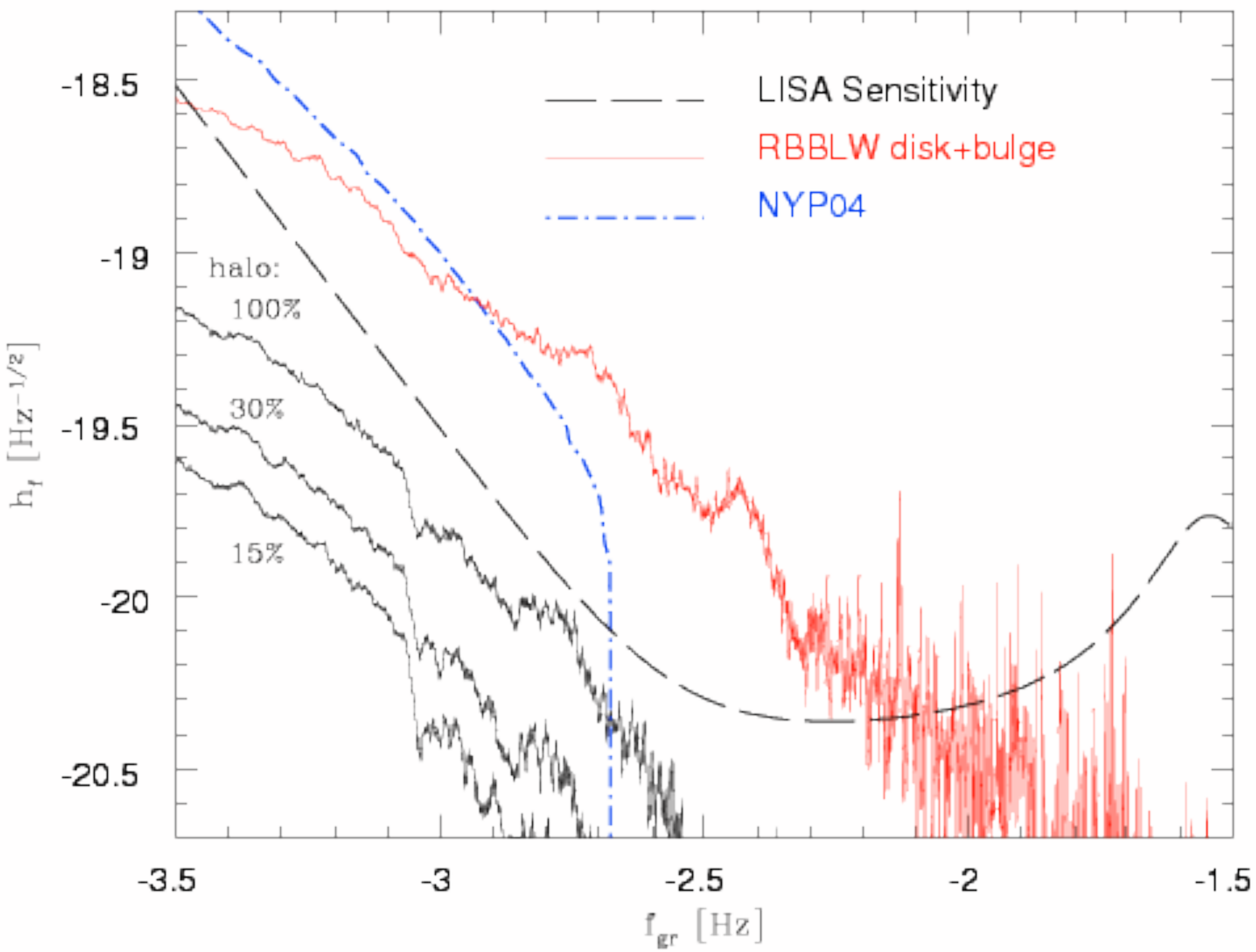
- Galactic white dwarfs
- Primordial backgrounds
- Supermassive binary black holes
- Capture orbits



10 -12
10 -14
10 -16
10 -18
10 -20
10 -22
10 -24
10 -26
Characteristic Strain
eLISA
Massive binaries
Resolvable galactic
binaries
Extreme mass
ratio inspirals
Type IA
supernovae
AdV
aLIGO
ET
Compact binary
inspirals
Core collapse
supernovae
Pulsars
10 -6
10 -4
10 -2
10 0
10 2
10 4
10 6
Frequency / Hz

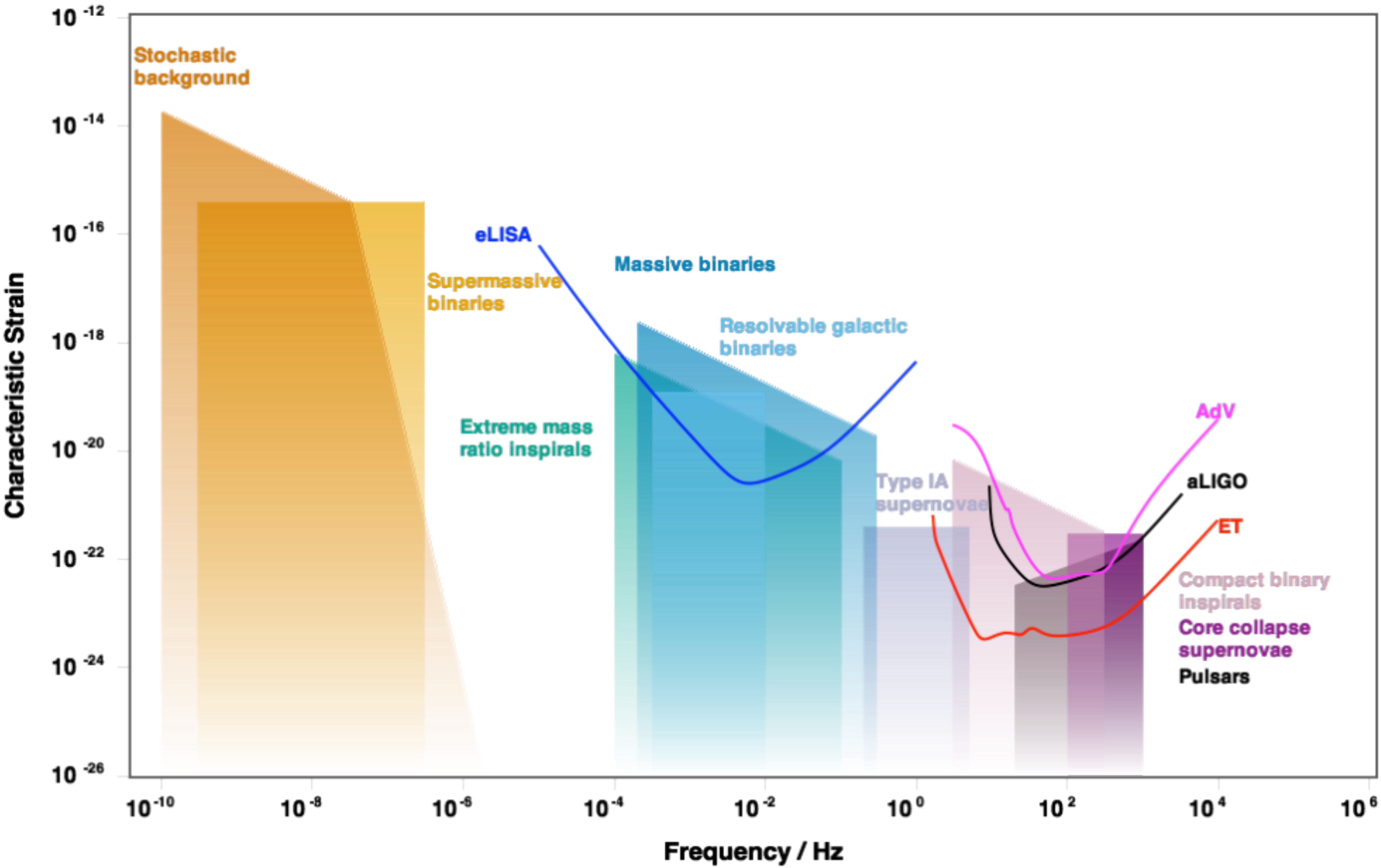
# LISA Gravitational Wave Background

- Produced by an extremely large number of weak, independent, and unresolved gravitational-wave sources. For LISA, this will be white dwarf binaries.

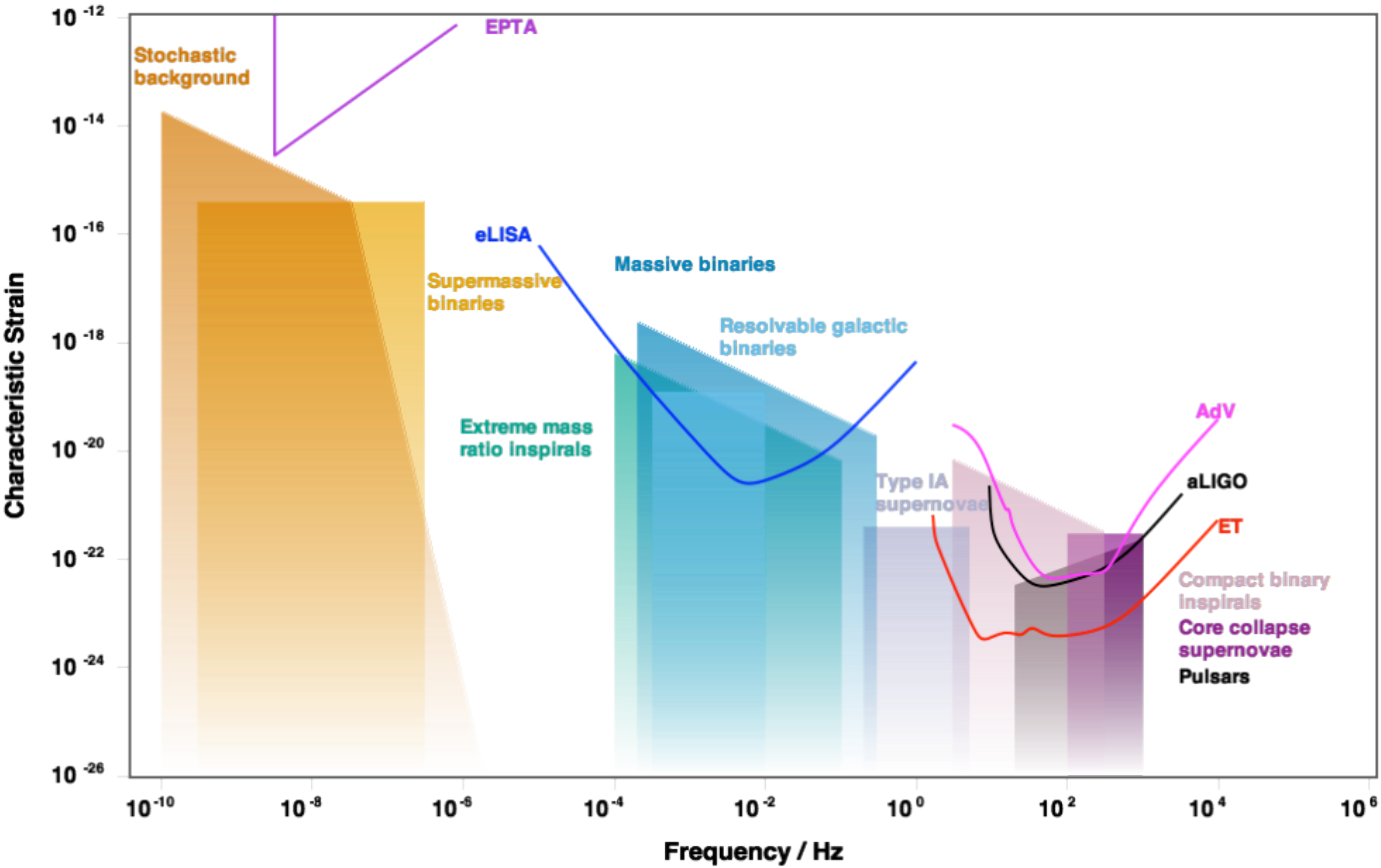


# Pulsar Timing Array Sources

- Also, supermassive binary black holes!



Stochastic background
Supermassive binaries
eLISA
Massive binaries
Resolvable galactic binaries
Extreme mass ratio inspirals
Type IA supernovae
AdV
aLIGO
ET
Compact binary inspirals
Core collapse supernovae
Pulsars
Characteristic Strain
Frequency / Hz
$10^{-12}$
$10^{-14}$
$10^{-16}$
$10^{-18}$
$10^{-20}$
$10^{-22}$
$10^{-24}$
$10^{-26}$
$10^{-10}$
$10^{-8}$
$10^{-6}$
$10^{-4}$
$10^{-2}$
$10^{0}$
$10^{2}$
$10^{4}$
$10^{6}$


EPTA
Stochastic background
Supermassive binaries
eLISA
Massive binaries
Resolvable galactic binaries
Extreme mass ratio inspirals
Type IA supernovae
AdV
aLIGO
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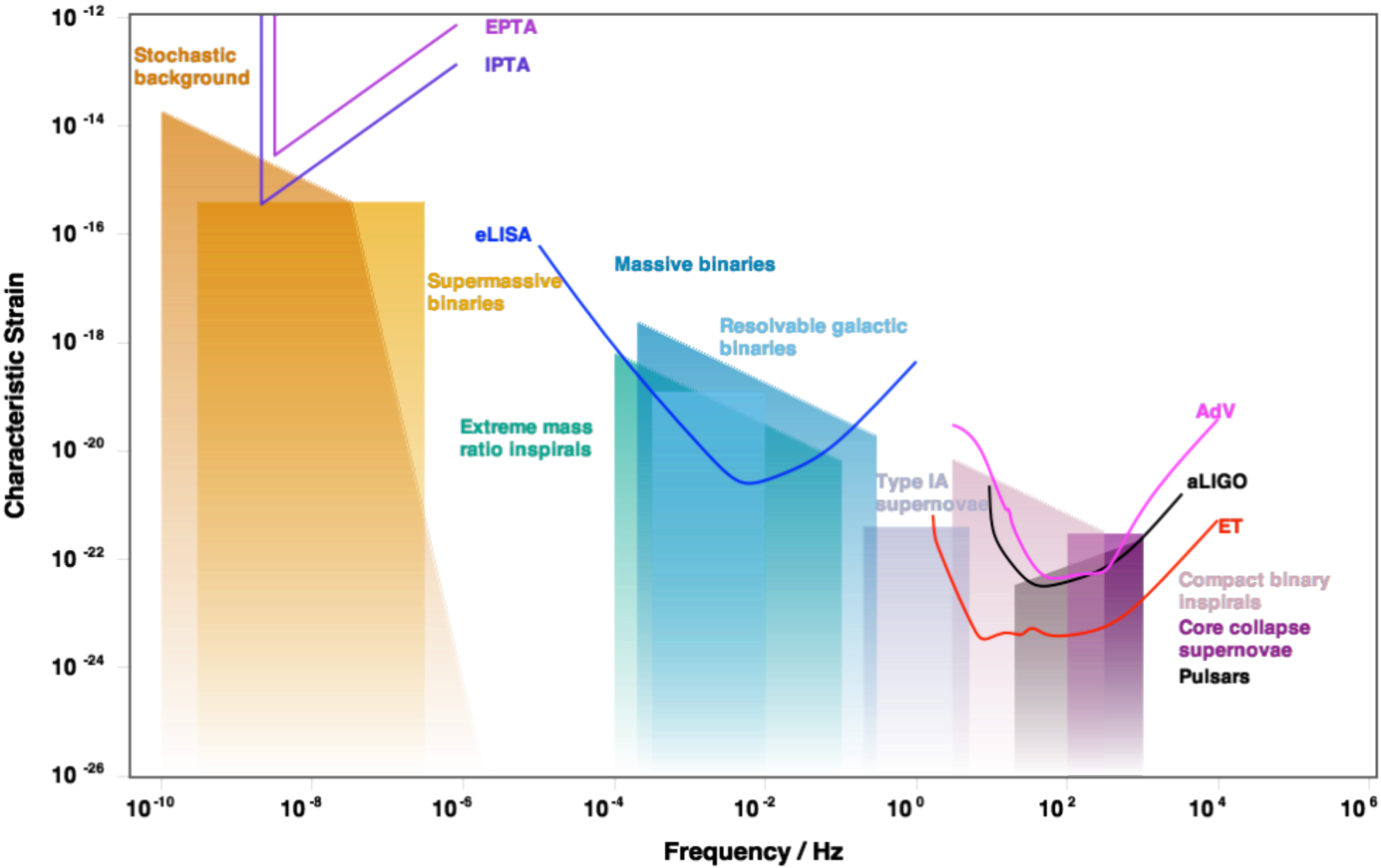
Core collapse supernovae
Pulsars
Characteristic Strain
Frequency / Hz



Characteristic Strain
Frequency / Hz
10 -12
10 -14
10 -16
10 -18
10 -20
10 -22
10 -24
10 -26
10-10
10 -8
10 -6
10 -4
10 -2
10 0
10 2
10 4
10 6
Stochastic background
EPTA
IPTA
eLISA
Supermassive binaries
Massive binaries
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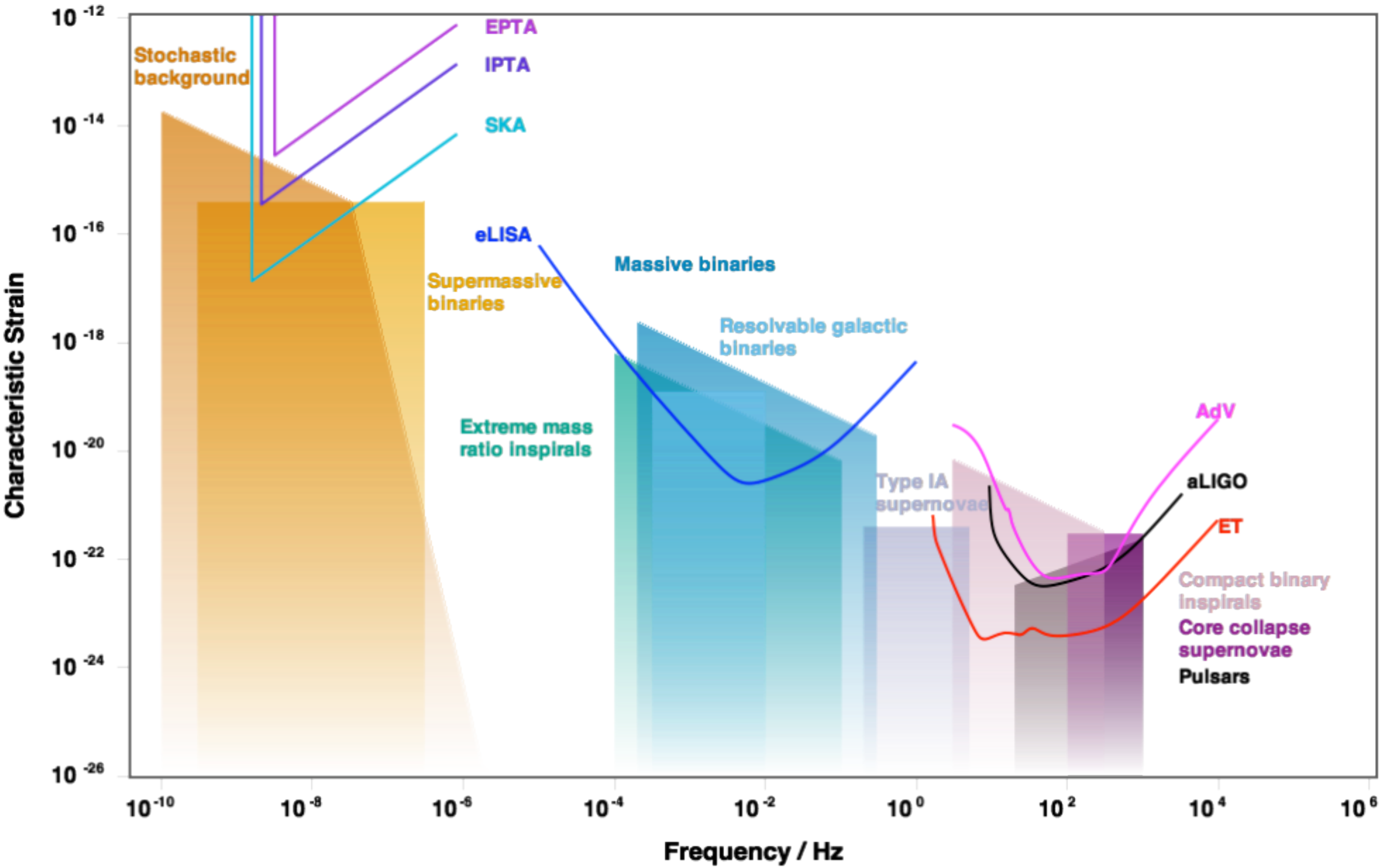

Characteristic Strain
Frequency / Hz
$10^{-12}$
$10^{-14}$
$10^{-16}$
$10^{-18}$
$10^{-20}$
$10^{-22}$
$10^{-24}$
$10^{-26}$
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$10^{-8}$
$10^{-6}$
$10^{-4}$
$10^{-2}$
$10^{0}$
$10^{2}$
$10^{4}$
$10^{6}$
Stochastic background
EPTA
IPTA
SKA
eLISA
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Pulsars

# Merging Supermassive Black Hole Binaries

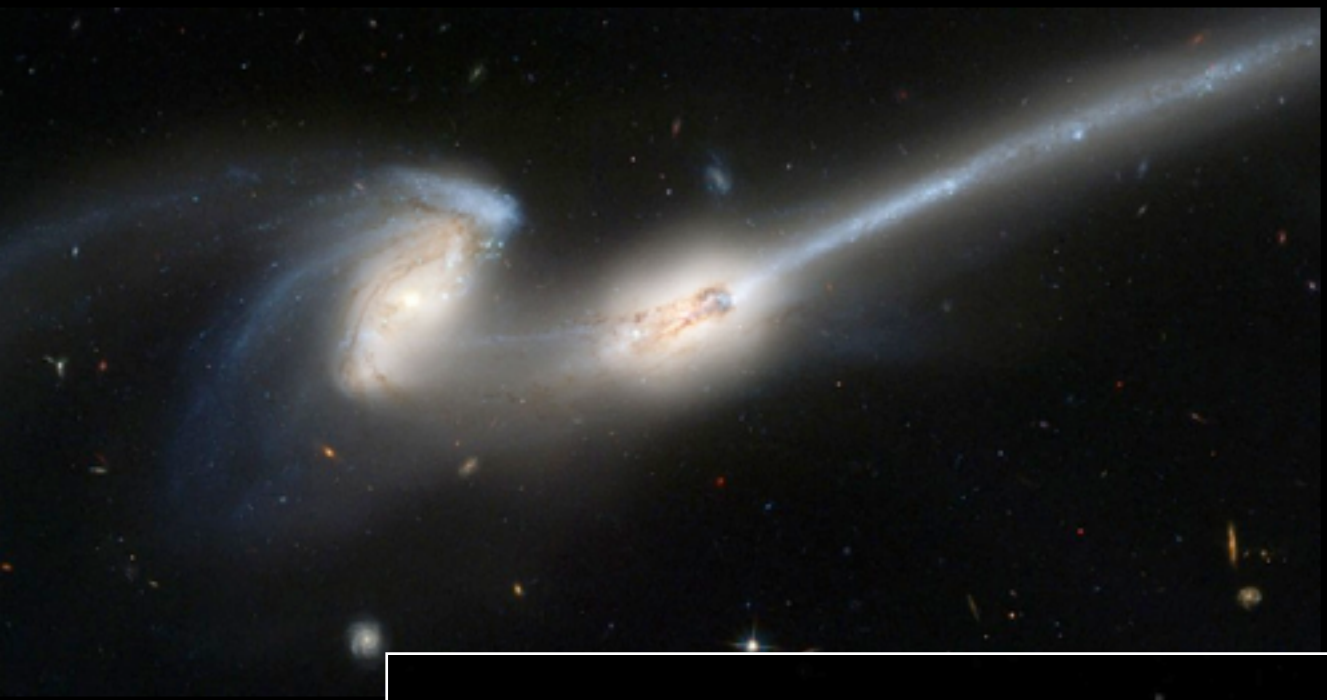


Image Credit: Debra Meloy Elmegreen (Vassar College) et al., & the Hubble Heritage Team (AURA/STScI/NASA)