Name (please print legibly!) ________________________________

Gravitational Waves: Assignment 1
Overview of the Field

1. A useful quantity to define when describing gravitational-wave radiation from a binary system is the chirp mass $\mathcal{M}$. If the two masses of the binary are m_1 and m_2, then the total mass is $M = m_1 + m_2$, the reduced mass is $\mu = m_1 m_2/M$, and the chirp mass is

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} = \left(\mu^3 M^2\right)^{1/5}. \tag{1}$$

Using the equations below for describing a two-body system orbiting around their center of mass, the chirp mass can also be written in terms of the frequency $f_{\rm GW}$ and frequency derivative $\dot{f}_{\rm GW}$ of emitted gravitational waves. Show that this is true.

$f_{\rm GW} = 2 f_{\rm orb}$: gravitational radiation has a frequency twice that of the orbital frequency

$\frac{\rm d}{{\rm d}t} E_{\rm GW} = \frac{32}{5}\frac{G}{c^5}\mu^2 r^4 \omega_{\rm orb}^6$: gravitational wave luminosity

$E_{\rm orb} = -\frac{GM\mu}{2r}$: orbital energy

$-\frac{\rm d}{{\rm d}t} E_{\rm GW} = \frac{\rm d}{{\rm d}t} E_{\rm orb}$: GW energy loss drains the orbital energy

$r^3 = \frac{GM}{\omega_{\rm orb}^2}$: Kepler's third law

2. At low frequencies, the evolution of the gravitational-wave signal emitted by two compact objects is characterized by the chirp mass. As you should have derived in Problem 1, this is given by

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} = \frac{c^3}{G}\left[\frac{5}{96}\pi^{-8/3} f_{\rm GW}^{-11/3} \dot{f}_{\rm GW}\right]^{3/5}. \tag{2}$$

Thus, by measuring the frequency evolution of a signal, you can obtain a measurement of a binary system's chirp mass parameter. We will perform an approximate measurement of the chirp mass for the gravitational-wave signal known as GW150914. You will need to use some programming and plotting. I will work mostly with `python` in this course but you are welcome to perform the analysis in any programming language you'd like.

A. Download the strain$\times 10^{21}$ versus time data measured from the Hanford detector here: `https://losc.ligo.org/s/events/GW150914/P150914/fig1-waveform-H.txt`

B. Plot the waveform. It should look like the waveform plotted in Fig. 1 below from the publication announcing GW150914.

C. Determine the evolution of $f_{\rm GW}$ from the data. An easy way to do this is to measure the time periods between zero-crossings in the strain data and to note that these zero-crossings are 1/2 of a cycle. Thus $f_{\rm GW} = 1/(2\Delta t)$. Plot $f_{\rm GW}$ versus time. Describe the evolution of the frequency and indicate the maximum instantaneous frequency.

D. Instead of trying to measure both $f_{\rm GW}$ and $\dot{f}_{\rm GW}$ from this data, we can instead use the integrated version of Eq. 2 so that we don't need to measure $\dot{f}_{\rm GW}$ explicitly:

$$f_{\rm GW}^{-8/3}(t) = \frac{(8\pi)^{8/3}}{5}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3}(t_c - t)\,, \tag{3}$$

where t_c is the coalescence time which can be absorbed into the y-intercept measurement. Use your frequency measurements from Part C. up until the frequency stops increasing to make a plot of $f_{\rm GW}^{-8/3}$ versus time. This is the part of the signal from which we can measure the chirp mass. Note also that the signal amplitude is pretty low for $t < 0.35$s so it may be useful to smooth or average the data in this region to get a measurement of adjacent zero crossings.

E. Fit a line to the $f_{\rm GW}^{-8/3}$ versus time data to determine the slope and use Eq. 3 to determine the chirp mass.

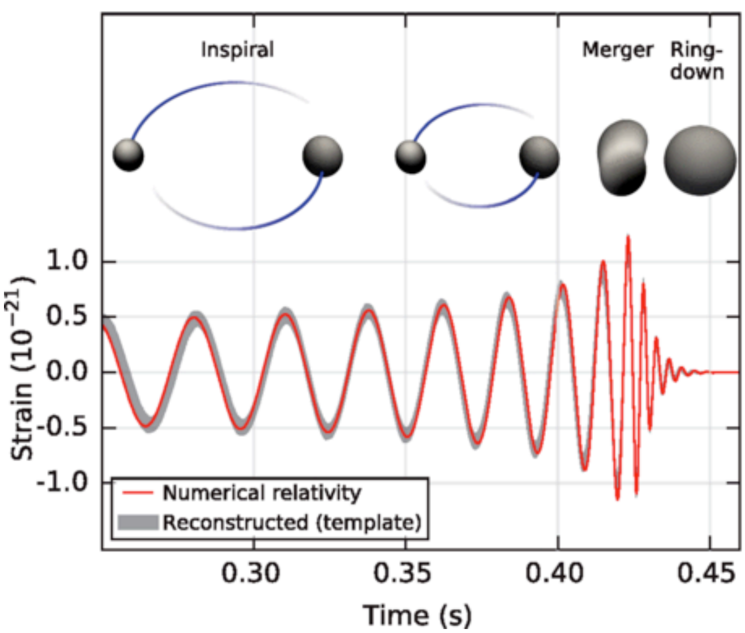

Figure 1

3. From your measurement of the chirp mass in Problem 2 and the expression for the chirp mass in Eq. 1, determine a lower bound on the total mass $M = m_1 + m_2$ for GW150914. It might help to consider when $m_1 = m_2$ and $m_1 > m_2$.

4. Explore the Gravitational Wave Sensitivity Curve Plotter available here: `http://rhcole.com/apps/GWplotter/`
    (a) Provide a brief description of each detector's acronym and what is its most sensitive frequency.
    (b) For each astrophysical source, what is the frequency band of the signals and which detectors are best suited for their detection.