2.1 Reference results for the evolution of parton distributions[1]

In this section we provide a new set of benchmark tables for the evolution of unpolarized parton distributions of hadrons in perturbative QCD. Unlike the only comparable previous study [1], we include results for unequal factorization and renormalization scales, $\mu_{\rm f} \neq \mu_{\rm r}$, and for the evolution with a variable number of partonic flavours $N_{\rm f}$. Besides the standard LO and NLO approximations, we also present the evolution including the (still approximate) NNLO splitting functions and the corresponding non-trivial second-order matching conditions at the heavy-quark thresholds. Our reference results are computed using two entirely independent and conceptually different evolution programs which, however, agree to better than 1 part in 10^5 for momentum fractions $10^{-8} < x < 0.9$.

2.11 Evolution equations and their solutions

At N$^{\rm m}$LO the scale dependence ('evolution') of the parton distributions $f_p(x,\mu_{\rm f}^2) \equiv p(x,\mu_{\rm f}^2)$, where $p = q_i,\, \bar{q}_i\,, g$ with $i = 1,\,\ldots, N_{\rm f}$, is governed by the $2N_{\rm f}+1$ coupled integro-differential equations

$$\frac{d\, f_p(x,\mu_{\rm f}^2)}{d\ln\mu_{\rm f}^2} \;=\; \sum_{l=0}^{m} a_{\rm s}^{l+1}(\mu_{\rm r}^2)\, \mathcal{P}^{(l)}_{pp'}\left(x, \frac{\mu_{\rm f}^2}{\mu_{\rm r}^2}\right) \otimes f_{p'}(x,\mu_{\rm f}^2) \ . \tag{1}$$

Here $\otimes$ denotes the Mellin convolution in the fractional-momentum variable x, and summation over p' is understood. The scale dependence of the strong coupling $a_{\rm s} \equiv \alpha_{\rm s}/(4\pi)$ is given by

$$\frac{d\, a_{\rm s}}{d\ln\mu_r^2} \;=\; \beta_{\rm N^mLO}(a_{\rm s}) \;=\; -\sum_{l=0}^{m} a_{\rm s}^{l+2}\, \beta_l \ . \tag{2}$$

The general splitting functions $\mathcal{P}^{(l)}$ in Eq. (1) can be reduced to the simpler expressions $P^{(l)}(x)$ at $\mu_{\rm r} = \mu_{\rm f}$. Up to NNLO ($\equiv$ N^2 LO) the corresponding relations read

$$\begin{aligned}
\mathcal{P}^{(0)}\left(x, \frac{\mu_{\rm f}^2}{\mu_{\rm r}^2}\right) &= P^{(0)}(x) \\
\mathcal{P}^{(1)}\left(x, \frac{\mu_{\rm f}^2}{\mu_{\rm r}^2}\right) &= P^{(1)}(x) - \beta_0 P^{(0)}(x)\, \ln\frac{\mu_{\rm f}^2}{\mu_{\rm r}^2} \\
\mathcal{P}^{(2)}\left(x, \frac{\mu_{\rm f}^2}{\mu_{\rm r}^2}\right) &= P^{(2)}(x) - \left\{\beta_1 P^{(0)}(x) + 2\beta_0 P^{(1)}(x)\right\} \ln\frac{\mu_{\rm f}^2}{\mu_{\rm r}^2} + \beta_0^2 P^{(0)}(x)\, \ln^2\frac{\mu_{\rm f}^2}{\mu_{\rm r}^2} \ .
\end{aligned} \tag{3}$$

The generalization to higher orders is straightforward but not required for the present calculations.

The LO and NLO coefficients β_0 and β_1 of the β-function in Eq. (2) and the corresponding splitting functions $P^{(0)}(x)$ and $P^{(1)}(x)$ in Eq. (3) have been known for a long time, see Ref. [2] and references therein. In the $\overline{\rm MS}$ scheme adopted here, the coefficient β_2 has been calculated in Refs. [3,4]. The NNLO quantities $P^{(2)}(x)$ have not been computed so far, but approximate expressions have been constructed [5–7] from the available partial results [8–16].

An obvious and widespread approach to Eq. (1) is the direct numerical solution by discretization in both x and $\mu_{\rm f}^2$. This method is also used by one of us (G.S.). The parton distributions are represented on a grid with n points uniformly spaced in $\ln 1/x$. Their values at arbitrary x are defined to be equal to a $p^{\rm th}$ order interpolation of neighbouring grid points. The splitting functions can then be represented as sparse matrices acting on the vector of grid points. At initialization the program takes a set of subroutines for the splitting functions and calculates the corresponding matrices with a Gaussian adaptive integrator. At each value of $\mu_{\rm f}^2$ the derivatives of the parton distributions are calculated through matrix multiplication and the evolution is carried out with a Runge-Kutta method. The algorithm has partial overlap with those of Refs. [17–20] and is described in more detail in Appendix F of Ref. [21].

[1]Contributing authors: G. Salam, A. Vogt

For the reference tables presented below, the program has been run with 6^{th} order interpolation in x and multiple x-grids: one for $10^{-8} < x < 1$ with 750 points, another for $0.135 < x < 1$ with 240 points and a third for $0.6 < x < 1$ with 180 points. A uniform $\mu_{\rm f}^2$ grid has been used with 220 points in the range $2\ \mathrm{GeV}^2 < \mu_{\rm f}^2 < 10^6\ \mathrm{GeV}^2$. Halving the density of points in both x and $\mu_{\rm f}^2$ leaves the results unchanged at the level of better than 1 part in 10^5 in the range $10^{-8} < x < 0.9$ (except close to sign-changes of parton distributions).

An important alternative to this direct numerical treatment of Eq. (1) is the Mellin-N moment solution in terms of a power expansion. This method is employed by the second author (A.V.). Here Eq. (1) is transformed to N-space (reducing the convolution to a simple product) and $\mu_{\rm f}^2$ is replaced by $a_{\rm s}$ as the independent variable, assuming that $\mu_{\rm f}/\mu_{\rm r}$ is a fixed number. Expanding the resulting r.h.s. into a power series in $a_{\rm s}$, one arrives at

$$\frac{d\, f_p(N,a_{\rm s})}{d\, a_{\rm s}} \;=\; -\sum_{l=0}^{\infty} a_{\rm s}^{l-1}\, R_{pp'}^{(l)}(N)\, f_{p'}(N,a_{\rm s}) \tag{4}$$

with

$$R_{pp'}^{(0)}(N) \;=\; \frac{1}{\beta_0}\, P_{pp'}^{(0)}(N)\ , \qquad R_{pp'}^{(k\geq 1)} \;=\; \frac{1}{\beta_0}\, \mathcal{P}_{pp'}^{(k)}(N) - \sum_{l=1}^{k} \frac{\beta_l}{\beta_0}\, R_{pp'}^{(k-l)}(N)\ . \tag{5}$$

At N$^{\rm m}$LO only the coefficients $\beta_{l\leq m}$ and $\mathcal{P}^{(l\leq m)}$ are retained in Eq. (5). The solution of Eq. (4) can be expressed as an expansion around the LO result

$$f(N,\mu_{\rm f}^2) \;=\; \left[1+\sum_{k=1}^{\infty} a_{\rm s}^k\, U_k(N)\right] \left(\frac{a_{\rm s}}{a_0}\right)^{-R_0(N)} \left[1+\sum_{k=1}^{\infty} a_0^k\, U_k(N)\right]^{-1} f(N,\mu_{\rm f,0}^2)\ , \tag{6}$$

where $\mu_{\rm f,0}^2$ is the initial scale for the evolution, and $a_0 \equiv a_{\rm s}(\mu_{\rm r}^2(\mu_{\rm f,0}^2))$. It is understood in Eq. (6) that the matrix structure is simplified by switching to the appropriate flavour singlet and non-singlet combinations. For the explicit construction of the remaining 2×2 matrices U_k the reader is referred to Section 5 of Ref. [22]. Finally the solutions $f_p(N,\mu_{\rm f}^2)$ are transformed back to x-space by

$$f_p(x,\mu_{\rm f}^2) \;=\; \frac{1}{\pi}\int_0^\infty dz\ \mathrm{Im}\left[e^{i\phi} x^{-c-z\exp(i\phi)} f_p(N\!=\!c\!+\!z\exp(i\phi),\, \mu_{\rm f}^2)\right]\ . \tag{7}$$

The Mellin inversions (7) can be performed with a sufficient accuracy using a fixed chain of Gauss quadratures. Hence the quantities $R_0(N)$ and $U_k(N)$ in Eq. (6) have to the computed only once for the corresponding support points at the initialization of the program, rendering the N-space evolution competitive in speed with fast x-space codes. Except where the parton distributions become very small, an accuracy of 1 part in 10^5 or better is achieved by including contributions up to $k = 15$ in Eq. (6) and using at most 18 eight-point Gauss quadratures with, e.g., $z_{\rm max} = 70$, $c = 2$, $\phi = 3\pi/4$ in Eq. (7).

The two methods for solving Eq. (1) discussed above are completely equivalent, i.e., they do not differ even by terms beyond N$^{\rm m}$LO, provided that the coupling $a_{\rm s}$ evolves exactly according to Eq. (2). This condition is fulfilled in the present calculations. Thus the results of our two programs can be compared directly, yielding a powerful check of the correctness and accuracy of the codes. Note that the only previously published high-precision comparison of NLO evolution programs [1] used the truncated expansion of $a_{\rm s}^2(\mu_{\rm r})$ in terms of inverse powers of $\ln \mu_{\rm r}^2/\Lambda^2$ which does not exactly conform to Eq. (2). Consequently such direct comparisons were not possible in Ref. [1].

Following Ref. [2], the N-space solution (6) has usually been subjected to a further expansion in the coupling constants, retaining only the terms up to order m in the product of the U-matrices. Only the terms thus kept in Eq. (6) are free from contributions by the higher-order coefficients $\beta_{k>m}$ and $P^{(k>m)}$, and only these terms combine to factorization-scheme independent quantities when the parton distributions are convoluted with the N$^{\rm m}$LO partonic cross sections. Reference results for the truncated solution will be presented elsewhere [23].

2.12 *Initial conditions and heavy-quark treatment*

The following initial conditions for the reference results have been set up at the Les Houches meeting: The evolution is started at

$$\mu_{\mathrm{f},0}^2 \;=\; 2\ \mathrm{GeV}^2 \ . \tag{8}$$

Roughly along the lines of the CTEQ5M parametrization [24], the input distributions are chosen as

$$\begin{aligned}
xu_v(x,\mu_{\mathrm{f},0}^2) &= 5.107200\, x^{0.8}\, (1-x)^3 \\
xd_v(x,\mu_{\mathrm{f},0}^2) &= 3.064320\, x^{0.8}\, (1-x)^4 \\
xg\,(x,\mu_{\mathrm{f},0}^2) &= 1.700000\, x^{-0.1} (1-x)^5 \\
x\bar{d}\,(x,\mu_{\mathrm{f},0}^2) &= .1939875\, x^{-0.1} (1-x)^6 \\
x\bar{u}\,(x,\mu_{\mathrm{f},0}^2) &= (1-x)\, x\bar{d}\,(x,\mu_{\mathrm{f},0}^2) \\
xs\,(x,\mu_{\mathrm{f},0}^2) &= x\bar{s}\,(x,\mu_{\mathrm{f},0}^2) \;=\; 0.2\, x(\bar{u}+\bar{d})(x,\mu_{\mathrm{f},0}^2)
\end{aligned} \tag{9}$$

where, as usual, $q_{i,v} \equiv q_i - \bar{q}_i$. The running couplings are specified via

$$\alpha_{\mathrm{s}}(\mu_{\mathrm{r}}^2 \!=\! 2\ \mathrm{GeV}^2) \;=\; 0.35 \ . \tag{10}$$

For simplicity these initial conditions are employed regardless of the order of the evolution and the ratio of the renormalization and factorization scales. At LO this ratio is fixed to unity, beyond LO we use

$$\mu_{\mathrm{r}}^2 \;=\; k_{\mathrm{r}} \mu_{\mathrm{f}}^2 \ , \qquad k_{\mathrm{r}} \;=\; 0.5\,,\ 1\,,\ 2 \ . \tag{11}$$

For the evolution with a fixed number $N_{\mathrm{f}} > 3$ of quark flavours the quark distributions not specified in Eq. (9) are assumed to vanish at $\mu_{\mathrm{f},0}^2$, and Eq. (10) is understood to refer to the chosen value of N_{f}. For the evolution with a variable $N_{\mathrm{f}} = 3 \ldots 6$, Eqs. (9) and (10) always refer to three flavours. N_{f} is then increased by one unit at the heavy-quark pole masses taken as

$$m_{\mathrm{c}} \;=\; \mu_{\mathrm{f},0}\ , \quad m_{\mathrm{b}} \;=\; 4.5\ \mathrm{GeV}^2\ , \quad m_{\mathrm{t}} \;=\; 175\ \mathrm{GeV}^2\ , \tag{12}$$

i.e., the evolution is performed as discussed in Section 2.11 between these thresholds, and the respective matching conditions are invoked at $\mu_{\mathrm{f}}^2 = m_{\mathrm{h}}^2$, $h = c,\, b,\, t$. For the parton distributions these conditions have been derived in Ref. [25]. Up to $\mathrm{N^{m=2}}$ LO they read

$$l_i^{(N_{\mathrm{f}}+1)}(x,m_h^2) \;=\; l_i^{(N_{\mathrm{f}})}(x,m_h^2) + \delta_{m2}\, a_{\mathrm{s}}^2\, A_{qq,h}^{\mathrm{NS},(2)}(x) \otimes l_i^{(N_{\mathrm{f}})}(x,m_h^2) \tag{13}$$

where $l = q,\, \bar{q}$ and $i = 1, \ldots N_{\mathrm{f}}$, and

$$\begin{aligned}
g^{\,(N_{\mathrm{f}}+1)}(x,m_h^2) \;&=\; g^{\,(N_{\mathrm{f}})}(x,m_h^2) + \\
&\quad \delta_{m2}\, a_{\mathrm{s}}^2 \left[A_{\mathrm{gq,h}}^{S,(2)}(x) \otimes \Sigma^{\,(N_{\mathrm{f}})}(x,m_h^2) + A_{\mathrm{gg,h}}^{S,(2)}(x) \otimes g^{\,(N_{\mathrm{f}})}(x,m_h^2) \right] \\
(h+\bar{h})^{\,(N_{\mathrm{f}}+1)}(x,m_h^2) \;&=\; \delta_{m2}\, a_{\mathrm{s}}^2 \left[\tilde{A}_{\mathrm{hq}}^{S,(2)}(x) \otimes \Sigma^{\,(N_{\mathrm{f}})}(x,m_h^2) + \tilde{A}_{\mathrm{hg}}^{S,(2)}(x) \otimes g^{\,(N_{\mathrm{f}})}(x,m_h^2) \right]
\end{aligned} \tag{14}$$

with $\Sigma^{\,(N_{\mathrm{f}})} \equiv \sum_{i=1}^{N_{\mathrm{f}}} (q_i + \bar{q}_i)$ and $h = \bar{h}$. The coefficients $A^{(2)}$ can be found in Appendix B of ref. [25] – due to our choice of $\mu_{\mathrm{f}}^2 = m_{\mathrm{h}}^2$ for the thresholds only the scale-independent parts of the expressions are needed here – from where the notation for these coefficients has been taken over. The corresponding $\mathrm{N^m}$ LO relation for the coupling constant [26, 27] is given by

$$a_{\mathrm{s}}^{(N_{\mathrm{f}}+1)}(k_{\mathrm{r}} m_h^2) \;=\; a_{\mathrm{s}}^{(N_{\mathrm{f}})}(k_{\mathrm{r}} m_h^2) + \sum_{n=1}^{m} \left(a_{\mathrm{s}}^{(N_{\mathrm{f}})}(k_{\mathrm{r}} m_h^2) \right)^{n+1} \sum_{l=0}^{n} c_{n,l}\, \ln k_{\mathrm{r}} \ . \tag{15}$$

The pole-mass coefficients $c_{n,l}$ in Eq. (15) can be inferred from Eq. (9) of Ref. [27], where $4\, a_{\mathrm{s}}^{(N_{\mathrm{f}}-1)}$ is expressed in terms of $4\, a_{\mathrm{s}}^{(N_{\mathrm{f}})}$. Note that we use $a_{\mathrm{s}}^{(N_{\mathrm{f}}+1)}(k_{\mathrm{r}} m_h^2)$ on the r.h.s. of Eq. (14).

2.13 *The benchmark results*

We have compared the results of our two evolution programs, under the conditions specified in Section 2.12, at 500 x-$\mu_{\rm f}^2$ points covering the range $10^{-8} \le x \le 0.9$ and $2\,{\rm GeV}^2 \le \mu_{\rm f}^2 \le 10^6\,{\rm GeV}^2$. A representative subset of our results at $\mu_{\rm f}^2 = 10^4$ GeV^4, a scale relevant to high-E_T jets at TEVATRON and close to $m_{\rm W}^2$, $m_{\rm Z}^2$ and, possibly, $m_{\rm Higgs}^2$, is presented in Tables 1–5. These results are given in terms of the valence distributions, defined below Eq. (9), $L_\pm \equiv \bar{d} \pm \bar{u}$, and the quark-antiquark sums $q_+ \equiv q - \bar{q}$ for $q = s$, c and, for the variable-$N_{\rm f}$ case, b.

For compactness an abbreviated notation is employed throughout the tables, i.e., all numbers $a \cdot 10^b$ are written as a^b. In the vast majority of the x-$\mu_{\rm f}^2$ points our results are found to agree to all five figures displayed, except for the tiny NLO and NNLO sea-quark distributions at $x = 0.9$, in the tables. In fact, the differences for $x < 0.9$ are not larger than ± 1 in the sixth digit, i.e, the offsets are smaller than 1 part in 10^5. Entries where these residual offsets lead to a different fifth digit after rounding are indicated by the subscript '$*$'. The number with the smaller modulus is then given in the tables, e.g., 1.1111^1_* should be read as $1.11115 \cdot 10^1$ with an uncertainty of ± 1 in the last figure.

As mentioned in Section 2.11, the three-loop (NNLO) splitting functions $P^{(2)}(x)$ in Eq. (3) are not yet exactly known. For the NNLO reference results in Tables 4 and 5 we have resorted to the average of the two extreme approximations constructed in Ref. [7]. The remaining uncertainties of these results can be estimated by instead employing the extreme approximations itselves. Their relative effects are illustrated, also at $\mu_{\rm f}^2 = 10^4$ GeV^4, in Fig. 1. The uncertainties of the NNLO evolution to this scale turn out to be virtually negligible at $x \ge 0.05$ and to amount to less than $\pm 1\%$ down to $x \simeq 10^{-4}$.

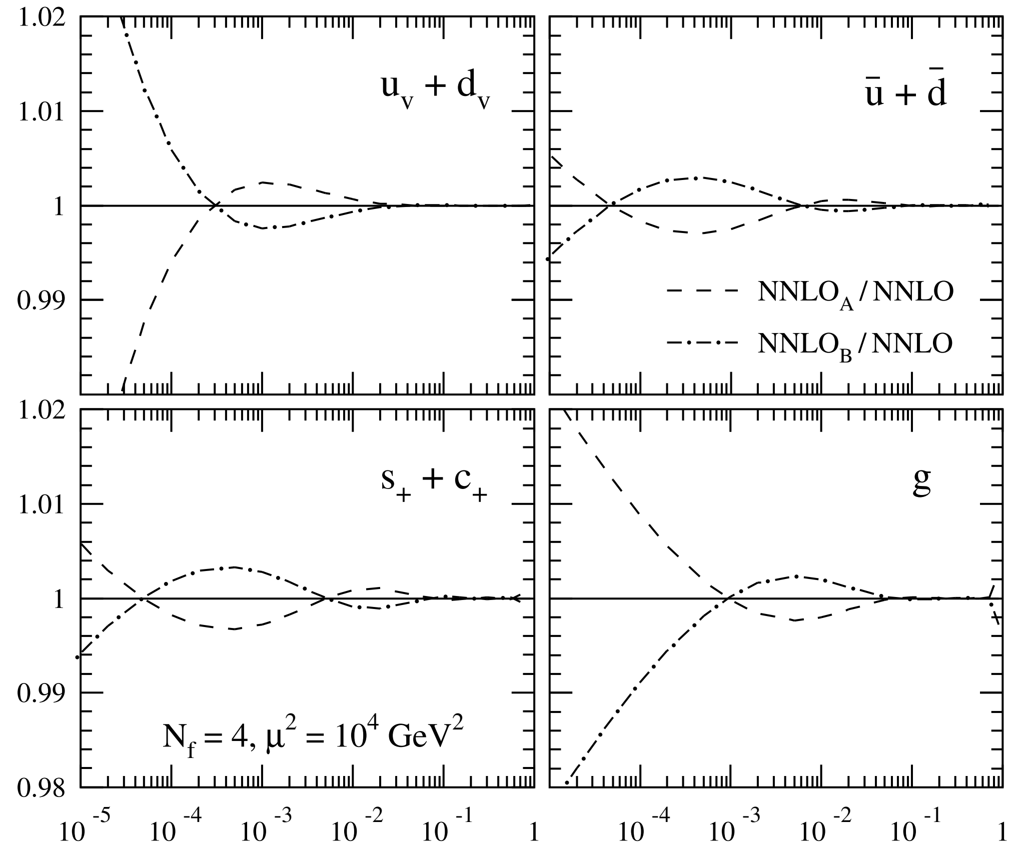

Fig. 1: The relative effects of the present uncertainties of the NNLO splitting functions on the evolution of the input (8) – (10) for $\mu_{\rm r} = \mu_{\rm f}$, estimated by using the extreme approximations 'A' and 'B' of Ref. [7] instead of their average.

Table 1: Reference results for the $N_{\rm f} = 4$ (FFN) and the variable-$N_{\rm f}$ (VFN) leading-order evolution for the initial conditions (8) – (10), shown together with the input parton distributions (9). The respective values for $\alpha_{\rm s}(\mu_{\rm r}^2 = \mu_{\rm f}^2 = 10^4\ {\rm GeV}^2)$ read 0.117374 (FFN) and 0.122306 (VFN). For the notation see the first two paragraphs of Section 2.13.

x	xu_v	xd_v	xL_-	xL_+	xs_+	xc_+	xb_+	xg
Input, $\mu_{\rm f}^2 = 2\ {\rm GeV}^2$								
10^{-7}	1.2829^{-5}	7.6972^{-6}	9.7224^{-8}	3.8890^{+0}	7.7779^{-1}	0.0^{+0}	0.0^{+0}	8.5202^{+0}
10^{-6}	8.0943^{-5}	4.8566^{-5}	7.7227^{-7}	3.0891^{+0}	6.1782^{-1}	0.0^{+0}	0.0^{+0}	6.7678^{+0}
10^{-5}	5.1070^{-4}	3.0642^{-4}	6.1341^{-6}	2.4536^{+0}	4.9072^{-1}	0.0^{+0}	0.0^{+0}	5.3756^{+0}
10^{-4}	3.2215^{-3}	1.9327^{-3}	4.8698^{-5}	1.9478^{+0}	3.8957^{-1}	0.0^{+0}	0.0^{+0}	4.2681^{+0}
10^{-3}	2.0271^{-2}	1.2151^{-2}	3.8474^{-4}	1.5382^{+0}	3.0764^{-1}	0.0^{+0}	0.0^{+0}	3.3750^{+0}
10^{-2}	1.2448^{-1}	7.3939^{-2}	2.8946^{-3}	1.1520^{+0}	2.3041^{-1}	0.0^{+0}	0.0^{+0}	2.5623^{+0}
0.1	5.9008^{-1}	3.1864^{-1}	1.2979^{-2}	4.9319^{-1}	9.8638^{-2}	0.0^{+0}	0.0^{+0}	1.2638^{+0}
0.3	6.6861^{-1}	2.8082^{-1}	7.7227^{-3}	8.7524^{-2}	1.7505^{-2}	0.0^{+0}	0.0^{+0}	3.2228^{-1}
0.5	3.6666^{-1}	1.1000^{-1}	1.6243^{-3}	9.7458^{-3}	1.9492^{-3}	0.0^{+0}	0.0^{+0}	5.6938^{-2}
0.7	1.0366^{-1}	1.8659^{-2}	1.0259^{-4}	3.8103^{-4}	7.6207^{-5}	0.0^{+0}	0.0^{+0}	4.2810^{-3}
0.9	4.6944^{-3}	2.8166^{-4}	1.7644^{-7}	4.3129^{-7}	8.6259^{-8}	0.0^{+0}	0.0^{+0}	1.7180^{-5}
LO, $N_{\rm f} = 4\,,\ \mu_{\rm f}^2 = 10^4\ {\rm GeV}^2$								
10^{-7}	5.7722^{-5}	3.4343^{-5}	7.6527^{-7}	9.9465^{+1}	4.8642^{+1}	4.7914^{+1}	0.0^{+0}	1.3162^{+3}
10^{-6}	3.3373^{-4}	1.9800^{-4}	5.0137^{-6}	5.0259^{+1}	2.4263^{+1}	2.3685^{+1}	0.0^{+0}	6.0008^{+2}
10^{-5}	1.8724^{-3}	1.1065^{-3}	3.1696^{-5}	2.4378^{+1}	1.1501^{+1}	1.1042^{+1}	0.0^{+0}	2.5419^{+2}
10^{-4}	1.0057^{-2}	5.9076^{-3}	1.9071^{-4}	1.1323^{+1}	5.1164^{+0}	4.7530^{+0}	0.0^{+0}	9.7371^{+1}
10^{-3}	5.0392^{-2}	2.9296^{-2}	1.0618^{-3}	5.0324^{+0}	2.0918^{+0}	1.8089^{+0}	0.0^{+0}	3.2078^{+1}
10^{-2}	2.1955^{-1}	1.2433^{-1}	4.9731^{-3}	2.0433^{+0}	7.2814^{-1}	5.3247^{-1}	0.0^{+0}	8.0546^{+0}
0.1	5.7267^{-1}	2.8413^{-1}	1.0470^{-2}	4.0832^{-1}	1.1698^{-1}	5.8864^{-2}	0.0^{+0}	8.8766^{-1}
0.3	3.7925^{-1}	1.4186^{-1}	3.3029^{-3}	4.0165^{-2}	1.0516^{-2}	4.1379^{-3}_{*}	0.0^{+0}	8.2676^{-2}
0.5	1.3476^{-1}	3.5364^{-2}	4.2815^{-4}	2.8624^{-3}	7.3138^{-4}	2.6481^{-4}	0.0^{+0}	7.9240^{-3}
0.7	2.3123^{-2}	3.5943^{-3}	1.5868^{-5}	6.8961^{-5}	1.7725^{-5}	6.5549^{-6}	0.0^{+0}	3.7311^{-4}
0.9	4.3443^{-4}	2.2287^{-5}	1.1042^{-8}	3.6293^{-8}	1.0192^{-8}	4.8893^{-9}_{*}	0.0^{+0}	1.0918^{-6}
LO, $N_{\rm f} = 3\ldots 5\,,\ \mu_{\rm f}^2 = 10^4\ {\rm GeV}^2$								
10^{-7}	5.8771^{-5}	3.4963^{-5}	7.8233^{-7}	1.0181^{+2}	4.9815^{+1}	4.9088^{+1}	4.6070^{+1}	1.3272^{+3}
10^{-6}	3.3933^{-4}	2.0129^{-4}	5.1142^{-6}	5.1182^{+1}	2.4725^{+1}	2.4148^{+1}	2.2239^{+1}	6.0117^{+2}
10^{-5}	1.9006^{-3}	1.1229^{-3}	3.2249^{-5}	2.4693^{+1}	1.1659^{+1}	1.1201^{+1}	1.0037^{+1}	2.5282^{+2}
10^{-4}	1.0186^{-2}	5.9819^{-3}	1.9345^{-4}	1.1406^{+1}	5.1583^{+0}	4.7953^{+0}	4.1222^{+0}	9.6048^{+1}
10^{-3}	5.0893^{-2}	2.9576^{-2}	1.0730^{-3}	5.0424^{+0}	2.0973^{+0}	1.8147^{+0}	1.4582^{+0}	3.1333^{+1}
10^{-2}	2.2080^{-1}	1.2497^{-1}	4.9985^{-3}	2.0381^{+0}	7.2625^{-1}	5.3107^{-1}	3.8106^{-1}	7.7728^{+0}
0.1	5.7166^{-1}	2.8334^{-1}	1.0428^{-2}	4.0496^{-1}	1.1596^{-1}	5.8288^{-2}	3.5056^{-2}	8.4358^{-1}
0.3	3.7597^{-1}	1.4044^{-1}	3.2629^{-3}	3.9592^{-2}	1.0363^{-2}	4.0740^{-3}	2.2039^{-3}	7.8026^{-2}
0.5	1.3284^{-1}	3.4802^{-2}	4.2031^{-4}	2.8066^{-3}	7.1707^{-4}	2.5958^{-4}	1.3522^{-4}	7.4719^{-3}
0.7	2.2643^{-2}	3.5134^{-3}	1.5468^{-5}	6.7201^{-5}	1.7278^{-5}	6.3958^{-6}	3.3996^{-6}	3.5241^{-4}
0.9	4.2047^{-4}	2.1529^{-5}	1.0635^{-8}	3.4998^{-8}	9.8394^{-9}	4.7330^{-9}	2.8903^{-9}	1.0307^{-6}

Table 2: Reference results for the $N_{\rm f} = 4$ next-to-leading-order evolution for the initial conditions (8) – (10). The corresponding value of the strong coupling is $\alpha_{\rm s}(\mu_{\rm r}^2 = 10^4 \text{ GeV}^2) = 0.110902$. As in the leading-order case, the valence distributions s_v and c_v vanish for the input (9). The notation is explained in the first two paragraphs of Section 2.13.

NLO, $N_{\rm f} = 4,\ \mu_{\rm f}^2 = 10^4 \text{ GeV}^2$							
x	xu_v	xd_v	xL_-	xL_+	xs_+	xc_+	xg
$\mu_{\rm r}^2 = \mu_{\rm f}^2$							
10^{-7}	1.0616^{-4}	6.2328^{-5}	4.2440^{-6}	1.3598^{+2}	6.6913^{+1}_{*}	6.6195^{+1}	1.1483^{+3}_{*}
10^{-6}	5.4177^{-4}	3.1719^{-4}	1.9241^{-5}	6.8396^{+1}	3.3342^{+1}	3.2771^{+1}	5.3911^{+2}
10^{-5}	2.6870^{-3}	1.5677^{-3}	8.3575^{-5}	3.2728^{+1}	1.5685^{+1}	1.5231^{+1}	2.3528^{+2}
10^{-4}	1.2841^{-2}	7.4558^{-3}	3.4911^{-4}	1.4746^{+1}	6.8355^{+0}	6.4769^{+0}	9.2872^{+1}_{*}
10^{-3}	5.7926^{-2}	3.3337^{-2}	1.4162^{-3}	6.1648^{+0}_{*}	2.6659^{+0}	2.3878^{+0}	3.1502^{+1}
10^{-2}	2.3026^{-1}	1.2928^{-1}	5.3251^{-3}	2.2527^{+0}	8.4220^{-1}_{*}	6.5246^{-1}	8.1066^{+0}
0.1	5.5452^{-1}	2.7336^{-1}	1.0011^{-2}	3.9336^{-1}	1.1489^{-1}	6.0351^{-2}	8.9867^{-1}
0.3	3.5393^{-1}	1.3158^{-1}	3.0362^{-3}	3.5848^{-2}	9.2030^{-3}	3.3890^{-3}	8.3451^{-2}
0.5	1.2271^{-1}	3.1967^{-2}	3.8265^{-4}	2.4126^{-3}	5.8424^{-4}	1.6955^{-4}	8.0473^{-3}
0.7	2.0429^{-2}	3.1473^{-3}_{*}	1.3701^{-5}	5.3622^{-5}	1.2393^{-5}	2.7807^{-6}	3.8721^{-4}
0.9	3.6096^{-4}	1.8317^{-5}	8.923^{-9}	2.092^{-8}	4.039^{-9}	-2.405^{-10}	1.2127^{-6}
$\mu_{\rm r}^2 = 2\,\mu_{\rm f}^2$							
10^{-7}	9.2960^{-5}	5.4699^{-5}	3.3861^{-6}	1.2214^{+2}	5.9987^{+1}	5.9265^{+1}	1.0911^{+3}
10^{-6}	4.8463^{-4}	2.8440^{-4}	1.5820^{-5}	6.1831^{+1}	3.0056^{+1}	2.9483^{+1}	5.1456^{+2}
10^{-5}	2.4578^{-3}	1.4374^{-3}	7.1265^{-5}	2.9845^{+1}	1.4240^{+1}	1.3785^{+1}	2.2580^{+2}
10^{-4}	1.2018^{-2}	6.9946^{-3}	3.1111^{-4}	1.3618^{+1}	6.2690^{+0}	5.9088^{+0}	8.9753^{+1}
10^{-3}	5.5483^{-2}	3.2009^{-2}	1.3254^{-3}	5.8076^{+0}	2.4848^{+0}	2.2050^{+0}	3.0729^{+1}
10^{-2}	2.2595^{-1}	1.2720^{-1}	5.2141^{-3}	2.1896^{+0}	8.0746^{-1}	6.1564^{-1}	8.0188^{+0}
0.1	5.6007^{-1}	2.7697^{-1}	1.0180^{-2}	3.9945^{-1}	1.1570^{-1}	5.9661^{-2}	9.1201^{-1}
0.3	3.6474^{-1}	1.3612^{-1}	3.1588^{-3}	3.7501^{-2}	9.6302^{-3}	3.5499^{-3}	8.6368^{-2}
0.5	1.2843^{-1}	3.3610^{-2}	4.0510^{-4}	2.5822^{-3}	6.3044^{-4}	1.8999^{-4}	8.4178^{-3}
0.7	2.1779^{-2}	3.3725^{-3}	1.4798^{-5}	5.9125^{-5}	1.3961^{-5}	3.5593^{-6}	4.0836^{-4}
0.9	3.9817^{-4}	2.0321^{-5}	9.987^{-9}	2.555^{-8}	5.586^{-9}	7.930^{-10}	1.3008^{-6}
$\mu_{\rm r}^2 = 1/2\,\mu_{\rm f}^2$							
10^{-7}	1.2438^{-4}	7.2817^{-5}	5.5568^{-6}	1.4556^{+2}	7.1706^{+1}	7.0990^{+1}	1.1468^{+3}
10^{-6}	6.1759^{-4}	3.6051^{-4}	2.4322^{-5}	7.3406^{+1}	3.5851^{+1}	3.5282^{+1}_{*}	5.4041^{+2}
10^{-5}	2.9770^{-3}	1.7316^{-3}	1.0121^{-5}_{*}	3.5158^{+1}_{*}	1.6903^{+1}	1.6452^{+1}	2.3663^{+2}
10^{-4}	1.3820^{-2}	7.9998^{-3}	4.0093^{-4}	1.5795^{+1}	7.3626^{+0}	7.0057^{+0}	9.3640^{+1}
10^{-3}	6.0585^{-2}	3.4766^{-2}	1.5300^{-3}	6.5284^{+0}	2.8504^{+0}	2.5740^{+0}	3.1795^{+1}
10^{-2}	2.3422^{-1}_{*}	1.3114^{-1}	5.4411^{-3}	2.3221^{+0}	8.8022^{-1}	6.9260^{-1}	8.1613^{+0}
0.1	5.4824^{-1}	2.6954^{-1}	9.8435^{-2}	3.8787^{-1}	1.1419^{-1}	6.0997^{-2}	8.9361^{-1}
0.3	3.4425^{-1}	1.2760^{-1}	2.9317^{-3}	3.4294^{-2}	8.7599^{-3}	3.1681^{-3}	8.2031^{-2}
0.5	1.1794^{-1}	3.0618^{-2}	3.6454^{-4}	2.2530^{-3}	5.3541^{-4}	1.4134^{-4}	7.8595^{-3}
0.7	1.9356^{-2}	2.9698^{-3}	1.2847^{-5}	4.8328^{-5}	1.0666^{-5}	1.6668^{-6}	3.7624^{-4}_{*}
0.9	3.3264^{-4}	1.6800^{-5}	8.124^{-9}	1.573^{-8}	2.024^{-9}	-1.870^{-9}	1.1647^{-6}

Table 3: As Table 2, but for the variable-$N_{\rm f}$ evolution using Eqs. (12) – (15). The corresponding values for the strong coupling $\alpha_{\rm s}(\mu_{\rm r}^2 = 10^4\ {\rm GeV}^2)$ are given by 0.116461, 0.116032 and 0.115663 for $\mu_{\rm r}^2/\mu_{\rm f}^2 = 0.5$, 1 and 2, respectively.

NLO, $N_{\rm f} = 3 \ldots 5$, $\mu_{\rm f}^2 = 10^4$ GeV^2								
x	xu_v	xd_v	xL_-	xL_+	xs_+	xc_+	xb_+	xg
$\mu_{\rm r}^2 = \mu_{\rm f}^2$								
10^{-7}	1.0927^{-4}	6.4125^{-5}	4.3925^{-6}	1.3787^{+2}	6.7857^{+1}	6.7139^{+1}	6.0071^{+1}	1.1167^{+3}
10^{-6}	5.5533^{-4}	3.2498^{-4}	1.9829^{-5}	6.9157^{+1}	3.3723^{+1}	3.3153^{+1}	2.8860^{+1}	5.2289^{+2}_{*}
10^{-5}	2.7419^{-3}	1.5989^{-3}	8.5701^{-5}	3.2996^{+1}	1.5819^{+1}	1.5367^{+1}	1.2892^{+1}	2.2753^{+2}
10^{-4}	1.3039^{-2}	7.5664^{-3}	3.5582^{-4}	1.4822^{+1}	6.8739^{+0}	6.5156^{+0}	5.1969^{+0}	8.9513^{+1}
10^{-3}	5.8507^{-2}	3.3652^{-2}	1.4329^{-3}	6.1772^{+0}	2.6726^{+0}	2.3949^{+0}	1.7801^{+0}	3.0245^{+1}
10^{-2}	2.3128^{-1}	1.2978^{-1}	5.3472^{-3}	2.2500^{+0}	8.4161^{-1}	6.5235^{-1}	4.3894^{-1}	7.7491^{+0}
0.1	5.5324^{-1}	2.7252^{-1}	9.9709^{-3}	3.9099^{-1}	1.1425^{-1}	6.0071^{-2}	3.5441^{-2}	8.5586^{-1}
0.3	3.5129^{-1}	1.3046^{-1}	3.0061^{-3}	3.5463^{-2}	9.1084^{-3}	3.3595^{-3}	1.9039^{-3}	7.9625^{-2}
0.5	1.2130^{-1}	3.1564^{-2}	3.7719^{-4}	2.3775^{-3}	5.7606^{-4}	1.6761^{-4}	1.0021^{-4}	7.7265^{-3}
0.7	2.0101^{-2}	3.0932^{-3}	1.3440^{-5}	5.2605^{-5}	1.2166^{-5}	2.7408^{-6}	2.0095^{-6}	3.7574^{-4}
0.9	3.5232^{-4}_{*}	1.7855^{-5}	8.680^{-9}	2.028^{-8}	3.896^{-9}	-2.666^{-10}	5.819^{-10}	1.1954^{-6}
$\mu_{\rm r}^2 = 2\,\mu_{\rm f}^2$								
10^{-7}	9.5154^{-5}	5.5970^{-5}	3.4869^{-6}	1.2301^{+2}	6.0424^{+1}	5.9703^{+1}	5.3916^{+1}	1.0568^{+3}
10^{-6}	4.9433^{-4}	2.8998^{-4}	1.6229^{-5}	6.2149^{+1}	3.0215^{+1}	2.9643^{+1}	2.6100^{+1}	4.9744^{+2}
10^{-5}	2.4974^{-3}	1.4600^{-3}	7.2776^{-5}	2.9936^{+1}	1.4286^{+1}	1.3831^{+1}	1.1768^{+1}	2.1783^{+2}
10^{-4}	1.2161^{-2}	7.0751^{-3}	3.1597^{-4}	1.3631^{+1}	6.2759^{+0}	5.9160^{+0}	4.7997^{+0}	8.6372^{+1}
10^{-3}	5.5907^{-2}	3.2239^{-2}	1.3377^{-3}_{*}	5.8020^{+0}	2.4824^{+0}	2.2029^{+0}	1.6701^{+0}	2.9488^{+1}
10^{-2}	2.2670^{-1}	1.2756^{-1}	5.2304^{-3}	2.1840^{+0}	8.0522^{-1}	6.1376^{-1}	4.2146^{-1}	7.6713^{+0}
0.1	5.5915^{-1}	2.7636^{-1}	1.0151^{-2}	3.9744^{-1}	1.1509^{-1}	5.9318^{-2}	3.5492^{-2}	8.7075^{-1}
0.3	3.6281^{-1}	1.3530^{-1}	3.1367^{-3}	3.7201^{-2}	9.5529^{-3}	3.5209^{-3}	1.9869^{-3}	8.2697^{-2}
0.5	1.2739^{-1}	3.3311^{-2}	4.0102^{-4}	2.5552^{-3}	6.2392^{-4}	1.8815^{-4}	1.0842^{-4}	8.1131^{-3}
0.7	2.1534^{-2}	3.3317^{-3}	1.4600^{-5}	5.8333^{-5}	1.3778^{-5}	3.5193^{-6}	2.2960^{-6}	3.9788^{-4}
0.9	3.9150^{-4}	1.9963^{-5}	9.797^{-9}	2.502^{-8}	5.459^{-9}	7.574^{-10}	1.002^{-9}	1.2895^{-6}
$\mu_{\rm r}^2 = 1/2\,\mu_{\rm f}^2$								
10^{-7}	1.2937^{-4}	7.5695^{-5}	5.8161^{-6}	1.4923^{+2}	7.3543^{+1}	7.2829^{+1}	6.4225^{+1}	1.1239^{+3}
10^{-6}	6.3890^{-4}	3.7272^{-4}	2.5317^{-5}	7.4973^{+1}	3.6635^{+1}	3.6068^{+1}	3.0846^{+1}	5.2763^{+2}
10^{-5}	3.0615^{-3}	1.7796^{-3}	1.0470^{-4}	3.5762^{+1}	1.7205^{+1}	1.6755^{+1}	1.3756^{+1}	2.3006^{+2}
10^{-4}	1.4120^{-2}	8.1674^{-3}	4.1155^{-4}	1.5994^{+1}	7.4628^{+0}	7.1064^{+0}	5.5232^{+0}_{*}	9.0599^{+1}
10^{-3}	6.1458^{-2}	3.5239^{-2}	1.5557^{-3}	6.5774^{+0}	2.8758^{+0}	2.5999^{+0}	1.8773^{+0}	3.0589^{+1}
10^{-2}	2.3574^{-1}	1.3187^{-1}	5.4739^{-3}	2.3254^{+0}	8.8301^{-1}	6.9613^{-1}	4.5587^{-1}	7.7996^{+0}
0.1	5.4630^{-1}	2.6827^{-1}	9.7828^{-3}	3.8485^{-1}	1.1348^{-1}	6.0825^{-2}	3.5524^{-2}	8.4801^{-1}
0.3	3.4035^{-1}	1.2596^{-1}	2.8877^{-3}	3.3752^{-2}	8.6313^{-3}	3.1345^{-3}	1.8308^{-3}	7.7817^{-2}
0.5	1.1589^{-1}	3.0034^{-2}	3.5665^{-4}	2.2032^{-3}	5.2398^{-4}	1.3891^{-4}	9.2307^{-5}	7.4943^{-3}
0.7	1.8886^{-2}	2.8923^{-3}	1.2475^{-5}	4.6898^{-5}	1.0350^{-5}	1.6179^{-6}	1.7069^{-6}	3.6221^{-4}
0.9	3.2053^{-4}	1.6155^{-5}	7.787^{-9}	1.489^{-8}	1.848^{-9}	-1.884^{-9}	6.129^{-11}	1.1353^{-6}

Table 4: Reference results for the $N_{\rm f} = 4$ next-next-to-leading-order evolution for the initial conditions (8) – (10). The corresponding value of the strong coupling is $\alpha_s(\mu_r^2 = 10^4 \text{ GeV}^2) = 0.110141$. The valence distributions s_v and c_v are equal for the input (9). The notation is explained in the first two paragraphs of Section 2.13.

NNLO, $N_{\rm f} = 4\,,\ \mu_{\rm f}^2 = 10^4 \text{ GeV}^2$								
x	xu_v	xd_v	xL_-	xL_+	xs_v	xs_+	xc_+	xg
$\mu_{\rm r}^2 = \mu_{\rm f}^2$								
10^{-7}	1.4425^{-4}	8.9516^{-5}_{*}	6.0836^{-6}	1.3492^{+2}	1.3116^{-5}	6.6385^{+1}	6.5669^{+1}	1.0105^{+3}
10^{-6}	6.7325^{-4}	4.1192^{-4}	2.6060^{-5}	6.9636^{+1}	4.8617^{-5}	3.3965^{+1}	3.3396^{+1}	4.9543^{+2}
10^{-5}	3.0782^{-3}	1.8510^{-3}	1.0652^{-4}	3.3826^{+1}	1.5280^{-4}	1.6236^{+1}	1.5784^{+1}	2.2406^{+2}
10^{-4}	1.3746^{-2}	8.0979^{-3}	4.1608^{-4}	1.5278^{+1}	3.3973^{-4}	7.1041^{+0}	6.7468^{+0}	9.0854^{+1}_{*}
10^{-3}	5.9222^{-2}	3.4137^{-2}	1.5718^{-3}	6.3249^{+0}	2.6800^{-4}	2.7483^{+0}	2.4713^{+0}	3.1353^{+1}
10^{-2}	2.3074^{-1}	1.2920^{-1}	5.5461^{-3}	2.2736^{+0}	-5.2053^{-4}	8.5420^{-1}	6.6542^{-1}	8.1341^{+0}
0.1	5.5178^{-1}	2.7165^{-1}	1.0024^{-2}	3.9020^{-1}	-3.0680^{-4}	1.1386^{-1}	5.9780^{-2}	9.0567^{-1}
0.3	3.5071^{-1}	1.3025^{-1}	3.0098^{-3}	3.5358^{-2}	-3.1905^{-5}	9.0479^{-3}	3.3060^{-3}	8.4184^{-2}
0.5	1.2117^{-1}	3.1528^{-2}	3.7743^{-4}	2.3866^{-3}	-2.7207^{-6}	5.7966^{-4}	1.7171^{-4}	8.1127^{-3}_{*}
0.7	2.0078^{-2}	3.0886^{-3}	1.3442^{-5}	5.4226^{-5}	-1.0121^{-7}	1.2936^{-5}	3.5305^{-6}	3.8946^{-4}
0.9	3.5111^{-4}	1.7783^{-5}	8.870^{-9}	2.630^{-8}	-1.450^{-10}	7.115^{-9}	2.973^{-9}	1.2147^{-6}
$\mu_{\rm r}^2 = 2\,\mu_{\rm f}^2$								
10^{-7}	1.2819^{-4}	7.8560^{-5}	5.2370^{-6}	1.3229^{+2}	8.8691^{-6}	6.5070^{+1}	6.4352^{+1}	1.0296^{+3}
10^{-6}	6.1508^{-4}	3.7250^{-4}	2.2897^{-5}	6.7704^{+1}	3.3358^{-5}	3.2997^{+1}	3.2427^{+1}	4.9907^{+2}
10^{-5}	2.8891^{-3}	1.7248^{-3}	9.5768^{-5}	3.2745^{+1}	1.0595^{-4}	1.5694^{+1}	1.5241^{+1}	2.2382^{+2}
10^{-4}	1.3223^{-2}	7.7642^{-3}	3.8437^{-4}	1.4803^{+1}	2.3744^{-4}	6.8647^{+0}	6.5065^{+0}	9.0293^{+1}
10^{-3}	5.8091^{-2}	3.3494^{-2}	1.4978^{-3}	6.1716^{+0}	1.8990^{-4}	2.6702^{+0}	2.3924^{+0}	3.1117^{+1}
10^{-2}	2.2927^{-1}	1.2858^{-1}	5.4465^{-3}_{*}	2.2481^{+0}	-3.6267^{-4}	8.4001^{-1}	6.5031^{-1}	8.0964^{+0}
0.1	5.5429^{-1}	2.7326^{-1}	1.0073^{-2}	3.9298^{-1}	-2.1631^{-4}	1.1440^{-1}	5.9718^{-2}	9.0854^{-1}
0.3	3.5501^{-1}	1.3205^{-1}	3.0556^{-3}	3.6008^{-2}	-2.2642^{-5}	9.2226^{-3}	3.3770^{-3}	8.5020^{-2}_{*}
0.5	1.2340^{-1}	3.2166^{-2}	3.8590^{-4}	2.4458^{-3}	-1.9414^{-6}	5.9487^{-4}	1.7699^{-4}	8.2294^{-3}_{*}
0.7	2.0597^{-2}	3.1751^{-3}	1.3854^{-5}	5.5708^{-5}	-7.2724^{-8}	1.3244^{-5}	3.5361^{-6}	3.9686^{-4}
0.9	3.6527^{-4}	1.8544^{-5}	9.204^{-9}	2.616^{-8}	-1.057^{-10}	6.700^{-9}	2.364^{-9}	1.2497^{-6}
$\mu_{\rm r}^2 = 1/2\,\mu_{\rm f}^2$								
10^{-7}	1.6566^{-4}	1.0501^{-4}	7.1003^{-6}	1.3181^{+2}	2.0996^{-5}	6.4832^{+1}	6.4117^{+1}	9.4582^{+2}
10^{-6}	7.4521^{-4}	4.6428^{-4}	2.9744^{-5}	6.9457^{+1}	7.6503^{-5}	3.3877^{+1}	3.3309^{+1}_{*}	4.7543^{+2}_{*}
10^{-5}	3.2879^{-3}	2.0041^{-3}	1.1872^{-4}	3.4193^{+1}	2.3809^{-4}	1.6421^{+1}	1.5970^{+1}	2.1922^{+2}
10^{-4}	1.4223^{-2}	8.4396^{-3}	4.5146^{-4}	1.5524^{+1}	5.2716^{-4}	7.2281^{+0}	6.8715^{+0}	9.0112^{+1}
10^{-3}	5.9889^{-2}	3.4553^{-2}	1.6545^{-3}	6.4073^{+0}	4.1428^{-4}	2.7908^{+0}	2.5144^{+0}	3.1340^{+1}
10^{-2}	2.3100^{-1}	1.2915^{-1}_{*}	5.6663^{-3}	2.2851^{+0}	-8.0979^{-4}	8.6079^{-1}	6.7251^{-1}	8.1526^{+0}
0.1	5.5039^{-1}	2.7076^{-1}	1.0032^{-2}	3.8851^{-1}_{*}	-4.7547^{-4}	1.1333^{-1}	5.9500^{-2}	9.0801^{-1}
0.3	3.4890^{-1}	1.2949^{-1}	2.9942^{-3}	3.5090^{-2}	-4.9326^{-5}	8.9666^{-3}	3.2669^{-3}	8.4307^{-2}
0.5	1.2026^{-1}	3.1269^{-2}	3.7428^{-4}_{*}	2.3728^{-3}	-4.1969^{-6}	5.7784^{-4}	1.7391^{-4}	8.1102^{-3}
0.7	1.9867^{-2}	3.0534^{-3}	1.3285^{-5}	5.4607^{-5}	-1.5564^{-7}	1.3275^{-5}_{*}	3.9931^{-6}	3.8822^{-4}_{*}
0.9	3.4524^{-4}	1.7466^{-5}	8.828^{-9}	2.930^{-8}	-2.216^{-10}	8.837^{-9}	4.777^{-9}	1.2041^{-6}

Table 5: As Table 4, but for the variable-$N_{\rm f}$ evolution using Eqs. (12) – (15). The corresponding values for the strong coupling $\alpha_{\rm s}(\mu_{\rm r}^2 = 10^4 \text{ GeV}^2)$ are given by 0.115818, 0.115605 and 0.115410 for $\mu_{\rm r}^2/\mu_{\rm f}^2 = 0.5$, 1 and 2, respectively. For brevity the small, but non-vanishing valence distributions s_v, c_v and b_v are not displayed.

NNLO, $N_{\rm f} = 3 \ldots 5$, $\mu_{\rm f}^2 = 10^4$ GeV^2								
x	xu_v	xd_v	xL_-	xL_+	xs_+	xc_+	xb_+	xg
$\mu_{\rm r}^2 = \mu_{\rm f}^2$								
10^{-7}	1.5053^{-4}	9.3396^{-5}	6.3350^{-6}	1.4236^{+2}	7.0107^{+1}	6.8526^{+1}	5.8645^{+1}	1.0101^{+3}
10^{-6}	6.9800^{-4}	4.2692^{-4}	2.7015^{-5}	7.2905^{+1}	3.5601^{+1}	3.4447^{+1}	2.8749^{+1}	4.9242^{+2}
10^{-5}	3.1691^{-3}	1.9048^{-3}	1.0987^{-4}	3.5102^{+1}	1.6875^{+1}	1.6059^{+1}	1.3011^{+1}	2.2122^{+2}
10^{-4}	1.4048^{-2}	8.2711^{-3}	4.2655^{-4}	1.5694^{+1}	7.3126^{+0}	6.7623^{+0}	5.2796^{+0}	8.8987^{+1}
10^{-3}	6.0085^{-2}	3.4610^{-2}	1.5988^{-3}	6.4190^{+0}	2.7963^{+0}	2.4502^{+0}	1.8138^{+0}	3.0407^{+1}
10^{-2}	2.3240^{-1}	1.3001^{-1}	5.5869^{-3}	2.2762^{+0}	8.5667^{-1}	6.6665^{-1}	4.5032^{-1}	7.7873^{+0}
0.1	5.4993^{-1}	2.7035^{-1}	9.9604^{-3}	3.8527^{-1}	1.1231^{-1}	6.4471^{-2}	3.7283^{-2}	8.5270^{-1}
0.3	3.4622^{-1}	1.2833^{-1}	2.9571^{-3}	3.4600^{-2}	8.8409^{-3}	4.0131^{-3}	2.1046^{-3}	7.8896^{-2}
0.5	1.1868^{-1}	3.0811^{-2}	3.6760^{-4}	2.3198^{-3}	5.6309^{-4}	2.3748^{-4}	1.2003^{-4}	7.6399^{-3}
0.7	1.9486^{-2}	2.9902^{-3}	1.2964^{-5}	5.2335^{-5}	1.2504^{-5}	5.6019^{-6}	2.8883^{-6}	3.7079^{-4}
0.9	3.3522^{-4}	1.6933^{-5}	8.4201^{-9}	2.5098^{-8}	6.8378^{-9}	4.3190^{-9}	2.6761^{-9}	1.1731^{-6}
$\mu_{\rm r}^2 = 2\,\mu_{\rm f}^2$								
10^{-7}	1.3305^{-4}	8.1537^{-5}	5.4285^{-6}	1.3770^{+2}	6.7776^{+1}	6.6452^{+1}	5.7708^{+1}	1.0172^{+3}
10^{-6}	6.3465^{-4}	3.8427^{-4}	2.3638^{-5}	7.0043^{+1}	3.4167^{+1}	3.3187^{+1}	2.8117^{+1}	4.9102^{+2}
10^{-5}	2.9622^{-3}	1.7679^{-3}	9.8415^{-5}	3.3642^{+1}	1.6144^{+1}	1.5435^{+1}	1.2685^{+1}	2.1913^{+2}
10^{-4}	1.3469^{-2}	7.9047^{-3}	3.9275^{-4}	1.5088^{+1}	7.0083^{+0}	6.5145^{+0}	5.1475^{+0}	8.7878^{+1}
10^{-3}	5.8793^{-2}	3.3879^{-2}	1.5195^{-3}	6.2333^{+0}	2.7018^{+0}	2.3753^{+0}	1.7742^{+0}	3.0063^{+1}
10^{-2}	2.3060^{-1}	1.2923^{-1}	5.4792^{-3}	2.2478^{+0}	8.4084^{-1}	6.5026^{-1}	4.4324^{-1}	7.7467^{+0}
0.1	5.5280^{-1}	2.7222^{-1}	1.0021^{-2}_{*}	3.8898^{-1}	1.1313^{-1}	6.2921^{-2}	3.7050^{-2}	8.5900^{-1}
0.3	3.5141^{-1}	1.3051^{-1}	3.0134^{-3}	3.5398^{-2}	9.0559^{-3}	3.8724^{-3}	2.0992^{-3}	8.0225^{-2}
0.5	1.2140^{-1}	3.1590^{-2}	3.7799^{-4}	2.3919^{-3}	5.8148^{-4}	2.2373^{-4}	1.1917^{-4}	7.8099^{-3}
0.7	2.0120^{-2}	3.0955^{-3}	1.3467^{-5}	5.4182^{-5}	1.2896^{-5}	5.0316^{-6}	2.8148^{-6}	3.8098^{-4}
0.9	3.5230^{-4}	1.7849^{-5}	8.835^{-9}	2.523^{-8}	6.499^{-9}	3.377^{-9}	2.404^{-9}	1.2196^{-6}
$\mu_{\rm r}^2 = 1/2\,\mu_{\rm f}^2$								
10^{-7}	1.7462^{-4}	1.1066^{-4}	7.4685^{-6}	1.4238^{+2}	7.0121^{+1}	6.8041^{+1}	5.7115^{+1}	9.6557^{+2}
10^{-6}	7.7945^{-4}	4.8542^{-4}	3.1111^{-5}_{*}	7.4239^{+1}	3.6270^{+1}	3.4778^{+1}	2.8492^{+1}	4.8135^{+2}
10^{-5}	3.4099^{-3}	2.0775^{-3}	1.2343^{-4}	3.6106^{+1}	1.7379^{+1}	1.6353^{+1}	1.3052^{+1}	2.1979^{+2}
10^{-4}	1.4619^{-2}	8.6691^{-3}	4.6591^{-4}	1.6161^{+1}	7.5479^{+0}	6.8868^{+0}	5.3331^{+0}	8.9300^{+1}
10^{-3}	6.1014^{-2}	3.5170^{-2}	1.6915^{-3}	6.5570^{+0}	2.8669^{+0}	2.4816^{+0}	1.8360^{+0}	3.0619^{+1}_{*}
10^{-2}	2.3320^{-1}	1.3024^{-1}	5.7229^{-3}	2.2924^{+0}	8.6595^{-1}	6.7561^{-1}	4.5538^{-1}	7.8182^{+0}
0.1	5.4799^{-1}	2.6905^{-1}	9.9481^{-3}_{*}	3.8193^{-1}	1.1125^{-1}	6.7100^{-2}	3.7702^{-2}	8.4914^{-1}
0.3	3.4291^{-1}	1.2693^{-1}	2.9238^{-3}	3.4069^{-2}	8.6866^{-3}	4.3918^{-3}	2.1434^{-3}	7.8107^{-2}_{*}
0.5	1.1694^{-1}	3.0310^{-2}	3.6112^{-4}	2.2827^{-3}	5.5538^{-4}	2.7737^{-4}	1.2414^{-4}	7.5373^{-3}
0.7	1.9076^{-2}	2.9217^{-3}	1.2646^{-5}	5.2035^{-5}	1.2677^{-5}	7.2055^{-6}	3.0900^{-6}	3.6439^{-4}_{*}
0.9	3.2404^{-4}	1.6333^{-5}	8.231^{-9}	2.752^{-8}	8.380^{-9}	6.769^{-9}	3.204^{-9}	1.1424^{-6}

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