hep-ph/0010146

INLO-PUB 10/00
October 2000

Next-to-Next-to-Leading Logarithmic Threshold Resummation for Deep-Inelastic Scattering and the Drell-Yan Process

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Abstract

The soft-gluon resummation exponents G^N in moment space are investigated for the quark coefficient functions in deep-inelastic structure functions and the quark-antiquark contribution to the Drell-Yan cross section $d\sigma/dM$. Employing results from two- and three-loop calculations we obtain the next-to-next-to-leading logarithmic terms $\alpha_s(\alpha_s \ln N)^n$ of G^N to all orders in the strong coupling constant α_s. These new contributions facilitate a reliable assessment of the numerical effect and the stability of the large-N expansion.

Deep-inelastic lepton-hadron scattering (DIS) and Drell-Yan (DY) lepton pair production in hadronic collisions are among the processes best suited for probing the short-distance structure of hadrons. The subprocess cross sections (coefficient functions) for these processes in perturbative QCD receive large logarithmic corrections, originating from soft-gluon radiation, at large values of the scaling variables x corresponding to large Mellin moments N. These corrections have been resummed [1, 2] to all orders in the strong coupling constant α_s up to next-to-leading logarithmic accuracy (technically defined in eq. (10) below). As the next-to-leading contributions are often hardly suppressed against the leading terms, it is important to extend the resummation to the next-to-next-to-leading logarithms. In this paper we present corresponding results for the DIS structure functions $F_{1,2,3}(x,Q^2)$ (where Q^2 represents the resolution scale) and for the Drell-Yan cross section $d\sigma/dQ^2$ (where Q stands for the invariant mass M of the lepton pair).

The soft-gluon resummation of the DIS ($P \equiv 1$) and DY ($P \equiv 2$) quark coefficient functions $C_P^N(Q^2)$ in N-space is given by [1, 2]

$$C_P^N(Q^2) \;=\; g_{P,0}(Q^2) \cdot \exp\left[G_P^N(Q^2)\right] \;+\; \mathcal{O}(N^{-1}\ln^n N) \;. \tag{1}$$

The functions C^N, g_0 and G^N also depend on the factorization scale μ_f and the renormalization scale μ_r, a dependence which we will often suppress for brevity. For the quantities under consideration the contributions g_0 collecting the N-independent terms are known up to second order in α_S from the two-loop calculations performed in refs. [3]. In the standard $\overline{\rm MS}$ renormalization and factorization scheme employed throughout this paper the exponents G_P^N in eq. (1) can be written as

$$\begin{aligned} G_1^N(Q^2) &= \ln\Delta_q(Q^2,\mu_f^2) + \ln J_q(Q^2) + \ln\Delta_1^{\rm int}(Q^2) \;, \\ G_2^N(Q^2) &= 2\,\ln\Delta_q(Q^2,\mu_f^2) + \ln\Delta_2^{\rm int}(Q^2) \;. \end{aligned} \tag{2}$$

Closely following the notations of ref. [4], the components entering eq. (2) are

$$\ln\Delta_q(Q^2,\mu_f^2) \;=\; \int_0^1 dz\,\frac{z^{N-1}-1}{1-z}\int_{\mu_f^2}^{(1-z)^2Q^2}\frac{dq^2}{q^2}\,A(a_s(q^2)) \tag{3}$$

collecting of effects of soft-gluon radiation collinear to initial-state partons,

$$\ln J_q(Q^2) \;=\; \int_0^1 dz\,\frac{z^{N-1}-1}{1-z}\left[\int_{(1-z)^2Q^2}^{(1-z)Q^2}\frac{dq^2}{q^2}\,A(a_s(q^2)) + B(a_s([1-z]Q^2))\right] \tag{4}$$

taking into account collinear final-state radiation, and the process-dependent piece

$$\ln\Delta_P^{\rm int}(Q^2) \;=\; \int_0^1 dz\,\frac{z^{N-1}-1}{1-z}\,D_P(a_s([1-z]^2Q^2)) \tag{5}$$

attributed to large-angle soft-gluon emissions. The integrands in eqs. (3)–(5) are given by

$$F(a_s) \;=\; \sum_{l=1}^{\infty} F_l\,a_s^l\;, \quad F = A,\,B,\,D_P \;, \tag{6}$$

where we normalize the expansion parameter as $a_s = \alpha_s/(4\pi)$.

The constants A_l in eq. (6) are the coefficients of the $1/[1-x]_+$ terms of l-loop quark-quark splitting functions $P_{qq}^{(l-1)}(x)$ — this actually completes the $\overline{\rm MS}$ definition of the coefficients B_l and $D_{P,l}$ [2] (see also ref. [5]). Thus A_1 and A_2 are well-known, reading

$$A_1 \;=\; 4C_F\,, \quad A_2 \;=\; 8C_F\left[\left(\frac{67}{18}-\zeta_2\right)C_A - \frac{5}{9}\,N_f\right] \;. \tag{7}$$

Here N_f denotes the number of effectively massless quark flavours, and the colour factors are $C_F = 4/3$ and $C_A = 3$ in QCD. The exact expression for $P_{qq}^{(2)}(x)$ has not been completed yet. However, recently rather accurate approximations have been derived [6] from the available partial results, most notably the lowest integer-N moments [7, 8]. Together with the exact N_f^2 term determined in ref. [9] the results of ref. [6] yield[1]

$$A_3 \;=\; (1178.8 \pm 11.5) - (183.95 \pm 0.85)N_f - \frac{16}{27}\,C_f N_f^2 \;. \tag{8}$$

For $N_f = 3 \ldots 5$ the (independent) errors in eq. (8) can be combined to an overall uncertainty of ± 12. For $\alpha_s < 0.3$ this uncertainty amounts to less than 0.1% of the total three-loop value of $A(a_s)$. Finally the constants B_1 and $D_{P,1}$ in eq. (6) are given by [2]

$$B_1 \;=\; -P_{q,\delta}^{(0)} \;=\; -3\,C_F\,, \quad D_{P,1} \;=\; 0 \;, \tag{9}$$

where $P_{q,\delta}^{(0)}$ is the coefficient of $\delta(1-x)$ in the one-loop quark-quark splitting function. The second-order terms B_2 and $D_{P,2}$ will be discussed below.

After the integrations in eqs. (3)–(5) are performed, the functions $G_P^N(Q^2)$ in eq. (2) take the form

$$G^N(Q^2) \;=\; L\,g_1(\lambda) + g_2(\lambda) + a_s\,g_3(\lambda) + \ldots \tag{10}$$

with $L = \ln N$, $\lambda = \beta_0 a_s L$ and

$$g_i(\lambda) \;=\; \sum_{k=1}^{\infty} g_{ik}\,(a_s L)^k \;. \tag{11}$$

The first term in eq. (10), which depend on A_1 only (see eq. (16) below), collects the leading logarithmic (LL) large-N contributions $L(a_sL)^n$. The coefficients A_2 and B_1 determine the functions g_2 resumming the next-to-leading logarithmic (NLL) terms $(a_sL)^n$. These functions have been determined in refs. [1, 2]. The next-to-next-to-leading logarithmic (NNLL) approximation includes g_3 which is correspondingly fixed by A_3, B_2 and $D_{P,2}$.

In order to calculate g_3 it is convenient to introduce

$$X = 1 + a_s(\mu_r^2)\beta_0 \ln\frac{q^2}{\mu_r^2} \tag{12}$$

[1]The error estimate (8) has been derived in ref. [6] under an assumption (no $\ln^3(1-x)$ terms) on the form of $P_{qq}^{(2)}(x)$. Abandoning this constraint does not lead to a significant modification of eq. (8).

and to write the next-to-next-to-leading order coupling constant as

$$\begin{aligned} a_s(q^2) \;&=\; a_s(\mu_r^2)\,\frac{1}{X} \;-\; a_s^2(\mu_r^2)\,\frac{\beta_1}{\beta_0}\,\frac{\ln X}{X^2} \\ &+\; a_s^3(\mu_r^2)\left(\frac{\beta_1^2}{\beta_0^2}\,\frac{\ln^2 X - \ln X - 1 + X}{X^3} + \frac{\beta_2}{\beta_0}\,\frac{1-X}{X^3}\right) \\ &+\; \mathcal{O}\Big(a_s^4(\mu_r^2)\Big[a_s(\mu_r^2)\ln(q^2/\mu_r^2)\Big]^n\Big)\;. \end{aligned} \tag{13}$$

Here β_0, β_1 and β_2 are the coefficients of the β-function of QCD up to three loops [10]. After inserting eq. (13) into eqs. (3) and (4), the inner integrations over $A(a_s)$ can be readily carried out. A straightforward method to perform the z-integral is to write the integrand as an infinite series in $\ln(1-x)$, to use

$$\begin{aligned} \int_0^1 dz\,\frac{z^{N-1}-1}{1-z}\,\ln^k(1-x) \;&=\; \frac{(-1)^{k+1}}{k+1}\left\{S_1^{k+1}(N) + \frac{1}{2}\,k(k+1)\,S_1^{k-1}(N)\,S_2(N)\right\} \\ &\quad + \mathcal{O}(S_1^{k-2}) \end{aligned} \tag{14}$$

(cf. ref. [11]) with $S_a(N) = \sum_{j=1}^N 1/j^a$ and

$$S_1(N) \;=\; \ln N + \gamma_e + \mathcal{O}(1/N)\,, \quad S_2(N) \;=\; \zeta_2 + \mathcal{O}(1/N)\;, \tag{15}$$

and to re-assemble the expansions in the end. In this way we arrive at

$$g_1^{\rm DIS}(\lambda) \;=\; \frac{A_1}{\beta_0\lambda}\Big[\lambda + (1-\lambda)\ln(1-\lambda)\Big] \tag{16}$$

$$\begin{aligned} g_2^{\rm DIS}(\lambda) \;&=\; -\,\frac{A_1\gamma_e - B_1}{\beta_0}\,\ln(1-\lambda) + \frac{A_1\beta_1}{\beta_0^3}\Big[\lambda + \ln(1-\lambda) + \frac{1}{2}\ln^2(1-\lambda)\Big] \\ &\quad -\,\frac{A_2}{\beta_0^2}\Big[\lambda + \ln(1-\lambda)\Big] + \ln\left(\frac{Q^2}{\mu_r^2}\right)\frac{A_1}{\beta_0}\ln(1-\lambda) + \ln\left(\frac{\mu_f^2}{\mu_r^2}\right)\frac{A_1}{\beta_0}\,\lambda \end{aligned} \tag{17}$$

and our new result

$$\begin{aligned} g_3^{\rm DIS}(\lambda) \;&=\; +\,A_1\left\{\frac{1}{2}(\gamma_e^2+\zeta_2)\,\frac{\lambda}{1-\lambda} \;+\; \frac{\beta_1^2}{\beta_0^4}\,\frac{1}{1-\lambda}\left[\frac{1}{2}\ln^2(1-\lambda) + \lambda\ln(1-\lambda) + \frac{1}{2}\lambda^2\right]\right. \\ &\qquad \left. -\;\frac{\beta_1\gamma_e}{\beta_0^2}\,\frac{1}{1-\lambda}\Big[\lambda + \ln(1-\lambda)\Big] \;+\; \frac{\beta_2}{\beta_0^3}\left[\frac{1}{2}\frac{\lambda^2}{1-\lambda} + \ln(1-\lambda) + \lambda\right]\right\} \\ &\quad +\,A_2\left\{\frac{\gamma_e}{\beta_0}\,\frac{\lambda}{1-\lambda} \;-\; \frac{\beta_1}{\beta_0^3}\,\frac{1}{1-\lambda}\Big[\ln(1-\lambda) + \lambda + \frac{1}{2}\lambda^2\Big]\right\} \;+\; \frac{A_3}{2\beta_0^2}\,\frac{\lambda^2}{1-\lambda} \\ &\quad -\,B_1\left\{\gamma_e\,\frac{\lambda}{1-\lambda} - \frac{\beta_1}{\beta_0^2}\,\frac{1}{1-\lambda}\Big[\lambda + \ln(1-\lambda)\Big]\right\} \;-\; \frac{B_2}{\beta_0}\,\frac{\lambda}{1-\lambda} \;-\; \frac{D_{1,2}}{\beta_0}\,\frac{\lambda}{1-2\lambda} \\ &\quad +\ln\left(\frac{Q^2}{\mu_r^2}\right)\left\{\frac{A_1\beta_1}{\beta_0^2}\,\frac{1}{1-\lambda}\Big[\lambda + \ln(1-\lambda)\Big] + \left(B_1 - A_1\gamma_e - \frac{A_2}{\beta_0}\right)\frac{\lambda}{1-\lambda}\right\} \\ &\quad +\ln^2\left(\frac{Q^2}{\mu_r^2}\right)\frac{A_1}{2}\,\frac{\lambda}{1-\lambda} \;+\; \ln\left(\frac{\mu_f^2}{\mu_r^2}\right)\frac{A_2}{\beta_0}\,\lambda \;-\; \ln^2\left(\frac{\mu_f^2}{\mu_r^2}\right)\frac{A_1}{2}\,\lambda\;. \end{aligned} \tag{18}$$

The functions $g_{2,3}^{\rm DY}$ are obtained from eqs. (17) and (18) by substituting $\lambda \to 2\lambda$, $\gamma_e \to 2\gamma_e$ and $\zeta_2 \to 4\zeta_2$ in the terms with A_i, removing the terms with B_l (recall eq. (2)), and replacing $D_{1,1}$ by $D_{2,1}$. The result corresponding to eq. (16) reads $g_1^{\rm DY}(\lambda) = 2\, g_1^{\rm DIS}(2\lambda)$. The generalization of eqs. (16)–(18) to other processes involving eqs. (3)–(5) is obvious.

Now we are ready to address the second-order coefficients B_2 and $D_{P,2}$. The first-order expansion coefficients $g_{31}^{\rm DIS}$ and $g_{31}^{\rm DY}$, as defined in eq. (11), are given by

$$g_{31}^{(P)} \;=\; 1/2\, P^3 A_1(\gamma_e^2+\zeta_2)\beta_0 + P^2 A_2 \gamma_e - \delta_{P1}(B_2 - B_1\gamma_e\beta_0) - D_{P,2} \tag{19}$$

with $\delta_{kj} = 1$ for $k = j$ and $\delta_{kj} = 0$ else. On the other hand $g_{31}^{(P)}$ can be determined by expanding eqs. (1) and (10) to order α_s^2 and comparing to the two-loop results of refs. [3, 12], as done for the DIS case in ref. [13][2]. The result for the Drell-Yan coefficient function reads

$$\begin{aligned} g_{31}^{\rm DY} \;=\;& + C_F C_A \left(\frac{1616}{27} - 56\,\zeta_3 - 32\,\zeta_2\gamma_e + \frac{176}{3}\,\gamma_e^2 + \frac{1072}{9}\,\gamma_e\right) \\ & - C_F N_f \left(\frac{224}{27} + \frac{32}{3}\,\gamma_e^2 + \frac{160}{9}\,\gamma_e\right) \;, \end{aligned} \tag{20}$$

yielding

$$D_2^{\rm DY} = C_F C_A \left(-\frac{1616}{27} + 56\,\zeta_3 + \frac{176}{3}\,\zeta_2\right) \;+\; C_F N_f \left(\frac{224}{27} - \frac{32}{3}\,\zeta_2\right) \;. \tag{21}$$

This term has already been derived in ref. [14], albeit without explicitly attributing it to the $\alpha_s((1-z)^2Q^2)$ contribution. Note that, as it has to be for a $\Delta^{\rm int}$ contribution [4], eq. (21) does not contain an Abelian (C_F^2) term. We consider this as a first concrete check of the correctness of the setup (3)-(5) at the NNLL level. Using eq. (12) of ref. [13], the corresponding constraint for the DIS case reads

$$\begin{aligned} B_2 + D_2^{\rm DIS} \;=\;& C_F^2\left(-\frac{3}{2} - 24\,\zeta_3 + 12\,\zeta_2\right) \;+\; C_F C_A\left(-\frac{3155}{54} + 40\,\zeta_3 + \frac{44}{3}\,\zeta_2\right) \\ & +\; C_F N_f\left(\frac{247}{27} - \frac{8}{3}\,\zeta_2\right) \;=\; -P_{q,\delta}^{(1)} + \frac{1}{2}\,D_2^{\rm DY} - 7\,\beta_0 C_F \;, \end{aligned} \tag{22}$$

where $P_{q,\delta}^{(1)}$ is the coefficient of $\delta(1-x)$ in the two-loop quark-quark splitting function. Unlike the DY case (21) two new constants can occur at the NNLL level in DIS, hence the consistency of the resummed and the two-loop coefficient functions does not completely specify $g_3^{\rm DIS}$. For that either an extension of the calculations of refs. [1, 2] to the next order, or the $\ln^2 N$ term of the three-loop coefficient function (fixing g_{32} which involves the combination $B_2 + 2D_{1,2}$) is required. An approximate result has been derived for the latter [15] from the constraints of refs. [7, 8, 13]. Within errors that result is consistent with $D_2^{\rm DIS} = 0$, but, for instance, also with $B_2 = -P_{q,\delta}^{(1)} + \xi\,\beta_0 C_F$ for $\xi \simeq 8 \ldots 13$.

[2]Note that for g_3 the convention (11) differs from that in ref. [13]: there the second index i in g_{3i} refers to the overall power of α_s in the expansion (10) of G^N. Hence g_{31} is called g_{32} in ref. [13] etc.

The integrals in eqs. (3)–(5) are not well-defined, as they involve the running coupling at arbitrarily low scales. This feature leads to the poles at $(2)\, a_s\beta_0 \ln N = 1$ in eqs. (16)–(18) and their DY counterparts, a problem which is usually dealt with by the 'minimal-prescription' contour for the Mellin inversion [16]. Another option, followed in ref. [13] for the DIS case, is to re-expand eq. (1) in powers of α_s and to keep only those terms $\alpha_s^n \ln^{2n+1-k} N$, $k = 1, \ldots l$ (i.e., the first l 'towers' of logarithms) which are completely fixed by the known terms in eq. (10) and in g_0 of eq. (1). In connection with a two-loop result for g_0 the NLL and NNLL resummations lead to $l = 4$ and $l = 5$, respectively. General expressions for the first four towers can be found in eq. (14) of ref. [13][2], the extension to higher terms is straightforward if lengthy. After Mellin inversion the NNLL resummation thus leads to the following prediction for the five leading $x \to 1$ terms of the three-loop (quark-antiquark annihilation) DY coefficient functions for $d\sigma/dQ^2$ at $\mu_f^2 = \mu_r^2 = Q^2$ (the other terms are fixed by renormalization-group constraints):

$$
\begin{aligned}
c_{q,\mathrm{DY}}^{(3)}(x) \;=\; & 512\, C_F^3 \left[\frac{\ln^5(1-x)}{1-x}\right]_+ - \left(\frac{7040}{9}\, C_F^2 C_A - \frac{1280}{9}\, C_F^2 N_f\right) \left[\frac{\ln^4(1-x)}{1-x}\right]_+ \\
& + \left\{\left(-2048 - 3072\,\zeta_2\right) C_F^3 + \left(\frac{17152}{9} - 512\,\zeta_2\right) C_F^2 C_A + \frac{7744}{27}\, C_F C_A^2 \right. \\
& \qquad \left. - \frac{2560}{9}\, C_F^2 N_f - \frac{2816}{27}\, C_F C_A N_f + \frac{256}{27}\, C_F N_f^2 \right\} \left[\frac{\ln^3(1-x)}{1-x}\right]_+ \\
& + \left\{10240\,\zeta_3\, C_F^3 + \left(-\frac{4480}{9} + 1344\,\zeta_3 + \frac{11264}{3}\zeta_2\right) C_F^2 C_A \right. \\
& \qquad + \left(-\frac{28480}{27} + \frac{704}{3}\,\zeta_2\right) C_F C_A^2 + \left(\frac{544}{9} - \frac{2048}{3}\,\zeta_2\right) C_F^2 N_f \\
& \qquad \left. + \left(\frac{9248}{27} - \frac{128}{3}\,\zeta_2\right) C_F C_A N_f - \frac{640}{27}\, C_F N_f^2 \right\} \left[\frac{\ln^2(1-x)}{1-x}\right]_+ \\
& - \left(77949.50 - 5886.63\, N_f + 32.888\, N_f^2 + 4A_3\right) \left[\frac{\ln(1-x)}{1-x}\right]_+ + \ldots \;\; .
\end{aligned}
\tag{23}
$$

Here $c_{q,\mathrm{DY}}^{(3)}(x)$ refers to $a_s = \alpha_s/(4\pi)$, as all other expansion parameters in this paper. Since A_3 is presently known just in the form (8), only the numerical result has been written down for the $[(1-x)^{-1}\ln(1-x)]_+$ term in eq. (23). In practice, however, the uncertainty of ± 12 for A_3 is irrelevant here and in the corresponding higher-order terms.

An improvement on the tower expansion for DIS in ref. [13][3] will only be possible once B_2^{DIS} in eq. (22) is exactly determined. However, the present results for g_3^{DIS} are directly relevant to the 'physical' evolution kernels K^N for the scaling violations of the non-singlet structure functions investigated beyond order α_s^2 in refs. [15, 17]. Identifying all scales for

[3]Note that the third term of eq. (15) has been misprinted in the original preprint as well as the journal version of ref. [13]: $16/27\, C_F^2 N_f$ has to be replaced by $16/27\, C_F N_f^2$, and $70/9\, C_F N_f^2$ by $280/9\, C_F^2 N_f$.

brevity these kernels can be written as

$$\frac{dF^N_{a,\rm NS}}{d\ln Q^2} \;=\; \left(P^N(a_s) + \frac{d\ln C^N_a(a_s)}{da_s}\beta(a_s)\right) F^N_{a,\rm NS}(Q^2) \;\equiv\; K^N(a_s)\, F^N_{a,\rm NS}(Q^2)\;, \qquad (24)$$

where $P^N(a_s)$ and $C^N_a(a_s)$ ($a = 1, 2, 3$) are the moments of the non-singlet splitting functions and coefficient functions, respectively. At large-x / large-N eq. (24) holds for the full structure functions up to corrections of order $1/N$. Inserting eqs. (1) and (10) into this equation one arrives at the NNLL resummation of this kernel,

$$\begin{aligned} K^N_{a,\rm res}(a_s) \;=\;& -\ln N\,(A_1\,a_s + A_2\,a_s^2 + A_3\,a_s^3) \;-\; \left(1 + \frac{\beta_1}{\beta_0}a_s + \frac{\beta_2}{\beta_0}a_s^2\right)\lambda^2\frac{dg_1}{d\lambda} \qquad (25)\\ & -\left(a_s\beta_0 + a_s^2\beta_1\right)\lambda\frac{dg_2}{d\lambda} \;-\; a_s^2\beta_0\,\frac{d}{d\lambda}\Big(\lambda g_3(\lambda)\Big) \;+\; \mathcal{O}(a_s^3(a_s\ln N)^n) \end{aligned}$$

with $\lambda = a_s\beta_0\ln N$, A_i of eqs. (7) and (8), and g_i from eqs. (16)–(18) with $\mu_f = \mu_r = Q$. Note that the leading large-N term of the quark-quark splitting function is given by $\ln N$ at all orders in α_s [18].

Finally we briefly illustrate the numerical impact of the NNLL corrections. For these illustrations we choose $\mu_r^2 = \mu_f^2 = Q^2$, $N_f = 4$ and $\alpha_s = 0.2$. Depending on the precise value of $\alpha_s(M_Z^2)$, the latter number corresponds to scales between about 25 and 50 GeV^2, a range typical for fixed-target experiments both on DIS and the Drell-Yan process. The corresponding results for the functions $G^N_{\rm DIS}$ and $G^N_{\rm DY}$ are presented in fig. 1. Their convolutions with a schematic, but typical input are shown in fig. 2, where the minimal-prescription Mellin inversion [16] has been employed. The remaining ambiguity in the DIS case (discussed below eq. (22)) is illustrated by the results for $D_2^{\rm DIS} = 0$ and for $B_2 = -P^{(1)}_{q,\delta} + 13\,\beta_0 C_F$ (denoted by $\xi = 13$ in the figures), the latter curves indicating the maximal NNLL effects consistent with the available three-loop information [7, 8, 13, 15]. It is obvious from both figures that knowledge of the $\ln N(\alpha_s\ln N)^n$ and $(\alpha_s\ln N)^n$ terms [1, 2] alone is not sufficient for reliably determining the functions G^N_P and their impact after convolution even for rather moderate values of N and x (often denoted τ in the DY case). On the other hand the NNLL corrections presented in this paper are rather small over a wide range, e.g., less than 10% and 20% at $x \le 0.85$ and $\sqrt{x} \le 0.75$ for the DIS and DY results of fig. 2, respectively. This stabilization indicates that the soft-gluon exponents G^N_P and their effects can now be reliably estimated for these processes.

Acknowledgements

It is a pleasure to thank E. Laenen and W.L. van Neerven, but in particular W. Vogelsang (who checked the non-ζ_2 parts of eq. (18) using a conventional NLL technique) for useful discussions. The author has also profited from brief conversations with S. Catani and G. Sterman. This work has been supported by the European Community TMR research network 'Quantum Chromodynamics and the Deep Structure of Elementary Particles' under contract No. FMRX–CT98–0194.

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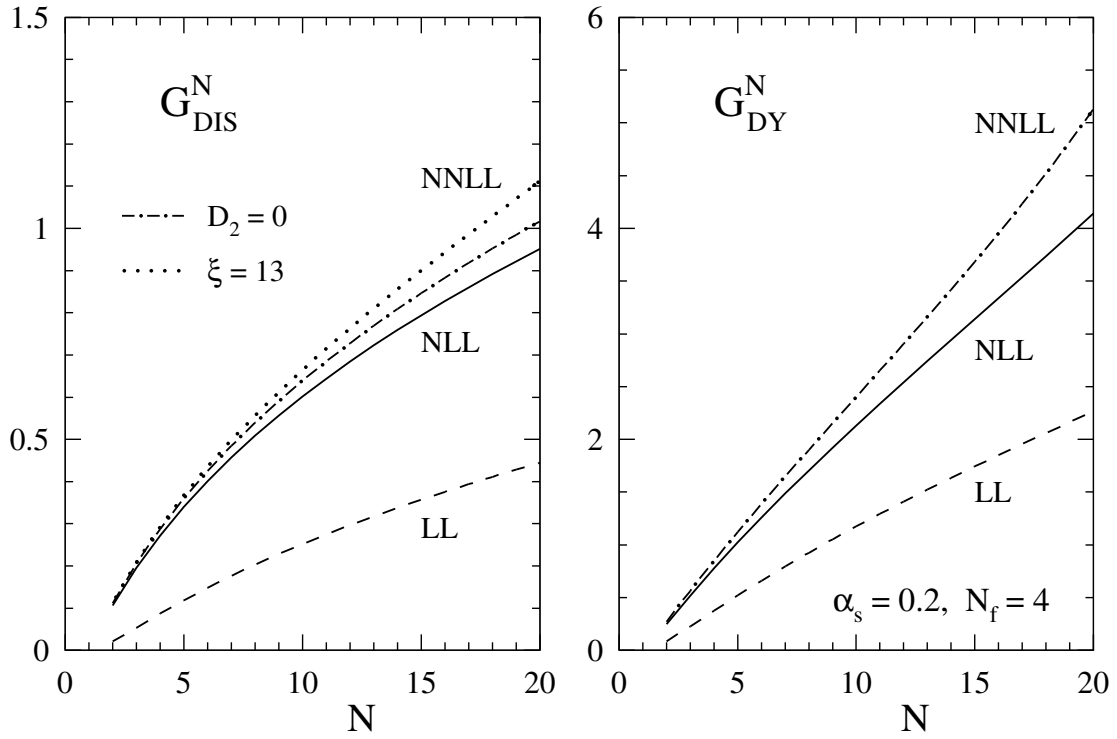

Fig. 1: The LL, NLL and NNLL approximations for the resummation exponents $G^N(Q^2)$ in eq. (10) at $\mu_r^2 = \mu_f^2 = Q^2$ for $\alpha_s(Q^2) = 0.2$ and four flavours. The two NNLL curves in the DIS case indicate the present uncertainty as discussed below eq. (22).

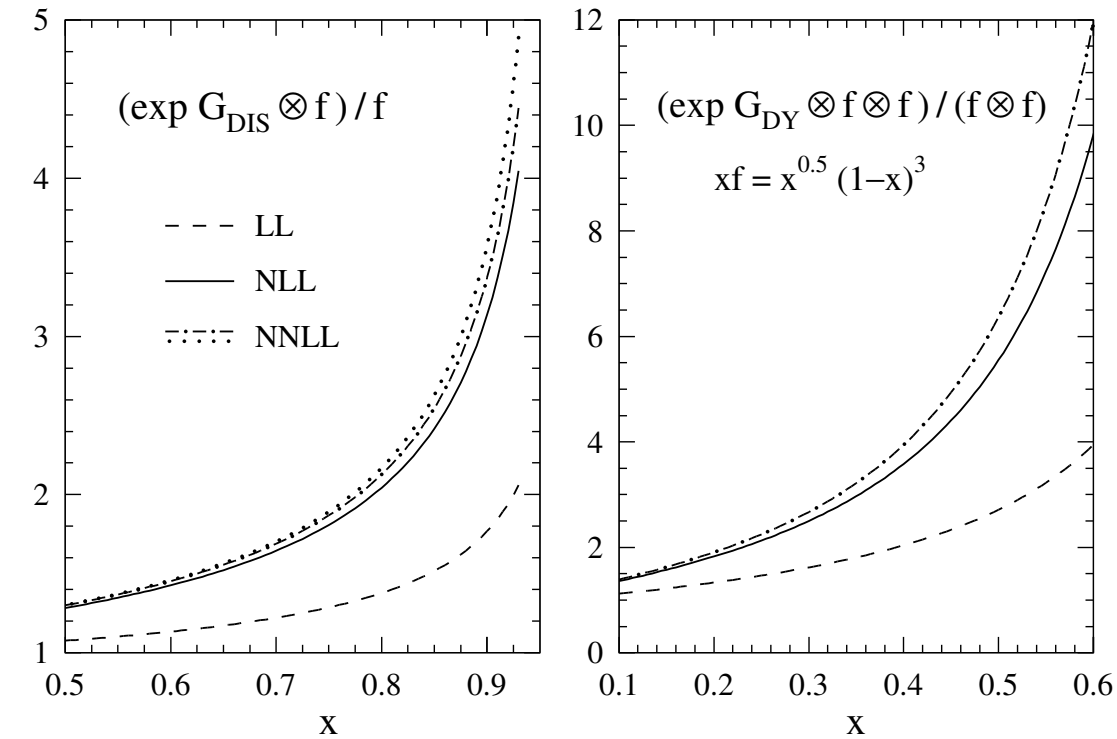

Fig. 2: The convolutions of the results shown in Fig. 1 with a typical input shape.