DESY 04-210, SFB/CPP-04-58
DCPT/04/130, IPPP/04/65
NIKHEF 04-013
November 2004

hep-ph/0411112

The Longitudinal Structure Function at the Third Order

S. Moch[a], J.A.M. Vermaseren[b] and A. Vogt[b,c]

[a] *Deutsches Elektronensynchrotron DESY*
Platanenallee 6, D–15735 Zeuthen, Germany

[b] *NIKHEF Theory Group*
Kruislaan 409, 1098 SJ Amsterdam, The Netherlands

[c] *IPPP, Department of Physics, University of Durham*
South Road, Durham DH1 3LE, United Kingdom

Abstract

We compute the complete third-order contributions to the coefficient functions for the longitudinal structure function F_L, thus completing the next-to-next-to-leading order (NNLO) description of unpolarized electromagnetic deep-inelastic scattering in massless perturbative QCD. Our exact results agree with determinations of low even-integer Mellin moments and of the leading small-x terms in the flavour-singlet sector. In this letter we present compact and accurate parametrizations of the results and illustrate the numerical impact of the NNLO corrections.

We have recently published the complete three-loop splitting functions $P^{(2)}$ for the evolution of unpolarized parton distributions of hadrons [1, 2]. Together with the second-order coefficient functions [3–7], these quantities form the next-to-next-to-leading order (NNLO) approximation of massless perturbative QCD for the structure functions F_1, F_2 and F_3 in deep-inelastic scattering (DIS). The situation is different, however, for the longitudinal structure function $F_L = F_2 - 2xF_1$. Here the leading contribution to the coefficient functions is of first order in the strong coupling constant α_s (instead of α_s^0 as in $F_{1,2,3}$), and consequently the scheme dependence of the splitting functions $P^{(2)}$ is compensated only by the third-order coefficient functions. The latter quantities are thus required for completing the NNLO description of the structure functions in DIS.

At this point there is only very limited (and no entirely model independent) experimental information on F_L in the most interesting region of very small values of the Bjorken variable x [8–10] (see ref. [11] for an overview over previous measurements at large x). Dedicated runs of HERA with lower proton energies would significantly improve this situation [12, 13] and provide valuable further constraints on the proton's gluon distribution at very small momentum fractions. The NNLO corrections are required to assess in which kinematical region of x and $Q^2 = -q^2$, with q being the momentum of the exchanged gauge boson, such constraints can be reliably obtained.

As discussed in refs. [1, 2], see also refs. [14–16], our determination of the NNLO splitting functions $P^{(2)}$ has been performed via a three-loop Mellin-N space calculation of DIS in dimensional regularization with $D = 4 - 2\varepsilon$. The splitting functions are then given by the coefficients of the $1/\varepsilon$ poles, while the ε^0 results include the third-order coefficient functions. Hence by keeping the terms of order ε^0 throughout the computations, and by adding the second Lorentz projection of the hadronic tensor required to disentangle F_2 and F_L, we are able to obtain the third-order coefficient functions as well. Our approach closely follows the calculation of the lowest even-integer moments in refs. [17–19]. Incidentally, the methods for obtaining all-N results at higher orders have been pioneered in the calculation of ref. [20] for the structure function F_L at second order.

In this letter we present our x-space results for the coefficient functions for F_L in electromagnetic DIS at three loops in a parametrized, but sufficiently accurate form. The lengthy exact formulae will be presented, together with the results for F_2, in a forthcoming publication [21].

Disregarding power corrections, the longitudinal structure function can be written as

$$x^{-1}F_L \,=\, C_{L,\mathrm{ns}} \otimes q_{\mathrm{ns}} + \langle e^2 \rangle \left(C_{L,\mathrm{q}} \otimes q_{\mathrm{s}} + C_{L,\mathrm{g}} \otimes g \right) \,. \tag{1}$$

Here q_i and g represent the number distributions of quarks and gluons, respectively, in the fractional hadron momentum. q_{s} stands for the flavour-singlet quark distribution, $q_{\mathrm{s}} = \sum_{i=1}^{n_f} (q_i + \bar{q}_i)$, where n_f denotes the number of effectively massless flavours. q_{ns} is the corresponding non-singlet combination. The average squared charge (= 5/18 for even n_f) is represented by $\langle e^2 \rangle$, and $\otimes$ denotes the Mellin convolution. We write the perturbative expansion of the coefficient functions as

$$C_{L,a}\left(\alpha_{\mathrm{s}}, x\right) \,=\, \sum_{n=1} \left(\frac{\alpha_{\mathrm{s}}}{4\pi}\right)^n c_{L,a}^{(n)}(x) \,. \tag{2}$$

Note that, as usual for F_L, the superscript of the coefficients on the right-hand-side represents the order in α_{s} and not, as for the splitting functions, the 'm' in N$^{\mathrm{m}}$LO. Throughout this article

we identify the renormalization and factorization scales with the physical scale Q^2. The results beyond the leading order (LO), $n = 1$ in eq. (2), are presented in the standard $\overline{\rm MS}$-scheme.

For the convenience of the reader we first recall the known first and second order results. The scheme-independent LO coefficient functions for F_L read

$$c_{L,\rm ns}^{(1)}(x) = 4C_F x \;, \quad c_{L,\rm ps}^{(1)}(x) = 0 \;, \quad c_{L,\rm g}^{(1)}(x) = 8\,n_f\,x(1-x) \;, \tag{3}$$

with $C_F = 4/3$ in QCD. Here and below, the singlet-quark coefficient function is decomposed into the non-singlet and a 'pure singlet' contribution, $c_{L,\rm q}^{(n)} = c_{L,\rm ns}^{(n)} + c_{L,\rm ps}^{(n)}$. To a sufficient accuracy, the NLO ($n = 2$) longitudinal coefficient functions [4, 7, 22] can be written as

$$\begin{aligned} c_{L,\rm ns}^{(2)}(x) \;\cong\; & 128/9\,xL_1^2 - 46.50\,xL_1 - 84.094\,L_0L_1 - 37.338 + 89.53\,x \\ & + 33.82\,x^2 + xL_0(32.90 + 18.41\,L_0) - 128/9\,L_0 - 0.012\,\delta(x_1) \\ & + 16/27\,n_f\,\{6xL_1 - 12xL_0 - 25x + 6\} \;, \end{aligned} \tag{4}$$

$$\begin{aligned} c_{L,\rm ps}^{(2)}(x) \;\cong\; & n_f\,\{\,(15.94 - 5.212\,x)\,x_1^2L_1 + (0.421 + 1.520\,x)\,L_0^2 + 28.09\,x_1L_0 \\ & - (2.370\,x^{-1} - 19.27)\,x_1^3\,\} \;, \end{aligned} \tag{5}$$

$$\begin{aligned} c_{L,g}^{(2)}(x) \;\cong\; & n_f\,\{\,(94.74 - 49.20\,x)x_1L_1^2 + 864.8\,x_1L_1 + 1161\,xL_1L_0 \\ & + 60.06\,xL_0^2 + 39.66\,x_1L_0 - 5.333\,(x^{-1} - 1)\,\} \;, \end{aligned} \tag{6}$$

where we have used the abbreviations

$$x_1 \,=\, 1 - x \;, \quad L_0 \,=\, \ln x \;, \quad L_1 \,=\, \ln x_1 \;. \tag{7}$$

Eq. (4) improves upon the previous parametrization in ref. [23], while eqs. (5) and (6) have been directly taken over from ref. [24]. The expressions (4) – (6), as well as their convolutions with typical parton distributions, deviate from the exact results by one part in a thousand or less.

Now we turn to our new third-order results. Again using the abbreviations (7) and inserting the numerical values for the colour factors, the non-singlet coefficient function can be parametrized as

$$\begin{aligned} c_{L,\rm ns}^{(3)}(x) \;\cong\; & 512/27\,L_1^4 - 177.40\,L_1^3 + 650.6\,L_1^2 - 2729\,L_1 - 2220.5 - 7884\,x \\ & + 4168\,x^2 - (844.7\,L_0 + 517.3\,L_1)L_0L_1 + (195.6\,L_1 - 125.3)\,x_1L_1^3 \\ & + 208.3\,xL_0^3 - 1355.7\,L_0 - 7456/27\,L_0^2 - 1280/81\,L_0^3 + 0.113\,\delta(x_1) \\ + \; & n_f\,\{1024/81\,L_1^3 - 112.35\,L_1^2 + 344.1\,L_1 + 408.4 - 9.345\,x - 919.3\,x^2 \\ & + (239.7 + 20.63\,L_1)\,x_1L_1^2 + (887.3 + 294.5\,L_0 - 59.14\,L_1)L_0L_1 \\ & - 1792/81\,xL_0^3 + 200.73\,L_0 + 64/3\,L_0^2 + 0.006\,\delta(x_1)\} \\ + \; & n_f^2\,\{3xL_1^2 + (6 - 25x)L_1 - 19 + (317/6 - 12\,\zeta_2)\,x - 6xL_0L_1 + 6x\,{\rm Li}_2(x) \\ & + 9xL_0^2 - (6 - 50x)L_0\}\,64/81 \\ + \; & fl_{11}^{\rm ns}\,n_f\,\{(107.0 + 321.05\,x - 54.62\,x^2)\,x_1 - 26.717 + 9.773\,L_0 \\ & + (363.8 + 68.32\,L_0)xL_0 - 320/81\,L_0^2(2 + L_0)\}\,x \;, \end{aligned} \tag{8}$$

where the n_f^2 part is exact. The (non-fl_{11}) fermionic contributions were already computed in ref. [14]. The NNLO pure-singlet coefficient function for F_L can be approximated by

$$\begin{aligned}
c_{L,\rm ps}^{(3)}(x) \;\cong\;& n_f\,\{(1568/27\,L_1^3 - 3968/9\,L_1^2 + 5124\,L_1)\,x_1^2 + (2184\,L_0 + 6059\,x_1)L_0L_1 \\
& - (795.6 + 1036\,x)x_1^2 - 143.6\,x_1L_0 + 2848/9\,L_0^2 - 1600/27\,L_0^3 \\
& - (885.53\,x_1 + 182.00\,L_0)x^{-1}x_1\} \\
+\;& n_f^2\,\{(-32/9\,L_1^2 + 29.52\,L_1)\,x_1^2 + (35.18\,L_0 + 73.06\,x_1)L_0L_1 - 35.24\,xL_0^2 \\
& - (14.16 - 69.84\,x)\,x_1^2 - 69.41\,x_1L_0 - 128/9\,L_0^2 + 40.239\,x^{-1}x_1^2\} \\
+\;& fl_{11}^{\rm ps}\,n_f\,\{(107.0 + 321.05\,x - 54.62\,x^2)x_1 - 26.717 + 9.773\,L_0 \\
& + (363.8 + 68.32\,L_0)xL_0 - 320/81\,L_0^2(2 + L_0)\}\,x\;. \qquad (9)
\end{aligned}$$

Finally the corresponding gluon coefficient function can be written as

$$\begin{aligned}
c_{L,\rm g}^{(3)}(x) \;\cong\;& n_f\,\{(144\,L_1^4 - 47024/27\,L_1^3 + 6319\,L_1^2 + 53160\,L_1)\,x_1 + 72549\,L_0L_1 \\
& + 88238\,L_0^2L_1 + (3709 - 33514\,x - 9533\,x^2)\,x_1 + 66773\,xL_0^2 \\
& - 1117\,L_0 + 45.37\,L_0^2 - 5360/27\,L_0^3 - (2044.70\,x_1 + 409.506\,L_0)\,x^{-1}\} \\
+\;& n_f^2\,\{(32/3\,L_1^3 - 1216/9\,L_1^2 - 592.3\,L_1 + 1511\,xL_1)\,x_1 + 311.3L_0L_1 \\
& + 14.24\,L_0^2L_1 + (577.3 - 729.0\,x)\,x_1 + 30.78\,xL_0^3 + 366.0\,L_0 \\
& + 1000/9\,L_0^2 + 160/9\,L_0^3 + 88.5037\,x^{-1}x_1\} \\
+\;& fl_{11}^{\rm g}\,n_f^2\,\{(-0.0105\,L_1^3 + 1.550\,L_1^2 + 19.72\,xL_1 - 66.745\,x + 0.615\,x^2)\,x_1 \\
& + 20/27\,xL_0^4 + (280/81 + 2.260\,x)xL_0^3 - (15.40 - 2.201\,x)\,xL_0^2 \\
& - (71.66 - 0.121\,x)\,xL_0\}\;. \qquad (10)
\end{aligned}$$

Eqs. (8) – (10) involve the new charge factors (see refs. [17, 18] for the corresponding diagrams)

$$fl_{11}^{\rm ns} = 3\langle e\rangle\;,\quad fl_{11}^{\rm g} = \langle e\rangle^2/\langle e^2\rangle\;,\quad fl_{11}^{\rm ps} = fl_{11}^{\rm g} - fl_{11}^{\rm ns} \qquad (11)$$

where $\langle e^k\rangle$ stand for the average of the charge e^k for the active quark flavours, $\langle e^k\rangle = n_f^{-1}\sum_{i=1}^{n_f} e_i^k$. The rational coefficients in eqs. (8) – (10) are exact, as are (up to the truncation) the irrational coefficients of L_1^2 in eq. (8) and of the $1/x$ terms in eqs. (9) – (10). The remaining coefficients have been fitted to the exact coefficient functions which we evaluated using a weight-five extension of the program [25] for the evaluation of the harmonic polylogarithms [26] (see ref. [27] for a new program without restrictions on the weight). Also the parametrizations (8) – (10) deviate from the exact results by less than one part in a thousand, except for x-values close to zeros of $c_{L,a}^{(3)}(x)$.

Our results agree with the lowest six/seven even moments computed in refs. [17–19]. The same holds for the sixteenth moments for both F_2 and F_L computed in ref. [28] as another check of our calculation. We also find agreement of the leading small-x terms $x^{-1}\ln x$ of $c_{L,\rm ps}^{(3)}$ and $^{(3)}_{L,\rm g}$ with the predictions obtained in ref. [29] from the small-x resummation. Moreover a threshold resummation has been derived [30, 31] for the large-x logarithms in $C_{L,\rm ns}$. We are however not

aware of any third-order predictions derived in this framework. Note that also at this order in α_s the coefficient of the leading large-x logarithm of $c^{(3)}_{L,\rm ns}$ ($= 8C_F^3$ in our notation – recall that we expand in terms of $\alpha_s/(4\pi)$) equals that of the leading +-distribution of the corresponding coefficient function for F_2 [18, 21]. We expect that this relation hold to all orders. Together with the prediction of the soft-gluon resummation for F_2 (see, e.g., ref. [32]) this leads to the conjecture

$$c^{(n)}_{L,\rm ns}(x)\Big|_{x\to 1} = c^{(n)}_{L,\rm q}(x)\Big|_{x\to 1} = \frac{2(2C_F)^n}{(n-1)!}\ln^{2n-2}(1-x) + O(\ln^{2n-3}(1-x))\ , \tag{12}$$

where, of course, the last term has to be replaced by $O(1-x)$ for $n = 1$. Eq. (12) will, e.g., be useful for approximate reconstructions of $c^{(4)}_{L,\rm q}(x)$ from a future four-loop generalization of ref. [17].

We now illustrate the size of the contributions (3) – (10) to eqs. (1) and (2), for brevity focusing on the flavour singlet sector. The perturbative expansions of $C_{L,\rm q} = C_{L,\rm ns} + C_{L,\rm ps}$ and of $C_{L,\rm g}$ are shown in fig. 1 for four flavours and a typical value $\alpha_s = 0.2$ of the strong coupling. Especially for $C_{L,\rm g}$, both the second-order and the third-order contributions are rather large over almost the whole x-range. Most striking, however, is the behaviour of both $C_{L,\rm q}$ and $C_{L,\rm g}$ at very small values of x. Here the anomalously small ($xc^{(0)}_{L,a} \sim x^2$) one-loop parts are negligible against the (negative) constant two-loop terms, which in turn are completely overwhelmed by the (positive) new three-loop corrections $xc^{(3)}_{L,a} \sim \ln x + const$. All we know beyond the third order are the leading small-x terms derived in ref. [29]. The resulting fourth-order (N^{3}LO) corrections in turn exceed our three-loop contributions (9) and (10). Recall, however, that the leading low-x terms often substantially overestimate the complete results at accessible values of x, as shown in fig. 1 for the third-order results, and that small-x information alone is usually insufficient for reliable estimates of the convolutions connecting the coefficient functions to observables as in eq. (1), see also refs. [1,2]. Hence it is not possible to draw any conclusions about the behaviour of F_L beyond NNLO at this point.

In figs. 2 and 3, $C_{L,\rm q}$ and $C_{L,\rm g}$ are convoluted with the sufficiently realistic model distributions

$$\begin{aligned} xq_{\rm s}(x) &= 0.6x^{-0.3}(1-x)^{3.5}(1+5.0x^{0.8})\ , \\ xg(x) &= 1.6x^{-0.3}(1-x)^{4.5}(1-0.6x^{0.3})\ , \end{aligned} \tag{13}$$

corresponding to a typical scale of about 30 GeV^2. A comparison of figs. 1 and 2 clearly reveals the smoothening effect of the Mellin convolutions. In fact, under the chosen conditions, the (mostly positive) NNLO corrections to the flavour-singlet F_L amount to less than 20% for $x < 0.3$ and to less than 10% for $5\cdot 10^{-5} < x < 0.1$. Note that these numbers refer to using the same α_s and the same parton distributions (13) irrespective of the order of the expansion. In data fits we expect a decrease of α_s^3 at NNLO of up to 10% with respect to the NLO value at about 30 GeV^2 [33–35]. Also the parton distributions will be affected at this level, mostly in directions stabilizing the overall NNLO/NLO ratio [24, 34, 35]. Thus such changes further support our conclusion of a rather good perturbative stability of F_L, for the x-values accessible at HERA, at least above about 10 GeV^2.

Definite conclusions for F_L at considerably lower scales, say $Q^2 \simeq 2$ GeV^2, require more detailed studies, since the larger value of α_s and the flatter small-x shape of the parton distributions

can lead to much larger NLO and NNLO corrections. This is roughly illustrated in Fig. 4, where we have employed (again independent of the order) the low-scale model distribution of ref. [2],

$$\begin{aligned} xq_{\rm s}(x) &= 0.6\,x^{-0.1}(1-x)^3\,(1+10\,x^{0.8})\;, \\ xg(x) &= 1.2\,x^{-0.1}(1-x)^4\,(1+1.5\,x)\;, \end{aligned} \tag{14}$$

together with $\alpha_{\rm s} = 0.35$ for three flavours. Under these conditions the NNLO corrections now exceed 20% outside the range $7 \cdot 10^{-4} < x < 0.1$, rising sharply for decreasing lower values of x. Note that the low-scale gluon distribution is in fact poorly known at small x. Some of the recent analyses [9,34–37] actually prefer a considerably smaller gluon density in this region, to the extend that at NLO F_L can become almost zero, or even negative at $Q^2 \lesssim 2$ GeV^2 and very small x (see also ref. [38]). More theoretical and experimental [12, 13] efforts are required to clarify whether this behaviour and the large corrections in fig. 4 are analysis artefacts (in our case due to the unphysical order-independence of the inputs), or indeed indicate an anomalous low-order behaviour of F_L, or even signal an early (as compared to F_2) breakdown of the perturbative expansion for this quantity.

FORTRAN subroutines of our parametrized coefficient functions can be obtained from the preprint server `http://arXiv.org` by downloading the source of this article. They are also available from the authors upon request. Routines for the exact results will be released with ref. [21].

Acknowledgments

We thank T. Gehrmann for providing a weight-five extension of the FORTRAN package [25] for the harmonic polylogarithms. The work of S.M. has been supported in part by Deutsche Forschungsgemeinschaft in Sonderforschungsbereich/Transregio 9. The work of J.V. and A.V. has been part of the research program of the Dutch Foundation for Fundamental Research of Matter (FOM).

References

[1] S. Moch, J.A.M. Vermaseren and A. Vogt, Nucl. Phys. B688 (2004) 101, hep-ph/0403192

[2] A. Vogt, S. Moch and J.A.M. Vermaseren, Nucl. Phys. B691 (2004) 129, hep-ph/0404111

[3] W.L. van Neerven and E.B. Zijlstra, Phys. Lett. B272 (1991) 127

[4] E.B. Zijlstra and W.L. van Neerven, Phys. Lett. B273 (1991) 476

[5] E.B. Zijlstra and W.L. van Neerven, Phys. Lett. B297 (1992) 377

[6] E.B. Zijlstra and W.L. van Neerven, Nucl. Phys. B383 (1992) 525

[7] S. Moch and J.A.M. Vermaseren, Nucl. Phys. B573 (2000) 853, hep-ph/9912355

[8] H1, C. Adloff et al., Eur. Phys. J. C21 (2001) 33, hep-ex/0012053

[9] H1, C. Adloff et al., Eur. Phys. J. C30 (2003) 1, hep-ex/0304003

[10] E.M. Lobodzinska, hep-ph/0311180, and proceedings of DIS 2004 (to appear)

[11] Particle Data Group, S. Eidelman et al., Phys. Lett. B592 (2004) 1

[12] L.A.T. Bauerdick, A. Glazov and M. Klein, hep-ex/9609017

[13] M. Klein, Nucl. Phys. Proc. Suppl. 116 (2003) 91, and proceedings of DIS 2004 (to appear)

[14] S. Moch, J.A.M. Vermaseren and A. Vogt, Nucl. Phys. B646 (2002) 181, hep-ph/0209100

[15] A. Vogt, S. Moch and J.A.M. Vermaseren, proceedings of DIS 2004 (to appear), hep-ph/0407321

[16] S. Moch, J.A.M. Vermaseren and A. Vogt, Nucl. Phys. Proc. Suppl. 135 (2004) 137, hep-ph/0408075

[17] S.A. Larin, T. van Ritbergen and J.A.M. Vermaseren, Nucl. Phys. B427 (1994) 41

[18] S.A. Larin et al., Nucl. Phys. B492 (1997) 338, hep-ph/9605317

[19] A. Retey and J.A.M. Vermaseren, Nucl. Phys. B604 (2001) 281, hep-ph/0007294

[20] D.I. Kazakov and A.V. Kotikov, Nucl. Phys. B307 (1988) 721, Erratum ibid. B345 (1990) 299

[21] J.A.M. Vermaseren, A. Vogt and S. Moch, to appear

[22] J.S. Guillen et al., Nucl. Phys. B353 (1991) 337

[23] W.L. van Neerven and A. Vogt, Nucl. Phys. B568 (2000) 263, hep-ph/9907472

[24] W.L. van Neerven and A. Vogt, Nucl. Phys. B588 (2000) 345, hep-ph/0006154

[25] T. Gehrmann and E. Remiddi, Comput. Phys. Commun. 141 (2001) 296, hep-ph/0107173

[26] E. Remiddi and J.A.M. Vermaseren, Int. J. Mod. Phys. A15 (2000) 725, hep-ph/9905237

[27] J. Vollinga and S. Weinzierl, hep-ph/0410259

[28] J. Blümlein and J.A.M. Vermaseren, hep-ph/0411111

[29] S. Catani and F. Hautmann, Nucl. Phys. B427 (1994) 475, hep-ph/9405388

[30] R. Akhoury, M.G. Sotiropoulos and G. Sterman, Phys. Rev. Lett. 81 (1998) 3819, hep-ph/9807330

[31] R. Akhoury and M.G. Sotiropoulos, hep-ph/0304131

[32] A. Vogt, Phys. Lett. B471 (1999) 97, hep-ph/9910545

[33] W.L. van Neerven and A. Vogt, Nucl. Phys. B603 (2001) 42, hep-ph/0103123

[34] A.D. Martin, R.G. Roberts, W.J. Stirling and R. Thorne, Phys. Lett. B531 (2002) 216, hep-ph/0201127

[35] S. Alekhin, Phys. Rev. D68 (2003) 014002, hep-ph/0211096

[36] CTEQ, J. Pumplin et al, JHEP 0207 (2002) 012, hep-ph/0201195

[37] ZEUS, S. Chekanov et al., Phys. Rev. D67 (2003) 012007, hep-ex/0208023

[38] A.V. Kotikov, Phys. Lett. B338 (1994) 349, JETP Lett. 59 (1995) 1

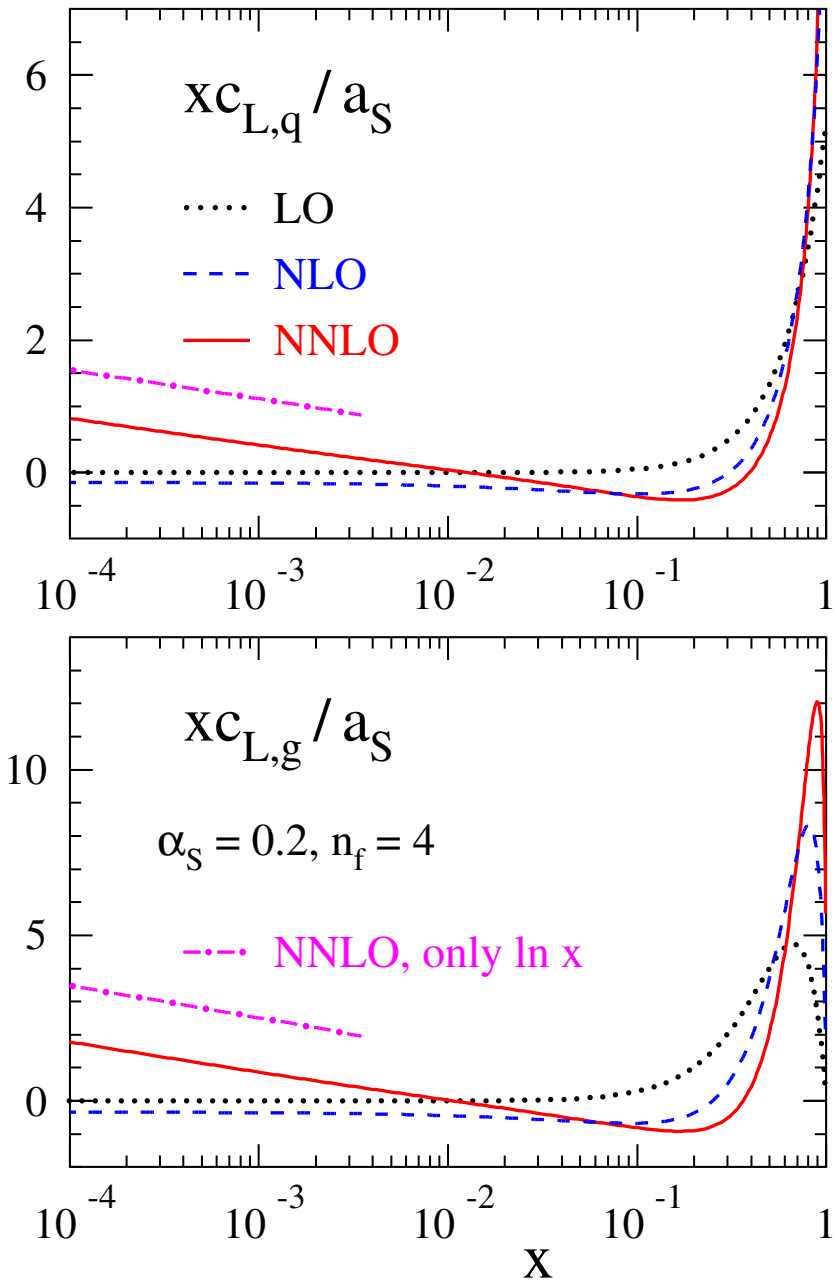

Figure 1: The perturbative expansion of the singlet-quark and gluon coefficient functions for F_L in electromagnetic DIS at a typical value of α_s. The results have been divided by $a_s = \alpha_s/(4\pi)$, i.e., the LO curves are directly given by eq. (3). Also shown are the NNLO results at small x obtained if only the leading low-x third-order terms [29], $xc^{(3)}_{L,a} \sim \ln x$, in eqs. (9) and (10) are included.

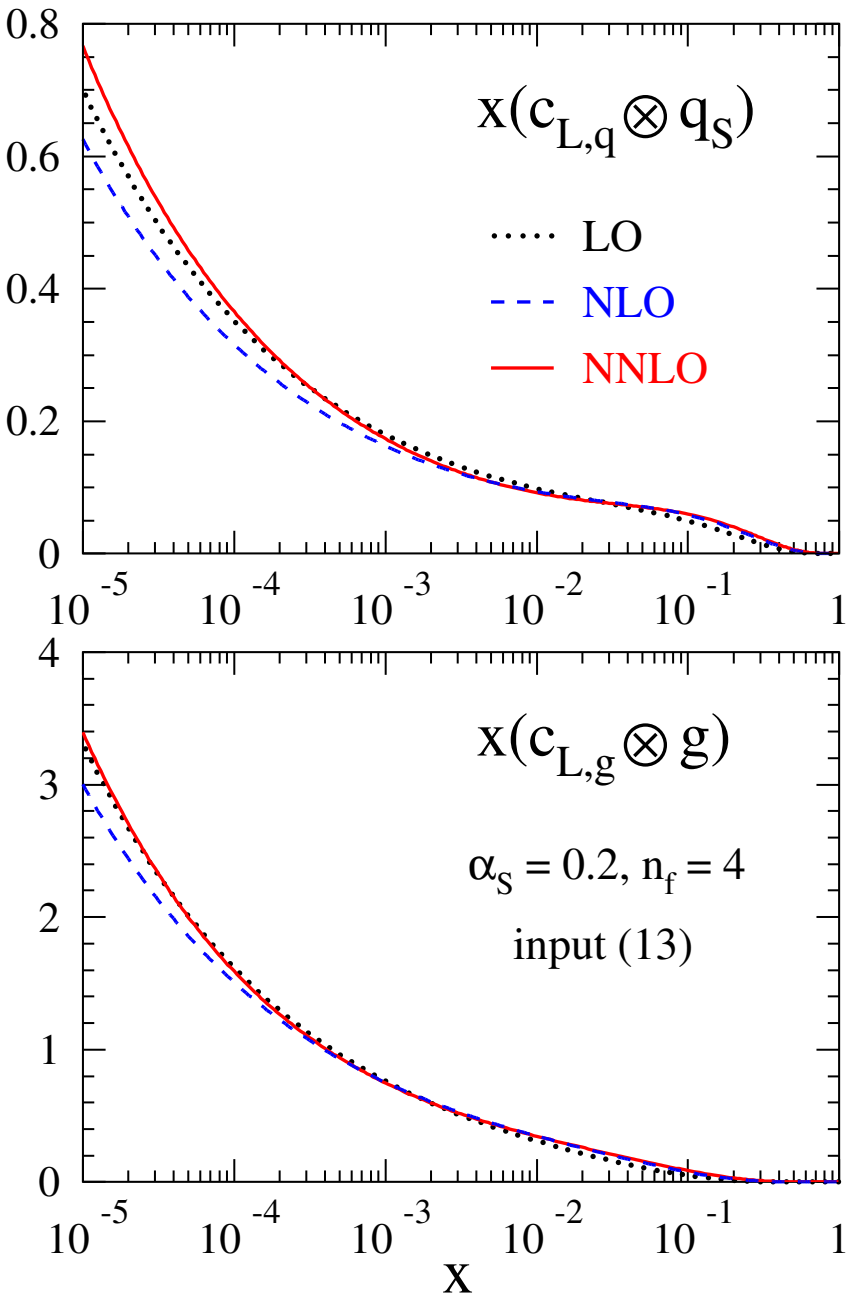

Figure 2: The perturbative expansion of the singlet-quark and gluon contributions (up to a factor $\langle e^2 \rangle$, see eq. (1)) to the longitudinal structure function F_L at $Q^2 \simeq 30\,\mathrm{GeV}^2$ for the typical (but order-independent) parton distributions (13) employed already for the illustrations in ref. [2].

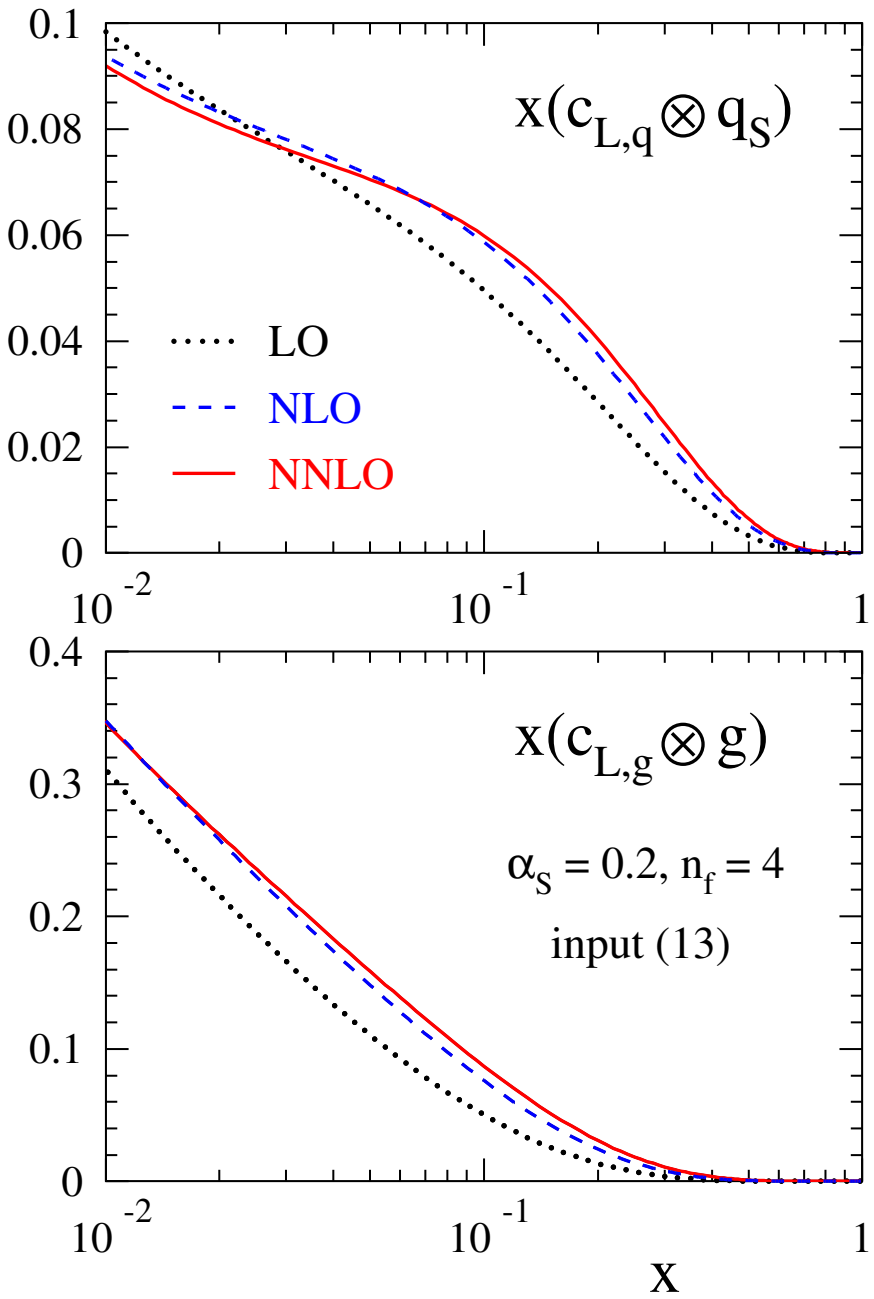

Figure 3: As figure 2, but showing only the results at large x on a correspondingly expanded scale.

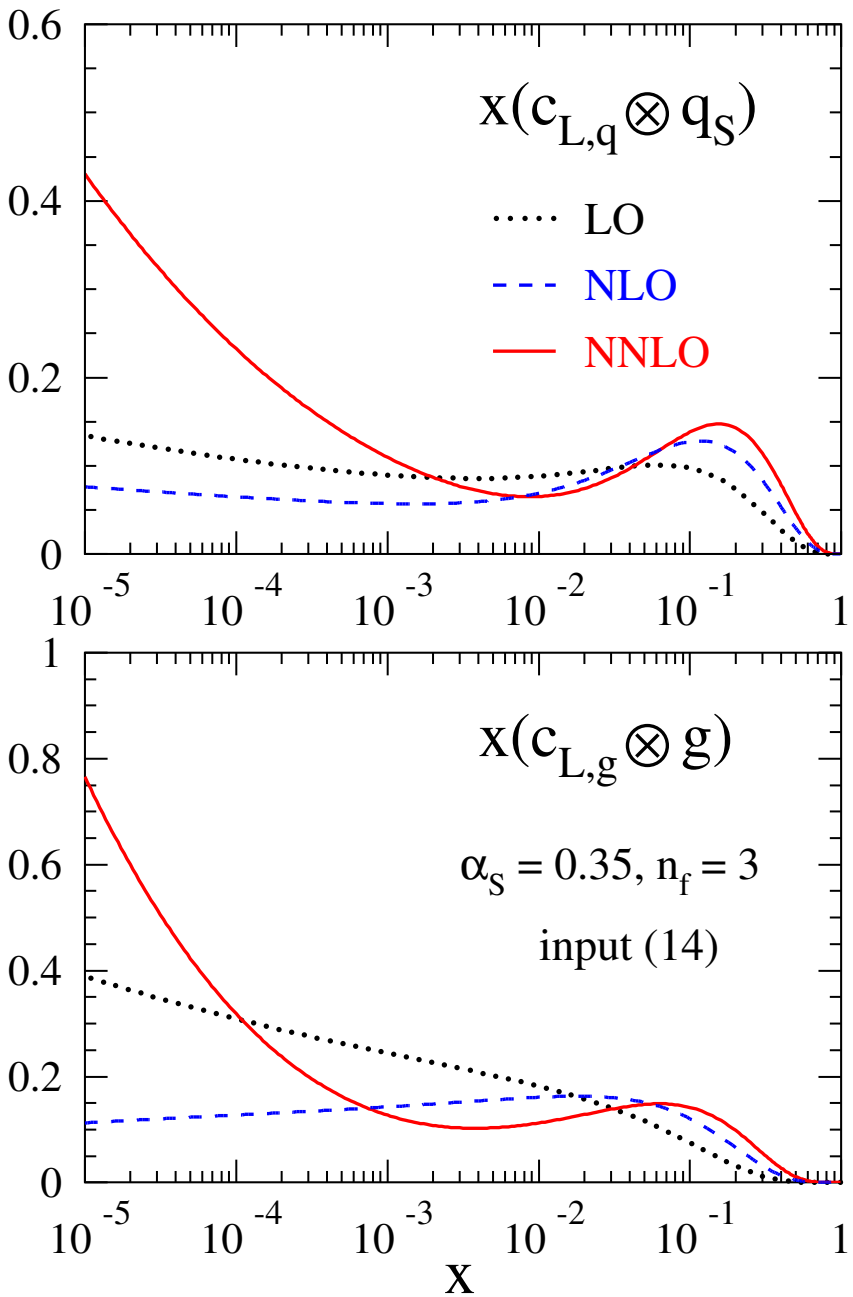

Figure 4: As figure 2, but showing the results at a low scale $Q^2 \simeq 2$ GeV^2 for accordingly modified values of α_s, n_f and the (again order-independent) model distributions (14) used already in ref. [2].