Measurement of Triple Gauge-Boson Couplings in $e^+e^-$ Collisions at LEP
Cover illustration: Sunset above lake Geneva, Montreux, Switzerland (1997)
Measurement of Triple Gauge-Boson Couplings in $e^+e^-$ Collisions at LEP

Een wetenschappelijke proeve op het gebied van de Natuurwetenschappen, Wiskunde en Informatica.

Proefschrift

ter verkrijging van de graad van doctor aan de Katholieke Universiteit Nijmegen, op gezag van de Rector Magnificus Prof. Dr. C. W. P. M. Blom, volgens besluit van het College van Decanen in het openbaar te verdedigen op dinsdag 13 januari 2004 des namiddags om 1.30 uur precies,

door

Mark Eugeen Theophila Dierckxsens

geboren op 15 januari 1976 te Mol (België)
The work described in this thesis is part of the research programme of the ‘Nationaal Instituut voor Kernfysica en Hoge-Energie Fysica (NIKHEF)’. The author was financially supported by the ‘Stichting Fundamenteel Onderzoek der Materie (FOM)’
## Contents

**Introduction** 1

1 Theoretical Framework 3
   1.1 The History of Electroweak Interactions 3
   1.2 The Standard Model of Electroweak Interactions 4
      1.2.1 Particle Fields 4
      1.2.2 Force Fields 6
      1.2.3 Boson Masses: the Higgs Mechanism 7
      1.2.4 Fermion masses: Yukawa Couplings 8
      1.2.5 Strong Interactions 8
   1.3 Triple Gauge-Boson Couplings 8
      1.3.1 General Parametrisation 8
      1.3.2 Gauge Invariant Parametrisation 10
      1.3.3 Signatures of New Physics 13
      1.3.4 Existing Limits 13
   1.4 W-pair production at LEP 14
      1.4.1 Helicity Amplitudes 15
      1.4.2 The Standard Model 16
      1.4.3 TGC Parametrisation 16
   1.5 W Decay 18
   1.6 Other Four-Fermion Processes 19
   1.7 WW Event Generators 20
      1.7.1 Electroweak Radiative Corrections 20
      1.7.2 Fragmentation and Hadronisation 23

2 W-pair Measurement 25
   2.1 The Experiment 25
      2.1.1 The LEP Collider 25
      2.1.2 The L3 Detector 26
      2.1.3 Event Reconstruction 29
      2.1.4 Data Sample 31
   2.2 WW Event Selection 32
      2.2.1 Signatures of W Decay Products 33
      2.2.2 Reconstruction of W Decay Products 34
      2.2.3 Background processes 36
## 2.3 The $qqqq$ Selection

2.3.1 Pre-selection ................................................. 40
2.3.2 Neural Network Selection ................................. 42
2.3.3 Performance of the Selection ............................. 43

## 2.4 The $qqe$ Selection .............................................. 44

2.4.1 Cut Based Selection ........................................ 44
2.4.2 Performance of the Selection ............................. 46

## 2.5 The $qq\mu$ Selection ........................................... 46

2.5.1 Cut Based Selection ........................................ 48
2.5.2 Performance of the Selection ............................. 50

## 2.6 The $qq\tau$ Selection .......................................... 50

2.6.1 Tau Identification ......................................... 50
2.6.2 Cut Based Selection ........................................ 51
2.6.3 Performances of the Selection ............................. 53

## 2.7 Cross Section Measurement .................................... 53

## 3 TGC Fit Method ..................................................... 55

3.1 The Phase Space Angles ...................................... 55
3.2 Kinematic Fit .................................................. 56
3.3 Reconstruction of the $W^-$ Production Angle ............... 57
  3.3.1 The $qq\ell\nu$ channel .................................. 57
  3.3.2 The $qqqq$ channel ..................................... 58
3.4 Reconstruction of the $W$ Decay Angles ....................... 62
3.5 Resolutions and Distributions ................................ 63
3.6 Fitting Procedure ............................................ 64
  3.6.1 Reweighting ............................................. 64
  3.6.2 Binned Maximum Likelihood Fit ......................... 69
3.7 Technical Tests .............................................. 72
  3.7.1 Bin Sizes ............................................... 72
  3.7.2 Bias and Linearity ...................................... 75
  3.7.3 Expected Errors ......................................... 77
3.8 Fit Results .................................................. 80

## 4 Systematic Checks ................................................ 87

4.1 Charge Measurement Studies ................................ 87
  4.1.1 Charge Confusion ....................................... 88
  4.1.2 Charge Confusion for $W \to \ell\nu$ ....................... 88
  4.1.3 Charge Confusion for $W \to qq$ ......................... 94
  4.1.4 Influence of the Charge Measurement .................. 96
4.2 Contribution from the Total Cross Section .................. 98
4.3 Background .................................................. 98
  4.3.1 $qq(\gamma)$ background in the $qqqq$ channel .......... 100
  4.3.2 Background Statistics .................................. 100
4.4 Energy Difference between Data and MC ..................... 102
4.5 Bin Size Dependence ........................................ 105
4.6 Comparison with Other Fit Methods ........................................... 106
4.6.1 Unbinned Phase Space Angles ............................................ 106
4.6.2 Optimal Observables .................................................. 107

5 Systematic Errors ................................................................. 109
5.1 Evaluation of Systematic Errors ........................................... 109
5.2 Signal Modelling ............................................................... 110
  5.2.1 Total Cross Section ................................................... 111
  5.2.2 Differential Cross Section ........................................ 111
5.3 Background Modelling ....................................................... 113
5.4 Initial and Final State Radiation ........................................ 116
5.5 Monte Carlo Statistics ....................................................... 116
5.6 Charge Confusion .............................................................. 119
5.7 Jet and Lepton Reconstruction ........................................... 120
5.8 W Mass and Width ............................................................. 120
5.9 Fragmentation and Hadronisation ...................................... 121
  5.9.1 Fragmentation .......................................................... 123
  5.9.2 Bose-Einstein Correlations ........................................ 123
  5.9.3 Colour Reconnection ................................................ 125
5.10 Combination of Systematic Errors ..................................... 125

6 Results and Discussion ........................................................ 129
6.1 Results ........................................................................... 129
6.2 Combination of L3 results ................................................ 132
  6.2.1 WW Results at 161, 172 and 183 GeV ......................... 132
  6.2.2 Single-W Results ..................................................... 132
  6.2.3 $Q^2$ Dependence .................................................... 133
  6.2.4 Combined Results .................................................. 133
6.3 W boson substructure ....................................................... 135
6.4 Comparison with Other Results .................................... 137

7 Conclusions and Outlook ....................................................... 139
7.1 Conclusions .................................................................. 139
7.2 Prospects for Future Experiments ................................. 140

A Performance of WW selections .............................................. 143

B Phase Space Distributions .................................................... 147

C Likelihood Distributions ....................................................... 157

Bibliography ........................................................................ 163

Summary ............................................................................ 175

Samenvatting .................................................................. 177
Dankwoord 181
Curriculum Vitae 183
Introduction

The previous century can be considered as a Golden Age for elementary particle physics. Vast achievements on both the experimental and theoretical side eventually lead to a unified theory for electromagnetic and weak interactions, the Standard Model, and to quantum chromodynamics, the theory describing the strong interactions. For the remaining fundamental force, gravity, no adequate quantum mechanical description exists at present. Although the Standard Model gives a very accurate description of all electroweak physics processes at energies currently accessible, it is commonly believed that it is just an effective low-energy approximation of a theory that unifies all four fundamental forces.

A very important aspect of the Standard Model is the requirement of local gauge invariance under $SU(2)_L \times U(1)_Y$ symmetry. This space and time dependent symmetry acts on the particle fields (fermions) and generates the interaction with the force fields (gauge-bosons). A consequence of the non-Abelian symmetry $SU(2)_L$ is the appearance of self-couplings between three and four gauge-bosons leading to extra diagrams for various physics processes. Due to cancellations between all possible contributing diagrams, a proper high energy behaviour is ensured for all processes. Such a property is one of the prerequisites for any theory to be valid at all energy scales.

Some of the most recent accomplishments in particle physics were realised by the four experiments at the LEP collider at CERN. After the first phase of the LEP physics programme, dedicated to the precision measurements of the properties of the Z boson, the centre-of-mass energy of the colliding beams was increased to run above the W-pair production threshold. The main goals of this phase were to accurately determine the mass and the width of the W boson and to search for the Higgs boson at higher masses. Another important feature was the possibility for a direct verification of the subtle cancellation between the diagrams contributing to W-pair production leading to a proper high energy behaviour. At LEP energies, the diagram containing the Higgs boson can be neglected and only three diagrams contribute significantly to the process. Two of these diagrams contain a triple gauge-boson vertex as a consequence of the non-Abelian symmetry. The measurement of the structure and strength of these couplings are the subject of this thesis.

The outline of this thesis is as follows. Chapter 1 will start with the history and the basic ingredients of the Standard Model of electroweak interactions. The part concerning the triple gauge-boson vertices will be described in more detail as well as the W-pair production and decay processes. The procedures for selecting these processes out of the events recorded with the L3 detector at LEP will be discussed in Chapter 2. From these events, a set of phase space variables describing a WW event will be derived. Their reconstruction is described in Chapter 3, together with the fit method used to extract
values for the couplings from these distributions. Systematics that might affect the phase space angles will be verified in Chapter 4, while the effects of the remaining uncertainties are turned into systematic errors in Chapter 5. The final results will be compared and combined with complementary measurements from L3 and the other LEP experiments in Chapter 6. A limit on a possible substructure of the W boson is also derived from the L3 results. Chapter 7 will give the concluding remarks about what can be learnt from the measurements presented in this thesis. Also more precise measurements of triple gauge-boson couplings attainable with future experiments will be discussed.
Chapter 1

Theoretical Framework

An overview of the theoretical background of the triple gauge-boson couplings will be given in this chapter. First a short historical overview of the establishment of what nowadays is considered as the Standard Model of electroweak interactions is given, after which the basic ingredients of this model are described. In Section 1.3, the parametrisation of the triple gauge-boson vertex is extensively described. Also the interpretation of these couplings, the effect of physics beyond the Standard Model as well as present limits on the couplings are discussed. Section 1.4 is devoted to the description of W-pair production in $e^+e^-$ collisions, with a special emphasis on the trilinear vertex. The subsequent two sections treat the decay of the W boson and the processes that contribute to the same final states as the W-pairs. Section 1.7 describes the theoretical predictions available for the processes of interest and in particular the uncertainties accompanied with this.

1.1 The History of Electroweak Interactions

The rather accidental discovery of spontaneous radioactivity in uranium decays by Becquerel [1] in 1896 started a new and exciting era in physics. One of the first — but certainly not the last — puzzling properties of the $\beta$ decay was the continuous spectrum of the emitted particles, first observed by Chadwick [2] in 1914. In 1930, Pauli [3] proposed the presence of a light, neutral and weakly interacting particle in $\beta$ decay. It was four years later that he explained the continuous $\beta$ spectrum with the introduction of this neutrino [4]. Fermi came in the same year with the first theory for $\beta$ decay, by proposing a four-point interaction between two vector currents with a strength proportional to $G_F$ [5]. During the same period, numerous experiments were conducted to get better insight in this weak interaction. For instance, following the suggestion by Lee and Yang that parity might not be conserved in weak decays [7], one of these experiments [8] observed that this was indeed the case. Another experiment measured that the neutrino has a negative helicity [9]. All these results lead to the conclusion that the interaction in weak decays should be of the type $\mathcal{V} - \mathcal{A}$, i.e. having a vector minus axial-vector current [10]. However, the cross sections in the Fermi-theory are typically proportional to the centre-of-mass energy squared, denoted as $s$. This will violate the partial wave unitarity bounds for the $s$-wave point-like interaction for energies $\sqrt{s} \approx 300$ GeV.
In 1957 Schwinger [13] and independently Lee and Yang [14] published the development of the idea of an intermediate vector boson (IVB) in weak interactions, proposed by Klein already 20 years earlier [15], in analogy to the photon exchange in quantum electrodynamics. In this model, the interaction is transmitted by a charged IVB, the W boson, which should be heavy to account for the short range of the weak interactions. In the low energy limit, this model should be equivalent to the Fermi theory and a relation between the parameters of the two models can be derived:

\[
\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2},
\]

where \( g \) is the coupling strength between the charged current and the W boson, and \( M_W \) denotes its mass. However, it should be noted that with the introduction of this W boson, the unitarity violation does not disappear, but is only delayed to centre-of-mass energies of \( \sqrt{s} \approx 1 \) TeV.

Due to the analogy between these weak charged current interactions and the electromagnetic interaction, numerous efforts were made in trying to unify the two theories. An important step forward in the development of an electroweak model was the introduction of the neutral intermediate weak boson by Glashow [16]. Using this idea together with spontaneous symmetry breaking through the Higgs mechanism [17], Weinberg [20] and Salam [21] were able to write down a Lagrangian for the electroweak interaction.

### 1.2 The Standard Model of Electroweak Interactions

This theory, also known as the Glashow-Weinberg-Salam model, is one of the major achievements in 20th century particle physics. The foundations of this theory are based on the belief that all interactions are governed by invariance of physics under local gauge symmetries, which in this case is the disjunct product \( SU(2)_L \times U(1)_Y \). The proof by 't Hooft and Veltman [22] that non-Abelian gauge theories with spontaneously broken gauge invariance are renormalizable, i.e. the calculated quantities that can be measured are finite, made the Standard Model a solid theory. A first important experimental proof of the Standard Model was the discovery of weak neutral currents in 1973 [24]. About 10 years later, the W boson [26] and Z boson [28] were observed directly, having masses in agreement with the prediction. Numerous experiments, like the precision measurements of the LEP experiments at the Z resonance peak [30], further confirmed the validity of the Standard Model.

In the next sections, the different constituents of the Standard Model Lagrangian for electroweak interactions will be explained in short. First, the Lagrangian for the matter fields containing kinematic terms and interactions with the force fields will be described. Then the part with the kinematic terms of these force fields will be explained. The next step will cover the generation of boson masses through the Higgs mechanism. The generation of fermion masses and the strong interactions will also be discussed briefly.
1.2. The Standard Model of Electroweak Interactions

<table>
<thead>
<tr>
<th>generations</th>
<th>$T^3$</th>
<th>$Y$</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>leptons</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu_e, \nu_\mu, \nu_\tau$</td>
<td>$+1/2$</td>
<td>$-1$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\nu_{e,R}, \nu_{\mu,R}, \nu_{\tau,R}$</td>
<td>$-1/2$</td>
<td>$-1$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$e_L, e_R$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\nu_{e,R}$</td>
<td>$0$</td>
<td>$-2$</td>
<td>$-1$</td>
</tr>
<tr>
<td>quarks</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$u, c, t$</td>
<td>$+1/2$</td>
<td>$+1/3$</td>
<td>$+2/3$</td>
</tr>
<tr>
<td>$d, s, b$</td>
<td>$-1/2$</td>
<td>$+1/3$</td>
<td>$-1/3$</td>
</tr>
<tr>
<td>$u_R, c_R, t_R$</td>
<td>$0$</td>
<td>$+4/3$</td>
<td>$+2/3$</td>
</tr>
<tr>
<td>$d_R, s_R, b_R$</td>
<td>$0$</td>
<td>$-2/3$</td>
<td>$-1/3$</td>
</tr>
</tbody>
</table>

Table 1.1: The fermions, leptons and quarks, in the standard model and their electroweak quantum numbers: the third component of the weak isospin, $T^3$, the weak hyper-charge $Y$ and the electromagnetic charge $Q$ given by $Q = T^3 + Y/2$. The indices $L$ and $R$ denote the left- and right-handed fermions. Due to the strong force, the quarks have an extra quantum number which can have three values referred to as colours: red, green and blue.

### 1.2.1 Particle Fields

The spectrum of fundamental particles is made up of spin-1/2 fields, fermions, where the left-handed particles are grouped in $SU(2)$ isospin doublets and the right-handed in singlets. The members can be divided into leptons and quarks, with the difference that the latter ones also undergo strong interactions (see Section 1.2.5). Furthermore, the particles can be grouped into families or generations and up to now, all the particles of three families have been detected while no other elementary particles have been observed. All these particles and their electroweak quantum numbers are listed in Table 1.1. The right-handed neutrinos were not included in the original theory, since the neutrinos were believed to be massless. However, recent results have shown compelling evidence for the existence of neutrino oscillations [31], requiring the neutrinos to have a non-vanishing mass.

Starting from the kinetic term in the Lagrangian for the free particles, the dynamics of the fermions can be obtained by replacing the partial derivatives with the covariant derivative which ensures that the equation of motion remains invariant under local gauge transformations:

$$\partial_\mu \rightarrow D_\mu = \partial_\mu - ig \frac{Y}{2} B_\mu - ig \vec{T} \cdot \vec{W}_\mu.$$  \hspace{1cm} (1.2)

The operators $Y$ and $\vec{T}$ are the generators of the groups of local gauge transformations, $U(1)_Y$ and $SU(2)_L$, respectively. The gauge-boson fields $B_\mu$ and $\vec{W}_\mu$ are the force carriers of the two interactions and will be described in the next section. After substitution, the Lagrangian for the fermions becomes:

$$\mathcal{L}_{\text{fermions}} = \bar{\Psi}_L i\gamma^\mu \left( \partial_\mu - ig' \frac{Y}{2} B_\mu - ig \vec{T} \cdot \vec{W}_\mu \right) \Psi_L$$

$$+ \bar{\Psi}_R i\gamma^\mu \left( \partial_\mu - ig' \frac{Y}{2} B_\mu \right) \Psi_R,$$  \hspace{1cm} (1.3)
where the right handed components do not couple to the vector fields $\bar{W}_\mu$ since these are $SU(2)_L$ singlets.

### 1.2.2 Force Fields

The kinematic terms in the Lagrangian for the gauge-boson fields, introduced by minimal coupling to the fermions, can be written as

$$L_{\text{bosons}} = -\frac{1}{4} B^{\mu\nu} B_{\mu\nu} - \frac{1}{4} \bar{W}^{\mu\nu} \cdot \bar{W}_{\mu\nu}, \quad (1.4)$$

where the field strength tensors are given by:

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \quad (1.5a)$$

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g e^{ijk} W^j_\mu W^k_\nu. \quad (1.5b)$$

The last term of Eq. (1.5b) is a direct consequence of the non-Abelian structure of the $SU(2)_L$ symmetry. This will give rise to triple and quadruple couplings among the gauge-bosons involving at most two identical bosons. The measurement of the former ones are the subject of this thesis and will be described in detail in Section 1.3. The presence of these types of interactions will also lead to a good high-energy behaviour of the process $e^+ e^- \rightarrow W^+ W^-$, as will be shown in Section 1.4.

The $\bar{W}_\mu$ and $B_\mu$ fields do not correspond to the physical boson fields. The first two components of $\bar{W}_\mu$ are rearranged into two oppositely charged bosons to explain the charged currents observed in Nature:

$$W^\pm_\mu = \frac{1}{\sqrt{2}} (W^1_\mu \mp W^2_\mu) \quad (1.6)$$

The two remaining bosons have identical quantum numbers and are therefore not prevented from mixing with each other. The resulting bosons can then be identified with the photon field $A_\mu$ and $Z$ field $Z_\mu$. This relation between the neutral gauge-bosons is given as:

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \sin \theta_w & \cos \theta_w \\ \cos \theta_w & -\sin \theta_w \end{pmatrix} \cdot \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} \quad (1.7)$$

where $\theta_w$ is the mixing angle referred to as the Weinberg angle with the following relation between the coupling constants:

$$\frac{g'}{g} = \tan \theta_w. \quad (1.8)$$

Up to this point, the electroweak theory predicts the existence of massless fermions and gauge-bosons. However, it is well known that all the fermions and most of the gauge-bosons are massive. Simply adding bilinear mass terms to the Lagrangian introduces irremovable terms that are not invariant under local $SU(2)_L \times U(1)_Y$ gauge transformations.
1.2.3 Boson Masses: the Higgs Mechanism

The solution to the mass problem lies in the introduction of an extra scalar weak isospin doublet $\Phi$, called the Higgs field. The Lagrangian for this self-interacting scalar field can be written as:

$$ \mathcal{L}_{\text{Higgs}} = \partial_\mu \Phi^\dagger \partial^\mu \Phi - \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2, \quad (1.9) $$

where the first term is the kinetic part and the two other terms form the potential. The gauge invariance can again be obtained by introducing the covariant derivative (1.2).

In the case of a negative value for $\mu^2$, the potential has a whole family of non-trivial minima. Choosing a specific minimum $v^2 = -\mu^2/\lambda$ and expanding around the minimum will lead to a state that breaks the symmetry. This Higgs field can be parametrised as

$$ \Phi = \frac{v + H}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad (1.10) $$

where only one out of the four degrees of freedom remain.

After introducing the covariant derivative and replacing the vector fields with the physical fields in the Lagrangian (1.9), terms quadratic in the force fields appear:

$$ \frac{g^2 v^2}{4} W^\mu W^{-\mu} + \frac{g^2 v^2}{8 \cos^2 \theta_w} Z_\mu Z^\mu + 0 A_\mu A^\mu $$

These terms can be interpreted as mass terms and thus

$$ M_W = \cos \theta_w M_Z = \frac{g v}{2}, \quad M_\gamma = 0. \quad (1.11) $$

Now it is also clear what has happened to the three degrees of freedom of the Higgs field that disappeared. They form the longitudinal degrees of freedom for the three massive vector bosons. The fact that the photon remains massless is not such a surprise. The gauge transformation to obtain the specific form for the scalar field in Eq. (1.10) is chosen such that the electromagnetic symmetry, $U(1)_{em}$, remains a symmetry of the vacuum (unitary gauge). The meaning of the Weinberg angle rotation of the $W^3_\mu$ and $B_\mu$ fields can also be better understood. This matrix diagonalises the mass matrix for the neutral gauge-bosons.

The remaining degree of freedom results in a neutral scalar particle, the so-called Higgs boson $H$. From the Lagrangian, the mass is deduced to be $\sqrt{-2\mu^2}$, but the Standard Model does not give any prediction for this value since $\mu^2$ is a parameter of the model and thus a priori unknown. Up to this date the Higgs boson remains the only particle in the Standard Model that is not yet observed. During the last year of running of the LEP2 programme, some excess of events having signatures compatible with the Higgs boson has been seen around 115 GeV [34]. However, the signal was not significant and experiments at Tevatron or LHC are expected to find the Higgs, if it exists.
1.2.4 Fermion masses: Yukawa Couplings

So far, the fermions are still considered to be massless, since just adding a simple term proportional to the square of the fermion fields would violate $SU(2)_L \times U(1)_Y$ symmetry. Also in this case, the scalar Higgs field comes as a saviour to introduce the fermion masses in a gauge invariant way. The left-handed fermion doublets and right-handed singlets are given Yukawa couplings with the Higgs field. This results in an extra term in the Standard Model Lagrangian, for each of the generations, of the form:

$$L_{\text{Yukawa}} = -\frac{v + H}{\sqrt{2}} (g_e \bar{e} e + g_u \bar{u} u + g_d \bar{d} d). \quad (1.12)$$

The mass term for each fermion can be identified as $M_f = v g_f / \sqrt{2}$, where the value of $g_f$ is arbitrarily. Note that also terms appear that predict couplings of the Higgs boson with fermions proportional to their masses. These couplings are predicted very accurately since the fermion masses are known to high precision and will therefore be a very important measurement for future experiments.

1.2.5 Strong Interactions

The electroweak theory can easily be extended to include also the strong interactions, described by Quantum Chromodynamics (QCD), by enlarging the symmetry group to $SU(3)_c \times SU(2)_L \times U(1)_Y$. Practically this means that 8 terms are added to the covariant derivative in (1.2) and to the boson terms, corresponding to the eight force carriers, called gluons. These bosons will remain massless since the Higgs field is a singlet under $SU(3)_c$ transformations. The only particles that interact with these gluons are the quarks and gluons themselves since they are triplets, respectively octets, under $SU(3)_c$. They usually referred to as being coloured (red, green or blue). All the other particles and bosons in the Standard model are singlets, or colourless, and will not undergo strong interactions. The strong interactions are not directly related to the physics topic of this thesis, but give important contributions through higher order radiative corrections to the processes studied.

1.3 Triple Gauge-Boson Couplings

As was already pointed out in Section 1.2.2, a direct consequence of the non-Abelian structure of the $SU(2)_L$ symmetry group is the appearance of couplings between three bosons. A closer look at the Lagrangian (1.4) reveals that only couplings of the form $\gamma W^+ W^-$ and $Z W^+ W^-$ are present.

1.3.1 General Parametrisation

The measurement of these triple gauge-boson couplings can be made quantitative by introducing arbitrary couplings for the different terms in the Lagrangian containing three boson fields. This expression can be further expanded by adding more general terms.
subject only to Lorentz invariance. This results in the following effective Lagrangian containing seven independent couplings for both the $Z$ boson and photon:

$$
i\mathcal{L}^{\text{eff}}_{\gamma WW}/g_{\gamma WW} = g_1^V V^\mu (W^-_\mu W^+ - W^+_\mu W^-)$$
$$+ \kappa_V W^+ W^- V^{\mu\nu} + \frac{\lambda_V}{m_W^2} V^{\mu\nu\rho\sigma} W^+ W^-$$
$$+ i g_5^V \varepsilon_{\mu\rho\sigma} ((\partial^\rho W^-) W^{+\nu} - W^{+\mu}(\partial^\rho W^+) V^\sigma$$
$$+ i g_4^V W^+ W^- (\partial^\mu V^\nu + \partial^\nu V^\mu)$$
$$- \frac{\bar{\kappa}_V}{2} W^-_\mu W^+_\nu \varepsilon^{\mu\rho\sigma\nu} V_{\rho\sigma} - \frac{\bar{\lambda}_V}{2m_W^2} W^-_\mu W^{+\mu} \varepsilon^{\mu\rho\sigma\nu} V_{\nu\beta}.$$  

(1.13)

In this expression, the physical fields are used and $V$ stands for either the photon field $A$ or the $Z$ field as defined in Eq. (1.7). All the field strength tensors are given by $F_{\mu\nu} = \partial_\mu F_\nu - \partial_\nu F_\mu$. The parameters appearing in front of each term in this Lagrangian control the deviation from the Standard Model case where $g_{\gamma WW} = e$ and $g_{ZWW} = e \cot \theta_w$. These factors are referred to as the triple gauge-boson couplings (TGC’s) and only four out of the 14 have a non-zero value in the Standard Model at tree level, namely $g_1^V = \kappa_V = 1$. For practical purposes, the deviation of these parameters from their Standard Model value is sometimes used and denoted by $\Delta g_1^V$ and $\Delta \kappa_V$. The terms in the Lagrangian that contain these two couplings, along with $\lambda_V$, are invariant under parity operation ($P$) and charge conjugation ($C$). On the contrary, the term containing $g_5^V$ violates both symmetries, and hence conserves $CP$. The other terms, with $g_4^V$, $\bar{\kappa}_V$ and $\bar{\lambda}_V$ are $CP$ violating.

From a comparison between the $\gamma WW$ Lagrangian and a multi-pole expansion of the $\gamma W$ Compton scattering process, a number of electromagnetic properties of the $W$ boson can be derived. First of all, the charge of the $W$ boson is given by $q_W = \pm e g_1^V$. If the Lagrangian (1.13) is required to be invariant under local electromagnetic gauge transformations, the charge will be fixed and hence $g_1^V = 1$. This invariance results also in the extra condition that $g_3^V = 0$. This assumption will be maintained throughout the rest of this thesis. The other properties that can be derived from this expansion are the magnetic dipole and electric quadrupole moments:

$$\mu_W = \frac{e}{2M_W}(1 + \kappa_\gamma + \lambda_\gamma),$$  

(1.14a)

$$Q_W = -\frac{e}{M_W^2}(\kappa_\gamma - \lambda_\gamma).$$  

(1.14b)

At this point, in total 12 couplings are present, 5 for the $\gamma WW$ vertex and 7 of the $ZWW$ type. This number is too large to be measured simultaneously with the present data and therefore extra constraints are desirable. Most theories describing physics beyond the standard model – some of them are described in the next section – affect mainly the $CP$-conserving couplings. The measurement will be restricted to these 6 couplings. A further
reduction of the remaining parameters can be obtained by requiring invariance under the custodial $SU(2)$ symmetry\(^1\). This results in the following relations between the weak and electromagnetic couplings [38]:

$$
\kappa_Z = g_1^2 - \tan^2 \theta_w (\kappa_\gamma - 1), \quad \lambda_Z = \lambda_\gamma.
$$

In general the couplings are complex numbers, for which the imaginary part is the absorptive part of the vertex function. These parts appear when the particles in loop corrections are produced on their mass shell and are proportional to the coupling constant of the theory. In a theory with a strongly interacting $W$ boson, these contributions can become large and would modify the complete $W$-pair production amplitude. This would also lead to large deviations in other electroweak tests, which have not been observed so far. Therefore, it is assumed that a weakly coupled theory is realised in Nature, and the imaginary part of the couplings can be neglected.

### 1.3.2 Gauge Invariant Parametrisation

The Lagrangian (1.13) is purely phenomenological and an interpretation of the couplings in terms of a new theory can be obtained by writing the low energy effective Lagrangian for the theory considered. The existence of anomalous couplings turns the Lagrangian into an expression which is not gauge invariant. In order to restore the gauge invariance under $SU(2)_L \times U(1)_Y$, additional bosons [39], would-be Goldstone bosons\(^2\) and the Higgs field are required [40, 41, 43]. When more couplings need to be reproduced, operators of higher dimension than the dimension four or six in Eq. (1.13) need to be considered. They can be generated at the new physics mass scale $\Lambda_{NP}$ with a strength that is generally suppressed by a factor $(\sqrt{s}/\Lambda_{NP})^{(d-4)}$ [44]. To be able to reproduce all the introduced couplings, operators up to dimension 12 have to be considered. The introduced scale is also interpreted as a cut-off, since the existence of anomalous couplings leads to a bad high-energy behaviour. The low energy approximation of the theory, i.e. $\sqrt{s} \ll \Lambda_{NP}$ and thus neglecting operators of higher dimension, then leads to relations among the various couplings.

To construct the effective Lagrangian, the general structure of the new physics theory participating at low energies has to be identified. This can essentially be done by specifying the particle contents and the symmetry. The $SU(2)_L \times U(1)_Y$ symmetry can be realised in two different ways depending on the particle content. If the Higgs boson is light, the new physics will be described in terms of a direct extension of the Standard Model formalism, called a linear realisation. On the other hand, when the Higgs boson is absent – or very heavy for that matter – the effective Lagrangian will be derived using a non-linear realisation.

---

\(^1\) In the limit of a decoupling hyper-weak $B_\mu$ field ($g' \to 0$) and in the absence of mass degeneracy within left- and right-handed doublets, the Standard Model Lagrangian is invariant under a global $SU(2)$ symmetry. This is called custodial symmetry and insures that the so-called $\rho$ parameter, which measures the relative strength of the neutral and charged currents, is equal to one at tree level.

\(^2\) A would-be Goldstone boson is a spin-0 particle that would contribute as the third degree of freedom of a gauge-boson when the symmetry is spontaneously broken.
Linear Realisation

In the first case considered, the Higgs doublet field $\Phi$, as defined in Section (1.2.3), is included in the low-energy particle content of the theory. In addition to this field, the effective Lagrangian contains its covariant derivatives, $D_\mu \Phi$, and the field strength tensors $B_{\mu\nu}$ and $W_{\mu\nu}$ of the $U(1)_Y$ and $SU(2)_L$ gauge fields as defined in Eqs. (1.5a) and (1.5b). With these ingredients, the lowest order three-particle operators that can be generated are of dimension six.

If the new theory is restricted to conserve local $SU(2)_L \times U(1)_Y$ gauge invariance and only $CP$-conserving interactions of dimension six are considered, 11 independent operators can be constructed. Four of them generate anomalous Higgs couplings which are not relevant for the present study. Another set of four operators affect the gauge-boson two-point functions and the coefficients of these operators are severely constrained by low energy data [45, 40, 41]. The remaining three operators give rise to anomalous TGC values and the corresponding effective Lagrangian has the form:

$$L = \frac{1}{\Lambda_{NP}^2} (f_{B\Phi} O_{B\Phi} + f_{W\Phi} O_{W\Phi} + f_{WWW} O_{WWW}),$$

(1.16)

where the operators $O$ are explicitly given by

$$O_{B\Phi} = (D_\mu \Phi)^\dagger \hat{B}^\mu\nu (D_\nu \Phi),$$

(1.17a)

$$O_{W\Phi} = (D_\mu \Phi)^\dagger \hat{W}^\mu\nu (D_\nu \Phi),$$

(1.17b)

$$O_{WWW} = \text{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\nu\lambda}\hat{W}^{\mu\lambda}].$$

(1.17c)

The field strength tensors are redefined as

$$\hat{B}_{\mu\nu} = \frac{g}{2} B_{\mu\nu}, \quad \hat{W}_{\mu\nu} = \frac{g}{2} \tau^a W^a_{\mu\nu},$$

(1.18)

where the Pauli matrices $\tau_i$ are a matrix representation of the generators of the $SU(2)_L$ group, and $B_{\mu\nu}$ and $W^a_{\mu\nu}$ are the field strength tensors from Eqs. (1.5).

When the symmetry is broken by choosing the minimum of the Higgs potential as in Eq. (1.10), the anomalous couplings can be identified with the parametrisation introduced in (1.13):

$$\Delta \kappa_\gamma = \frac{(f_{B\Phi} + f_{W\Phi}) M_W^2}{2 \Lambda_{NP}^2},$$

(1.19a)

$$\Delta \kappa_Z = \frac{(f_{W\Phi} - \sin^2 \theta_w (f_{B\Phi} + f_{W\Phi})) M_Z^2}{2 \Lambda_{NP}^2},$$

(1.19b)

$$\Delta g_1^Z = f_{W\Phi} \frac{M_Z^2}{2 \Lambda_{NP}^2} = \Delta \kappa_Z + \tan^2 \theta_w \Delta \kappa_\gamma,$$

(1.19c)

$$\lambda_\gamma = \frac{3 M_W^2 g_2^2}{2 \Lambda_{NP}^2} f_{WWW}.$$  

(1.19d)

The anomalous couplings are suppressed by a factor of $(M_W/\Lambda_{NP})^2$ and will vanish for $\Lambda_{NP} \to \infty$, i.e. the Standard Model. The above assumptions introduce relations among the couplings that reduce the number of independent $CP$-conserving couplings to three.
However, there is no reason to believe that dimension eight operators are suppressed when the scale of new physics is rather low. When the full theory is a gauge theory, the dimension six operators that affect the triple gauge-boson vertex can not be generated at the tree level \[46\]. Since they can only be generated at the loop level, the size of the couplings is suppressed by an extra factor of \(16\pi^2\). However, some of the relevant dimension eight operators may be produced at tree level and are suppressed by a factor of \((M_W/\Lambda_{NP})^4\). These operators will thus be more important than the dimension six operators up to scales of \(\Lambda \approx 2\) TeV. Since the relations (1.19c) and (1.19d) were obtained by truncating the Lagrangian at the level of dimension six, they will not be valid anymore when higher dimension operators are considered.

**Non-linear Realisation**

When the Higgs does not exist or is very heavy, the interaction terms have to be rendered gauge invariant in a non-linear way. The Lagrangian is built in a manner similar to the linear case, but instead of the scalar doublet \(\Phi\), the matrix \(U \equiv \exp(i\vec{\omega} \cdot \vec{\tau}/v^2)\) is used. The \(\omega_i\) are the would-be Goldstone bosons that generate the mass of the \(W\) and \(Z\) bosons through the Higgs mechanism. Introducing this \(U\) field generates operators of dimension six and eight. To determine the effective dimension of the terms in Eq. (1.13), the so-called “naive dimensional analysis” is applied \[47\]. The coupling constants \(\Delta g_1^Z\) and \(\Delta \kappa_V\) are, just like in the linear realisation, of the order \((M_W/\Lambda_{NP})^2\) and thus the corresponding terms of dimension six. The terms of the type \(\Lambda_V\) on the other side are effectively of dimension eight since these couplings are expected to be of the order \((M_W/\Lambda_{NP})^4\). These couplings will be substantially smaller than \(\Delta g_1^Z\), \(\Delta \kappa_Z\) and \(\Delta \kappa_\gamma\), if the scale of new physics is large enough.

**Form Factor Behaviour**

The parameters that were introduced in Eq. (1.13) were treated so far as constants. However, terms containing higher order derivatives in the boson fields can enter the Lagrangian, but proportional to \(\Lambda_{NP}^n\), where \(n\) is the number of extra derivatives. When the centre-of-mass energy approaches the scale of new physics, these terms will become equally important as the lowest order terms and a description of the physics by an effective Lagrangian will not be adequate anymore.

These higher-order terms can be absorbed in the definition of the couplings by assuming a dependence on the momentum \(Q^2\). This is commonly done by introducing a form factor of the kind

\[
\alpha(Q^2) = \frac{\alpha(0)}{(1 + Q^2/\Lambda^2_F)^n},
\]

where both the form factor scale \(\Lambda_F\) and the exponent \(n\) depend on the new physics introduced. For the lowest order couplings, the dipole form factor, with \(n = 2\), is used.

The analysis of this thesis will use the data provided by LEP, where the momentum transfer is given by the centre-of-mass energy, \(Q^2 = s\). Under the assumption that the new physics scale is large enough, the \(Q^2\) dependence between the different energies at LEP
can be neglected. The couplings will therefore be treated as constants in the following. However, the $Q^2$ dependence will be checked explicitly in Section 6.2.3.

### 1.3.3 Signatures of New Physics

Numerous models that modify the triple gauge-boson vertex can be found in the literature. The different manners in which they affect the processes containing these vertices can basically be grouped into two categories. Either the yet unknown fermions and/or bosons are created on-shell and contribute at tree level to the same final states as the processes containing the triple gauge-boson vertex, or the particles modify the properties of the SM bosons through radiative loop corrections. The Standard Model itself gives rise to deviations from the tree level TGC values through the latter effect which are of the order $10^{-3}$ [48]. The largest deviations from models describing possible new physics are of the same order, but often for specific values of certain parameters. These models are: supersymmetry [50], models containing two Higgs doublets [52], fourth generation lepton family with Majorana neutrinos [54], $E(6)$ vector leptons [55], technicolor hadrons [56], an additional neutral gauge-boson [57] and a composite W boson [59]. However, it should be pointed out that all these expected deviations are much smaller than the expected LEP2 sensitivity of the order $10^{-1}$–$10^{-2}$ [38].

### 1.3.4 Existing Limits

Indirect constraints can be obtained by investigating the effect of radiative corrections on low-energy processes containing loops with VWW vertices. When the centre-of-mass energy is high enough to produce interactions containing these vertices, it is possible to put direct limits on the couplings.

**Indirect Limits**

The most stringent indirect constraints on TGC’s are obtained from the electroweak precision measurements at the Z-resonance [62]. These limits have been derived under the assumption of a linearly realised symmetry and neglecting operators with higher dimension than six, thus assuming the relations (1.19). The new physics scale has been set to $\Lambda_{NP} = 1$ TeV and the Higgs mass to $M_H = 300$ GeV/c$^2$. By fixing all but one coupling at a time, the resulting limits on the affected couplings are $|\Delta g_1^Z| \simeq |\Delta \alpha | \lesssim 0.02$. When both couplings are allowed to vary at the same time, the limit increases to 0.03.

When this sort of analysis is restricted to $Z \to b\bar{b}$ [63], the following limits can also be derived: $|\lambda_{\gamma}| = |\lambda_Z| \lesssim 0.8$ and $|g_5^Z| \lesssim 0.2$. The same value for the new physics scale is used and the result does not depend on the Higgs mass. A much tighter allowed range for the $\lambda$ couplings can be obtained from atomic parity violation measurements. A bound of $|\lambda_{\gamma Z}| \lesssim 0.02$ is estimated [64] when the current error of 0.4 on the measurement of the weak charge of Cesium atoms is used [65].

Much weaker bounds can be obtained from numerous other experiments, like the measurement of $(g-2)_\mu$ [66], inclusive decays $b \to s\gamma$ [67], the decay $B \to K^{(*)}\mu^+\mu^-$ [68] and many others.
1.4 W-pair production at LEP

The indirect limits are below the expected LEP2 sensitivity, but it is important to realise that these constraints rely heavily on the SM and are obtained under very specific assumptions of the new physics (like linear realisation, neglecting higher order operators, scale of new physics, ...). These types of measurements do not in fact establish whether the triple gauge-boson vertices are realised in Nature. To obtain a proof of their existence, direct measurements are required.

Direct Limits

The first direct measurements of the couplings were performed using the single-photon process $e^+e^- \to \nu\bar{\nu}\gamma$ produced in $e^+e^-$ collisions at PEP and PETRA [69]. Only a weak bound of $-73.5 < \Delta\kappa_\gamma < 37$ at 90% C.L. could be derived. The UA2 experiment was the first to measure $\lambda_\gamma$ and improved the limits on $\kappa_\gamma$ by an order of magnitude [70].

The current best direct limits besides the LEP measurements come from $p\bar{p}$ interactions at Tevatron. The DØ collaboration performed a combined fit [71] to the photon $p_T$ distribution in $W\gamma$ events [72], the $p_T^{\ell\nu}$ spectrum in the $WW/WZ \to e\nu jj$ data [73] and lepton $p_T$ distribution from $WW \to \ell\nu\ell'\nu'$ data [74]. Since a wide range of $Q^2$ values is covered at hadron colliders, an explicit form factor behaviour like Eq. (1.20) needs to be taken into account. The DØ limits are derived using a dipole form factor and the scale is set at $\Lambda_F = 2$ TeV. In this kind of analyses, the assumption is usually that the $\gamma WW$ and $ZWW$ couplings are equal ($g_1^\gamma g_1^Z = 1$, $\Delta\kappa_\gamma = \Delta\kappa_Z$ and $\lambda_\gamma = \lambda_Z$). However, the analysis is also performed under the assumption of a linear realisation and neglecting higher dimensions, thus the relations (1.19). The resulting combined limits at 95% C.L., varying one coupling at a time, are:

\begin{align*}
-0.29 < \Delta g_1^Z < 0.57 \\
-0.22 < \Delta\kappa_\gamma < 0.44 \\
-0.20 < \lambda_\gamma < 0.20
\end{align*} \tag{1.21}

Limits on these couplings were also obtained by CDF [75], but are much weaker than the ones quoted above.

Limits on the TGC’s can also be derived from the single-W process in ep colliders, like HERA. The ZEUS experiment has analysed the process $e^+p \to e^+W^\pm X$ at a centre-of-mass energy of $\sqrt{s} = 300$ GeV and finds at 95% C.L. [78]:

\begin{align*}
-4.7 < \Delta\kappa_\gamma < 1.5 \\
-3.2 < \lambda_\gamma < 3.2
\end{align*} \tag{1.22}

Although these limits are much weaker, they give a completely independent measurement of these couplings.

1.4 W-pair production at LEP

An ideal process to study the trilinear vertex is W-pair production at an $e^+e^-$ collider, since these couplings appear in the $s$-channel $\gamma$ and $Z$ exchange. A third diagram exists, where a neutrino is exchanged through the $t$-channel. These three diagrams which
1.4. W-pair production at LEP

Figure 1.1: The three Feynman diagrams, referred to as CC03, which contribute at tree level to the process \( e^+e^- \rightarrow W^+W^- \). Two diagrams contain a triple gauge-boson vertex of the type VWW, indicated by the shaded circles.

contribute at tree level to the process \( e^+e^- \rightarrow W^+W^- \), are referred to as CC03 diagrams. They are shown in Fig 1.1. The matrix element for W-pair production at tree level is the sum of the matrix elements for these three diagrams separately. Actually, a fourth diagram exists at tree level in the SM where a Higgs boson is exchanged through the s-channel, but its amplitude is proportional to the electron mass and can thus be neglected. However, at very high energies this diagram needs to be taken into account to ensure a proper behaviour of the cross section.

1.4.1 Helicity Amplitudes

To study the effect of anomalous couplings on the W-pair production process, it is instructive to express the matrix elements in terms of the helicity states of the two W bosons, \( \mathcal{M}(\sigma, \lambda, \lambda') \). The helicities of the W\(^-\) and W\(^+\) are given by \( \lambda \) and \( \lambda' \), incoming \( e^- \) and \( e^+ \) helicities are \( \sigma/2 \) and \( -\sigma/2 \), with the assumption that the electrons are massless.

It is convenient to define reduced matrix elements by extracting some common factors:

\[
\mathcal{M}(\sigma, \lambda, \lambda'; \Theta) = \sqrt{2} e^2 \epsilon \tilde{\mathcal{M}}_{\sigma,\lambda,\lambda'}(\Theta) d^{J_0}_{\sigma,\Delta\lambda}(\Theta).
\]

The angle \( \Theta \) is the production angle of the W\(^-\) with respect to the incoming e\(^-\). The leading angular dependence is given in terms of the \( d \)-functions \( d^{J_0}_{\sigma,\Delta\lambda} \), where \( J_0 = \max(|\sigma|, |\Delta\lambda|) \) gives the lowest angular momentum contributing to a given helicity combination. Two out of the nine possible helicity combinations give \( J_0 = 2 \), with both W’s oppositely, transversely polarised \((\pm, \mp)\) thus \( |\Delta\lambda| = 2 \). The other seven possible helicity configurations all have \( J_0 = 1 \). The explicit form of the \( d \) functions for all possible helicity combinations is given in the last column of Table 1.2.

The reduced matrix elements are not partial wave amplitudes since they can still have a \( \Theta \) dependence due to partial waves with \( J > J_0 \). The two s-channel diagrams only contribute to the seven helicity contributions that have \( J_0 = 1 \), since angular momentum conservation in the decay of a spin-1 particle dictates that \( J = 1 \). The t-channel diagram on the other hand, can form all nine possible helicity combinations and contributions from
1.4. W-pair production at LEP

Theoretical Framework

Partial waves with $J \geq J_0$ are allowed. The reduced amplitudes can be expressed as [36]:

\[ \tilde{\mathcal{M}}_{\gamma} = -\beta A_{\lambda}^\gamma, \]  
\[ \tilde{\mathcal{M}}_Z = \beta \left(1 - \frac{1}{2\sin^2 \theta_w} \delta_{\sigma,-1}\right) \frac{s}{s - M_Z^2} A_{\lambda}^Z, \]  
\[ \tilde{\mathcal{M}}_\nu = \frac{1}{2\sin^2 \theta_w \beta} \delta_{\sigma,-1} \left(B_{\lambda} - \frac{1}{1 + \beta^2 - 2\beta \cos \Theta} C_{\lambda} \right). \]

The $W^\pm$ velocity is given by $\beta = \sqrt{1 - 4M_W^2/s}$, with $\sqrt{s}$ the centre-of-mass energy. The coefficients $A_{\lambda}^\gamma$ give the contributions from the $s$-channel containing the $VWW$ vertex, while $B_{\lambda}$ and $C_{\lambda}$ result from the $ffW$ couplings in the $t$-channel neutrino exchange diagram.

1.4.2 The Standard Model

The subtle cancellation of the different diagrams appearing in the $e^+e^-\rightarrow W^+W^-$ process is one of the interesting properties of the Standard Model. The expression of the helicity amplitude coefficients in the Standard Model are given in the top part of Table 1.2. A first thing to note is that for both the $\gamma$ and $Z$ exchange diagrams, the coefficients are equal, $A_{\lambda}^\gamma = A_{\lambda}^Z$. In the high energy limit, they will also become equal to $B_{\lambda}$. Some of the coefficients $A$ and $B$ grow with increasing energy since they are proportional to the Lorentz factor $\gamma$. This is not a problem however, since for sufficiently high energies, $\sqrt{s} \gg M_Z$, the $Z$ propagator in (1.24b) will disappear and the terms in this equation will be cancelled by (1.24a) and the first term in (1.24c). Thus at high energies, the $(0,0)$ configuration will get a constant contribution from the “weak” part of the $Z$ exchange. However, all helicity configurations will have non-vanishing contributions from the coefficients $C_{\lambda}$ in the neutrino exchange amplitude.

Going to very high energies, $\gamma \gg 1, \beta \rightarrow 1$, the only contributions will come from the $(0,0)$ $Z$ channel and the $\Delta \lambda = \pm 2$ amplitudes, since all other helicity combinations for the $t$ channel are proportional to at least $\gamma^{-1}$. The contributions from the $(0,0)$ and $(+, -)$ configurations are softened by an extra $\sin \Theta$ factor from the $d$-functions. The $(-, +)$ combination is enhanced by a factor $(1 + \cos \Theta)/(1 - \cos \Theta)$ and peaked in the forward direction, but an extra $\sin \Theta$ factor ensures a finite value at $\cos \Theta = +1$.

As can be seen in Fig. 1.2, all the helicity combinations will contribute to the lower energy processes. As the energy increases, the cancellations come into play, and ultimately the final state will be dominated by the strongly forward peaked transverse configuration $(-, +)$.

1.4.3 TGC Parametrisation

The introduction of the parametrisation of the general $VWW$ Lagrangian in Eq. (1.13) will change the values of the $A_{\lambda}^\gamma$ coefficients as indicated in the bottom part of Table 1.2. The number of independent couplings that were introduced comes from the fact that only seven helicity configurations can be formed in the $s$-channel, as pointed out above. Since the coefficients are linear in the couplings, the cross section will depend at most
Theoretical Framework 1.4. W-pair production at LEP

Standard Model

<table>
<thead>
<tr>
<th>Δλ</th>
<th>(λλ′)</th>
<th>(A^V_{λλ′})</th>
<th>(B_{λλ′})</th>
<th>(C_{λλ′})</th>
<th>(d_{σ,Δλ}^{J_0})</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>(+, −)</td>
<td>0</td>
<td>0</td>
<td>2√2β</td>
<td>+ sin Θ(1 − cos Θ)/2</td>
</tr>
<tr>
<td>1</td>
<td>(+, 0)</td>
<td>2γ</td>
<td>2γ</td>
<td>2(1 + β)/γ</td>
<td>(1 + σ cos Θ)/2</td>
</tr>
<tr>
<td>1</td>
<td>(0, −)</td>
<td>2γ</td>
<td>2γ</td>
<td>2(1 + β)/γ</td>
<td>(1 + σ cos Θ)/2</td>
</tr>
<tr>
<td>0</td>
<td>(+, +)</td>
<td>1</td>
<td>1</td>
<td>1/γ²</td>
<td>−σ sin Θ/2</td>
</tr>
<tr>
<td>0</td>
<td>(0, 0)</td>
<td>1 + 2γ²</td>
<td>2γ²</td>
<td>2/γ²</td>
<td>−σ sin Θ/2</td>
</tr>
<tr>
<td>0</td>
<td>(−, −)</td>
<td>1</td>
<td>1</td>
<td>1/γ²</td>
<td>−σ sin Θ/2</td>
</tr>
<tr>
<td>−1</td>
<td>(0, +)</td>
<td>2γ</td>
<td>2γ</td>
<td>2(1 − β)/γ</td>
<td>(1 − σ cos Θ)/2</td>
</tr>
<tr>
<td>−1</td>
<td>(−, 0)</td>
<td>2γ</td>
<td>2γ</td>
<td>2(1 − β)/γ</td>
<td>(1 − σ cos Θ)/2</td>
</tr>
<tr>
<td>−2</td>
<td>(−, +)</td>
<td>0</td>
<td>0</td>
<td>2√2β</td>
<td>− sin Θ(1 + cos Θ)/2</td>
</tr>
</tbody>
</table>

TGC Parametrisation

<table>
<thead>
<tr>
<th>Δλ</th>
<th>(λλ′)</th>
<th>(A^V_{λλ′})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(+, 0)</td>
<td>γ(g_Y^V + κ_V + λ_V - i g_{g_Y}^Y + β g_{g_Y}^Y + \frac{i}{2}(κ_V - λ_V))</td>
</tr>
<tr>
<td>1</td>
<td>(0, −)</td>
<td>γ(g_Y^V + κ_V + λ_V + i g_{g_Y}^Y + β g_{g_Y}^Y - \frac{i}{2}(κ_V - λ_V))</td>
</tr>
<tr>
<td>0</td>
<td>(+, +)</td>
<td>g_Y^V + 2γ²λ_V + \frac{i}{2}(κ_V - λ_V)</td>
</tr>
<tr>
<td>0</td>
<td>(0, 0)</td>
<td>g_Y^V + 2γ²κ_V</td>
</tr>
<tr>
<td>0</td>
<td>(−, −)</td>
<td>g_Y^V + 2γ²λ_V - \frac{i}{2}(κ_V - λ_V)</td>
</tr>
<tr>
<td>−1</td>
<td>(0, +)</td>
<td>γ(g_Y^V + κ_V + λ_V + i g_{g_Y}^Y - β g_{g_Y}^Y - \frac{i}{2}(κ_V - λ_V))</td>
</tr>
<tr>
<td>−1</td>
<td>(−, 0)</td>
<td>γ(g_Y^V + κ_V + λ_V - i g_{g_Y}^Y - β g_{g_Y}^Y - \frac{i}{2}(κ_V - λ_V))</td>
</tr>
</tbody>
</table>

Table 1.2: The top table shows the values for the coefficients \(A^V_{λλ′} = A^V_{λλ′}, B_{λλ′}\) and \(C_{λλ′}\) for the different helicity combinations of the W bosons, (λ, λ′), in the standard model. Here, γ = 1/√1 − β² and the values \(J_0 = 2\) have been used for the two |Δλ| = 2 cases, while \(J_0 = 1\) for the other cases. The bottom part shows the coefficients \(A^V_{λλ′}\) in the case of the parametrisation introduced in equation (1.13).
The presence of anomalous couplings will destroy the subtle cancellation between the diagrams and this will violate unitarity bounds at a certain energy scale. However, the introduction of a form factor behaviour of the couplings in Section 2 ensures that the coupling values fall off rapidly once the threshold of new physics is crossed.

At energies just above the W-pair production threshold (like at LEP, see Table 2.1), the cancellations are not yet fully operative and thus linear combinations of couplings enter the matrix elements quite differently than in the asymptotic form. In the threshold region, the reaction is not very sensitive to anomalous couplings, since the $s$-channel is suppressed by at least a factor $\beta$. The $\gamma^2$ enhancement factors are still small so that none of the helicity contributions are fully dominated by individual couplings. But the interference between the $s$- and $t$-channel diagrams will give an important contribution in the presence of anomalous couplings.

An interesting study of the minimal cross section for W-pair production as function of the TGC’s is performed in [80]. It was shown that over a centre-of-mass energy range from 180 to 700 GeV, it is only possible to lower the SM cross section for very specific values of the couplings with a maximum of 4%.

1.5 W Decay

Up to now, only the production of stable W bosons has been considered, but in fact they have a very short lifetime. The W bosons in the pair production process should be described as resonances having a non-zero width. The decay is very well understood and proceeds via the left-handed charged weak current ($V - A$). The W boson can thus...
Theoretical Framework 1.6. Other Four-Fermion Processes

decay into either a lepton and neutrino pair ($e\nu_e$, $\mu\nu_\mu$ or $\tau\nu_\tau$) or into a quark anti-quark pair ($ud$ or $cs$). The former one is referred to as leptonic W decay with a branching fraction of 10.8%, for each possible lepton under the assumption of lepton universality. The latter one is called hadronic W decay and has a branching fraction of 67.5% for the decay into all possible quarks. Since the quarks are not weak eigenstates, it is also possible to form the final states $us$, $cd$, $cb$ and $ub$. Their contribution will be very small, since it is proportional to the corresponding off-diagonal element of the mixing matrix, also known as the Cabibo-Kobayashi-Maskawa (CKM) matrix. Depending on the decay mode of the two W bosons in the pair production, the final states are usually divided into three classes:

- **hadronic channel**: $q_1\bar{q}_1\bar{q}_2\bar{q}_2$ (qqqq) 45.6%
- **semi-leptonic channel**: $qq\ell\nu_\ell$ (qq$\ell\nu$) 43.8%
- **leptonic channel**: $\ell\nu_\ell\nu_\ell$ ($\ell\nu\ell\nu$) 10.6%

Between brackets is the short notation that will be used throughout this thesis to indicate the different decay channels. Sometimes these channels are subdivided according to the decay lepton. This is not done for different quarks, since no quark flavour tagging will be performed in this thesis.

1.6 Other Four-Fermion Processes

The measurement of a W-pair requires the identification of four fermions in the final state but this requisite alone is not sufficient for the measurement. In fact, many other diagrams exist with the same initial and final state, but with different intermediate states which form an irreducible background. In order to have a correct description of the considered signal, all the relevant diagrams need to be identified and taken into account. A complete description of all the diagrams contribution to four-fermion (4f) production can be found in [81].

The simplest case is when no electron and electron-neutrino is present in the final state, nor a fermion anti-fermion pair. Then the only type of graphs contributing besides the CC03 diagrams from Fig. 1.1 is the singly resonant W process. This diagram is given in Fig. 1.3: a fermion pair is created through the s-channel and one of these fermions radiates a W boson. The decay channels which get only contributions from this sort of diagrams are $\mu\nu_\tau\nu_\tau$ (9 diagrams in total), $qq\mu\nu$ and $qq\tau\nu$ (10) and $udsc$ (11).

The next step is to look at the processes that contain one $e\nu_e$ pair, where three extra types of diagrams play a role, shown in Fig. 1.4. The first is of a bremsstrahlung type, where a W is radiated off from an electron (in initial or final state) and a boson is exchanged through the t-channel. The so-called non-Abelian diagrams form another type, where two bosons are radiated off the incoming electrons and interact to form another boson. This type of diagram is of special interest, since the process involves a triple gauge-boson vertex of the type $VWW$. The third type of diagram that contributes is called multi-peripheral scattering. Here, the two radiated bosons are not on-shell and do not interact with each other directly, but rather exchange a fermion in the t-channel.
The leptonic final states $e\nu\mu\nu$ and $e\nu\tau\nu$ can be produced through 18 diagrams, while the semi-leptonic channel $qq\ell\nu$ gets contributions from 20 different channels.

The last group of diagrams contributes when the final state contains fermion pairs. One type of diagrams is obtained when a photon or Z boson is radiated by the final state fermions. Another type is the production of two neutral bosons, photons or Z’s, through conversion diagrams. Both diagrams are drawn in Fig. 1.5. The processes that also get contributions from this type of diagrams are $\mu\nu\mu\nu$ and $\tau\nu\tau\nu$ that can be produced by 19 diagrams, the $dudu$ and $scsc$ by 43 diagrams and the $e\nu\nu\nu$ by 56 diagrams in total.

For the comparison between data and generated events, only the CC03 diagrams are used to predict the signal distributions. The missing four-fermion diagrams are taken into account by a reweighting procedure explained in Section 3.6.1. When two quark anti-quark pairs are present in the final state, also QCD diagrams, where quark pairs can be produced by gluon interaction, have to be taken into account.

Single-W process

An important subset of the diagrams discussed in the previous paragraphs are the ones that contribute to the single-W process, $e^+e^- \rightarrow We\nu$. The cross section for this process has been measured by the L3 experiment and limits on the triple gauge-boson couplings have been derived [82]. A combination of these complementary results with the outcome of the analysis described in this thesis will be performed in Section 6.2.

1.7 WW Event Generators

In order to perform a measurement of the triple gauge-boson couplings, the relevant reconstructed quantities have to be compared to a theoretical prediction. These predictions have uncertainties arising from certain assumptions that have to be made to make it possible to perform these calculations. The most important uncertainties are due to neglecting higher order electroweak radiative corrections and the description of the fragmentation and hadronisation of the produced quarks.

1.7.1 Electroweak Radiative Corrections

Not only can the initial and final state fermions emit real photons, but also virtual contributions exist. Here, a photon is emitted and absorbed again during the process, connecting any pair of charged particles throughout the entire process. If the photon links two particles from the same production or decay subprocess, the correction can be very well approximated by multiplying the Born cross section with a radiator function and is therefore called factorisable. However, when the photon links different subprocesses, this treatment is not possible anymore and the corrections will thus be non-factorisable.

At the start of the LEP2 programme in 1996, the available theoretical predictions for the W-pair production process [83] included only the universal weak corrections. Typically, these consisted of photon radiation from the initial state (ISR) or final state (FSR)
1.7. WW Event Generators

Figure 1.3: An example of the singly resonant diagram contributing to the same four-fermion final states as the W-pair production.

Figure 1.4: The three types of diagrams resulting in the same four-fermion final states as in W-pair production when at least one \((e, \nu_e)\)-pair is present. In the bremsstrahlung diagram (left) a W is radiated from a final state fermion. The W can also come from one of the incoming electrons. The non-Abelian type of diagram (middle) contains a triple gauge-boson vertex, where two of the three interacting bosons is a W boson. The two produced bosons in the multi-peripheral diagrams (right) interact with each other through the exchange of a fermion.

Figure 1.5: Examples of diagrams with two fermion anti-fermion pairs in the final state. In the bremsstrahlung type (left), a photon or Z boson is radiated by one of the initial or final state fermions. The conversion diagram (right) produces a pair of neutral bosons.
and their interference, and the factorisable virtual corrections, like the Coulomb correction, i.e. the exchange of a photon between the two W bosons. By neglecting the non-factorisable corrections, the theoretical error on the W-pair cross section was estimated to be about 2%, but increasing with centre-of-mass energy. At that time, the combined error on the measured cross section at the end of LEP2 running was estimated to be of the order of 1%. Having such a large theoretical error would thus mean a loss of analysing power for the LEP experiments. It was therefore necessary to reduce the uncertainties on the theoretical predictions coming from the missing EW radiative corrections. Ideally, the full $\mathcal{O}(\alpha)$ EW corrections to off-shell $e^+e^- \rightarrow WW \rightarrow 4f$ production as well as ISR has to be taken into account, but this is a very complex problem. Nevertheless, a step in this direction has been made in more recent calculations by using the so-called double pole approximation (DPA).

The complete amplitude for a process can be expanded around the poles $1/(p^2 - M^2)$ that arise in physical observables when the unstable resonances are treated as stable particles. This description remains gauge invariant when a finite width is taken into account and can then be viewed as an expansion in terms of $\Gamma/M$. It should be noted that in the case of W-pair production, two unstable resonances are present and the off-shell phase space has to be carefully mapped on the on-shell phase space. The expansion is then approximated by keeping only the terms with the highest degree of resonance, which are, in the case of W-pair production, the double-pole residues. The intrinsic error of this method is $\lesssim 0.5\%$ at centre-of-mass energies attainable at LEP2.

However, the definition of the DPA is problematic for bremsstrahlung corrections, where a photon is radiated from an initial or final state fermion, since the W propagators become resonant in different regions of phase space. When a hard photon is emitted, the regions are well separated and the photon can unambiguously be assigned to the subprocess it is radiated from. The soft photons do not pose a problem either, since the definition of the DPA will be identical to the one without the radiation. The problem comes however from the semi-soft radiation ($E_\gamma \sim \Gamma_\gamma$), where the resonant propagators overlap and it is then not so obvious how the DPA should be defined.

The method of DPA has been implemented in two Monte Carlo event generators: racoonww [84] and yfsww3 [85]. The racoonww approach is to treat the virtual $\mathcal{O}(\alpha)$ corrections to off-shell $e^+e^- \rightarrow WW \rightarrow 4f$ in DPA. The real $\mathcal{O}(\alpha)$ corrections are based on the minimal gauge invariant subset of the $4f + \gamma$ matrix elements. The generator yfsww3 applies the DPA only to the on-shell W-pair production combined with YFS exponentiation for the initial fermions and intermediate WW states and leading-logarithm approximation for the photon radiation in W decays.

During the LEP200 MC workshop, these two generators were extensively cross-checked with each other [86] and with a semi-analytical program, referred to as BBC [87]. The predicted values for the CC03 cross section were in agreement with each other up to a level of about 0.2–0.3%. This can also be seen in the left plot of Fig. 1.6, where the prediction for the W production angle from racoonww is compared with yfsww3. The agreement between the total cross section (offset of the linear fit) obtained from both generators is about 0.2%, while the shape of the distributions (slope of the fit) is compatible up to 0.4%. The effect of the inclusion of the non-factorisable electroweak radiative corrections using DPA can be seen in the right plot of Fig. 1.6. A first thing to notice is that the
Theoretical Framework

1.7. WW Event Generators

The cross section is about 2.3% lower than was estimated without the $O(\alpha)$ correction. Also the $W^-$ production angle is affected and becomes less peaked forward by about 0.7%.

Another generator that will be used in this analysis, mainly for testing purposes, is Kandy [88]. This program combines the four-fermion generator KORALW [89] with the DPA calculations implemented in YFSWW3, by running them concurrently as separate processes. First, the four-fermion phase space is generated with KORALW for massive fermions including all possible diagrams. The momenta of the particles are then passed to YFSWW3 which calculates the $O(\alpha)$ corrections described above. The resulting weight is given back to the KORALW program which processes the event further.

1.7.2 Fragmentation and Hadronisation

Any quark produced in the aforementioned processes can radiate a gluon, which in turn can create a quark anti-quark pair. However, these particles are not colour singlets and cannot be observed as free particles in Nature due to colour confinement in QCD. The partons (quarks and gluons) are moving away from each other due to momentum conservation while at the same time the potential energy from the attraction due to the strong force increases. At a certain point, it will be energetically more favourable to produce a quark pair out of the vacuum in such a way that the effective separation between the quarks reduces. This process of splitting up and dividing the energy over a number of partons is known as fragmentation, while at the same time the hadronisation takes place, a recombination of quarks to form colour singlets, mesons and baryons.

In the beginning of the fragmentation process, the partons are highly energetic and the process can in principle be described perturbatively. During the process the energy is divided over more and more partons and the strong coupling strength $\alpha_s$ increases. At a certain point, the perturbative description will not be adequate anymore and phenomenological models have to be used.

The Monte Carlo programs JETSET [90], HERWIG [91] and ARIADNE [92] all simulate the perturbative phase using exact matrix elements (ME) in combination with parton
showers (PS). These PS contain the leading logarithms of all orders of the parton splitting processes. The ME are used only for the lowest order, since the matching between ME and PS to avoid double counting is not performed for higher orders.

The Lund string model, one of the models describing the non-perturbative phase, is implemented in JETSET. In this picture, the partons are connected by strings representing the colour flux. As the partons move away from each other, the potential in the string increases and eventually, the string breaks up such that hadrons are formed. The JETSET approach is also used for the non-perturbative part in ARIADNE.

The cluster model used in HERWIG starts by splitting remaining gluons non-perturbatively into quark pairs. These pairs form together with the already present quarks colourless clusters of various masses, which eventually all decay, possibly via intermediate lighter clusters, to hadron pairs.

At the end of the fragmentation and hadronisation phase, a large number of hadrons is present, which then decay into stable particles according to decay tables of branching fractions included in the Monte Carlo generators.

All the phenomenological models above contain a number of parameters that can be determined (“tuned”) using the large statistics available for hadronic decaying Z bosons from LEP1. The tunings of the Monte Carlo generators that are used in this thesis are described in [93].
Chapter 2

W-pair Measurement

The aim of this chapter is to describe how a sample of W-pairs is obtained. For this purpose, among others, the Large Electron Positron (LEP) accelerator and collider has been built to deliver electron and positron beams and collide them with centre-of-mass energies above the W-pair production threshold ($\sqrt{s} > 2M_W$). This apparatus is described in the next section after which an overview of the L3 detector is given. This was one of the four devices that were built around the interaction points of the LEP collider to detect the particles produced in the collisions. Subsequently, common features of all selection procedures are explained, such as the signatures and reconstruction of the different decay particles and the different types of background. The major part of this chapter is devoted to the description of the different selection procedures that try to separate genuine events from various background sources for the WW decay channels used in this analysis.

2.1 The Experiment

After a brief introduction to the LEP collider, the different constituents that form the L3 detector will be described. Also the manner in which the signals in these parts are combined and transformed into event objects and the available data sample will be discussed.

2.1.1 The LEP Collider

The several stages of the CERN accelerator complex employed to bring electrons and positrons to the desired energy are shown in the top part of Fig. 2.1. The positrons were created in a tungsten converter target hit by a 200 MeV e$^-$ beam coming from the high intensity linac $LEP$ $Injecteur$ $Linéaire$ (LIL). Both types of particles, after being separately accelerated by a second linac to 600 MeV, were stored in the Electron Positron Accumulator (EPA) before they were transferred to the Proton Synchrotron (PS). In this machine, the particles reached energies of 3.5 GeV and were then passed to the Super Proton Synchrotron (SPS) which acted as a 22 GeV injector for the LEP collider, where the final stage of the acceleration to the desired energy took place.

From the startup in 1989 until 1995, the particles were colliding at beam energies around 45 GeV to study the Z-pole resonance. After phase I, from 1996 onwards, LEP started to run at centre-of-mass energies just above the W-pair production threshold and
gradually increased the beam energies up to $\sqrt{s} = 209.2$ GeV during 2000, the last year of data taking.

The LEP ring has a circumference of 26.7 km and actually consists of eight straight sections as well as bending sections. At four points, where the electron and positron beams were focused, large detectors were built to study the $e^+e^-$ interactions: ALEPH, DELPHI, L3 and OPAL. Technical details about the LEP collider and the accelerator complex can be found in [96].

### 2.1.2 The L3 Detector

For the measurement described in this thesis, data taken with the L3 detector during phase II of LEP are analysed. Like most present-day high-energy physics detectors, L3 consisted of several layers of sub-detectors, each having a dedicated purpose. A solenoid surrounded the entire detector and provided a 0.5 T magnetic field parallel to the beam axis to give charged particles a curvature from which their momentum and charge can be derived. A return yoke in the forward and backward direction provided a toroidal field.

The L3 reference frame, also shown in Fig. 2.1, is defined as follows: the origin lies at the interaction point, the $z$-axis is in the direction of the electron beam, the $x$-axis is pointing towards the centre of the LEP ring and the $y$-axis is pointing upwards in order to form a right-handed coordinate system. The polar angle $\theta$ is the angle with respect to the $z$-axis and the azimuthal angle $\phi$ is the angle between the $x$-axis and the projection on the $(x, y)$-plane. The central part of the detector is usually referred to as the barrel, while the forward and backward parts are called end-caps. The boundary between the two regions depends on the specific sub-detector, but lies around $|\cos \theta| \approx 0.72 - 0.82$.

A brief overview of the different parts of the L3 detector used in the WW event selections is given in the order in which a particle coming from the interaction point would have encountered them. The different sub-detectors are also indicated in Fig. 2.1. Detailed description of the construction can be found in [97] and references therein, while upgrades performed after the startup are discussed in [98].

### Tracking detector

The inner part of the detector served to measure the path of a charged particle near the event vertex. From the curvature of this track, the momentum transverse to the $z$-axis and the charge could be extracted.

Closest to the beam pipe was the Silicon Microvertex Detector (SMD). It was made out of two concentric layers of double-sided silicon ladders, 35 cm in length and situated at 6.0 cm and 7.7 cm from the beam axis. The polar coverage was from $22^\circ$ to $158^\circ$ and the single track resolution was 6 $\mu$m in $r - \phi$ and $20 - 25$ $\mu$m in $z$.

The SMD was surrounded by the Time Expansion Chamber (TEC), a multi-wire proportional chamber operating in time expansion mode. This 98 cm long drift chamber had an inner and outer radius of 8.5 cm and 47 cm, respectively. All of the 62 wires could be used to determine the track of a charged particle if it was produced at a polar angle between $44^\circ$ and $136^\circ$. Outside of this angular interval the number of wires would be less, becoming zero for angles lower than $10^\circ$ or larger than $170^\circ$. Due to the low drift velocities
Figure 2.1: The top part shows a schematic overview of the LEP collider with the four experiments and the accelerator complex. The bottom part is a perspective view of the L3 detector where the different components, described in Section 2.1.2, are indicated. Also the right-handed coordinate system \((x, y, z)\) and the definition of the polar coordinates \((r, \theta, \phi)\) are shown.
of 6 μm/nm, a spatial resolution of 50 μm was obtained using the timing of the signal. To obtain such a resolution, a complex iterative calibration needs to be performed, involving the measurement of the drift times for each wire using Z peak e- and μ-pair production.

The Z-chambers, two additional proportional wire chambers mounted on the outside of the TEC, covered polar angles between 45° and 135° and had a single track resolution of 300 μm. The Forward Tracking Chambers (FTC) provided additional position measurement in the region 12° < θ < 34° and 146° < θ < 168°.

**Electromagnetic calorimeter**

The tracking devices were enclosed by an electromagnetic calorimeter (ECAL) which was designed to measure the energy and position of electrons and photons. It was made out of 24 cm long bismuth germanate (BGO) crystals, serving both as a showering and detecting medium. These crystals corresponded to about 22 radiation lengths. The scintillation light was read out by two photodiodes glued to the outer end of the crystals. The barrel region of the detector contained 7680 crystals and covered the polar region between 42° and 138°. The two end-caps, each having 1527 crystals, covered 10° < θ < 35° and 145° < θ < 170°. This calorimeter had an energy resolution of 5% for electrons and photons at 100 MeV and better than 2% for energies between 1 GeV and 100 GeV. The angular resolution was about 2 mrad.

To improve the angular coverage, blocks of lead threaded with plastic scintillating fibres were installed in the gap between the barrel and end-cap parts of the BGO detector. This so-called spaghetti calorimeter (SPACAL) equalled about 21 radiation lengths and had an energy resolution of roughly 5% at 45 GeV.

**Scintillation counters**

Mounted between the electromagnetic and hadron calorimeters, the plastic scintillation counters served for timing purposes to reject cosmic background and for triggering hadronic events. They covered 93% of the azimuthal angle for a polar region between 25° and 155° and had a timing resolution of 0.5 ns.

**Hadron calorimeter**

Hadronic particles would start showering in the hadron calorimeter (HCAL) through nuclear interaction with the plates of depleted uranium, which were interlaced with proportional wire chambers to detect these showers. The barrel part extended from 35° to 145° in polar angle and two end-caps covered the regions 5.5° < θ < 35° and 145° < θ < 174.5°. The resolution of this detector was \( \sigma/E = (55/\sqrt{E} + 5)\% \) and the angular resolution for the determination of a jet axis was about 2.5°. The total thickness of the HCAL including the ECAL and support structures corresponded to about 6 to 7 nuclear interaction lengths.

A muon filter was located around the barrel HCAL and was made from brass plates interleaved with proportional tubes. This filter added an additional nuclear interaction length ensuring that all hadronic particles were absorbed and thus did not reach the muon chambers.
Muon detector

The muon chambers (MUCH) served to measure the momenta of muons, the only detectable particles which could pass all the detector material described above. The barrel part, covering \(43^\circ < \theta < 137^\circ\), was radially subdivided in 8 octants, each having three P-layers of wire chambers, measuring \(r - \phi\) coordinates. The \(z\) coordinate was determined using two Z-layers located in the inner and outer layer. To cover the forward direction down to \(22^\circ\), one layer of chambers had been installed inside the magnet door, and two layers had been mounted outside. The momentum resolution for a muon of 45 GeV was about 3\% for the barrel part, while it ranged from 3\% to 30\% for the end-caps.

Luminosity detector

The luminosity was determined by measuring the process \(e^+e^- \rightarrow e^+e^-\), also called Bhabha scattering, under very small angles. For this purpose, the luminosity monitor (LUMI), made out of BGO crystals, was placed in the forward and backward direction, covering the polar angle region between \(30\) mrad \(< \theta < 62\) mrad. A silicon strip detector (SLUM) was mounted in front of it to ameliorate the position measurement. The luminosity \(L\) can be directly derived from the measured number of Bhabha events \(N\), using the relation \(L = N / (\epsilon \sigma)\). The large Bhabha cross section \(\sigma\) can be calculated to a high precision. Background contamination is also taken into account. The selection efficiency \(\epsilon\) is estimated from Monte Carlo events. The method, described in detail in \([102]\), resulted in a relative error of 0.36\% on the luminosity of the total data sample used for the analysis presented in this thesis.

Trigger and Data Acquisition

The beams crossed each other at the interaction point at a rate of 45 kHz, but interesting events did not occur at this rate. The trigger system was designed to decide whether activity occurring in the detector originated from a genuine \(e^+e^-\) collision. It consisted of three levels of decisions of increasing complexity. The level 1 trigger was the logical OR of trigger conditions from several sub-detectors: TEC, energy, luminosity, scintillator, and muon trigger. After a positive decision, the detector was read out and the data was digitised. During this process, the detector was inactive for about 500 ms. Events with more than one positive level 1 sub-trigger were automatically accepted by higher level triggers. After this stage, the trigger rate was 15-25 Hz. The level 2 trigger took this data and some extra information not available to the level 1 trigger, reducing the trigger rate with 20-30\%. The third level used the complete digitised raw data available for the event. Finally, the data were written to storage with a rate of about 5 Hz. The dead time was kept below 3\%.

2.1.3 Event Reconstruction

The first step in the reconstruction of an event is the transformation of the signals measured in the sub-detectors to physical quantities, like position and energy, using calibration
data. This information is translated into tracks and energy clusters which are combined to form event objects.

**Tracks.** The points resulting from the hits in the TEC wires are combined to form tracks using a pattern recognition algorithm. This track is first extrapolated to the Z-chamber and then to the SMD to look for associated hits. Due to the solenoidal magnetic field, the charged particles follow a helix around the beam pipe. Projected on the transverse plane, this path looks like a circle. This transverse track is then fit with a circle to obtain the curvature \( \rho \), the reciprocal of the radius of the track. This curvature is proportional to the charge \( q \) of the particle and to the inverse of the momentum in the transverse plane, \( p_T \):

\[
\rho = 0.3 q B \frac{1}{p_T},
\]

with \( \rho \) expressed in meters, \( q \) in electron charges, \( B \) in Tesla and \( p_T \) in GeV/c. Also the distance of closest approach to the interaction vertex, DCA, and the azimuthal angle \( \phi \) are derived.

**Electromagnetic energy clusters.** Adjacent BGO crystals with an energy deposit of more than 10 MeV are grouped together and are kept as clusters if the total energy surpasses 40 MeV. Bumps are defined as local maxima of more than 40 MeV in a cluster.

**Hadronic energy clusters.** Three dimensional clusters are composed from hits in the HCAL associated to an energy deposit of more than 9 MeV.

**Muon tracks.** For each layer of the muon chambers, a pattern recognition algorithm is applied to form track segments. If at least two P-segments can be connected, a helix is fit to all associated segments, providing a measurement of the momentum and charge of the muon track.

**Event objects.** In the second phase, the tracks and clusters of the sub-detectors are combined into objects which roughly correspond to real particles. These objects are the following:

- **ATRK ("A TRacK"):** a TEC track that is associated with calorimetric clusters;
- **ASRC ("A Smallest Resolvable Cluster"):** energy deposits in ECAL and HCAL are combined to form one cluster;
- **AMUI ("A MUon Identified"):** a muon track is associated with a TEC track and calorimetric clusters.

**Energy measurement.** The combined clusters described above originate from a particle traversing the detector. Despite the calibrations of the different sub-detectors, the total energy measured in such a cluster does not correspond exactly to the energy of that particle, in particular for hadrons and jets. First of all, these calibrations are performed
under certain assumptions, e.g. a particle loses energy in the ECAL through an electromagnetic shower. There is a small amount of energy loss in non-detecting materials, like wires, supports and amplifiers. The energy can also leak away into gaps between the active material of the detectors or can be carried away by neutrinos.

To take all these effects into account, gain-factors ($g$-factors) are introduced and the total visible energy is determined as:

$$E_{\text{vis}} = \sum_{i=1}^{12} g_i E_i,$$

(2.2)

where the sum runs over 12 different detector regions, including the energy measurements from calorimetric clusters as well as tracks. The factors are defined such that they equal unity for identified particles, i.e. electrons, photons and muons, used in the calibration of the corresponding sub-detector.

The $g$-factors are determined in nine bins of $\cos \theta$ using a high statistics $q\bar{q}$ sample having high purity and selection efficiency. The values of the factors are varied to minimise the resolution of the visible energy, while keeping its value fixed to the centre-of-mass energy. The resulting errors on the $g$-factors are less than 5% for the high-energy data.

### 2.1.4 Data Sample

The analysis described in this thesis is performed using the data collected during the last three years of the running of LEP. In 1998, LEP delivered beams with one fixed centre-of-mass energy, 189 GeV, while in 1999 the electrons and positrons were collided at four distinct energies: 192 GeV, 196 GeV, 200 GeV and 202 GeV. The exact centre-of-mass energies $\sqrt{s}$ and the corresponding delivered integrated luminosities $\mathcal{L}$ are given in Table 2.1. The same table also shows the details of the 2000 run, where the focus was to produce interactions at energies as high as possible. A wide range of energies was delivered, from 202 GeV up to 209 GeV. For this reason, three energy bins have been defined, referred to as 205 GeV, 206 GeV and 208 GeV.

Besides these high-energy runs, LEP also delivered in these years collisions at centre-of-mass energies around the $Z$ resonance ($\approx 91$ GeV). These $Z$ peak data are used to calibrate the detector using the processes $e^+e^- \rightarrow Z \rightarrow ff$ measured very accurately during phase I. These data are also used for certain systematic studies.

The energy of the colliding beams is determined by the LEP Energy Working Group, using the magnetic extrapolation method [103]. This technique is based on the relation between the beam energy and the magnetic field strength in the dipoles. For this purpose, 16 nuclear magnetic resonance (NMR) probes were installed in several of the main bend dipole magnets in order to continually measure the magnetic field strength. The measurement of the beam energy is calibrated by resonant depolarisation in the region between 41 and 61 GeV and results are extrapolated to the high-energy regime. The resulting error on the centre-of-mass energy varies between 40 and 50 MeV, depending on the year of running.
### 2.2 WW Event Selection

The selection procedures for the different WW decay channels are based on the typical topology of the various types of events and try to select as many events as possible. However, other processes can leave similar traces in the detector and will therefore be a source of background. To parametrise these quantities, the selection efficiency $\epsilon$ is defined as the probability to select a signal event and the sample purity $P$ as the probability that a selected event is indeed a signal event:

\[
\epsilon = \frac{\sigma_{acc}^{sig}}{\sigma_{tot}^{sig}} = \frac{N_{acc}^{sig}}{N_{tot}^{sig}},
\]

\[
P = \frac{\sigma_{acc}^{sig}}{\sigma_{acc}^{all}} = \frac{N_{acc}^{sig}}{N_{acc}^{all}},
\]

where the indices $sig$ refers to the WW signal events, $all$ to the sum of signal and background events, $tot$ means before and $acc$ after the selection has been performed. All these quantities are determined using Monte Carlo generated event samples. The selection is

<table>
<thead>
<tr>
<th>year</th>
<th>reference</th>
<th>$&lt;\sqrt{s}&gt;$ [GeV]</th>
<th>$\sqrt{s}$ range</th>
<th>$\mathcal{L}$ [pb$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998</td>
<td>189 GeV</td>
<td>188.64</td>
<td>fixed</td>
<td>176.77 ± 0.12</td>
</tr>
<tr>
<td>1999</td>
<td>192 GeV</td>
<td>191.60</td>
<td>fixed</td>
<td>29.826 ± 0.048</td>
</tr>
<tr>
<td></td>
<td>196 GeV</td>
<td>195.54</td>
<td>fixed</td>
<td>84.146 ± 0.082</td>
</tr>
<tr>
<td></td>
<td>200 GeV</td>
<td>199.55</td>
<td>fixed</td>
<td>83.314 ± 0.084</td>
</tr>
<tr>
<td>1999</td>
<td>202 GeV</td>
<td>201.75</td>
<td>fixed</td>
<td>37.139 ± 0.056</td>
</tr>
<tr>
<td>2000</td>
<td>205 GeV</td>
<td>204.82</td>
<td>$201.6 \leq \sqrt{s} \leq 205.8$ GeV</td>
<td>79.046 ± 0.084</td>
</tr>
<tr>
<td></td>
<td>206 GeV</td>
<td>206.48</td>
<td>$205.8 \leq \sqrt{s} \leq 207.2$ GeV</td>
<td>130.54 ± 0.11</td>
</tr>
<tr>
<td>2000</td>
<td>208 GeV</td>
<td>208.00</td>
<td>$207.2 \leq \sqrt{s} \leq 209.0$ GeV</td>
<td>8.582 ± 0.028</td>
</tr>
</tbody>
</table>

Total integrated luminosity: 629.36 ± 0.23

<table>
<thead>
<tr>
<th>year</th>
<th>reference</th>
<th>$&lt;\sqrt{s}&gt;$ [GeV]</th>
<th>$\sqrt{s}$ range</th>
<th>$\mathcal{L}$ [pb$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998</td>
<td>Z peak</td>
<td>91.31</td>
<td>fixed</td>
<td>3.0055 ± 0.0073</td>
</tr>
<tr>
<td>1999</td>
<td>Z peak</td>
<td>91.25</td>
<td>fixed</td>
<td>4.0232 ± 0.0084</td>
</tr>
<tr>
<td>2000</td>
<td>Z peak</td>
<td>91.26</td>
<td>fixed</td>
<td>4.0536 ± 0.0085</td>
</tr>
</tbody>
</table>

Total integrated luminosity: 11.082 ± 0.014

Table 2.1: The centre-of-mass energies, $\sqrt{s}$, and the corresponding integrated luminosities, $\mathcal{L}$, during the last three years of running for the high-energy data and the Z peak calibration runs. The third column shows the actual centre-of-mass energies for the 1998 and 1999 runs and the average energies of each energy bin, indicated in the fourth column, during the 2000 run.
then optimised to maximise the product of the selection efficiency and the sample purity, which is inversely proportional to the relative error on the measured signal cross section. This procedure is usually started by applying a pre-selection to the data which removes only background. Then either further refined cuts are applied or a neural network is used. The choice of variables and cut positions are always determined using large samples of Monte Carlo events, and their distributions are compared with real data.

The $ℓνν$ events are not considered in this analysis purely for technical reasons. In the original production of the baseline Monte Carlo samples with YFSWW3, the branching fractions of the $τ$ were wrongly implemented. New samples of $qqτν$ events with correct branching fractions were produced and replaced the original samples. For $ℓννν$ events, these samples were not available in time to perform a complete analysis. The leptonic decay channel, with a branching ratio of about 11% is the least significant decay mode. When at least one of the W bosons in a $ℓνν$ event decays into a $τντ$-pair, the relevant phase space angles (see Section 3.1) cannot be reconstructed due to the presence of at least three neutrinos. This reduces the useful branching ratio by a factor of two. For the remaining events, most of these angles can only be determined up to a two-fold ambiguity. All these factors together result in a decay channel that is far less sensitive to couplings than other WW decay channels. The measurement of the triple gauge-boson couplings with $ℓνν$ events would therefore not have a significant impact on the total combined result, although it is a completely complementary measurement.

### 2.2.1 Signatures of W Decay Products

From the point of view of an experimentalist, a particle is considered stable if it lives long enough to travel distances larger than the dimensions of the detector. As already described in Section 1.4, the W boson is not a stable particle and decays into a lepton and a neutrino or a quark anti-quark pair. Again, not all of these particles are stable and some can only be detected through their decay products. Each of the particles produced in W decays will therefore give a typical signature on which the W-pair selections are based.

**Electrons.** Since these particles are charged, they leave a trace in the central tracking devices before they reach the ECAL. In this part of the detector, they lose all their energy, due to the low critical energy\(^1\) of $E_c ≈ 10$ MeV and the length of the BGO crystals ($≈ 22$ times the radiation length). The total energy of the electrons can be determined from the amount of light detected in the ECAL.

**Photons.** When a photon reaches the ECAL, an electron-positron pair is created near a nucleus of the crystal, after which all the energy is absorbed in the same way as for the

---

\(^1\)The critical energy is the value above which the dominant processes for energy loss in a material are bremsstrahlung and $e^+e^−$ pair creation. Particles with lower energy do not feel the nucleus and scatter off the atoms, leaving them in an ionised or excited state. In the ECAL, scintillation light is emitted when the excited state falls back into the ground state. The value of the critical energy depends strongly on the material and the particle.
2.2. WW Event Selection W-pair Measurement

electrons. The only difference between the signatures of these two particles is that the photon does not leave a track in the TEC.

**Quarks and hadrons.** As discussed in Section 1.7.2, a quark produces a jet of particles containing mostly hadrons. Most of these hadrons are not stable and decay even before the first detector is encountered. If the particles are charged, they leave a track in the TEC. Most of the hadrons that reach the ECAL only scatter off the crystal atoms since their critical energy is hundreds of GeV. Only pions can start showering in this part of the detector but not all their energy will be contained. In the HCAL however, all the hadrons produce showers and the length of the material is sufficient to stop even the most energetic hadrons.

The typical signature of a quark will thus be a large number of tracks and calorimetric clusters which follow closely the direction of the original quark. The collection of these objects is usually referred to as a jet.

**Muons.** After leaving a track in the central tracking devices, a muon passes through the ECAL and HCAL, since the energy of the muons will be far too low to produce showers in either part of the detector. They only lose some energy through ionisation and excitation of the atoms of the material. Most of the muons have enough momentum (> 2.6 GeV/c) to arrive at the MUCH where they leave a track.

**Taus.** Due to its very short lifetime and its typical decay length $c\tau$ of 87.11 μm [79], the tau lepton decays into an odd number of charged particles and at least one neutrino before it reaches the tracking device. The most common decay modes are called the 1- and 3-prong events, according to the number of charged particles in the final state. The decay particles can be electrons, muons and hadrons, and will give signatures in the detector accordingly. In all these cases, the measured particle(s) are referred to as a $\tau$ jet.

**Neutrinos.** Since they can only undergo weak interactions, the probability of neutrinos leaving a mark in the detector is quasi zero. Hence, the signature of these particles will be missing energy and momentum.

### 2.2.2 Reconstruction of W Decay Products

The W decay products are identified using their typical signatures described above.

**Electron and Photon Identification**

The identification of an electron or photon is based on the narrow and symmetric shape of the shower it creates in the electromagnetic calorimeter. The energy deposit from a hadron that started showering in the ECAL will have a much broader and more irregular spatial distribution.

One of the variables used to quantify this difference is $E_9/E_{25}$, the ratio between the energy deposited in arrays of $3 \times 3$ and $5 \times 5$ crystals centred around the highest energetic crystal. This value peaks close to unity for electrons and photons, while it shows a very
broad distribution for the other particles. Therefore, it is required to be larger than 0.95 for the data taken at $\sqrt{s} = 189$ GeV and larger than 0.98 for the higher centre-of-mass energies. Before calculating this ratio, the values $E_9$ and $E_{25}$ are corrected for energy leakage between the crystals.

A second variable used for the identification is $\chi_{em}^2$, a measure of electron or photon resemblance of the energy deposition in the BGO calorimeter. It is calculated from a comparison of the shower profile measured in a $3 \times 3$ array of crystals around the most energetic crystal and the results obtained from test beam measurements using electrons and pions. This variable is not used for the 1998 data but has to be smaller than 35.0 for the last two years of data taking.

The shape of the shower is not used to identify an electron or photon in the SPACAL. Instead, the candidate is chosen as the highest energy deposition with less than 0.2 GeV measured in the HCAL directly behind the SPACAL cluster.

The further requirement of an associated good TEC track separates the electrons from the photons. A TEC track is considered good when the number of wires hit divided by the span is larger than 0.5, where the span is the difference in the number of the outer-and innermost wire hit. Furthermore, the distance of closest approach (DCA) of the reconstructed track to the event vertex should be less than 5 mm and the transverse momentum should be larger than 0.5 GeV/$c$. The accepted track is then extrapolated to the edge of the ECAL or SPACAL and the difference in azimuthal angle between the two objects is required to be smaller than 20 mrad.

**Muon Identification**

Two different classes of muons can be reconstructed depending on whether or not the particle was seen in the muon chambers. A muon can be missed when it passes through a gap or dead cell in the muon chambers.

- **“AMUI”**. Further requirements are imposed on the AMUI object, as defined in Section 2.1.3, to accept it as a muon candidate. The DCA of the track is required to be smaller than 500 mm in the $z$-direction and 100 mm in the $r - \phi$ plane. When it fails these criteria, the object is still accepted if the DCA of the matched TEC track is less than 10 mm.

- **“MIP”**. Muons not reconstructed in the muon chambers can be identified by the signature of a minimum ionising particle (MIP) in the other detector parts. This requires a well measured track in the TEC, with a DCA smaller than 2 mm and at least 30 hits if the track is in the barrel or 10 hits otherwise. The best matching ECAL and HCAL clusters are assigned to the MIP candidate, where the cluster energy must be consistent with the typical energy loss of a MIP. For the ECAL this means more than 0.1 GeV but less than 2 GeV and for the HCAL more than 0.5 GeV but less than 8 GeV.

The selection of MIP’s is completely independent of the AMUI reconstruction. Large overlap between the two samples shows how well muons can be recovered as MIP’s.
Jet Reconstruction

As described previously, quarks are seen as a jet containing many particles, resulting in a large number of calorimetric clusters and tracks (combined to form event objects) in the detector. The assignment of these objects to a certain jet is performed using the Durham jet clustering algorithm [104].

First, the variable \( y \) for every pair of objects \((i, j)\) in the event is defined which is a measure of the distance between the two particles:

\[
y_{ij} = \frac{2 \min(E_i^2, E_j^2)}{E_{\text{vis}}} (1 - \cos \theta_{ij}),
\]

where \( E_{\text{vis}} \) is the total visible energy in the event, \( E_i \) and \( E_j \) are the energies of the two objects and \( \theta_{ij} \) is the angle between them. Then the two objects with the smallest \( y \)-value are combined to form one object. This procedure is repeated until the number of objects has decreased to a specified value or all the remaining \( y_{ij} \) values are above a certain cut, \( y_{\text{cut}} \). The 4-momenta of the quarks and gluons are calculated from the reconstructed 4-momenta of the particles that are grouped together, resulting in massive jets.

A variable often used when this algorithm is applied is \( y_{34} \), the smallest of the \( y_{ij} \) values in the case when four objects are left. Due to this definition, an event containing four well separated and energetic jets will have large values of \( y_{34} \).

For the reconstruction of a hadronic \( \tau \) jet, another clustering algorithm is used based on the combination of the particles inside a cone of 15° half-opening angle [106].

Neutrino Reconstruction

A produced neutrino will not be measured in the detector and carries away a fraction of the total energy and momentum. Under the assumption that only one energetic neutrino is produced, as in \( q\bar{q}\nu \) and \( q\bar{q}\mu\nu \) events\(^2\), the momentum of this neutrino can be identified with the total missing momentum of the event:

\[
\vec{p}_\nu = \vec{p}_{\text{miss}} \equiv -\sum_i \vec{p}_i,
\]

where the sum runs over all the objects measured in the detector.

2.2.3 Background processes

This section describes the known background sources that can contaminate the various WW selections.

WW production itself

Since the different decay channels can leave similar traces in the detector, one channel can form a background for another. This can happen when a particle is misidentified or when

\(^2\)Jets typically contain also a number of low energetic neutrinos from decaying particles. This missing energy is already corrected for in the energy calibrations.
a particle decays and some decay products are not detected or got lost in the beam pipe. As will be shown in the next sections, this background can be kept low and certain cut variables are introduced to minimise the overlap between the different decay channels.

$e^+e^- \rightarrow ZZ$

At centre-of-mass energies above twice the mass of the Z boson, i.e. $\sqrt{s} \geq 182$ GeV, it is possible to produce a pair of Z bosons. The dominant production channels for this process are two conversion diagrams, shown in Fig. 2.2. The total cross section for this process ranges from 0.97 pb at 189 GeV to 1.34 pb at 208 GeV. Although these values are an order of magnitude smaller than the W-pair production cross section, this background source is difficult to reduce since the resulting four-fermion final states can look quite similar to WW events. This process has been observed by L3 [107] and the measured cross sections are in agreement with the SM prediction within a precision of 11%. To estimate the contamination caused by this background, ZZ events were generated with PYTHIA [90].

$e^+e^- \rightarrow Zee$

The so-called “single-Z” process is possible through t-channel diagram production of a Z boson. The common process is the radiation of a virtual photon by an incoming electron or positron. The $e^+$ or $e^-$ scatters under low polar angles and is often not detected. The photon is absorbed by the other incoming particle which in turn radiates a Z boson. Two such diagrams are shown in Fig. 2.2. All events of the single-Z type were also generated with PYTHIA to estimate the level of background. The cross section for this process varies only slightly over the studied energy range from 3.35 pb to 3.64 pb. These numbers have been calculated after applying a number of phase space cuts: the final state positron (electron) is generated at zero polar angle, while the electron (positron) must satisfy $\theta > 15$ mrad ($\pi - \theta > 15$ mrad) and $E > 1$ GeV. The cross section of this process has been measured by the L3 collaboration [108].

**Singly and non-resonant four-fermion production**

These processes, described in detail in Section 1.6, will also form a non-reducible background. They are taken into account by reweighting the CC03 generated WW events using the EXCALIBUR [109] program. This mainly affects qqe events. Details about the calculation of these weights can be found in Chapter 3. Events that do not have a WW-like topology are partially incorporated in the ZZ and Zee samples described in the previous sections. However, event samples containing all possible 4f final states are generated with KANDY [88] to cross check this approach.

$e^+e^- \rightarrow ff(\gamma)$

This is the process where a fermion pair is created through the annihilation of the incoming electron and positron. For the production of an $e^+e^-$ pair, called Bhabha events, the diagram with a t-channel photon exchange is also possible. Both types of lowest order
diagrams are shown in Fig. 2.2. In the case of the s-channel process, an interesting effect occurs when $\sqrt{s} \geq M_Z$. The incoming particles can radiate a number of hard photons which reduces the effective centre-of-mass energy to a value where the cross section is larger, especially towards the strongly resonant Z peak. This process is called radiative return to the Z. In most cases, only one hard photon is emitted, having a typical energy of:

$$E_{\gamma\rightarrow Z} = \frac{\sqrt{s}}{2} \left(1 - \frac{M_Z^2}{s}\right)$$  \hspace{1cm} (2.7)

When a quark pair is produced, also gluons can be emitted. This type of event forms the major background for the qqqq channel. The cross sections for the different types of final states at the lowest and highest centre-of-mass energies used in this analysis are given in Table 2.2. All these processes have been extensively studied at L3 [110]. The generator kk2f [111] was used for the $\mu^+\mu^-(\gamma)$, $\tau^+\tau^-(\gamma)$ and $q\bar{q}(\gamma)$ processes, while for the last one, events have also been generated with PYTHIA as a cross check. To simulate the Bhabha process, the following programs were used: BHAGENE [112], BHWIDTH [113] and TEEGG [114].

<table>
<thead>
<tr>
<th>$\sqrt{s}$</th>
<th>$e^+e^-\rightarrow q\bar{q}(\gamma)$</th>
<th>$e^+e^-\rightarrow e^+e^- (\gamma)$</th>
<th>$e^+e^-\rightarrow \mu^+\mu^-(\gamma)$</th>
<th>$e^+e^-\rightarrow \tau^+\tau^-(\gamma)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>189 GeV</td>
<td>97</td>
<td>1532</td>
<td>8.4</td>
<td>8.2</td>
</tr>
<tr>
<td>208 GeV</td>
<td>80</td>
<td>1262</td>
<td>6.7</td>
<td>6.6</td>
</tr>
</tbody>
</table>

Table 2.2: The cross sections for the different $e^+e^-\rightarrow f\bar{f}(\gamma)$ processes at the lowest and highest centre-of-mass energy used in this analysis. A phase space cut ($8^\circ < \theta_e < 172^\circ$) for the Bhabha process has been applied.

$e^+e^- \rightarrow e^+e^- \gamma \gamma \rightarrow e^+e^- X$

The incoming electrons and positrons constantly radiate and reabsorb virtual photons. It often happens that two such photons interact with each other to form a final state $X$. When this final state is a lepton anti-lepton pair, the dominant diagram is the multi-peripheral process as shown in Fig. 2.2. In the case of hadronic final states, also other diagrams need to be considered, such as the vector meson dominance model diagram, also given in Fig. 2.2. The incoming particles continue along the beam pipe carrying away a large fraction of the energy, resulting in small visible energy. But the tails of the distributions might overlap with the signal since the cross section for these processes is very large (see Table 2.3). The leptonic two-photon processes are generated with DIAG36 [115], while PHOJET [116] is used for the hadronic events. All these processes have also been measured in L3 [117].

**Cosmic Ray Background**

Energetic cosmic ray muons can travel through the earth above L3 and reach the detector. Most of the tracks of the cosmic ray particles will pass far from the interaction
Figure 2.2: The dominant lowest order Feynman diagrams for the main background sources: $e^+e^- \rightarrow ZZ$ (first row), $e^+e^- \rightarrow Zee$ (second row), $e^+e^- \rightarrow ff(\gamma)$ (third row) and $e^+e^- \rightarrow e^+e^-\gamma\gamma \rightarrow e^+e^-X$ (last row).
2.3. The $qqqq$ Selection

At least four jets are expected to be seen in the detector giving rise to a large number of particles. A $qqqq$ candidate is shown in Fig. 2.3. It obviously contains four jets. There is also no, or very small, missing energy and low momentum imbalance. Since the decaying quarks are produced back-to-back in the W rest frame which is not boosted by a large amount\(^3\), the event is evenly spread out in phase space and looks quite spherical.

First a pre-selection consisting of loose cuts is applied to reduce the large number of events detected in L3. Then a neural network is used to separate signal from background events.

2.3.1 Pre-selection

The pre-selection is based on the general event topology and is the same at all centre-of-mass energies. The requirements are:

<table>
<thead>
<tr>
<th>$\sqrt{s}$ (GeV)</th>
<th>$\gamma\gamma \rightarrow \text{hadrons}$</th>
<th>$\gamma\gamma \rightarrow e^+e^-$</th>
<th>$\gamma\gamma \rightarrow \mu^+\mu^-$</th>
<th>$\gamma\gamma \rightarrow \tau^+\tau^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>189</td>
<td>4190</td>
<td>640</td>
<td>570</td>
<td>380</td>
</tr>
<tr>
<td>208</td>
<td>4660</td>
<td>710</td>
<td>680</td>
<td>430</td>
</tr>
</tbody>
</table>

Table 2.3: The cross sections for the different $\gamma\gamma \rightarrow X$ processes at the lowest and highest centre-of-mass energy used in this analysis. A phase space cut is applied by requiring a minimum mass for the leptonic channels ($M_{ee}, M_{\mu\mu} > 3.0 \text{ GeV}/c^2$ and $M_{\tau\tau} > 4.0 \text{ GeV}/c^2$) and a minimum energy for the hadronic channel ($E_{qq} > 15 \text{ GeV}$).

Machine Background

Two kinds of interactions can take place in the beam pipe which have nothing to do with the physics of interest. Since the vacuum in the beam pipe is not perfect, the beam can interact with residual gas. These remnants are randomly distributed in the beam pipe and have low momenta. The second type of interactions are between the tails of the beam and the beam pipe itself. Due to the limits of the focusing system, the electron bunches have a finite transverse size. The accelerated electrons also lose energy through synchrotron radiation resulting in a worsening of the resolution of the beam size. As a consequence, electrons in the beam will occasionally hit the beam pipe.

Both type of events have typically low momentum and are produced under low polar angles. Furthermore, the reconstructed tracks will generally not pass close to the interaction point. Therefore, this type of background is quite easy to eliminate.
2.3. The qqqq Selection

- Four jets containing mostly hadronic particles:
  - \( N_{\text{ASRC}} \geq 20 \) with \( E_{\text{ASRC}} \geq 0.3 \) GeV
  - particles forced to 4 jets using the Durham algorithm
  - \( y_{34} \geq 0.0015 \)
  - \( E_{\text{ECAL}}/E_{\text{vis}} > 0.2 \)

By requiring a large number of clusters, background with low multiplicities like
Bhabhas, cosmic ray muons and \( \gamma\gamma \) events is reduced. The particles of the event are
forced into 4 jets using the Durham algorithm. Events with small values of \( y_{34} \) are
mainly due to events with two or three jets. Electronic noise can fake large energy
depositions in the HCAL. Such events are removed by the requirement that at least
a certain fraction of the visible energy is measured in the ECAL.

- No missing energy:
  - \( E_{\text{vis}} \geq 0.75 \sqrt{s} \)
  - \( |E_{\parallel}| < 0.3 E_{\text{vis}} \)

A large fraction of the total centre-of-mass energy must be measured since only few
particles in qqqq events are not detected. This cut removes a large part of the q\( \bar{q}(\gamma) \),
\( \gamma\gamma \) and semi-leptonic WW events. Further rejection of q\( \bar{q}\gamma \) and \( \gamma\gamma \) events is achieved
by requiring a small longitudinal energy imbalance \( E_{\parallel} \), defined as \( \sum E_{\text{clus}} \cos \theta_{\text{clus}} \).

- Identified \( \gamma \) and e must have small energy and \( \mu \) must have small momentum:
  - \( E_{\gamma,e} < 25 \) GeV
  - \( |p_\mu| < 25 \) GeV/c

Figure 2.3: A transverse (left) and longitudinal (right) view of a qqqq candidate recorded
during the 1999 run at \( \sqrt{s} = 200 \) GeV.
When these types of particles are produced in qqqq events, they typically have energies of a few GeV, unlike such particles in q\bar{q}(\gamma) and semi-leptonic WW events.

After applying these pre-selection cuts, the only significant backgrounds remaining are about 6% of the q\bar{q}(\gamma) events, 10% of the semi-leptonic WW events and most of the hadronic ZZ decays. The purity of this sample is roughly 50%, while the efficiency for selecting hadronic WW events is around 95%.

### 2.3.2 Neural Network Selection

The neural network used for this selection is a Multi-Layer Perceptron [119], where every neuron gets its input from the output of every neuron in the previous layer. There is one input layer with one neuron for each variable used as input, one hidden layer with three neurons and one output layer consisting of one node. For each value of \sqrt{s}, the network is trained on qqqq, q\bar{q}(\gamma) and ZZ Monte Carlo events to obtain an output close to 1 for signal events and close to 0 for background.

The variables fed into the neural network can be classified into three types and are the following:

- There should be four high multiplicity jets uniformly distributed in space:
  - Spherocity:
    \[ S = \min \frac{\left( \sum_i |\hat{n} \times p_i| \right)^2}{\left( \sum_i |p_i| \right)^2}, \]
    where \( p_i \) are the momenta of the particles in the event and the minimisation is done varying the direction of the unit vector \( \hat{n} \). This variable can be understood as the normalised transverse momentum relative to the axis \( \hat{n} \) and ranges between 0 for two exactly back-to-back jets of particles and 1 for completely isotropic events. This separates hadronic WW and ZZ events from q\bar{q}(\gamma) like events.
  - Multiplicity of the jet with the lowest multiplicity: this helps to recognise events containing particles misidentified as jets, such as an energetic ISR photon in q\bar{q}\gamma events.
  - Sum of the cosines of the 6 inter-jet angles: this selects events which are distributed uniformly in the detector.
  - \( y_{34} \): used for the same reason as in the pre-selection.

- Since the reconstructed jets produced in a qqqq event do not differ that much from each other\(^4\), some properties of the most and least energetic jet are used in the neural network:
  - Energy of the most and least energetic jet after the 4C kinematic fit (see Section 3.2).

\(^4\)In principle there is a difference in the energy distribution between the \( d \)- and \( u \)-type quarks in W decays. After jet reconstruction, this difference is typically smaller than jet energy differences in background events.
W-pair Measurement 2.3. The qqqq Selection

The qqqq Selection

Data output

\[ 0.04 \leq \sqrt{s} \leq 209 \text{ GeV} \]

Figure 2.4: The neural network output of the qqqq selection after the pre-selection cuts for all centre-of-mass energies between 189 GeV and 209 GeV combined. The expected contributions from signal and background processes are calculated using Standard Model cross sections. The signal events are peaked towards 1, while the q\bar{q}(\gamma) background is peaked at 0. For further analyses, only events with an output greater than 0.6 are kept.

- Difference in energy of second and third most energetic jets after the 4C kinematic fit.
- Jet broadening of the most and least energetic jet, defined as

\[ \frac{\sum |p_T|^\alpha}{\sum |p|^\alpha}, \]

with \( p \) and \( p_T \) the total and transverse momentum with respect to the jet axis. The sum runs over all the particles in the jet and a value of 0.5 is used for \( \alpha \).

- Good reconstruction of four jets:

  - Probability of the 4C kinematic fit: this must not be small so that the event is kinematically compatible with a WW event.

Fig. 2.4 shows the distribution of the neural network output for all centre-of-mass energies combined. The distribution does not peak at 1 since ZZ events, imitating the signal, are also used to train the network. A cut on the neural network output is placed at 0.6 since this maximises the product of the efficiency and purity.

2.3.3 Performance of the Selection

The selection efficiency decreases from 88.1% at 189 GeV to 83.4% at 208 GeV, while the purity increases from 79.0% to 81.3%. The remaining significant background sources are q\bar{q}(\gamma) and ZZ events, as can be seen in Fig. 2.5. The details of this selection at all centre-of-mass energies are given in Appendix A.
2.4 The qqν Selection

The typical signatures of a qqν event, namely two jets and one energetic lepton identified as an electron, can be seen in the event display shown in Fig. 2.6. From both the transverse and longitudinal view it is clear that there is some missing momentum.

2.4.1 Cut Based Selection

Dedicated cut variables and cut values have been determined to reduce the background while keeping as many signal events as possible. The selection for the 1998 data is slightly modified for 1999 and 2000 data by introducing a new cut variable and reoptimising the other cut values. The latter ones are listed below, while those used for the selection of 1998 data are given between brackets, if they differ.

- An isolated energetic electron:
  - $N_e \geq 1$ with $E_e \geq 20$ GeV
  - $E_e/E_{\text{ECAL}}^{15^\circ} \geq 0.80$ (0.85)

At least one electron, identified as described in Section 2.2.2, with energy greater than 20 GeV must be present. More than 80%(85%) of the electron energy must be in a cone of 15° half-opening angle around the electron direction. This ensures that the electron is well isolated and is unlikely to come from, for instance, the decay of a hadron inside a jet.
2.4. The \(q\bar{q}e\nu\) Selection

- Significant hadronic activity:
  \[N_{\text{ASRC}} \geq 14 \ (15) \text{ with } E_{\text{ASRC}} \geq 0.1 \text{ GeV}\]
  - particles forced to 2 jets using the Durham algorithm

Requiring a considerable number of clusters in the detector ensures that low multiplicity background like lepton-pair production and non-hadronic \(\gamma\gamma\) events are not selected. After removing the clusters and tracks associated with the identified electron, the remaining objects are forced into two jets using the Durham algorithm.

- Events consistent with on-shell W-pair production:
  \[M_{jj} \geq 45 \text{ GeV}/c^2 \ (44 \text{ GeV}/c^2)\]
  \[M_{e\nu} \geq 63 \text{ GeV}/c^2\]

The hadronic mass \(M_{jj}\) is calculated from the 4-momenta of the two jets, while the leptonic mass \(M_{e\nu}\) is calculated from the electron and missing momentum. Demanding that those masses are not too far from the W mass ensures that the events are compatible with W-pair production. The cut on the hadronic mass reduces most of the hadronic \(\gamma\gamma\) events, while low leptonic masses are mainly due to \(qq\tau\nu (\tau \rightarrow e\nu\bar{\nu})\) events since the neutrinos carry away a large part of the momentum. The cut value on the latter variable has been chosen to be complementary to the value used in the \(qq\tau\nu\) selection to prevent an overlap between the two selected samples.

- If a \(\mu\) is detected, it must be close to a jet:
  \[p_{T}(\mu, \text{jet}) \leq 12 \text{ GeV}/c \ (16 \text{ GeV}/c)\]
The decay of a hadron inside one of the jets can produce electrons or muons which generally follow closely the direction of the jet. However, it is possible that the angle between one of those decay leptons and the jet is large enough so that it is identified as an isolated lepton. For example, when a $qq\mu\nu$ event is produced, it is possible that an electron escapes from one of the jets. To reduce this background, the event is only accepted when a reconstructed muon is close to a jet by demanding that the transverse momentum of that muon with respect to the nearest jet, $p_T(\mu, \text{jet})$, is smaller than the indicated value.

- No missing energy in the direction of the beam pipe:
  \[ |\cos\theta_{\text{miss}}| \leq 0.96 \]
  \[ \alpha_{ej} \cdot |\sin\theta_{\text{miss}}| \geq 5^{\circ}(0^{\circ}) \]

Here, $\theta_{\text{miss}}$ is the polar angle of the missing momentum vector. A $q\bar{q}\gamma$ event with an energetic photon can resemble a $qq\nu$ event when the emitted photon escapes through the beam pipe and an electron escapes from one of the jets. For this reason, events with missing momentum along the beam pipe are not selected. For the 1999 and 2000 data, an extra variable is used to separate this kind of background, namely the separation angle between the electron and the nearest jet, $\alpha_{ej}$. This value is multiplied with $|\sin\theta_{\text{miss}}|$, since this is also small for this kind of background and will enhance the separation power of this cut.

- Difference between electron energy and momentum of the associated track:
  \[ E_e - c|\vec{p}_{\text{ATRK}}| \leq 68 \text{ GeV (66 GeV)} \]

The main purpose of this cut variable is to reduce the $q\bar{q}\gamma$ events where the photon has converted into an electron-positron pair.

### 2.4.2 Performance of the Selection

The number of selected $qq\nu$ candidates at each value of $\sqrt{s}$ can be seen in Fig. 2.7, where it is compared with the number of expected signal and background events. The efficiency varies over the analysed centre-of-mass energy range from 78.1% down to 72.5% at the highest energy, while the purity of the sample is very high, varying slightly from 91.3% to 89.9%. The largest remaining contributions to the background come from $q\bar{q}(\gamma)$, $Zee$ and the other $qq\ell\nu$ channels. The details of this selection at all centre-of-mass energies are given in Appendix A.

### 2.5 The $qq\mu\nu$ Selection

The signatures of a $qq\mu\nu$ event look very similar to those of a $qq\nu$ event, except that the energetic lepton must be consistent with the signature of a muon. Two events, one containing a muon reconstructed as an AMUI and one reconstructed as a MIP, are displayed in Fig. 2.8.
2.5. The $qq\mu\nu$ Selection

Figure 2.7: The total number of $qq\mu\nu$ candidates selected at the different centre-of-mass energies compared with the number of expected signal and background events, assuming Standard Model cross sections.

Figure 2.8: A transverse view (left) of a $qq\mu\nu$ candidate where the muon is reconstructed as an AMUI at $\sqrt{s} = 204.76$ GeV and a longitudinal view (right) of an event containing a MIP (lower left) recorded during the 2000 run at $\sqrt{s} = 205.29$ GeV.
2.5.1 Cut Based Selection

The muon candidate is chosen as the most energetic AMUI and if no such object is present, as the MIP with largest TEC momentum. Although the muon reconstruction is different for AMUI’s and MIP’s, the variables used to separate signal from background events are mostly the same. However, the cut values might differ since they are optimised for the two classes separately. All the cut values have been evaluated at all centre-of-mass energies, leading to the conclusion that only the cut on $W$ shows a significant dependence on the energy.

- An identified muon and hadronic activity:
  - $N_{\text{AMUI}} \geq 1$ or $N_{\text{MIP}} \geq 1$
  - $N_{\text{ASRC}} \geq 10$ with $E_{\text{ASRC}} \geq 0.1$ GeV
  - particles forced to 2 jets using the Durham algorithm

Of course there must be at least one muon reconstructed as an AMUI or a MIP, as described in Section 2.2.2. The number of clusters in the event must be large enough to remove all the background events with low multiplicity like lepton pair production and non-hadronic $\gamma\gamma$ events. Before the particles are forced into two jets using the Durham algorithm, the tracks and clusters associated with the identified muon(s) are removed from the event.

- Events consistent with on-shell W-pair production:
  - AMUI: $25 \text{ GeV}/c^2 \leq M_{jj} \leq 125 \text{ GeV}/c^2$ and $M_{\mu\nu} \geq 53 \text{ GeV}/c^2$
  - MIP: $50 \text{ GeV}/c^2 \leq M_{jj} \leq 98 \text{ GeV}/c^2$

The hadronic mass $M_{jj}$ is calculated from the 4-momenta of the two jets, while the leptonic mass $M_{\mu\nu}$ is calculated from the muon and missing momentum. The cut on the hadronic mass ensures that the event is consistent with the production of a W boson and removes most of the hadronic $\gamma\gamma$ events, which typically have low masses. The cut on the leptonic mass for AMUI events is chosen to be complementary to the value used in the $qq\mu\nu$ selection to avoid an overlap between the two channels.

- Momentum and decay angle of the muon:
  - AMUI: $P^* \geq 18.5$ GeV/$c$
  - MIP: $P^* \geq 15$ GeV/$c$

where $P^* = |\vec{p}_\mu| - 10(\cos \theta^* + 1)$ (GeV/$c$), with $\theta^*$ the decay angle of the muon in the W rest-frame. There exists a correlation between the muon momentum and the decay angle in the W rest-frame due to the W polarisation: higher momentum muons tend to go more often in the direction of the W boson. The use of this variable strongly reduces the background from qq$\tau\nu$ events, with $\tau \rightarrow \mu \nu_\tau \bar{\nu}_\mu$, since the muons in these events have lower momenta. This correlation is shown in Fig. 2.9 for both qq$\mu\nu$ and qq$\tau\nu$ events. At the same time, this cut will also remove a significant amount of the qq$(\gamma)$ background.
A well separated muon with no missing momentum along the beam pipe and no energetic photon present:

- AMUI: $\alpha_{ij} \cdot \sin \theta_{\text{miss}} \geq 5.5^\circ$ and $E_\gamma/E_\gamma^{\rightarrow Z} \leq 0.7$
- MIP: $\alpha_{ij} \cdot \sin \theta_{\text{miss}} \geq 20^\circ$

As for $qq\gamma$ events, a $qq\gamma$ event can fake a $qq\mu\nu$ event when the emitted photon escapes along the beam pipe and a muon emerges from one of the jets. The same variable is used to reduce this type of background, namely the product of the angle between the muon and the closest jet, $\alpha_{ij}$, and the sine of the polar angle of the missing momentum vector. The background for the AMUI class can be further reduced by requiring that the energy of an isolated photon must be considerably smaller than the energy of a Z return ISR photon, $E_\gamma^{\rightarrow Z}$ (see Eq. 2.7).

Events should not be compatible with ZZ events:

- AMUI: $M_{\mu\mu} \leq 73$ GeV/$c^2$
- MIP: $\beta_W > 0.34 - 0.49$ (189 – 208 GeV)

Most ZZ background comes from the $q\bar{q}\mu^+\mu^-$ channel, where one Z decayed into a muon pair. Therefore, when two muons are completely reconstructed, their invariant mass is required to be considerably smaller than the Z mass. For MIP’s, there are no completely reconstructed muons, and instead the velocity of the two-jet system is used. This velocity is defined as $\beta_W = (c|\vec{p}_{j1} + \vec{p}_{j2}|)/(E_{j1} + E_{j2})$. 

Figure 2.9: The correlation between the muon momentum, $p_\mu$, and the muon decay angle in the W rest-frame, $\cos \theta_\mu^*$, for $qq\mu\nu$ (top) and $qq\tau\nu$ (bottom) generated events. For the latter, the average muon momentum is lower and the correlation is less pronounced due to the extra neutrino in the event. The solid line represents the cut variable $P^* = 18.5$ GeV/$c$. 

- Most ZZ background comes from the $q\bar{q}\mu^+\mu^-$ channel, where one Z decayed into a muon pair. Therefore, when two muons are completely reconstructed, their invariant mass is required to be considerably smaller than the Z mass. For MIP’s, there are no completely reconstructed muons, and instead the velocity of the two-jet system is used. This velocity is defined as $\beta_W = (c|\vec{p}_{j1} + \vec{p}_{j2}|)/(E_{j1} + E_{j2})$. 

2.6. The $qq\tau\nu$ Selection

The $qq\tau\nu$ events can be classified in two classes according to the $\tau$ decay. The first class is the leptonic decay, where either an electron or a muon is produced, in 17.8% and 17.4% of the cases, respectively. The hadronic decay is the second class, with a branching fraction of 64.8%. An example of both type of events can be seen in Fig. 2.11. The $\tau$ decay always involves at least one neutrino and together with the neutrino from W decay, it will carry away a large fraction of the momentum. The selection of $qq\tau\nu$ events is based on these properties.

2.6.1 Tau Identification

First, the event is searched for the presence of an isolated electron or muon candidate and if none is found, a hadronic $\tau$ jet is reconstructed. Depending on the outcome, the event...
is classified as either a leptonic or hadronic tau decay.

**Leptonic tau decays.** The identification of electrons and muons is described in Section 2.2.2. Although muons having a MIP signature are reconstructed, they are discarded in the selection, as will be discussed in the next section.

**Hadronic tau decays.** To identify the hadronic $\tau$ jet, the cone algorithm described in Section 2.2.2 is used. There must be at least three reconstructed jets in the event and the three most energetic ones are used to identify the $\tau$ jet using a neural network with five input variables: the number of TEC tracks and calorimetric clusters in the jet, the electromagnetic energy, the mass and half opening angle of the jet. The probability of misidentification of the hadronic $\tau$ jet using this neural network is less than 20%. Since the branching ratio for the decay to more than 3 charged particles is less than 0.1% and since it would be difficult to separate such events from $q\bar{q}(\gamma)$ and $qqqq$, the number of TEC tracks in a $\tau$ jet is required to be less than 4.

### 2.6.2 Cut Based Selection

The cut variables common for all types of events are described first, while the ones specific for each final state will be described afterwards. The variables are based on the following characteristics:

- High multiplicity:
  - $N_{\text{ATRK}} > 5$
  - $N_{\text{ASRC}} > 15$ with $E_{\text{ASRC}} \geq 0.1$ GeV
2.6. The qqτν Selection

Requiring a lot of activity in the tracking detector and both calorimeters reduces the background coming from low multiplicity events, such as the purely leptonic final states, considerably.

- **Neutrinos carrying away energy and momentum:**
  - \( E_\perp > 10 \text{ GeV} \)
  - \( E_{\text{vis}} - c|\vec{p}_{\text{miss}}| < 135 \text{ GeV} \)
  - \( |\vec{p}_{\text{miss}}| + cM_{\text{vis}} > 110 \text{ GeV}/c \)

By requiring a large transverse energy imbalance, \( E_\perp = \sum E_{\text{clus}} \sin \theta_{\text{clus}} \), a not too large difference between the visible energy and missing momentum, \( E_{\text{vis}} - c|\vec{p}_{\text{miss}}| \) and a large sum of missing momentum and visible mass, \( |\vec{p}_{\text{miss}}| + cM_{\text{vis}} \), the contamination from q\overline{q}(\gamma) and qqqq events is largely reduced. The visible mass \( M_{\text{vis}} \) is the invariant mass of all the reconstructed objects combined.

- **Low lepton-neutrino masses:**
  - \( \tau \rightarrow e: M_{\nu e} < 63 \text{ GeV}/c^2 \)
  - \( \tau \rightarrow \mu: M_{\nu \mu} < 53 \text{ GeV}/c^2 \)
  - \( 50 \text{ GeV}/c^2 < M_{jj} < 110 \text{ GeV}/c^2 \)

Since the leptonic decay chain \( W \rightarrow \tau \nu \rightarrow l\nu
\nu(l = e, \mu) \) involves three neutrinos, the invariant mass of the system composed of the lepton and missing momentum, here denoted with \( \nu \), will be significantly lower than the same mass in the case of qq\overline{q}ν and qqqq events. The position of the cut is opposite to the values used in those selections to avoid an overlap. The selection of a certain mass region of the two jet system ensures that the event is compatible with the production of a W boson.

- **Production of a τ jet for hadronic τ decay events:**
  - \( \alpha_{q\overline{q}\tau} < 6 \text{ srad} \) and \( \theta_{q_1} - \theta_{q_2} < 2.5 \text{ rad} \)
  - \( |\cos \theta_{\text{miss}}| < 0.91 \)
  - \( E_{\tau,\text{ECAL}} < 35 \text{ GeV} \) if \( E_{\tau,\text{HCAL}} < 2 \text{ GeV} \)
  - τ jet not compatible with MIP

In order to separate the qqτν events with hadronic τ decay from qqqq events, the solid angle spanned by the jets arising from the two quarks and the tau, \( \alpha_{q\overline{q}\tau} \), and the difference in polar angle between the two quark jets, \( \theta_{q_1} - \theta_{q_2} \), must be smaller than the values indicated. To get rid of the q\overline{q}γ events where the photon escapes along the beam pipe, the cosine of the polar angle of the missing momentum, \( |\cos \theta_{\text{miss}}| \), must not be too large. To reduce the contamination of q\overline{q}ν events when the electron is not identified, a maximum on the energy of the τ jet measured in the ECAL is imposed when the energy deposit in the HCAL is small. If the τ jet is compatible with a MIP signature, the event is rejected in order to reduce background coming from qqμν events where the μ is not reconstructed in the muon chambers.
• Events with $\tau \to h$ consistent with on-shell W-pair production:

\[ \tau \to h: \quad M_{\tau\nu} > 40 \text{ GeV}/c^2 \text{ and } 50 \text{ GeV}/c^2 < M_{jj} < 110 \text{ GeV}/c^2 \]

By requiring the invariant mass of the hadronic $\tau$ jet plus neutrino to be large and the invariant mass of the two hadronic jets to be around the W mass, most of the $\gamma\gamma$ events and some $q\bar{q}(\gamma)$ events will be removed.

### 2.6.3 Performances of the Selection

As can be seen in Fig. 2.12, the purity of the selected sample is rather low compared to the other WW channels, ranging from 62.3% to 66.2%. The main contamination comes from the other $q\bar{q}\ell\nu$ channels and $q\bar{q}(\gamma)$ events, while there are small contributions from ZZ, Zee, $\gamma\gamma$ and $\tau\tau$ events. The efficiency for selecting a $q\bar{q}\tau\nu$ event varies between 55.1% and 50.3%. The details of this selection at all centre-of-mass energies are given in Appendix A.

![Figure 2.12: The total number of $q\bar{q}\tau\nu$ candidates selected at the different centre-of-mass energies compared with the number of expected signal and background events, calculated using Standard Model cross sections.](image)

### 2.7 Cross Section Measurement

From the events selected as described in the previous sections, the total CC03 W-pair production cross section, $\sigma_{WW}$, can be derived. A total likelihood is constructed as the product of the likelihoods from the different decay channels.
2.7. Cross Section Measurement

For a qqℓν channel $i$, the likelihood is defined as the Poisson probability of detecting $N_i$ events while expecting $\mu_i = \sigma_i^{\exp} \mathcal{L}$ events. The expected cross section is given by:

$$\sigma_i^{\exp} = \sum_j \epsilon_{ij} B_j \sigma_{WW} + \sigma_{\text{acc, backgr},i},$$

(2.8)

where $\epsilon_{ij}$ is the efficiency for accepting events from channel $j$ in the selection of channel $i$, $B_j$ are the WW branching fractions of the different decay channels and $\sigma_{\text{acc, backgr},i}$ is the sum of the accepted cross sections from all background sources contaminating channel $i$. In the case of qqℓν events, the signal cross section $\sigma_{WW}$ is modified by a CC03 to 4f conversion factor to take into account contributions from other four-fermion processes.

The derivation of the total cross section is different for the qqqq channel. A likelihood as function of the signal cross section is constructed by fitting the neural network output after the pre-selection (see Fig. 2.4).

More details of this cross section measurement can be found in [120] and references therein along with the final results. These are shown in Fig. 2.13 from which can be seen that the measurement is in agreement with the Standard Model prediction. It should be pointed out that for this measurement the leptonic channels are also included using the same likelihood based on Poisson probability as for the qqℓν channels.

Figure 2.13: The cross section of the process $e^+e^- \rightarrow WW \rightarrow fff(\gamma)$ measured by the L3 collaboration as a function of the centre-of-mass energy. This measurement is performed using all the possible WW decay channels and systematic errors are included. The solid curve shows the prediction of YFSWW3 and RACOONWW, which are in agreement up to 0.3% and which both have an uncertainty of 0.5% for $\sqrt{s} \geq 170$ GeV (see Section 1.7), indicated by the width of the band.
Chapter 3
TGC Fit Method

This chapter will describe how values for the triple gauge-boson couplings are extracted from the WW events obtained with the selection criteria explained in the previous chapter. First, a set of variables sensitive to anomalous couplings has to be identified. The definition of these phase space angles will be given in the next section. The reconstruction of these variables is explained in detail in Sections 3.3 and 3.4, preceded by a short explanation of kinematic fitting to improve the resolutions. These resolutions, as well as a comparison of the actual distributions between the data and Monte Carlo predictions are shown in Section 3.5. These comparisons are used to determine the TGC values with a fit method that is formulated in Section 3.6, followed by a section describing extensive technical tests. The last section gives the fit values for the TGC’s using all these ingredients.

3.1 The Phase Space Angles

Each of the four decay particles in a $W^+W^- \rightarrow 4f$ event is described by three variables, since the particle masses are known. Only eight out of these 12 variables are independent since the total energy and momentum is conserved in $e^+e^-$ collisions. In the absence of beam polarisation, the overall azimuthal angle of the event carries no information and in the narrow-width approximation the $W$ masses are fixed to their on-shell values. This gives three extra constraints. Under these assumptions, five variables are required to completely describe a $W^+W^-$ event.

Besides a change in total cross section, the existence of anomalous couplings will affect the relative contribution of each helicity state of the $W$ boson (see Section 1.4.3). This would directly manifest itself in a change of the angular distribution of the produced $W$ bosons. Also the angular distribution of the decay products will be affected since these angles are excellent polarisation analysers due to the $'V-A'$ structure of the $W$-fermion vertex. Consequently, the following natural angles are used in the measurement of the TGC’s:

- $\cos \Theta_{W^-}$: the cosine of the angle between the incoming electron, $e^-$, and the outgoing $W^-$ boson, which corresponds to the polar angle of the $W$ since the positive $z$-direction coincides with the direction of the $e^-$ beam.
3.2 Kinematic Fit

The resolutions of the measured energies and angles of the reconstructed jets and leptons can be improved by putting extra constraints on the event, based on general physical concepts. A $\chi^2$ minimisation is used to find the energies and momenta of the particles that satisfy these constraints and are closest to the measured quantities given their errors.

The energies and momenta of the incoming electrons and positrons are known very
accurately and the event can be constrained by imposing energy and momentum conservation:

\[ \sum_{i=1}^{4} E_i = \sqrt{s}, \quad \sum_{i=1}^{4} \vec{p}_i = 0, \]  

(3.1)

where the sum runs over the reconstructed jets and/or leptons produced in W-pair decays. These equations neglect possible initial state radiation. This is not a problem as long as the MC generator treats ISR correctly and the MC events are treated in the same way as data.

An extra constraint can be applied by requiring that the two reconstructed masses of the W’s produced in the event are equal:

\[ M_{W^+} = M_{W^-}. \]  

(3.2)

This constraint can be relaxed by requiring for instance that the difference between the two masses is smaller than the W width, but this has no visible effect on the final resolutions.

These kinematic fits are applied to channels that contain at most one undetected neutrino. All four or five constraints will be used in the fit applied to the hadronic channel (4C or 5C fit). However, for the qqeν and qqμν channels the total momentum is automatically conserved since it is used to calculate the neutrino momentum. This means that only one or two constraints can be used for these channels (1C or 2C fit). Since there are at least two neutrinos in a qqτν event, it is not possible to use a kinematic fit. Nevertheless, the hadronic jet energies are rescaled with a common factor so that their total energy equals the beam energy. This can be done since they are well separated from the tau jet.

The largest improvements are obtained for the resolutions of the jet energies. A comparison of the jet energy resolutions before and after kinematic fits can be seen in Fig. 3.2 for qqqq events. An improvement in the resolution of the energies and angles of the jets and leptons leads to an improvement in the resolution of the quantities reconstructed from them, such as the W mass, or the production and decay angles.

### 3.3 Reconstruction of the W^- Production Angle

The first step in the reconstruction of the above defined phase space angles is the determination of the W^- production angle, \( \cos \Theta_{W^-} \). Once this angle is known, the momenta of the decay particles can be boosted to their parent rest frame to obtain the decay angles.

#### 3.3.1 The qqℓν channel

In the case of W-pair events, the reconstruction of the W^- production angle is the simplest for semi-leptonic decays, where two jets and an energetic lepton are detected. The W angle is just determined from the hadronic part of the event while the W charge is resolved by looking at the lepton charge, which should be the same as the charge of its parent W. If the charge of this lepton is negative, the angle of the W obtained from the hadronic part is shifted by 180°, to obtain the distribution of the negative W. The measurement of the
3.3. Reconstruction of the W− Production Angle

The energy resolution of the jets in qqqq events before (shaded histogram) and after a 4C (dashed histogram) and 5C (solid histogram) kinematic fit. The WW events were generated at a centre-of-mass energy of 205 GeV and passed the standard hadronic selection.

lepton charge is described in Section 2.1.3, while a detailed study of its performance is given in Section 4.1.2.

3.3.2 The qqqq channel

For the purely hadronic decays, the reconstruction is complicated due to the presence of four rather similar jets in the detector. These jets can be combined into two pairs in three different ways and a dedicated algorithm is used to decide which combination is the correct one. The pairing algorithm used for this analysis, described in the next section, gives the correct pairing for about 77% of the events at all centre-of-mass energies. As can be seen in the left plot of Fig. 3.3, a wrong pairing worsens the cosθW− resolution and constitutes an irreducible background.

Then, the charge of each W is determined by adding the jet charges, the measurement of which will be described after the pairing algorithm. The negatively charged W is chosen to be the W with the smallest sum of jet charges. The distribution of the difference between the charges of the two pairs is shown in the right plot of Fig. 3.3. This method yields an efficiency for correct charge determination of about 70% for correctly paired events, which remains constant over the analysed centre-of-mass energy range.

Jet Pairing

The pairing method that has formerly been used [121] is based on the smallest mass difference of the two pairs after a 4C kinematic fit with a veto on the smallest sum of the two masses. The problem with this method is the drastic decrease in efficiency to get the correct pairing for higher centre-of-mass energies, as can be seen in Fig. 3.4.
Figure 3.3: The resolution of the $W^{-}$ production angle at 189 GeV (left) and the charge difference between the two $W$'s reconstructed with the jet charge technique at 196 GeV (right), both for $qqqq$ generated events. The shaded histogram is the distribution for correctly paired events, while the hatched one results from wrongly paired jets.

Figure 3.4: The pairing efficiency as a function of the centre-of-mass energy for the pairing algorithm based on the mass difference of the pairs only (dots) and on a neural network using the variables described in Section 3.3.2 (stars). The efficiency is determined from Monte Carlo events.
3.3. Reconstruction of the W^- Production Angle TGC Fit Method

In order to solve this problem, a detailed study of the jet pairing has been performed \[122\], combining a larger number of variables and comparing different methods. This study has shown that the best performance is obtained using a neural network (NN) which has one hidden layer of 25 nodes and the following input variables:

- Difference of pair masses: $\Delta M = |M(\text{pair 1}) - M(\text{pair 2})|$
  The mass of a pair of jets, $M(\text{pair } i)$, is obtained by adding the 4-momenta of the jets after applying a 4C fit. Since the choice of order of the pairs is arbitrary, the absolute value for the difference is used.

- Sum of pair masses: $\Sigma M = M(\text{pair 1}) + M(\text{pair 2})$

- Sum of di-jet angles: $\Sigma \alpha = \alpha(\text{pair 1}) + \alpha(\text{pair 2})$
  The di-jet angle, $\alpha(\text{pair } i)$, is the angle between the 2 jets forming pair $i$.

- Minimum di-jet angle: $\min \alpha = \min(\alpha(\text{pair 1}), \alpha(\text{pair 2}))$

- Difference of pair energies: $\Delta E = |E(\text{pair 1}) - E(\text{pair 2})|$
  The energy of a pair, $E(\text{pair } i)$, is the sum of the energy of the jets that form this pair.

- Minimum jet energy difference: $\min \Delta E = \min(\Delta E(\text{pair 1}), \Delta E(\text{pair 2}))$
  The jet energy difference, $\Delta E(\text{pair } i)$, is the absolute difference of the energy of the two jets in pair $i$.

- Matrix elements of combination: $|M(\text{pair 1, pair 2, } M_W)|^2$
  The matrix elements for W-pair production, $|M|^2$, are calculated with EXCALIBUR. A value of 80.50 GeV is taken for $M_W$, but a variation of this parameter does not influence the efficiency of the algorithm.

- Difference of pair charges: $\Delta Q = Q(\text{pair 1}) - Q(\text{pair 2})$
  The charge of a pair, $Q(\text{pair } i)$, is the sum of the charges of the two jets forming this pair. The determination of the jet charge is described in the next section.

The network is trained at each centre-of-mass energy using 40000 qqqq generated events in such a way that the correct combination has an output close to one and the wrong combination an output close to zero. The NN output for the correct and wrong pairs for $WW \rightarrow qqqq$ events generated at 207 GeV is shown in the left plot of Fig. 3.5. The small peak for the correctly paired events towards zero is due to badly reconstructed events. Also certain event topologies exist for which it is difficult to separate between two combinations. In most of these cases, all possible combinations will have small values of the NN output.

The combination having the highest NN output is then chosen to be the pairing used in the TGC analysis. Using this method, a pairing efficiency of about 77% is obtained at all centre-of-mass energies as can be seen in Fig. 3.4. The distribution of the NN output of the pairing with the highest output, for all the data between 189 and 209 GeV combined, is shown in the right plot of Fig. 3.5.
Figure 3.5: The neural network output of the pairing algorithm. The left plot shows the distribution for correctly and wrongly paired $qqqq$ events generated at 207 GeV. The right plot compares the highest neural network output between data and MC prediction for all the centre-of-mass energies between 189 and 209 GeV combined.

Jet Charge Assignment

The charge determination in the hadronic channel is a bit more complicated than for single tracks since each of the four jets contains a number of charged and neutral particles. The charge of the original quark can be estimated from the charge distribution of the hadronisation particles inside the jet [123]. The charge of the jet is defined as a rapidity weighted average of the charges of the particles in the jet:

$$Q_{jet} = \frac{\sum_i q_i (p_i^\parallel)^\kappa}{\sum_i (p_i^\parallel)^\kappa},$$

(3.3)

where $q_i$ is the charge of a particle with momentum $p_i^\parallel$ parallel to the jet axis,

$$p_\parallel = \vec{p} \cdot \frac{\vec{p}_{jet}}{||\vec{p}_{jet}||}$$

with $\vec{p}$ the momentum vector of the particle in the jet with total momentum vector $\vec{p}_{jet}$. A Monte Carlo study showed that the value $\kappa = 0.5$ gives the optimal efficiency for the reconstruction of the charge of the original quark [124].

The result of this method can be seen in Fig. 3.6, where the jet charge for up- and down-type quarks from WW generated events at 202 GeV are shown. The resulting charge confusion, the probability to reconstruct the wrong charge, is about $39.21 \pm 0.01\%$ for the
3.4. Reconstruction of the W Decay Angles

Once the production angle of the negative W boson has been determined, the momenta of the decay products are boosted to their parent’s rest frames, which are moving with known velocities $\beta(W^\pm) = (1 - 4m^2_W/s)^{1/2}$. At this stage, it is necessary to separate between the particle and anti-particle coming from one W.

The angular dependence of the decay (anti-)particles is identical for the $W^-$ and the $W^+$ decays [125]. This means that the following transformation between the decay angles of the fermion and the anti-fermion exists:

$$
\theta_f \leftrightarrow \pi - \theta_f, \quad \phi_f \leftrightarrow \pi + \phi_f. $$

(3.4)

For the leptonically decaying W, it is easy to determine whether the lepton is the particle or anti-particle by looking at its charge. For the hadronic decays however, it is very difficult to distinguish between the quark and the anti-quark, since the charge determination for a single jet is poor and no adequate flavour tagging can be performed. Although charm tagging could be performed by identifying open charm mesons in the jet, efficiencies in L3 are in general low and would not lead to a significant increase in sensitivity. Therefore, an ambiguity according to (3.4) exists between the hadronic decay

Figure 3.6: The distribution of the jet charges for down-type quarks with charge $-1/3$ (shaded histogram) and up-type quarks with charge $2/3$ (hatched histogram) in $WW \rightarrow qqqq$ events generated at 202 GeV. The charge of each jet has been determined using Eq. (3.3) and the opposite of the measured charge has been taken for the anti-quarks.

down-type quarks and $36.77 \pm 0.01\%$ for the up-type quarks. It is clear that it is difficult to make a separation between the two types of quarks. For the analysis presented here, only the sum of the jet charges is of interest, as this determines the charge of the parent W. The charge confusion of the hadronically decaying W bosons is studied in detail in Section 4.1.3.

3.4 Reconstruction of the W Decay Angles

Once the production angle of the negative W boson has been determined, the momenta of the decay products are boosted to their parent’s rest frames, which are moving with known velocities $\beta(W^\pm) = (1 - 4m^2_W/s)^{1/2}$. At this stage, it is necessary to separate between the particle and anti-particle coming from one W.

The angular dependence of the decay (anti-)particles is identical for the $W^-$ and the $W^+$ decays [125]. This means that the following transformation between the decay angles of the fermion and the anti-fermion exists:

$$
\theta_f \leftrightarrow \pi - \theta_f, \quad \phi_f \leftrightarrow \pi + \phi_f. $$

(3.4)

For the leptonically decaying W, it is easy to determine whether the lepton is the particle or anti-particle by looking at its charge. For the hadronic decays however, it is very difficult to distinguish between the quark and the anti-quark, since the charge determination for a single jet is poor and no adequate flavour tagging can be performed. Although charm tagging could be performed by identifying open charm mesons in the jet, efficiencies in L3 are in general low and would not lead to a significant increase in sensitivity. Therefore, an ambiguity according to (3.4) exists between the hadronic decay
angles. The distributions of these angles are folded in the following way:

\[
\frac{dN}{d\theta_{fold}} = \frac{dN}{d\theta} + \frac{dN}{d(\pi - \theta)} \quad 0 < \theta < \pi, \quad (3.5)
\]

and

\[
\frac{dN}{d\phi_{fold}} = \frac{dN}{d\phi} + \frac{dN}{d(\pi + \phi)} \quad 0 < \phi < 2\pi. \quad (3.6)
\]

The folded angles are restricted to \(0 < \theta_{fold} < \pi/2\) and \(0 < \phi_{fold} < \pi\).

This means that for the hadronic channel two sets of folded quark decay angles are used. In the specific case when speaking about the quark decay angles from qqqq events, these angles are referred to as \((\cos \theta_{q1}, \phi_{q1})\) and \((\cos \theta_{q2}, \phi_{q2})\).

Since for semi-leptonic events there are folded and unfolded angles which cannot be combined, the choice is made to use the angles of the negative lepton from the \(W^-\) decays and the quark angles for the \(W^+\) decays. This means that when a positive lepton is identified, the decay angles need to be transformed, assuming of course that the \(W^-\) and \(W^+\) behave in the same way. No extra transformation for the quark angles is necessary since these angles are already folded. These angles are referred to as \((\cos \theta_l, \phi_l)\) and \((\cos \theta_q, \phi_q)\).

### 3.5 Resolutions and Distributions

The angular information used in the different WW decay channels is summarised in Table 3.1. Note that for none of the decay channels a full reconstruction of the five phase space angles is possible. All the channels suffer from at least one ambiguity, due to the inability to determine the quark flavours.

The resolutions of the reconstruction of the phase space angles can be determined with MC events by taking the difference between the reconstructed and generated values. The resulting distributions are shown in Fig. 3.7 for the qqqq and qq\(\ell\nu\) channels at a centre-of-mass energy of 205 GeV. The region around zero is then fitted with a Gaussian to obtain the resolution. The tails are fitted with a polynomial, the exact choice of this description has little influence on the value of the resolution.

The energy dependence of the resolution of the \(W^-\) production angle for the different decay channels can be seen in Fig. 3.8. The resolution becomes better with increasing energy, but seems to saturate at the highest centre-of-mass energies. It is interesting to
see that the resolution for the $qq\tau\nu$ events is much worse than for the other channels, which follows from the inability to perform a kinematic fit. The energy dependence of the resolutions for the decay angles is less pronounced.

The distributions of the $W^-$ production angle for the $qqqq$ and combined $qq\ell\nu$ events are shown in Fig. 3.9. The combination is performed by adding the contributions from the different centre-of-mass energies, ignoring any energy dependence in the shape of this distribution (see Section 1.4). It is clear that the $qq\ell\nu$ channels suffer much less from background contamination than the $qqqq$ channel. The distribution for the latter one is also less forward peaked due to the pairing ambiguity and much worse charge assignment. The decay angles from the hadronic channel are shown in Fig. 3.10, while those from the semi-leptonic channels are shown in Fig. 3.11.

All these combined distributions are for illustrative purposes only. The extraction of the couplings will be performed on the 5-dimensional distributions for each channel at each energy separately. The projection of these individual distributions are given in Appendix B.

### 3.6 Fitting Procedure

The extraction of the couplings is done by comparing the five-dimensional differential distributions of the phase space angles in data with Monte Carlo predictions. Such predictions can be obtained by generating many MC samples with different coupling values. However, full MC simulation is a very time-consuming task and therefore this method is generally not favoured. Instead, a reweighting technique is used.

#### 3.6.1 Reweighting

The events of a reference Monte Carlo generated at certain TGC values, usually referred to as baseline MC, are reweighted to other coupling values. Each event $n$ is assigned a weight which is the ratio between the probability $P_n(\psi)$ of this event to occur at coupling values $\psi$ and the probability $P_n(\psi_{\text{ref}})$ for the event to occur at the reference value $\psi_{\text{ref}}$:

$$w_n(\psi) = \frac{P_n(\psi)}{P_n(\psi_{\text{ref}})}.$$  

This weight thus indicates how much more or less probable it is to find this event at coupling values $\psi$ than at $\psi_{\text{ref}}$. The probability of an event to occur at coupling $\psi$ is given by the normalised differential cross section:

$$P_n(\psi) = \frac{1}{\sigma(\psi)} \frac{d\sigma}{d\Omega}(\Omega_n, \psi),$$

where $\Omega$ is the phase space of interest, here it consists of the five angles described in Section 3.1. The normalised cross section can be factorised into a matrix element squared times a coupling-independent factor. The latter part cancels out in the ratio, and the expression for the weight reduces to a ratio of matrix elements:

$$w_n(\psi) = \frac{|\mathcal{M}^{4f}(p_n, \psi)|^2}{|\mathcal{M}^{\text{CC}03}(p_n, \psi_{\text{MC}})|^2},$$  \hspace{1cm} (3.7)
Figure 3.7: The resolution of the reconstructed phase space angles used in the TGC analysis: the $W^-$ production angle $\cos \Theta_{W^-}$ (upper plot), the hadronic decay angles $\cos \theta_q$ and $\phi_q$ (middle and bottom plot on the left) and the leptonic decay angles $\cos \theta_l$ and $\phi_l$ (middle and bottom plot on the right). The shaded histograms are obtained from $qqqq$, the solid from $qqe\nu$, the dashed from $qq\mu\nu$ and the dotted from $qq\tau\nu$ reconstructed $WW$ events generated at 205 GeV with YFSWW3.
3.6. Fitting Procedure

Figure 3.8: The resolution of the $W^-$ production angle $\cos^{}\Theta_{W^-}$ as a function of the centre-of-mass energy for the $qqqq$ and $qql\nu$ channels.

Figure 3.9: The $W^-$ production angle for the $qqqq$ (left) and $qql\nu$ (right) channels combining all centre-of-mass energies. Any energy dependence of this angle has been ignored. The dots represent the data points, the light shaded histogram is the predicted contribution from all WW events, and the dark histogram is the contribution from non-WW background. The MC histograms are normalised to the total integrated luminosity and the WW events are generated with SM couplings. The dashed and dotted histograms show the expectations for anomalous TGC's $g_{1}^{Z} = 2$ and $g_{1}^{Z} = 0$, respectively.
Figure 3.10: The folded decay angles for the qqqq channel for all centre-of-mass energies combined. The top row shows the polar and the bottom row the azimuthal angles. On the left are the angles of the quark that is believed to come from the reconstructed W⁻, while on the right are the angles of the quark from the W⁺. The different MC contributions are normalised to the total integrated luminosity and are obtained using SM couplings. The dashed and dotted histograms show the expectations for anomalous TGC’s $g_1^Z = 2$ and $g_1^Z = 0$, respectively.
Figure 3.11: The decay angles for the combined $qq\ell\nu$ channels for all centre-of-mass energies combined. The leptonic decay angles are shown on the left while the folded hadronic decay angles are on the right. The top plots depict the cosine of the polar angles and the bottom plots the azimuthal angles. Also here the MC histograms are normalised to the total integrated luminosity and are generated assuming SM couplings. The dashed and dotted histograms show the expectations for anomalous TGC’s $g^Z_1 = 2$ and $g^Z_1 = 0$, respectively.
where \( p_n \) are the 4-momenta of the initial and final state particles in the event.

The events used for the baseline MC are generated with \( \text{YFSWW3} \) at Standard Model coupling values and the weights are calculated with the matrix elements from \( \text{EXCALIBUR} \). Since the events of the baseline MC are generated using the CC03 diagrams only, the matrix element for the full 4f process is taken in the numerator to take into account the missing diagrams (see Section 1.6). Using this method of reweighting, the coupling dependence for the 4f background (single-W) is taken into account automatically. This particular choice was made because \( \text{YFSWW3} \) was the only available MC event generator that included the \( \mathcal{O}(\alpha) \) corrections.\(^1\)

The distribution of these event weights can be seen in Fig. 3.12 for the four WW decay channels used in this analysis. The spread of the distribution of the weights increases when more diagrams need to be incorporated. This effect is largest for the \( \text{qqe} \) channel, where 17 extra diagrams need to be taken into account. The \( \text{qq} \) channels receive the smallest corrections, since only 8 extra diagrams are needed. However, the spread of the weights for the reconstructed \( \text{qq} \) events is larger due to the contamination of \( \text{qqe} \) events in the selection. The weights can become very large for non-WW-like events which have more the characteristics of a singly or non-resonant event. The plots show that such events are not present and are thus efficiently removed by the WW event selections. On the same figure, the distributions of the weights for various anomalous couplings are also shown.

A disadvantage of the reweighting technique is encountered when the event weight for the new coupling values differs significantly from the original weight. In this case, poorly populated phase space regions become important and the MC statistical error is large. From the variance of the weights, the number of effective MC events can be calculated:

\[
N_{\text{eff}} = \frac{(\sum w_n)^2}{\sum w_n^2}.
\]

The dependence of this number as a function of several couplings is shown in Fig. 3.13. Its value is always smaller than the true number of MC events and drops rapidly for larger deviations of the couplings from the generated values. Also for larger weight values, phase space regions that are less occupied than other regions can get delusively important due to statistical fluctuations. These effects result in the deterioration of the statistical significance of the fit result. One way to overcome this problem is to generate extra events in poorly populated phase space regions. But when the measurement of the couplings leads to a large deviation from the values used to generate the baseline MC, it is better to regenerate events at coupling values close to the measured ones and to repeat the fit.

### 3.6.2 Binned Maximum Likelihood Fit

For the comparison between the data and the theory prediction, the binned maximum likelihood method is used (for a detailed description, see e.g. [126]). The five-dimensional phase space spanned by the W\(^-\) production and decay angles is divided into bins. For

---

\(^1\)At present, two generators are available that include both four-fermion diagrams and the radiative corrections in DPA. These are \( \text{RACOONWW} \) and \( \text{Kandy} \), both explained in Section 1.7.1. The latter one is used further in this thesis for testing purposes.
3.6. Fitting Procedure

**TGC Fit Method**

**Event weight**

- **qqqq**
  - SM value
  - $g_1^Z = -0.2$
  - $g_1^Z = +0.2$

- **qqeμν**
  - SM value
  - $\kappa_\gamma = -0.5$
  - $\kappa_\gamma = +0.5$

- **qqτν**
  - SM value
  - $\kappa_\gamma = -0.5$
  - $\lambda_Z = +0.2$

**SM value**

- **g_1^Z**
- **λ_γ**
- **κ_γ**
- **κ_γ**

**Figure 3.12:** The distributions of the weights obtained from Excalibur for the YFSWW3 generated events that are reconstructed by the selection procedures of the four channels at 202 GeV. The shaded histogram represents the CC03 to 4f correction at Standard Model coupling values, while the dashed and dotted lines include also the changes due to various anomalous couplings.

For each channel and centre-of-mass energy, a coupling dependent likelihood is defined as the product of Poisson probabilities of occupation in each bin:

$$L_{j,k}(\psi) = \prod_i e^{-\mu_{j,k}(\psi)} \frac{N_{j,k}^i}{N_{j,k}^i!},$$  \hspace{1cm} (3.9)

where $N_{j,k}^i$ is the number of observed events in bin $i$ for channel $j$ at centre-of-mass energy $k$, and $\mu_{j,k}^i$ the corresponding number of expected events.

Since the total cross section is also proportional to the square of the matrix element, the total number of expected events as a function of the coupling values can be obtained by adding all the weights of the accepted MC events. Limited to the $i$-th bin, this results in:

$$\mu_i(\psi) = \sum_{sources} \left( \frac{\sigma_{gen} \mathcal{L}}{N_{gen}} \sum_{n \in bin \ i} w_n(\psi) \right).$$  \hspace{1cm} (3.10)
The first sum runs over all the signal and background sources contributing to the selection, while the second runs over all accepted events \( n \) falling into bin \( i \). The generated cross section \( \sigma_{\text{gen}} \) and the resulting number of events \( N_{\text{gen}} \) are for the concerned process and \( \mathcal{L} \) is the integrated luminosity of the data sample. The events coming from background sources that are coupling independent are assigned a weight equal to one.

The combination of the different channels and centre-of-mass energies is then the product of each individual likelihood:

\[
L(\psi) = \prod_{j,k} L_{j,k}(\psi).
\]

The number of expected events is varied as a function of the couplings, and according to the principle of maximum likelihood, the best estimate for the couplings \( \psi \) are those values that maximise the likelihood function.

### Multi-parameter Fits

In general, fits are performed leaving one or several couplings free to vary, while all other couplings are fixed to their Standard Model expectation.

The effects of the different couplings on the observables are not uncorrelated and therefore their measurements are also correlated. Assume that one coupling has an anomalous value. The fits to the other couplings will then also show a deviation. Investigating the differences between multi-parameter fits will give more insight into which one is really anomalous. This can be done in the following way. First a fit is performed leaving all couplings free. Then the fit is repeated several times fixing one coupling at a time to its SM value. If the real physical value of this fixed coupling is indeed at the SM expectation,
it will hardly affect the location of the maximum. On the other hand, if this fixed coupling was really anomalous, significant changes will be observed in the fit values of the others.

It is also imaginable that the effect of one anomalous coupling is cancelled by another, in which case the anomalous couplings would be obscured in the one-parameter fits. Performing multi-parameter fits would reveal such situations. These multi-parameter fits are thus very convenient to look for the presence of anomalous couplings.

**Fit Results and Errors**

In practice, the natural logarithm of the likelihood function is taken and the products of the different contributions become sums. In order to find the best estimate for the couplings, the software package *minuit* [127] is used. This program is designed to search for minima and thus the negative of the log-likelihood is taken. Since the absolute normalisation of the likelihood is independent of the couplings, the value of the negative log-likelihood at the minimum is subtracted and the difference denoted as $-\Delta \ln L$.

The errors on the fit values from one-parameter fits are obtained from the coupling values where the negative log-likelihood is 0.5 above its minimum. For a Gaussian likelihood, this corresponds to the one standard deviation error and represents the 68% confidence interval. The coupling values at which the negative log-likelihood is 1.92 above its minimum yield the 95% confidence interval.

In quoting the errors on single couplings obtained from multi-parameter fits, the same changes in likelihood are taken. This results in the projected error for a certain coupling, i.e. the error on this coupling when all the other couplings in the fit are left free to vary. The two-dimensional contours at 68% and 95% confidence intervals are determined by the coupling values that change the negative-log likelihood by 1.15 and 3.00, respectively.

The likelihood defined in Eq. (3.9) is constructed from Poisson probabilities. For some samples, the likelihood may acquire an asymmetric form and can have a double minimum. Care should be taken when interpreting the confidence intervals obtained from such likelihoods. These features can be the consequence of statistical fluctuations or due to a systematic effect. In the former case, the likelihood will approach a Gaussian distribution for larger samples.

### 3.7 Technical Tests

Before the results using the fit method described in the previous section are listed, a number of technical tests have to be performed in order to prove the validity of the fit method. First of all, the sizes of the bins used for each of the phase space angles have to be chosen. The next step is to check whether the fit results are good estimators for the real physical values and that the fit errors are correct.

#### 3.7.1 Bin Sizes

In the ideal case, the bins should be as small as possible, especially in regions that are very sensitive to anomalous couplings. However, in order to be able to calculate the number of expected events needed for the likelihood, each bin should contain at least one MC
event. Preferably, the number of MC events in a bin should be large enough not to be sensitive to statistical fluctuations. Since only a limited number of events are available in the baseline MC, the bins can consequently not be too small. Moreover, if the bins are smaller than the experimental resolution, the bin-to-bin migration becomes large and no gain of information is expected.

Bearing in mind the above considerations, it is natural to choose more bins for angles that are more affected by anomalous couplings. To determine the sensitivity of a certain angle, a variable is constructed from the difference in the expected distributions with and without anomalous couplings:

\[
S = \sum_{i}^{\text{bins}} \frac{(N_{i}^{\text{norm}}(\psi) - N_{i}(\psi_{SM}))^{2}}{N_{i}(\psi_{SM})},
\]

where the number of expected events \(N_{i}^{\text{norm}}(\psi)\) at coupling values \(\psi\) is evaluated using the reweighting procedure described above. The label \(\text{norm}\) means that the total number of expected events, thus summed over all bins, is normalised to the total number of expected events at Standard Model values in order to eliminate the cross section dependence.

The sensitivity \(S\) has been determined using the baseline MC events generated at 200 GeV and with a sample with luminosity comparable with the data at this centre-of-mass energy. For the reconstructed phase space angles of the analysed channels, six different numbers of bins between 2 and 30 have been used. The variation is performed for each angle separately to check the sensitivity of the angles with respect to each other and to observe where no more gain can be expected.

The top part of Table 3.2 shows the results for the \(qq\mu\nu\) channel for two anomalous values of \(g_1^Z\). As can be seen, when the bin sizes become smaller, the sensitivity increases. However, at a certain point, there is no significant gain in sensitivity anymore. This is a consequence of the bin-to-bin migration that becomes significant, and which eliminates the benefit of dividing the phase space angle into finer bins.

The same exercise has been repeated for the other decay channels resulting in similar observations. To make a comparison between the channels, the sensitivity using 10 bins for each of the phase space angles is listed in the central part of Table 3.2. It is clear that for negative coupling value — that is, lower than the Standard Model expectation — and for all the channels, the \(W^-\) production angle is much more sensitive than the other angles. However, the sensitivity drops for positive couplings and the relative importance of the decay angles is much larger.

The difference between the \(qq\nu\nu\) and \(qq\mu\nu\) channels is due to the smaller charge confusion for the latter one. Although the charge confusion is much larger for the hadronic channel, the sensitivity of the production angle from hadronic events is similar to the \(qq\nu\nu\) and \(qq\mu\nu\) channels due to the branching ratio which is about three times larger for the \(qqqq\) channel. For the \(qqqq\) channel, the polar decay angles are more sensitive than the azimuthal angles, both for negative and positive couplings. The sensitivity of the angles of the two different decay quarks are also compatible with each other, just like their resolutions. The two leptonic decay angles in the semi-leptonic channels always have comparable sensitivity. This is also the case for the polar angle of the quarks in \(qq\ell\nu\) events for negative coupling values, but the sensitivity decreases for positive values. The hadronic azimuthal decay angle is always the least sensitive variable.
### 3.7. Technical Tests

bin sizes

<table>
<thead>
<tr>
<th>$N_{\text{bins}}$</th>
<th>$g_1^Z = 0.0$</th>
<th>$g_1^Z = +2.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\cos \Theta_W$</td>
<td>$\cos \theta_1$</td>
</tr>
<tr>
<td>2</td>
<td>27.4</td>
<td>0.93</td>
</tr>
<tr>
<td>5</td>
<td>35.1</td>
<td>1.49</td>
</tr>
<tr>
<td>10</td>
<td>36.4</td>
<td>1.65</td>
</tr>
<tr>
<td>15</td>
<td>36.6</td>
<td>1.71</td>
</tr>
<tr>
<td>20</td>
<td>36.7</td>
<td>1.70</td>
</tr>
<tr>
<td>30</td>
<td>36.8</td>
<td>1.77</td>
</tr>
</tbody>
</table>

channels

<table>
<thead>
<tr>
<th>channel</th>
<th>$g_1^Z = 0.0$</th>
<th>$g_1^Z = +2.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\cos \Theta_W$</td>
<td>$\cos \theta_1$</td>
</tr>
<tr>
<td>qqqq</td>
<td>32.2</td>
<td>2.68</td>
</tr>
<tr>
<td>qqeν</td>
<td>24.8</td>
<td>1.98</td>
</tr>
<tr>
<td>qqμν</td>
<td>36.4</td>
<td>1.65</td>
</tr>
<tr>
<td>qqτν</td>
<td>9.00</td>
<td>0.52</td>
</tr>
</tbody>
</table>

couplings

<table>
<thead>
<tr>
<th>$\psi$</th>
<th>$\psi - \psi_{\text{SM}} = -1.0$</th>
<th>$\psi - \psi_{\text{SM}} = +1.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\cos \Theta_W$</td>
<td>$\cos \theta_1$</td>
</tr>
<tr>
<td>$g_1^Z$</td>
<td>24.8</td>
<td>1.98</td>
</tr>
<tr>
<td>$\kappa_\gamma$</td>
<td>3.12</td>
<td>0.59</td>
</tr>
<tr>
<td>$\lambda_\gamma$</td>
<td>26.3</td>
<td>1.79</td>
</tr>
</tbody>
</table>

Table 3.2: The sensitivity $S$ of the five phase space angles, determined using the baseline MC sample generated at 200 GeV. The top part shows the values for different bin sizes at $g_1^Z$ values of 0.0 and 2.0 for the qqqq channel. The central table shows the sensitivity for the different decay channels at two different values of $g_1^Z$ using 10 bins for each of the phase space variables. The bottom part lists the values for different couplings using the qqeν channel and also 10 bins for the angles.

So far, the sensitivities have been discussed using only the coupling $g_1^Z$, but the test has been performed with all the couplings that will be measured. The values derived from the qqeν channel for three different couplings are given in the bottom part of Table 3.2. It is immediately clear that the sensitivity for $\kappa_\gamma$ is much smaller than for $g_1^Z$ and $\lambda_\gamma$. The latter two show comparable sensitivities, although $\lambda_\gamma$ is slightly more sensitive to the production angle. Similar effects are observed for the other couplings $g_5^Z$, $\kappa_Z$ and $\lambda_Z$, as
well as for the different decay channels.

In order not to be affected by bin-to-bin migration, the bin sizes are chosen such that they are always larger than four times the resolution of the reconstructed angle (see Figs. 3.7 and 3.8). This assures that about 95% of the events at certain phase space angles will also be reconstructed in the corresponding bin. These values also roughly correspond to the bin sizes in Table 3.2 where the sensitivity is not significantly increasing anymore. This rule is first applied to the $W^-$ production angle, the most sensitive angle.

The choice of number of bins for the decay angles is mainly dominated by the limited amount of events in the baseline MC. For consistency, the same number of bins have been used for all the hadronic decay angles for all decay channels. The same principle is applied to the leptonic decay angles. Only the hadronic polar angle is limited by its resolution, but the bin sizes for other angles could in principle be made smaller if larger baseline MC samples would be available. However, this would merely lead to a small increase in overall sensitivity.

After all the above considerations, the following number of bins are used for the different phase space angles in the different channels:

<table>
<thead>
<tr>
<th></th>
<th>$\cos \Theta_W$</th>
<th>$\cos \theta_1$</th>
<th>$\phi_1$</th>
<th>$\cos \theta_2$</th>
<th>$\phi_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$qqqq$</td>
<td>12</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$qq\nu$</td>
<td>10</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$qq\mu\nu$</td>
<td>10</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$qq\tau\nu$</td>
<td>8</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

It must be noted that in the fit to the $qq\tau\nu$ channel at both 192 GeV and 202 GeV with the above bin choices, bins containing no events occur. In these two cases, the number of bins used for both the azimuthal decay angles is reduced by one.

### 3.7.2 Bias and Linearity

After choosing the bin sizes, the fit method is checked for the presence of a bias. This is done by performing the fit with the same baseline MC as for the data, generated with \texttt{yfsww3}, but using a large sample of MC events generated with \texttt{Kandy} at Standard Model values for the couplings as pseudo-data. Since these samples contain all four-fermion events, this will also be a test of the 4f reweighting by \texttt{EXCALIBUR} and the inclusion of ZZ and Zee background.

In total, a sample corresponding to an integrated luminosity $\mathcal{L} = 16 \text{ fb}^{-1}$ generated at 200 GeV is used, which is about 25 times the size of the actual total data sample. Fig. 3.14 shows the results from this test using the $qqqq$ and $qq\tau\nu$ events. As can be seen, there is a good agreement between the fitted and Standard Model values at which the samples were generated. The results for some of the couplings using the $qq\nu$ and $qq\mu\nu$ channels can be seen as triangles in Fig. 3.15. These measurements are also compatible with the Standard Model generated values. All these results show that the fit method is bias free at the few percent level.

The next step is to check whether the measurements of anomalous values would represent the real physical values. In order to test this aspect, often called linearity of the
3.7. Technical Tests

Figure 3.14: The results of the bias tests for the $qqqq$ (left) and $qq\tau\nu$ (right) channel. A large sample generated with Kandy has been used as pseudo-data and is fitted with the standard YFSWW3 sample taking into account all four-fermion background. The dashed line shows the Standard Model values for the couplings, used in both generators.

Figure 3.15: The fit results as a function of the generated coupling values using the $qqe\nu$ channel for $g_1^Z$ (top left) and $\kappa_\gamma$ (top right) and using $qq\mu\nu$ events for $\lambda_\gamma$ (bottom left) and $g_5^Z$ (bottom right). The dots show the results from the EXCALIBUR samples generated with anomalous couplings, fitted with KORALW as baseline. The triangle is obtained using events generated with Kandy at Standard Model coupling values as pseudo-data and YFSWW3 as baseline and including all 4f background. The dashed line indicates where the generated and fitted couplings are equal.
**3.7. Technical Tests**

### Table 3.3: The fit results of the linearity tests using $qq\mu\nu$ samples with two or three anomalous values for the couplings $g_1^Z$, $\kappa_\gamma$, and $\lambda_\gamma$. These events, generated with EXCALIBUR, are used as pseudo-data and fitted with KORALW as baseline.

<table>
<thead>
<tr>
<th></th>
<th>$g_1^Z$ (SM = 1)</th>
<th>$\kappa_\gamma$ (SM = 1)</th>
<th>$\lambda_\gamma$ (SM = 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>generated</td>
<td>1.25</td>
<td>—</td>
<td>−0.30</td>
</tr>
<tr>
<td>fitted</td>
<td>1.16 ± 0.07</td>
<td>—</td>
<td>−0.25 ± 0.07</td>
</tr>
<tr>
<td>generated</td>
<td>0.75</td>
<td>1.50</td>
<td>—</td>
</tr>
<tr>
<td>fitted</td>
<td>0.95$^{+0.06}_{-0.18}$</td>
<td>0.85$^{+0.84}_{-0.12}$</td>
<td>—</td>
</tr>
<tr>
<td>generated</td>
<td>—</td>
<td>0.50</td>
<td>0.30</td>
</tr>
<tr>
<td>fitted</td>
<td>—</td>
<td>0.58 ± 0.11</td>
<td>0.25 ± 0.06</td>
</tr>
<tr>
<td>generated</td>
<td>1.25</td>
<td>1.50</td>
<td>0.30</td>
</tr>
<tr>
<td>fitted</td>
<td>1.19 ± 0.04</td>
<td>1.62 ± 0.10</td>
<td>0.28 ± 0.03</td>
</tr>
<tr>
<td>generated</td>
<td>0.75</td>
<td>0.50</td>
<td>−0.30</td>
</tr>
<tr>
<td>fitted</td>
<td>0.75 ± 0.03</td>
<td>0.59 ± 0.07</td>
<td>−0.28 ± 0.03</td>
</tr>
</tbody>
</table>

### Expected Errors

Now that it has been shown that the fit method reproduces the actual coupling value, it has to be checked that the obtained fit error has its usual statistical meaning. For each channel at each centre-of-mass energy, a number of samples is randomly drawn from the baseline MC, each one corresponding to the actual available integrated data luminosity. The events ending up in this pseudo-data sample are removed from the baseline MC and are then fit with this reduced sample. The resulting root mean square (RMS) of the distribution of the fit values is then compared with the mean of the distribution of the errors. If these numbers agree with each other, it means that the error obtained from the fit is a good estimator for the statistical error.

The error on the RMS of the fit values is determined by approximating the distribution by a Gaussian. This is a good approximation, except for the $\kappa$ type of couplings in certain cases. The reason for this is that the cross section is not at its minimum for the SM value of $\kappa$. 

3.7.3 **Expected Errors**

Now that it has been shown that the fit method reproduces the actual coupling value, it has to be checked that the obtained fit error has its usual statistical meaning. For each channel at each centre-of-mass energy, a number of samples is randomly drawn from the baseline MC, each one corresponding to the actual available integrated data luminosity. The events ending up in this pseudo-data sample are removed from the baseline MC and are then fit with this reduced sample. The resulting root mean square (RMS) of the distribution of the fit values is then compared with the mean of the distribution of the errors. If these numbers agree with each other, it means that the error obtained from the fit is a good estimator for the statistical error.

The error on the RMS of the fit values is determined by approximating the distribution by a Gaussian. This is a good approximation, except for the $\kappa$ type of couplings in certain cases. The reason for this is that the cross section is not at its minimum for the SM value of $\kappa$. 

77
Figure 3.16: Comparison between the RMS of the fit results (large plot) and the mean of the corresponding fit errors (small plot) for the different couplings using the $qg\mu\nu$ channel at 200 GeV. The relative error on the RMS varies between 5% ($g^Z_1$) and 10% ($\kappa_{\gamma}$).
Figure 3.17: The fit results and errors for $g_1^Z$ using the $q\bar{q}e\nu$ channel at three different centre-of-mass energies, where an equal number of samples are drawn from the baseline MC at the corresponding energy. The error on all the three RMS values is about 7%.

Figure 3.18: Comparison between the distributions of the fit results and errors for $\lambda_\gamma$ using the different decay channels at 206 GeV. The relative error on the RMS is 8% for the $q\bar{q}\tau\nu$ channel and less than 6% for the others.
3.8. Fit Results

All the results that are quoted in this section are, unless otherwise stated, a combination of all hadronic and semi-leptonic decay channels using the data taken at centre-of-mass energies between 189 and 209 GeV. This sample corresponds to a total integrated luminosity of $\mathcal{L} = 629.36$ pb$^{-1}$ (see Table 2.1).

A first set of measurements is performed under the assumption of the custodial $SU(2)_c$ symmetry. This introduces relations among the coupling parameters as given in Eq. (1.15). These coincide with the constraints obtained under a linear realisation of the $SU(2)_L \times U(1)_Y$ symmetry, see Eq. (1.19c). The three couplings that are chosen to be measured under these assumptions are $g_1^Z$, $\kappa_\gamma$ and $\lambda_\gamma$ and will be referred to as set 1. In the Standard Model, the latter coupling is expected to be 0 and the other two 1.

The one-parameter fits are obtained by varying one of the couplings while fixing all

<table>
<thead>
<tr>
<th>coupling</th>
<th>RMS</th>
<th>MEAN</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_1^Z$</td>
<td>0.031 ± 0.001</td>
<td>0.031</td>
</tr>
<tr>
<td>$\kappa_\gamma$</td>
<td>0.084 ± 0.002</td>
<td>0.084</td>
</tr>
<tr>
<td>$\lambda_\gamma$</td>
<td>0.034 ± 0.001</td>
<td>0.032</td>
</tr>
<tr>
<td>$g_5^Z$</td>
<td>0.122 ± 0.002</td>
<td>0.121</td>
</tr>
<tr>
<td>$\kappa_Z$</td>
<td>0.054 ± 0.001</td>
<td>0.052</td>
</tr>
<tr>
<td>$\lambda_Z$</td>
<td>0.047 ± 0.001</td>
<td>0.045</td>
</tr>
</tbody>
</table>

Table 3.4: The combination between all the analysed channels and centre-of-mass energies of the RMS of the fit results and the means of the fit errors. The combination has been done under the assumption that these values are distributed as a Gaussian.

$\kappa$, but around $\kappa_\gamma = 1.4$. It can thus happen that two minima are found, and sometimes, the non Standard Model minimum is preferred by the fit. This effect can be seen in Fig. 3.16 around $\kappa_\gamma = 2$. The same figure also shows the distributions for the other couplings for the $q\bar{q} \mu \nu$ channel at 200 GeV. A good agreement between the RMS of the fit results and the mean of the fit errors is observed. Figs. 3.17 and 3.18 show some of the distributions at different centre-of-mass energies and for different channels, respectively. Also from these plots it can be seen that there is a good agreement between the RMS and mean values.

Both the resulting means of the fit errors and RMS values of the fit results have been combined between the different centre-of-mass energies and channels under the assumptions that they are distributed as a Gaussian. The result of this combination is given in Table 3.4. Also the combined numbers show good agreement between the RMS of the fit results and the mean of the fit errors, indicating that the error from the fit is a good estimator.

These tests also show that the mean of the distribution of the fit results is always close to the Standard Model expectation. This indicates that the fit method is also bias free for finite sampling, confirming the conclusion from the previous section.

3.8 Fit Results

All the results that are quoted in this section are, unless otherwise stated, a combination of all hadronic and semi-leptonic decay channels using the data taken at centre-of-mass energies between 189 and 209 GeV. This sample corresponds to a total integrated luminosity of $\mathcal{L} = 629.36$ pb$^{-1}$ (see Table 2.1).
the other couplings at their SM expectation. The results are:

\[ g_1^Z = 0.932^{+0.036}_{-0.034} \quad \text{(SM = 1)}, \]
\[ \kappa_\gamma = 0.854^{+0.067}_{-0.062} \quad \text{(SM = 1)}, \]
\[ \lambda_\gamma = -0.056^{+0.039}_{-0.036} \quad \text{(SM = 0)}, \]

with \( \kappa_Z = g_1^Z - \tan^2 \theta_w (\kappa_\gamma - 1) \) and \( \lambda_Z = \lambda_\gamma. \)

The errors shown are only statistical and reflect the 68\% C.L. intervals. The 95\% C.L. intervals of the combined results for the three-parameter fit to these couplings, thus allowing all three to vary simultaneously, are:

\[ 0.79 < g_1^Z < 1.06 \quad \rho(g_1^Z, \kappa_\gamma) = -0.23, \]
\[ 0.73 < \kappa_\gamma < 1.35 \quad \rho(\kappa_\gamma, \lambda_\gamma) = -0.13, \]
\[ -0.10 < \lambda_\gamma < 0.17 \quad \rho(\lambda_\gamma, g_1^Z) = -0.75. \]

The numbers on the right are the correlation coefficients between two parameters obtained from the fit.

The fit has been repeated relaxing the constraints for the couplings \( g_5^Z, \kappa_Z \) and \( \lambda_Z \). They will be referred to as set 2 and their Standard Model expectations are respectively 0, 1 and 0. This results in the following combined one-parameter fit results:

\[ g_5^Z = 0.00^{+0.15}_{-0.15} \quad \text{(SM = 0)}, \]
\[ \kappa_Z = 0.877^{+0.061}_{-0.057} \quad \text{(SM = 1)}, \]
\[ \lambda_Z = -0.097^{+0.071}_{-0.062} \quad \text{(SM = 0)}. \]

The last type of fit performed is a five-parameter fit to all the couplings that are affected by the aforementioned constraints, but without actual applying them. The combination gives the following result at 95\% C.L.:

\[ 0.88 < g_1^Z < 1.86 \quad \text{correlation coefficients :} \]
\[ 0.68 < \kappa_\gamma < 1.20 \]
\[ 0.56 < \kappa_Z < 1.57 \]
\[ -0.23 < \lambda_\gamma < 0.44 \]
\[ -0.50 < \lambda_Z < 0.43 \]

\[ \begin{array}{cccccc}
  g_1^Z & \kappa_\gamma & \kappa_Z & \lambda_\gamma & \lambda_Z \\
  1.00 & 0.00 & -0.91 & 0.02 & -0.89 \\
  \kappa_\gamma & 1.00 & 0.17 & 0.20 & -0.16 \\
  \kappa_Z & 1.00 & -0.10 & 0.76 \\
  \lambda_\gamma & 1.00 & 0.27 \\
  \lambda_Z & 1.00 
\end{array} \]
3.8. Fit Results

The negative log-likelihood distributions for the combined one-parameter fits are shown in Fig. 3.19. It is clear that these likelihood distributions show a parabolic behaviour around the minimum. The results for the different years of data taking, shown on the same plot, are in good agreement with each other. The same conclusions can be drawn comparing the results from the different WW decay channels, which can be seen in Fig. 3.20. The results for the \( \lambda \) couplings from the \( qq\tau\nu \) channel at first sight deviate from the other fits. However, the corresponding likelihoods have an asymmetric form, although they do not possess a second minimum. The individual likelihood distributions from the one-parameter fits for both sets of couplings from each channel at each centre-of-mass energy are given in Appendix C. Some likelihoods have double minima which are due to statistical fluctuations, since these shapes disappear when the full data sample is combined.

In Fig. 3.21, a comparison for the set 1 couplings is made between the fit results obtained using a different number of parameters. It can be seen that the SM expectations lie just outside 68% C.L. but well within the 95% C.L. contours of the two-parameter fit. It is interesting to compare the 68% C.L. contour from the two-parameter fit with the projection of the three-parameter fit onto the corresponding plane. To obtain the latter one, the same value of the negative log-likelihood above its minimum has been chosen as for the two-parameter fit contours. A good agreement between the two contours can be observed. The contours containing \( \kappa_\gamma \) are stretched out slightly for most of the fits, which is due to the appearance of a double minimum. The areas of the three-parameter fits are always a bit larger than the corresponding two-parameter contours. This can be expected since the same amount of information has to be divided over three instead of two couplings. Furthermore, it can be seen that fixing a second coupling to its predicted value also has little effect on the fit results. These observations are a strong indication that any deviation observed in the fit results is merely due to a statistical fluctuation. But before making any conclusion, the systematic errors on these measurements need to be determined. The final results will be discussed in Chapter 6.
Figure 3.19: The negative log-likelihood distributions from the one-parameter fits with the minimum subtracted, $-\Delta \ln L$, for both the set 1 and set 2 couplings. The combined result is given by the solid line. These distributions are also shown separately for the different years of data taking: 1998 (dashed line), 1999 (dotted) and 2000 (dashed-dotted). The horizontal line at 0.5 above the minima indicates the 68% C.L. intervals.
Figure 3.20: A comparison between the fit results obtained from the different WW decay channels. The errors are statistical only. The band represents the combined result while the line shows the Standard Model expectation.
Figure 3.21: A comparison between the one-, two- and three-parameter fit results for the couplings $g_1^Z$, $\kappa_\gamma$, and $\lambda_\gamma$, the errors are only statistical. The star indicates the Standard Model expectation at tree level. The solid lines show the result when two couplings are fixed to the SM expectation, while the shaded areas are the 68% and 95% C.L. regions when two couplings are varied simultaneously. The dashed line is obtained from the projection of the three-parameter fit using the same value above the minimum as for the 68% C.L. contours of the two-parameter fit. The closed and open dots are the minima obtained from the two- and three-parameter fits, respectively.
Chapter 4

Systematic Checks

The fit method presented in the previous chapter is based on the comparison between experimental measurements and theoretical predictions. It is therefore necessary to check that quantities that might affect the relevant distributions are adequately described by the Monte Carlo generation and simulation programs. If this is not the case, a bias or incorrect error may result.

A very important component of the TGC measurement is the assignment of the charge to the reconstructed W boson. This assignment is studied in great detail in Section 4.1 for both the semi-leptonic and the hadronic decay channels. Another cause of potentially significant effects on the phase space angles is a bad modelling of the background contribution. This modelling will be verified in Section 4.3. Further checks that are performed are the study of the contribution from the total cross section (Section 4.2), the effect of differences in centre-of-mass energies between data and MC (Section 4.4) and the stability of the fit result with respect to the sizes of the bins (Section 4.5). Finally, a comparison with other fit methods is performed as a cross check in Section 4.6.

4.1 Charge Measurement Studies

It is necessary to have a good W charge determination because the angle most sensitive to anomalous couplings is the reconstructed production angle of the negative W boson (see Section 3.7.1). Since the W bosons are produced back-to-back, a wrongly reconstructed charge will shift this angle by 180 degrees, i.e. the cosine of this angle will flip sign.

When the charge confusion, the probability to measure the wrong charge, increases, the separation power between the positive and negative W decreases, leading to a loss of sensitivity. An uncertainty on the charge confusion leads to an uncertainty on the predicted angular distributions resulting in a systematic error. Moreover, a difference between the real charge confusion and the value used in the Monte Carlo samples, leads to a W production angular distribution in the MC different from what one really expects to see in the detector, and hence introduces a bias. For these reasons, the charge confusion in W-pair events has been studied in detail.
4.1. Charge Measurement Studies

4.1.1 Charge Confusion

The charge of charged particles can be determined from the tracks reconstructed in the TEC. Since this detector has a finite spatial resolution, it is possible to reconstruct the wrong charge. This is especially the case for high momentum tracks, which follow an almost straight path. On the other hand, low momentum tracks undergo more multiple scattering, also resulting in an increased charge confusion. Since the charge assignment depends on the measurement of the transverse momentum of the track, the efficiency for the charge determination will depend on the energy and polar angle of the particle. With decreasing polar angle, fewer and fewer TEC wires are available for the momentum measurement. This leads to a poorer momentum resolution and consequently to a larger probability to misreconstruct the charge.

When a muon track is reconstructed in the barrel part of the muon detector with at least two segments (AMUI), the charge measured in this part of the detector is taken because of its very good momentum resolution. This results in a charge confusion which is less than 1\% [128]. When a track is reconstructed in the forward or backward chambers or if a muon is reconstructed as a MIP, the charge is determined using the TEC track.

The measurements of the charge confusion of the W boson for both the $qqq\bar{q}$ and $qq\ell\nu$ channels are extracted directly from the data. This value is compared to the one obtained from the MC, since a difference between these two numbers would be the source of bias in the couplings measurement. In the case of semi-leptonic events, the charge of the W boson is determined by the leptonically decaying W. Therefore, the W charge confusion is directly related to the lepton charge confusion, which will be discussed in Section 4.1.2. For the hadronic channel, the decision on the charge of the W is based on the comparison of the jet charges. The charge confusion of a hadronically decaying W boson is described in Section 4.1.3. The results obtained in the following sections will be used in Section 5.6 to determine the systematic error on the TGC measurement due to the charge confusion. Possible differences will be corrected by shifting the charge confusion in the MC to the values measured in the data.

4.1.2 Charge Confusion for $W \rightarrow \ell\nu$

The charge confusion for the lepton can be determined from lepton-pair production, i.e. $e^+e^- \rightarrow Z/\gamma^* \rightarrow \ell^+\ell^-$, by counting the events with two equal or different charges. The probability to get an event with two times the same charge is

$$P_{\text{equal}} = \frac{N_e}{N_e + N_d},$$

with $N_e$ ($N_d$) the number of events where both leptons have the same (different) charge. These events can be directly related to the charge confusion of a single lepton, $C_l$, since

$$P_{\text{equal}} = 2C_l(1 - C_l).$$

Solving this quadratic equation for $C_l$ and rejecting the non-physical solution having a probability greater than 1, the charge confusion for a lepton is

$$C_l = \frac{1}{2}(1 - \sqrt{1 - 2P_{\text{equal}}}), \quad (4.1)$$
with a corresponding error of

$$\sigma(C_l) = \frac{1}{2}\sqrt{\frac{P_{\text{equal}}(1 - P_{\text{equal}})}{N_d - N_e}}. \quad (4.2)$$

Since the reconstructed charge in the MC samples can be compared directly to the generated value, the charge confusion can also be calculated as

$$C_l^{\text{MC}} = \frac{N_w}{N_w + N_c} \quad (4.3)$$

where $N_c$ and $N_w$ are respectively the number of correctly and wrongly reconstructed charges. The error on this quantity is

$$\sigma(C_l^{\text{MC}}) = \sqrt{\frac{C_l^{\text{MC}}(1 - C_l^{\text{MC}})}{N_w + N_c}}. \quad (4.4)$$

The charge confusion for the electrons has been studied using e-pair (Bhabha) and $\tau$-pair production, while the measurement of the muon charge has been checked with $\mu$-pair samples. The calibration runs at the Z resonance peak have been used in order to have a large sample of events. The corresponding integrated luminosities for the different years of data taking are given in Table 2.1.

**Charge Confusion Study with Bhabha Events**

For this study, a sample of a total integrated luminosity $\mathcal{L} = 29.80 \text{ pb}^{-1}$ is generated at a centre-of-mass energy $\sqrt{s} = 91.25 \text{ GeV}$ with BHWIDTH for polar angles of the outgoing $e^+$ and $e^-$ satisfying $8^\circ < \theta < 172^\circ$. In both the data and the MC sample, two back-to-back charged tracks are selected using the same criteria as used for the identification of an electron in the $qq\nu\nu$ channel (see Section 2.2.2). In order to reject practically all two-photon background, the sum of the energies of the electrons are further required to be compatible with the Z boson mass, i.e. $88 \text{ GeV} \leq E_{\text{tot}} \leq 94 \text{ GeV}$.

As can be seen from Eq. (2.1), the charge assignment depends on the measurement of the transverse momentum of the track. Since the selected electrons are quasi mono-energetic, their transverse momentum is related to the polar angle by $p_T = p_e \sin \theta_e \approx 0.5\sqrt{s} \sin \theta_e$. Therefore the charge assignment is measured as function of the cosine of the polar angle of the track. Three regions in the barrel part of the detector ($|\cos \theta| < 0.25$, $0.25 \leq |\cos \theta| < 0.50$ and $0.50 \leq |\cos \theta| \leq 0.75$) and one in the end-caps ($0.80 \leq |\cos \theta| \leq 0.98$) are considered. The distribution of the charge divided by the momentum of the tracks in these four regions is shown for the 1999 Z peak data in Fig. 4.1. In the barrel part of the detector, the resolution improves for decreasing polar angles. This is expected, since the distance travelled trough the TEC increases, while the energies are equal. The resolution in the end-caps is much worse since fewer wires can be used to determine the track curvature.

Using Eq. (4.1), the charge confusion can be extracted by counting the equally and unequally charged pairs and the results are listed in Table 4.1. The values obtained from
4.1. Charge Measurement Studies

Systematic Checks

Figure 4.1: The charge $q$ divided the momentum $p_e$ for both tracks of the Bhabha sample selected in the 1999 Z peak data. The distribution is shown for the four separate regions of the polar angle of the tracks.

The charge confusion values obtained for the 2000 data are larger than for the other two years. During this year, the high voltages on the cathode wires of the TEC were set to lower values than the nominal ones. This regime had been chosen in order to cope with the higher level of backgrounds associated with the specific mode of LEP running that year. The effect is seen both in the data and MC samples, indicating that this is taken into account in the detector simulation.

Taking the ratio $R$ between the values obtained from data and MC allows to compare the different years. These values, given in the same table, are shown in Fig. 4.2. It can be seen that for all parts of the detector the real charge confusion is significantly larger than the one modelled in the MC samples. The ratios for the barrel part of the detector are all in agreement with each other and a constant fit to all the values results in $R_{\text{barrel}} = 1.535 \pm 0.060$. For the end-caps, the difference is smaller and a combination of the different years gives a value of $R_{\text{end-caps}} = 1.282 \pm 0.040$. 

---

The MC are in agreement with the results obtained by comparing the measured charge with the generated one.
Table 4.1: The results from the charge confusion study with Bhabha events at the Z peak from both the data and MC samples. The last column contains the ratio $R$ between data and MC charge confusion, indicating the level of agreement with each other.

| $| \cos \theta_e |$ range | $C_{\text{data}}$ [%] | $C_{\text{MC}}$ [%] | $R = C_{\text{data}}/C_{\text{MC}}$ |
|----------------|----------------|----------------|-----------------|
| 1998 barrel [0.00, 0.25] | 5.86 ± 0.74 | 4.60 ± 0.19 | 1.27 ± 0.17 |
| 1998 [0.25, 0.50] | 5.69 ± 0.64 | 3.08 ± 0.14 | 1.85 ± 0.22 |
| 1998 [0.50, 0.75] | 3.66 ± 0.45 | 2.42 ± 0.11 | 1.51 ± 0.20 |
| 1998 end-caps [0.80, 0.98] | 26.8 ± 1.4 | 20.85 ± 0.29 | 1.287 ± 0.068 |
| 1999 barrel [0.00, 0.25] | 6.13 ± 0.62 | 4.26 ± 0.18 | 1.44 ± 0.16 |
| 1999 [0.25, 0.50] | 4.80 ± 0.51 | 3.14 ± 0.14 | 1.53 ± 0.18 |
| 1999 [0.50, 0.75] | 3.50 ± 0.39 | 2.22 ± 0.11 | 1.58 ± 0.19 |
| 1999 end-caps [0.80, 0.98] | 26.6 ± 1.3 | 20.84 ± 0.29 | 1.276 ± 0.065 |
| 2000 barrel [0.00, 0.25] | 8.25 ± 0.80 | 5.17 ± 0.20 | 1.60 ± 0.17 |
| 2000 [0.25, 0.50] | 6.43 ± 0.60 | 3.57 ± 0.15 | 1.80 ± 0.18 |
| 2000 [0.50, 0.75] | 4.28 ± 0.45 | 3.01 ± 0.13 | 1.42 ± 0.16 |
| 2000 end-caps [0.80, 0.98] | 28.1 ± 1.5 | 21.83 ± 0.31 | 1.287 ± 0.073 |

Figure 4.2: The ratio between the charge confusion in Bhabha events measured in the data and MC as a function of the absolute value of the cosine of the polar angle of the electron track, $| \cos \theta_e |$. The results from the years 1998, 1999 and 2000 are shown separately. The two lines represent the average of the ratios from all the years for the entire barrel and end-caps regions, respectively.
Charge Measurement Studies

4.1. Charge Measurement Studies

Systematic Checks

The transverse momentum of the electron track, \( p_T \), measured in the TEC as a function of the cosine of the polar angle of this track, \( \cos \theta_e \), for reconstructed \( q\bar{q}e\nu \) events generated as W-pair events at 189 GeV. The dashed line indicates values of \( p_T = 4 \) GeV/c.

**Figure 4.3:** The transverse momentum of the electron track, \( p_T \), measured in the TEC as a function of the cosine of the polar angle of this track, \( \cos \theta_e \), for reconstructed \( q\bar{q}e\nu \) events generated as W-pair events at 189 GeV. The dashed line indicates values of \( p_T = 4 \) GeV/c.

**Charge Confusion Study with \( \tau \)-pair Events**

The drawback of using Bhabha events is the fact that the charge confusion is only measured at lepton energies around 45 GeV, thus having a fixed transverse momentum at a certain polar angle. However, the transverse momentum of the lepton in semi-leptonic W-pair events can have different values for all directions, as is shown in Fig. 4.3.

The charge confusion for tracks with a broader \( p_T \) spectrum can be studied using \( \tau^+\tau^- \) events, where both \( \tau \)-leptons decay into one charged particle. In such a 1-prong \( \tau \) decay, which happens in about 85% of the cases [79], there is always at least one \( \nu \) produced, which carries away a fraction of the energy. The energy and momentum of the charged particle will therefore cover a wide range.

The \( \tau \)-pair production was studied for the years 1998, 1999 and 2000 using the Z peak data [129]. The requirement of two back-to-back \( \tau \) jets, with similar definition as for the q\q\tau\nu selection (see Section 2.6), and missing momentum carried away by neutrinos, selects \( \tau \)-pairs. Further stringent cuts are imposed on the number of tracks and clusters and on the momenta of the tracks. This removes as much background as possible from the sample without losing too many signal events and results in an efficiency of 62.0% and purity of 86.7%. The efficiency is the same for events with equally and unequally charged tracks.

The charge confusion is studied as function of the inverse of the transverse momentum of the particles and separately for the barrel and end-caps. The results from this study are given in Table 4.2. As can be seen in Fig. 4.3, the transverse momentum of the electron in W-pair events is always larger than 4 GeV/c. Therefore only the results from this study using events with an inverse transverse momentum smaller than 0.25 (GeV/c)^{-1} are considered.

As expected, the charge confusion is higher for particles with high transverse momen-
<table>
<thead>
<tr>
<th>1/(p_T) range ([GeV/c]⁻¹)</th>
<th>barrel</th>
<th>end-caps</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(C_{\text{data}}) [%]</td>
<td>(C_{\text{MC}}) [%]</td>
</tr>
<tr>
<td>[0; 0.035]</td>
<td>4.80 ± 0.88</td>
<td>5.14 ± 0.64</td>
</tr>
<tr>
<td>[0.035; 0.060]</td>
<td>2.36 ± 0.38</td>
<td>1.24 ± 0.13</td>
</tr>
<tr>
<td>[0.060; 0.10]</td>
<td>1.05 ± 0.19</td>
<td>0.500 ± 0.039</td>
</tr>
<tr>
<td>[0.10; 0.25]</td>
<td>1.33 ± 0.18</td>
<td>0.729 ± 0.040</td>
</tr>
</tbody>
</table>

Table 4.2: The combined results from the charge confusion study using 1-prong \(\tau\) decays in \(\tau\)-pair production for all the years considered. This measurement is performed as a function of 1/\(p_T\) for barrel and end-caps separately and only results for tracks with \(p_T \geq 4\) GeV/c are shown.

The ratio between the data and MC, shown in Fig. 4.4, is fitted with a constant resulting in \(R_{\text{barrel}} = 1.46 \pm 0.14\) and \(R_{\text{end-caps}} = 1.70 \pm 0.22\). A comparison with the results from Bhabha events can be made by comparing with the appropriate \(p_T\) ranges.

The Bhabha events reconstructed in the barrel part have transverse momenta ranging from 30 to 45 GeV/c. This means that these events should be compared with the results from the first 1/\(p_T\)-bin from the \(\tau\)-pair analysis. The ratios between data and MC charge confusion obtained from the two methods differ significantly by about 2.8\(\sigma\). However, the fit to the ratio values \(R\) over the different 1/\(p_T\) bins in the \(\tau\)-pair analysis is in good agreement with the \(R\) value from the \(e^+e^-\) analysis.

The transverse momenta of the Bhabha events reconstructed in the end-caps roughly correspond to the second and third 1/\(p_T\)-bin from Table 4.2. Combining these two numbers, a value \(R = 1.39 \pm 0.32\) is obtained which is compatible with the Bhabha results. However, the \(R\) value obtained from a fit through the 1/\(p_T\)-bins differs by about 1.9\(\sigma\) with the Bhabha measurement. The reason might be that for the two types of events, the number of wires that are available for the track reconstruction as a function of the momentum spectra changes quite differently.

The charge confusion in the baseline MC is corrected to the measured one for semileptonic events that obtain their charge information from the TEC track. This correction is done using the results from the \(\tau\)-pair analysis given in Table 4.2, thus depending on the transverse momentum of the track and whether it was reconstructed in the barrel or end-caps. This correction is already applied to the distributions shown in Figs. 3.9 and 3.11.

### Charge Confusion Study with \(\mu\)-pair Events

Although the charge confusion for the muons reconstructed in the barrel part of the muon chambers is small, it has been cross checked using a similar approach as for the electrons. The Z peak data from 1998, 1999 and 2000 has been analysed by selecting \(\mu\)-pair events with two muons identified as an AMUI (see Section 2.2.2). Extra requirements on the timing of the scintillators ensures a rejection of all the cosmic muon background. Further, the two tracks have to be back-to-back and their momenta should be consistent with the
4.1. Charge Measurement Studies Systematic Checks

4.1.1 Charge Measurement Studies

Figure 4.4: The ratio between the charge confusion from data and MC for the barrel (dots) and end-caps (triangles) in $\tau$-pair events with two one-prong decays. The solid (dashed) line represents a constant fit to the ratios in the barrel (end-caps) region.

decay of a $Z$ boson.

The measurement is performed for different polar angle regions of the barrel detector. All the results are consistent with each other, also over the various years. A combination of these numbers results in the charge confusions $C_{\text{data}} = 0.47 \pm 0.06\%$ and $C_{\text{MC}} = 0.49 \pm 0.02\%$. The charge confusion for muons reconstructed in the barrel part of the muon detector is indeed very low and well modelled by the detector simulation.

4.1.3 Charge Confusion for $W \rightarrow q\bar{q}$

Although the charge determination for the individual quarks using the jet charge algorithm, described in Section 3.3.2, has already been checked in detail [130], it has never been checked for hadronic $W$ decays, where only the sum of the two jet charges is of interest. This charge confusion can again easily be measured from data using $qq\mu\nu$ events, since the sum of the jet charges can be compared to the muon charge.

Charge Confusion Study with $qq\mu\nu$ Events

The standard $qq\mu\nu$ selection (see Section 2.5) is applied to all the data with $\sqrt{s} \geq 189$ GeV and the same MC samples for $WW$, $q\bar{q}(\gamma)$ and $ZZ$ events are used. The identified muon is further required to have at least 2 out of 3 possible P-segments in the barrel part of the muon chambers.

The charge of the muon is determined and compared to the charge distribution of the sum of the two jets. When a negatively charged muon is detected, the sign of the sum of jet charges is reversed and added to the distributions obtained with positive muons. This distribution now represents the charge measurement for a negatively charged hadronically decaying $W$ boson, which can be seen in Fig. 4.5 for all the data combined.
4.1. Charge Measurement Studies

Figure 4.5: The sum of the jet charges for hadronically decaying $W^-$ bosons in $qq\mu\nu$ events measured at $189 \text{ GeV} \leq \sqrt{s} \leq 209 \text{ GeV}$. Both the results for positively and negatively charged muons are used. For the latter one the opposite of the sum of jet charges is taken to obtain the charge of the negative $W$ boson.

The charge confusion can be extracted by determining the fraction of events that have the same sign for the muon charge and the sum of jet charges. This number is derived by integrating over a Gaussian fit to this distribution. Due to the very high statistics of the MC sample, the tails in this distribution cannot be neglected. Therefore, the sum of the jet charges extracted from MC is fitted with a double Gaussian. The result applying this method on the full data set is given in Table 4.3. Since changes in the performance of the TEC will have an effect on the jet charge measurement, the results are also separated into the different years. The same table shows that no large differences are observed. The results obtained from positively and negatively charged muons are also in agreement with each other.

<table>
<thead>
<tr>
<th></th>
<th>$C_{\text{data}}$ [%]</th>
<th>$C_{\text{MC}}$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998</td>
<td>$36.7 \pm 2.4$</td>
<td>$31.45 \pm 0.25$</td>
</tr>
<tr>
<td>1999</td>
<td>$32.7 \pm 2.2$</td>
<td>$30.50 \pm 0.14$</td>
</tr>
<tr>
<td>2000</td>
<td>$34.4 \pm 2.2$</td>
<td>$31.78 \pm 0.18$</td>
</tr>
<tr>
<td>combined</td>
<td>$34.5 \pm 1.3$</td>
<td>$31.03 \pm 0.12$</td>
</tr>
</tbody>
</table>

Table 4.3: The charge confusion for $W \rightarrow qq$ extracted from data and Monte Carlo measured with $qq\mu\nu$ events at $189 \text{ GeV} \leq \sqrt{s} \leq 209 \text{ GeV}$. The separate contributions of the three years of data taking considered are also given.

Since this charge assignment determines the sign of the cosine of the production angle of the negative $W$ for $qqqq$ events, the charge confusion is studied as a function of this angle. From Fig. 4.6, it can be seen that the charge confusion is higher when the $W$ is produced in the forward or backward direction. This can easily be understood, since
the decay fermion follows preferentially the direction of the $W^-$ boson in its rest frame. The effect is enhanced when the momenta are boosted to the lab frame, although at LEP2 energies, this effect is not large. When the $W$ boson is produced in the direction of the end-caps, one of the decay quarks will most likely also point in this direction. A large fraction of particles in the jet produced by this quark obtain their charge from the reconstruction of their track in this region of the TEC, which has a much higher charge confusion. Consequently, the inefficiency for the determination of the charge of the jet, based on the charges of the particles in the jet, will also increase. For this reason, the data and MC are fitted with a quadratic function, shown in the same plot with a solid and dashed line respectively.

For the TGC analysis, the charge confusion in the baseline MC will be shifted to the measured value depending on $\cos \theta_{W^-}$. This correction is already applied to the distributions shown in Figs. 3.9 and 3.10. The error on the combined result is taken to evaluate the systematic error on the couplings measurement.

![Figure 4.6](image.png)

**Figure 4.6:** The $W \rightarrow qq$ charge confusion as a function of the cosine of the $W^-$ production angle, determined from $qq\mu\nu$ events with $189 \text{ GeV} \leq \sqrt{s} \leq 209 \text{ GeV}$. They grey band and line represent the measurement for data and MC from the full data sample. The solid and dashed line is the result from a fit with a quadratic function to data and MC respectively.

### 4.1.4 Influence of the Charge Measurement

When the charge is completely ignored, only the absolute value of the cosine of the $W$ production polar angle is known, and no possibility exists to differentiate between the fermion and anti-fermion. To determine the effect of the charge measurement on the fit values and errors, the fit is repeated without the charge information. In practice this is
done by folding all the phase space angles. The sizes of the bins are kept the same as in the standard fit.

The results from this fit are given in Table 4.4 together with the results from the standard fit. Notice that almost all the central values are in agreement between the fits with an without charge information. A large difference is observed for $g_1^Z$ and $\kappa_2$ derived from $qq\tau\nu$ events. The likelihood curves for those couplings have an asymmetric form which largely disappears when the charge information is included.

The channel with the best charge assignment, $qq\mu\nu$, also shows the largest reduction of the errors on the measurement when the charge is included. Both the $qqqq$ and $qq\nu$ channels also benefit considerably from the charge measurement. For the $qq\tau\nu$ channel, the improvement is marginal due to the worse resolution of the $W$ production angle. The error on $\lambda_\gamma$ for the $qq\tau\nu$ events seems to increase considerably when the charge is included. This is a bit misleading since also in this case, the likelihood without the charge information is asymmetric while the one with the charge information is not, and the error given in the table is the average of the positive and negative value.

Combining the different decay channels, it can be seen that the largest improvement using the charge information is obtained for the couplings of the $\lambda$-type and is about 45%. This is not surprising since these couplings are most sensitive to the $W$ production angle (see Table 3.2). The errors on the other couplings decrease by about 20–30%, except for $g_5^Z$ where the gain is only 5%.

<table>
<thead>
<tr>
<th></th>
<th>$qqqq$</th>
<th>$qq\nu$</th>
<th>$qq\mu\nu$</th>
<th>$qq\tau\nu$</th>
<th>combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_1^Z$ no</td>
<td>0.905 ± 0.066</td>
<td>0.927 ± 0.091</td>
<td>0.975 ± 0.091</td>
<td>1.27 ± 0.15</td>
<td>0.942 ± 0.048</td>
</tr>
<tr>
<td>yes</td>
<td>0.873 ± 0.055</td>
<td>0.992 ± 0.069</td>
<td>0.926 ± 0.055</td>
<td>1.02 ± 0.15</td>
<td>0.932 ± 0.035</td>
</tr>
<tr>
<td>$\kappa_\gamma$ no</td>
<td>0.87 ± 0.11</td>
<td>0.88 ± 0.18</td>
<td>1.03 ± 0.25</td>
<td>0.87 ± 0.22</td>
<td>0.900 ± 0.080</td>
</tr>
<tr>
<td>yes</td>
<td>0.82 ± 0.10</td>
<td>0.92 ± 0.17</td>
<td>0.88 ± 0.12</td>
<td>0.813 ± 0.17</td>
<td>0.854 ± 0.064</td>
</tr>
<tr>
<td>$\lambda_\gamma$ no</td>
<td>−0.082 ± 0.091</td>
<td>−0.065 ± 0.100</td>
<td>+0.033 ± 0.097</td>
<td>+0.34 ± 0.11</td>
<td>−0.016 ± 0.067</td>
</tr>
<tr>
<td>yes</td>
<td>−0.118 ± 0.065</td>
<td>−0.060 ± 0.067</td>
<td>−0.032 ± 0.055</td>
<td>+0.22 ± 0.16</td>
<td>−0.056 ± 0.037</td>
</tr>
<tr>
<td>$g_5^Z$ no</td>
<td>+0.27 ± 0.22</td>
<td>+0.12 ± 0.32</td>
<td>+0.14 ± 0.30</td>
<td>−0.43 ± 0.83</td>
<td>+0.17 ± 0.15</td>
</tr>
<tr>
<td>yes</td>
<td>+0.16 ± 0.23</td>
<td>+0.09 ± 0.28</td>
<td>−0.22 ± 0.26</td>
<td>−0.19 ± 0.65</td>
<td>+0.00 ± 0.15</td>
</tr>
<tr>
<td>$\kappa_2$ no</td>
<td>0.838 ± 0.105</td>
<td>0.87 ± 0.15</td>
<td>0.946 ± 0.160</td>
<td>1.34 ± 0.33</td>
<td>0.892 ± 0.077</td>
</tr>
<tr>
<td>yes</td>
<td>0.887 ± 0.087</td>
<td>1.02 ± 0.13</td>
<td>0.872 ± 0.095</td>
<td>0.93 ± 0.22</td>
<td>0.877 ± 0.059</td>
</tr>
<tr>
<td>$\lambda_2$ no</td>
<td>−0.16 ± 0.28</td>
<td>−0.13 ± 0.15</td>
<td>+0.01 ± 0.14</td>
<td>+0.37 ± 0.14</td>
<td>−0.026 ± 0.131</td>
</tr>
<tr>
<td>yes</td>
<td>−0.21 ± 0.10</td>
<td>−0.12 ± 0.11</td>
<td>−0.05 ± 0.09</td>
<td>+0.28 ± 0.15</td>
<td>−0.097 ± 0.066</td>
</tr>
</tbody>
</table>

Table 4.4: A comparison between the fit results obtained when ignoring the charge measurement (no) and the standard fit (yes). The quoted errors are the averages of the positive and negative errors.
4.2 Contribution from the Total Cross Section

The cross section depends quadratically on the couplings. If the measured cross section is larger than the predicted one, the negative log-likelihood resulting from the TGC fit when only the total cross section information is used has a double minimum and the errors appear to be much smaller than expected. This is especially so for the \( \kappa \) type of couplings for which the minimum cross section is not at the standard model coupling value. Combining such a likelihood with the results obtained from the normally more sensitive angles, has a significant effect on the location of the minimum. However, if these higher cross sections are just due to statistical fluctuations, this effect has to diminish when all the data is combined. To study the contribution from the total cross section, the fit can be repeated using only the shape of the distributions of the phase space angles. This is done by normalising the Monte Carlo prediction to the total number of measured events.

As an example, the results of the \( \kappa_\gamma \) measurement from \( q\bar{q}q\bar{q} \) events at 196 and 200 GeV are shown in Fig. 4.7. The parabolas in the top plots show the dependence of the cross section on \( \kappa_\gamma \). As mentioned before, the minimum of the cross section is not at the standard model expectation, which is at \( \kappa_\gamma = 1 \), but rather around \( \kappa_\gamma = 1.4 \). The shaded bands are the measured cross sections including only the statistical errors. The negative log-likelihoods obtained from the shape-only fit and those using all the information are plotted in the bottom part using dashed and solid lines, respectively. Both shape-only likelihoods have a double minimum structure. The second minimum almost disappears when the total cross section is included in the measurement at 196 GeV and the overall error – at larger values above the minimum – decreases significantly. This is due to the measured cross section being about one standard deviation smaller than the expected value. At 200 GeV on the other hand, the measured cross section is about one standard deviation higher than the SM value. As can be derived from the top part of Fig. 4.7, this results in two distinct 68% C.L. intervals for \( \kappa_\gamma \), corresponding to a likelihood with a double minimum. When this information is added to the measurement, the already existing double minimum is further enhanced.

The effect of including the total cross section in the fit on the measurement of \( \kappa_\gamma \) from the four decay channels is listed in Table 4.5. Although some differences between the values with and without the rate information are observed in the different decay channels, the overall combined result is in very good agreement. For the other couplings, similar or smaller effects are seen.

4.3 Background

The first part of this section will treat the verification of the modelling of the \( q\bar{q}(\gamma) \) background that passes the \( q\bar{q}q\bar{q} \) selection criteria. This is the largest contamination that occurs in any of the decay channels. The second part of this section will check whether the sizes of the available background MC samples are adequate to be used in the TGC analysis.
Figure 4.7: The solid lines on the top plots show the dependence of the cross section on the coupling $\kappa_\gamma$ for the CC03 process $e^+e^- \rightarrow WW \rightarrow qqqq$ at centre-of-mass energies of 196 GeV (left) and 200 GeV (right). The corresponding 68% C.L. allowed regions from the cross section measurements including only statistical errors are indicated by the shaded bands. The negative log-likelihood distributions for the same coupling and channel at the corresponding energies are shown in the bottom plots. The dashed line is from the shape-only fit while the solid line is the standard fit, including the total cross section.

Table 4.5: The fit results obtained using only the shape of the phase space angles (shape) and the standard fit (full) where also the total cross section is included. All the centre-of-mass energies are combined and the errors quoted are the averages of the positive and negative errors.
4.3.1 \( q\bar{q}(\gamma) \) background in the qqqq channel

In order to check the background modelling in the hadronic channel, a background-rich but WW-like sample should be selected from the data. This can be done by investigating the events that pass the pre-selection, but that do not have a neural network output larger than 0.6. This has been applied to the 1998 and 1999 data for which a total of 2677 such events are selected. At 189 GeV, this sample consists for 85% out of \( q\bar{q}(\gamma) \) events. This fraction decreases to about 76% at 202 GeV due to the decreasing cross section. The second largest contribution, from 12 to 19%, comes from WW events. These are in 70% of the cases real qqqq events, while the remainder consists mainly of qq\(\tau\nu \) events. Small contributions come from hadronic ZZ (1–2%) and 2-photon events (2%).

The distributions of phase space angles for this sample are shown in Fig. 4.8, where the distributions for the decay quarks are summed. For all the distributions good agreement is observed between the data and MC predictions. Most of the \( q\bar{q}(\gamma) \) events that are selected in the hadronic channel are events where the quarks radiate gluons. This is done preferentially at small angles with respect to the original quarks, as can also be seen in the distribution of the polar angle of the decay jet which is peaked forward.

The distributions of the \( q\bar{q}(\gamma) \) samples are also compared between the predictions of the generators KK2f and PYTHIA. The ratio between the two MC distributions of the quantity reconstructed as the W production angle for the events that pass the standard qqqq selection at 189 GeV is shown in Fig. 4.9. As can be seen, a good agreement exists between the two predictions, which is also the case for the other angles and at other centre-of-mass energies.

4.3.2 Background Statistics

When the number of background events selected in the MC samples is not sufficient to describe the five-dimensional phase space, statistical fluctuations become important. This may introduce a bias in the measurement, especially if the fluctuations affect regions where few signal events are expected. However, in order to have a significant effect on the distributions, the expected background contribution has to be large enough.

To check the stability of the fit method with respect to the background statistics, the fit has been repeated using smaller MC samples for the background. This is simply done by removing a fraction of the original background sample in order to obtain the desired number of events.

The efficiencies for selecting a certain type of background event in each of the decay modes are listed in Table 4.6. These values differ considerably between the various channels. In order to compare the minimum number of background events required in each channel for the fit results to be stable, the total number of generated events is always quoted rather than the selected ones.

The sizes of the available background samples differ for the various centre-of-mass energies. Usually there is a very large sample available at one energy. The effect of limited background statistics are first checked with these large background samples. They are then combined with other energies where smaller background samples are available. The largest relative shifts are always observed for the coupling \( \lambda_\gamma \). Therefore the results are
4.3. Background

Figure 4.8: The phase space distributions for the events that pass the pre-selection requirements for the $qqqq$ selection, but have a neural network output smaller than 0.6. The 1998 and 1999 data are combined and the decay angles of the two quarks are summed. The dots represent the data, and the MC predictions from the various contributions are given by the shaded histograms.

Figure 4.9: The ratio between the prediction of the $W$ production angle obtained from $q\bar{q}(\gamma)$ events generated with PYTHIA and KK2f. The events are generated at 200 GeV and passed the standard $qqqq$ selection.
4.4. Energy Difference between Data and MC Systematic Checks

<table>
<thead>
<tr>
<th></th>
<th>$\epsilon_{\text{qq}(\gamma)}$ [%]</th>
<th>$\epsilon_{\text{ZZ}}$ [%]</th>
<th>$\epsilon_{\text{Zee}}$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>qqqq</td>
<td>1.3</td>
<td>36</td>
<td>-</td>
</tr>
<tr>
<td>qqe$\nu$</td>
<td>0.072</td>
<td>0.52</td>
<td>0.46</td>
</tr>
<tr>
<td>qq$\mu\nu$</td>
<td>0.027</td>
<td>2.7</td>
<td>-</td>
</tr>
<tr>
<td>qq$\tau\nu$</td>
<td>0.30</td>
<td>6.0</td>
<td>0.77</td>
</tr>
</tbody>
</table>

Table 4.6: The selection efficiencies for the various types of background events (columns) in the various WW decay channels (rows). The values are determined at 200 GeV and depend only slightly on the centre-of-mass energy.

only quoted for this coupling, but it should be noted that the effect is studied for all the couplings measured.

**qq(\gamma) background**

For the large background sample, the 189 GeV data is used, where 3·10^6 generated qq(\gamma) events are available. For the full sample, the data taken at 189, 196, 200 and 206 GeV is combined and the number of generated qq(\gamma) events is varied from 5·10^5 events down to 2.5·10^4. The effect on the measurement of $\lambda_\gamma$ is shown in Fig. 4.10. It is clear that reducing the background statistics introduces a bias in the measurements in the qqqq and qq$\tau\nu$ channels. This effect is absent in the qqe$\nu$ and qq$\mu\nu$ channels due to the low contamination of qq(\gamma) events in these samples. The sizes of the qq(\gamma) samples used in the TGC fits always contain more than 4·10^5 generated events.

**ZZ background**

The effect of reducing the ZZ background statistics on the combined 189, 196 and 206 GeV data is shown in Fig. 4.11. It can be seen that no bias is introduced in any of the channels when small samples are used.

**Zee background**

Fig. 4.12 shows the same data sample but now varying the size of the Zee background MC samples. Also the contamination of this type of events in the qqe$\nu$ and qq$\tau\nu$ selections is too low to have a significant effect on the measurement.

**4.4 Energy Difference between Data and MC**

As is explained in Section 1.4, the W production angle, the cross section and thus the sensitivity to the couplings depend on the centre-of-mass energy. At the time at which most of the Monte Carlo samples were generated, the exact beam energy was not known. Furthermore, during the 2000 data taking period, a wide range of energies was covered. Hence, differences up to 0.4 GeV between the centre-of-mass energies in the MC samples and the actual data exist.
Figure 4.10: The effect on the fit result of $\lambda_\gamma$ when the number of generated events used to model the qq(\gamma) background is reduced. For each of the decay channels, the top plot is obtained with the 189 GeV data sample, while the bottom one includes also the results from the data at 196, 200 and 206 GeV.
4.4. Energy Difference between Data and MC Systematic Checks

Figure 4.11: The effect on the measurement of $\lambda_\gamma$ when the number of generated events used to model the $ZZ$ background is reduced. The combination of the data taken at 189, 196 and 206 GeV is shown for each of the decay channels.

Figure 4.12: The effect on the fit result of $\lambda_\gamma$ when the number of events used to model the Zee background is reduced. The combination of the data taken at 189, 196 and 206 GeV is shown for the $qqe\nu$ and $qq\tau\nu$ channels.
Figure 4.13: The effect on the measurement of $g_Z^1$ as function of the centre-of-mass energy difference between the pseudo-data and the baseline MC. This sample was generated at 200 GeV and the shifts measured in the different decay channels are combined.

The effect of using a MC sample with a different centre-of-mass energy than the data sample is studied by using the sample generated at 200 GeV as the baseline MC and the other available samples as pseudo-data. All MC samples are generated with Standard Model couplings. The shifts, i.e. fit results minus SM values, seen in the $q\bar{q}q\bar{q}$ and $qq\tau\nu$ channels are slightly larger than those observed in the other channels. Combining all the shifts for the different decay channels at a certain energy difference reveals a linear dependence on this difference. This can be seen in Fig. 4.13, where the effect on $g_Z^1$ is shown. Only for $\kappa_3$ is this effect non-linear, but for the largest energy differences, the relative shifts are about the same size as for the other couplings.

The effect observed for a $\sqrt{s}$ difference of 10 GeV is, depending on the coupling, about 2.7 to 3.4 times larger than the final statistical error. Both the MC samples at 189 GeV and at 205 GeV have a difference in centre-of-mass energy with the data of about 0.4 GeV. This results in a shift of one tenth of the statistical error at those energies, which could in principle be corrected for. This will however not have a visible effect on the final combined result since for the other centre-of-mass energies, the differences between data and MC $\sqrt{s}$ are much smaller than 0.4 GeV. Therefore, the shifts due to the centre-of-mass energy difference between the data and signal MC samples are neglected.

### 4.5 Bin Size Dependence

Since no strict prescription exists to determine the size of the bins and thus the choice is not unique, it is good to check that the fit results do not depend on the size of the bins. For each of the phase space angles, the fit is repeated increasing and decreasing the number of bins. For the $W^-$ production angle, the number of bins is varied by three units in the hadronic channel and by two units in the semi-leptonic channels. The number of bins used for the decay angles are all varied by one unit.

The effect on the measurement of some of the couplings for the different decay channels can be seen in Fig. 4.14. The odd (even) numbers indicate an increase (decrease) in the bin size. The order of the angles is the same as defined in Section 3.1: $\cos \Theta_W^-$, $\cos \theta_1$, $\phi_1$, $\cos \theta_2$ and $\phi_2$. It can be seen that all the central values and errors are in agreement with the standard fit. The same conclusions can be drawn for the other couplings and decay channels.
Figure 4.14: The difference between the fit result obtained when varying the bin size of a certain phase space angle and the standard fit. The shaded area indicates the errors on the fit results using standard bin sizes. Four different coupling measurements, each determined from different decay channels are shown. The variation of the bin sizes are explained in the text.

4.6 Comparison with Other Fit Methods

In the past, also two unbinned maximum likelihood fit methods were used to extract the TGC parameters. One of these uses the same phase space angles as described in this thesis, while the other constructs optimal observables from the available kinematic information.

In both methods, the likelihood for each event is calculated using the normalised differential cross section, which is determined with a Monte Carlo numerical evaluation [131]. This is done by taking the average of the differential cross section of the Monte Carlo events that lie in a region around the data point. Since the MC events are treated in the same way as the data, this approach takes automatically all selection and detector effects properly into account. The dependence on the couplings is obtained using the same reweighting technique as described in this thesis.

4.6.1 Unbinned Phase Space Angles

This method has been extensively described in [132], where it was shown that the unbinned fit performs slightly better than the binned fit when the same angular information is
used. It was also demonstrated that taking more angles into account always increases the sensitivity, regardless of whether the fit was binned or not. The disadvantage of the unbinned method is the fact that a sufficiently large MC sample needs to be generated to describe the entire phase space. In fact, with the available number of MC events, a maximum of only three angles can be used in this unbinned fit. Using more angles would require an unrealistically large amount of MC events, and therefore a binned fit was preferred over the unbinned method.

### 4.6.2 Optimal Observables

The basic idea of this method [133] is to construct one or two variables from the kinematic information that are optimally sensitive to the couplings. Since the Lagrangian (1.13) is at most linear in the couplings, the differential cross section can be parametrised as:

$$\frac{d\sigma}{d\Omega} = c_0(\Omega) + \sum_i c_{1,i}(\Omega)\psi_i + \sum_{i,j} c_{2,ij}(\Omega)\psi_i\psi_j,$$

where the sum runs over all possible couplings $\psi_i$ and $\Omega$ is the phase space. The second order term is generally neglected and for each coupling $i$, a variable is defined as $O_{1,i} = c_{1,i}/c_0$. These variables contain all the information from the differential cross section and should be maximally sensitive to the couplings. They are therefore called optimal observables. The total cross section is included in the fit by multiplying the likelihood with the Poisson probability of observing the number of measured events, given the coupling-dependent expectation.

The results for the couplings $g_1^Z$, $\Delta\kappa_{\gamma}$ and $\lambda_{\gamma}$ using the semi-leptonic data with $\sqrt{s} \geq 189$ GeV are compared in Table 4.7 between the optimal observables method and the binned fit to the five phase space angles. Both the central values and the obtained errors are in good agreement with each other.

<table>
<thead>
<tr>
<th></th>
<th>OO</th>
<th>5D BIN</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_1^Z$</td>
<td>$0.952^{+0.040}_{-0.034}$</td>
<td>$0.950^{+0.043}_{-0.041}$</td>
</tr>
<tr>
<td>$\kappa_{\gamma}$</td>
<td>$0.856^{+0.086}_{-0.078}$</td>
<td>$0.838^{+0.084}_{-0.077}$</td>
</tr>
<tr>
<td>$\lambda_{\gamma}$</td>
<td>$-0.062^{+0.040}_{-0.036}$</td>
<td>$-0.041^{+0.045}_{-0.042}$</td>
</tr>
</tbody>
</table>

Table 4.7: Comparison between the results for the couplings $g_1^Z$, $\kappa_{\gamma}$ and $\lambda_{\gamma}$ using the optimal observables method (OO) and the binned fit to the five phase space angles (5D BIN). The values are obtained using the semi-leptonic events selected at $\sqrt{s} \geq 189$ GeV.
Chapter 5
Systematic Errors

In the previous chapters, a number of verifications were performed to check both the validity of the fit method and the reconstruction of the relevant phase space angles. The remaining uncertainties on the quantities used to reconstruct these angles and calculate the reweighting factors are a source of systematic errors. Besides these sources of experimental origin, also uncertainties in the theoretical predictions contribute significantly to the systematic error.

The next section will explain in a general way how the systematic errors coming from the various possible sources are determined. The succeeding sections will describe in more detail how this error is extracted in each specific case for significantly contributing sources. Sections 5.2 and 5.3 will treat the uncertainties in the predictions for the signal and background processes. The error coming from the differences in the description of initial and final state radiation between the baseline MC and the program used to calculate the weights will be specified in Section 5.4. The next source investigated is the finite amount of available events in the signal MC which can be a limiting factor in the extraction of the couplings. Then follows the treatment of the errors resulting from the uncertainties on the charge confusion (Section 5.6), on the reconstruction of the jet and lepton quantities (Section 5.7) and on the values of the W mass and width used in the calculation of the matrix elements (Section 5.8). The theoretical description of the production of, and interactions between, the particles in the jets will be addressed in Section 5.9. In the last section, the combination of the systematic errors from all these sources will be discussed.

5.1 Evaluation of Systematic Errors

Systematic errors follow from the fact that the theoretical and experimental parameters used in the predictions for the relevant distributions themselves possess uncertainties. The effect on the TGC measurement is derived either by changing a parameter value in the baseline MC and repeating the fit to the data, or by using a large MC sample generated at different parameter values as pseudo-data, maintaining the original baseline MC. The advantage of the latter method is that the observed differences will be quasi-free of statistical fluctuations, while the shifts in the data might have a large statistical component. On the other hand, systematic errors can affect not only the central value,
but may also change the derived errors. These errors depend on the couplings and the likelihoods may acquire non-Gaussian shapes. The sensitivity of the fit can thus change and this should be taken into account in the determination of the systematic error. This can only be derived by observing changes in the fit to the data.

When the effect is determined on the data, the resulting systematic error is extracted in the following way. The fit result for coupling $\psi$, where a parameter value $\xi$ is used in the baseline MC for the source of systematic error in question, is

$$\psi(\xi) = x_0 \pm \sigma_0,$$

where $x_0$ is the central value of the fit and the error $\sigma_0$ is taken as half the asymmetric 68% C.L. interval. When the value $\xi$ of the systematic error source is now varied with $\pm \sigma_\xi$, its either theoretically or experimentally determined error, this results in

$$\psi(\xi + \sigma_\xi) = x_+ \pm \sigma_+,$$

$$\psi(\xi - \sigma_\xi) = x_- \pm \sigma_-.$$

The indices $+$ and $-$ refer to the corresponding variation of the systematic error source. The shift in the fit result, $\Delta x$, and the change of variance, $\Delta \sigma^2$, are determined as:

$$\Delta x_+ = x_+ - x_0,$$

$$\Delta \sigma^2_+ = \sigma^2_+ - \sigma^2_0,$$

$$\Delta x_- = x_- - x_0,$$

$$\Delta \sigma^2_- = \sigma^2_- - \sigma^2_0.$$

Assuming that a change of one standard deviation in the concerned parameter results in a change of approximately one standard deviation in the values of the coupling, the total systematic error is then defined as

$$\delta^2 = \left(\frac{|\Delta x_+| + |\Delta x_-|}{2}\right)^2 + \max \left(\frac{\Delta \sigma^2_+ + \Delta \sigma^2_-}{2}, 0\right)$$

(5.1)

The first term is just the average of the shifts in the central value, while the second term takes the average change in variance into account. The max operator is introduced since no systematic error from the variance change will be considered if an increase of variance from varying the parameter to one side is compensated by a decrease of variance from the variation to the other side, or if there is a decrease of variance on both sides. A Monte Carlo study showed that this expression gives a good estimate for the upper limit for the 68% C.L. interval. Whenever this method is applied to determine systematic errors that are correlated between centre-of-mass energies and/or decay channels, the information is extracted from the effect on the combined likelihood.

**5.2 Signal Modelling**

The TGC measurement is based on the comparison between data and predictions from a Monte Carlo program. These predictions themselves have uncertainties due to the theoretical calculations. This section will investigate the consequences of the uncertainties in the prediction of the total and differential cross section for W-pair production. The Monte Carlo programs that have become available recently include the electroweak radiative corrections up to $\mathcal{O}(\alpha)$ using the approach of the double pole approximation (DPA). They were discussed in more detail in Section 1.7.1.
5.2.1 Total Cross Section

The total cross section for WW production is used in the estimation of the number of expected events and depends on the couplings values. The theoretical uncertainty on this quantity is estimated to be about 0.5% [86].

To evaluate the corresponding error on the TGC measurement, the fit to the data is repeated varying the predicted total cross section by ±0.5%. The resulting systematic errors on the couplings are given in Table 5.1. The combined error is obtained by assuming this source to be fully correlated between the different channels and energies.

<table>
<thead>
<tr>
<th>channel</th>
<th>$\delta(g^Z)$</th>
<th>$\delta(\kappa)$</th>
<th>$\delta(\lambda)$</th>
<th>$\delta(g^Z)$</th>
<th>$\delta(\kappa)$</th>
<th>$\delta(\lambda)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>qqqq</td>
<td>0.008</td>
<td>0.022</td>
<td>0.010</td>
<td>0.018</td>
<td>0.013</td>
<td>0.015</td>
</tr>
<tr>
<td>qqe$\nu$</td>
<td>0.005</td>
<td>0.017</td>
<td>0.006</td>
<td>0.026</td>
<td>0.012</td>
<td>0.013</td>
</tr>
<tr>
<td>qq$\mu\nu$</td>
<td>0.002</td>
<td>0.012</td>
<td>0.002</td>
<td>0.015</td>
<td>0.004</td>
<td>0.004</td>
</tr>
<tr>
<td>qqt$\nu$</td>
<td>0.016</td>
<td>0.013</td>
<td>0.013</td>
<td>0.021</td>
<td>0.018</td>
<td>0.013</td>
</tr>
<tr>
<td>combined</td>
<td>0.004</td>
<td>0.016</td>
<td>0.004</td>
<td>0.013</td>
<td>0.009</td>
<td>0.007</td>
</tr>
</tbody>
</table>

Table 5.1: The systematic errors on the measurement of the various couplings (columns) for the different channels (rows) due to a 0.5% theoretical uncertainty on the WW cross section. The combination between the channels is obtained by treating the errors as fully correlated.

5.2.2 Differential Cross Section

Most of the information on TGC’s is distilled from the five-dimensional distribution of the production and decay angles. None of the authors of the various available Monte Carlo programs quote an error on the prediction of the differential cross section. There is also no clear prescription available on how this uncertainty can be derived, but it is very reasonable to assume that the remaining uncertainty is smaller than the full effect of including the missing $O(\alpha)$ corrections in DPA. Therefore the change in the differential distributions due to these corrections is investigated.

The effect on the distribution of the $W^-$ production angle is shown in Fig. 1.6. The slope in the ratio between the distributions without and with DPA corrections is of the order of 0.7%. The same figure shows a comparison between two different generators, with different implementations of the DPA approach. This can give an indication of the size of the uncertainty on the prediction. The level of agreement is about half the size of the full effect.

The distributions of Fig 1.6 were obtained by comparing Monte Carlo events at generator level. In the measurement however, the angles are also distorted by resolution effects and ambiguities. For this reason, events generated with KandY were passed through the detector simulation and selection procedures to make a comparison at reconstruction level. The distributions for the phase space angles were corrected to the CC03 level with and without the $O(\alpha)$ correction in DPA, using the KandY weights.

The ratio between the distribution of the $W^-$ production angle without and with the
Figure 5.1: The ratio between the distribution of the W⁻ production angle at CC03 level without and with the O(α) electroweak radiative corrections using the DPA approach as function of this angle. The distributions for the four analysed decay channels are shown at a centre-of-mass energy of 189 GeV. The slope following from a fit with a straight line to this ratio is also given.

corrections¹ is shown in Fig. 5.1 for the four analysed decay channels. The effect is largest for the qqμν channel, where it is slightly larger than the slope at generator level. For the qqeν channel, the effect is a bit smaller due to the higher charge confusion which flattens out the distribution of the production angle. This is also the cause for the drop of the ratio towards cos Θ_W⁻ values of −1. The smaller slope for the qqτν channel is due to the worse resolution and the large contamination from other WW decay channels. The qqqq channel undergoes the smallest changes at reconstruction level due to the pairing ambiguity and worse charge assignment efficiency. Table 5.2 shows the values for the slopes determined at different centre-of-mass energies. For both the qqeν and qqμν channel, the slopes remain constant, while for the qqqq and qqτν channels, an energy dependence is observed.

Half of the size of the effect is taken as the theoretical uncertainty on the W⁻ production angle since this was the observed level of agreement between YFSWW3 and RACOONWWW at generator level. The slope of the cos Θ_W⁻ distribution was varied with a magnitude depending on the decay channel and energy as shown in Table 5.3. The resulting systematic errors are listed in Table 5.2. Also in this case, the errors are taken

¹In practice, the average of the ratio between the two weights in a particular bin is plotted with an error that equals the root mean square divided by the square root of the number of events in that bin.
Table 5.2: The slopes of the linear fits to the ratio between the \( W^- \) production angle without and with \( \mathcal{O}(\alpha) \) electroweak corrections in DPA as a function of this angle for the different decay channels (columns) at different centre-of-mass energies (rows).

<table>
<thead>
<tr>
<th>channel</th>
<th>qqqq</th>
<th>qqe(\nu)</th>
<th>qq(\mu)(\nu)</th>
<th>qq(\tau)(\nu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>189 GeV</td>
<td>0.0028 ± 0.0009</td>
<td>0.0065 ± 0.0015</td>
<td>0.0091 ± 0.0015</td>
<td>0.0044 ± 0.0018</td>
</tr>
<tr>
<td>200 GeV</td>
<td>0.0039 ± 0.0011</td>
<td>0.0058 ± 0.0018</td>
<td>0.0111 ± 0.0019</td>
<td>0.0105 ± 0.0022</td>
</tr>
<tr>
<td>206 GeV</td>
<td>0.0054 ± 0.0012</td>
<td>0.0069 ± 0.0019</td>
<td>0.0092 ± 0.0021</td>
<td>0.0137 ± 0.0024</td>
</tr>
</tbody>
</table>

Table 5.3: The systematic error on the measured couplings (columns) for the different decay channels (rows) due to the uncertainty on the prediction of the \( W^- \) production angle. Half of the effect of the inclusion of the \( \mathcal{O}(\alpha) \) radiative corrections in DPA was taken as the uncertainty. All errors are assumed to be fully correlated between the energies and channels.

To be fully correlated between the channels and energies.

Although no significant changes in the decay angles are expected at generator level, a similar test as for the production angle is carried out. The effect of the DPA calculations on the decay angles at reconstruction level can be seen in Fig. 5.2 for the qqqq channel at 200 GeV. The change in the muon decay angle \( \cos \theta_{l^-} \) is one of the largest effects observed. The leptonic polar decay angles from the other channels and at other centre-of-mass energies show similar or smaller changes. No significant changes are observed for the leptonic azimuthal decay angles, nor for the hadronic polar decay angles. The hadronic azimuthal decay angles show sometimes significant changes, but no general tendency is found.

To test any possible uncertainty in the fit results due to the decay angles, the slope of the distributions of the decay angles was varied separately with \( \pm 1\% \), treating their effect as correlated between the channels and different centre-of-mass energies. All effects on the measured couplings were found to be smaller than 0.001 and were therefore neglected in this analysis.

5.3 Background Modelling

The expected background contributions are obtained using events generated with the MC programs listed in Section 2.2.3. Also in this case, the predictions for the total and differential cross sections contain uncertainties. These predictions are used to calculate
5.3. Background Modelling Systematic Errors

The normalisation of the expected background is varied by the estimated theoretical uncertainty on the total cross section, which is ±5\% for the q\bar{q}(\gamma) and Zee events and ±2\% for the ZZ background. To check the error from the differential cross section, only the distribution of the most sensitive angle, \cos \Theta_{W^-} is varied. This is done by reweighting each event with a factor \( (1 \pm 0.05 \cos \Theta) \), so that the slope of the distribution changes by ±5\%.

The resulting effect on the TGC measurements due to the uncertainty on the Zee cross section in the q\bar{q}\nu and q\bar{q}\tau\nu channels varies between 0.001 and 0.007, depending on the coupling. The other two channels are not contaminated by this background source. In the combined result, the remaining uncertainties are of the order 0.001 for all the couplings. The variation of the W^- production angle obtained from Zee events has no significant effect on the measurement. The systematic errors on the couplings obtained from the other two background sources are given in Table 5.4. The largest contribution comes from the uncertainty on the q\bar{q}(\gamma) cross section and in particular in the q\bar{q}qqq channel, where it forms a large background.

Figure 5.2: The ratio between the distributions of decay angles at CC03 level without and with the \( \mathcal{O}(\alpha) \) electroweak radiative corrections using the DPA approach as function of these angles. The distributions for the q\bar{q}\mu\nu channel are shown at a centre-of-mass energy of 200 GeV. The slope resulting from a linear fit to this ratio is also given.
### Systematic uncertainty due to $q\bar{q}(\gamma)$ normalisation

<table>
<thead>
<tr>
<th>channel</th>
<th>$\delta(g_1^Z)$</th>
<th>$\delta(\kappa_\gamma)$</th>
<th>$\delta(\lambda_\gamma)$</th>
<th>$\delta(g_5^Z)$</th>
<th>$\delta(\kappa_Z)$</th>
<th>$\delta(\lambda_Z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>qqqq</td>
<td>0.014</td>
<td>0.031</td>
<td>0.021</td>
<td>0.038</td>
<td>0.021</td>
<td>0.033</td>
</tr>
<tr>
<td>qqeν</td>
<td>0.002</td>
<td>0.004</td>
<td>0.001</td>
<td>0.003</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td>qqμν</td>
<td>0.001</td>
<td>0.003</td>
<td>0.001</td>
<td>0.004</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>qqτν</td>
<td>0.013</td>
<td>0.008</td>
<td>0.009</td>
<td>0.011</td>
<td>0.009</td>
<td>0.009</td>
</tr>
<tr>
<td>combined</td>
<td>0.005</td>
<td>0.014</td>
<td>0.006</td>
<td>0.010</td>
<td>0.009</td>
<td>0.012</td>
</tr>
</tbody>
</table>

### Systematic uncertainty due to $q\bar{q}(\gamma)$ shape

<table>
<thead>
<tr>
<th>channel</th>
<th>$\delta(g_1^Z)$</th>
<th>$\delta(\kappa_\gamma)$</th>
<th>$\delta(\lambda_\gamma)$</th>
<th>$\delta(g_5^Z)$</th>
<th>$\delta(\kappa_Z)$</th>
<th>$\delta(\lambda_Z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>qqqq</td>
<td>0.004</td>
<td>0.004</td>
<td>0.006</td>
<td>0.006</td>
<td>0.005</td>
<td>0.008</td>
</tr>
<tr>
<td>qqeν</td>
<td>0.001</td>
<td>0.002</td>
<td>0.001</td>
<td>0.003</td>
<td>0.002</td>
<td>0.001</td>
</tr>
<tr>
<td>qqμν</td>
<td>0.001</td>
<td>0.003</td>
<td>0.001</td>
<td>0.004</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td>qqτν</td>
<td>0.010</td>
<td>0.001</td>
<td>0.005</td>
<td>0.022</td>
<td>0.018</td>
<td>0.003</td>
</tr>
<tr>
<td>combined</td>
<td>0.003</td>
<td>0.002</td>
<td>0.003</td>
<td>0.006</td>
<td>0.004</td>
<td>0.006</td>
</tr>
</tbody>
</table>

### Systematic uncertainty due to ZZ normalisation

<table>
<thead>
<tr>
<th>channel</th>
<th>$\delta(g_1^Z)$</th>
<th>$\delta(\kappa_\gamma)$</th>
<th>$\delta(\lambda_\gamma)$</th>
<th>$\delta(g_5^Z)$</th>
<th>$\delta(\kappa_Z)$</th>
<th>$\delta(\lambda_Z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>qqqq</td>
<td>0.004</td>
<td>0.009</td>
<td>0.005</td>
<td>0.011</td>
<td>0.007</td>
<td>0.008</td>
</tr>
<tr>
<td>qqeν</td>
<td>0.001</td>
<td>0.002</td>
<td>–</td>
<td>0.002</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>qqμν</td>
<td>0.001</td>
<td>0.003</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.002</td>
</tr>
<tr>
<td>qqτν</td>
<td>0.004</td>
<td>0.003</td>
<td>0.003</td>
<td>0.004</td>
<td>0.005</td>
<td>0.004</td>
</tr>
<tr>
<td>combined</td>
<td>0.002</td>
<td>0.005</td>
<td>0.002</td>
<td>0.004</td>
<td>0.003</td>
<td>0.003</td>
</tr>
</tbody>
</table>

### Systematic uncertainty due to ZZ shape

<table>
<thead>
<tr>
<th>channel</th>
<th>$\delta(g_1^Z)$</th>
<th>$\delta(\kappa_\gamma)$</th>
<th>$\delta(\lambda_\gamma)$</th>
<th>$\delta(g_5^Z)$</th>
<th>$\delta(\kappa_Z)$</th>
<th>$\delta(\lambda_Z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>qqqq</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
</tr>
<tr>
<td>qqeν</td>
<td>0.001</td>
<td>0.002</td>
<td>–</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>qqμν</td>
<td>0.001</td>
<td>0.003</td>
<td>0.001</td>
<td>0.006</td>
<td>0.001</td>
<td>0.002</td>
</tr>
<tr>
<td>qqτν</td>
<td>0.007</td>
<td>0.002</td>
<td>0.003</td>
<td>0.004</td>
<td>0.003</td>
<td>–</td>
</tr>
<tr>
<td>combined</td>
<td>0.001</td>
<td>0.002</td>
<td>0.002</td>
<td>0.004</td>
<td>0.002</td>
<td>0.003</td>
</tr>
</tbody>
</table>

Table 5.4: The systematic error on the TGC measurements due to the uncertainties in the modelling of the two most important background sources, $q\bar{q}(\gamma)$ and ZZ events. The results for the separate channels and the combination (rows) are shown for each measured coupling (columns). Both the results from the variation of the total cross section and from the variation of the shape of the reconstructed $\cos \Theta_W$ distribution are shown for each background source.
5.4 Initial and Final State Radiation

The radiation of one or more photons from the initial and final state particles is an important source of uncertainty in LEP processes. The energy distribution of this radiation is strongly peaked towards zero and the angular distribution is peaked at small polar angles with respect to the originating particle. Initial state radiation (ISR) is the process, where photons are radiated from the incoming electrons and positrons before the interaction. The result of ISR is thus a reduction of the effective centre-of-mass energy and the introduction of a longitudinal energy imbalance. Also the charged particles in the intermediate (W) or final state can radiate photons, which is called final state radiation (FSR). This clearly affects the reconstruction of the phase space angles. The main source of systematic uncertainty comes from the difference between the description of these effects in the generator used to describe the signal events, YFSWW3, and the program to calculate the coupling dependent weights, EXCALIBUR.

The ISR photons in EXCALIBUR are always emitted collinear with the incoming particles, while YFSWW3 has a better treatment of ISR with photons having a non-zero transverse momentum. This is taken into account by recalculating the momenta of the incoming electron and positron after emission of the ISR photons before the weights are calculated. In order to have an idea of the accuracy of the ISR modelling, the energy spectra of the total energy emitted as ISR can be compared between the two programs. It can be seen in Fig. 5.3 that there is a very good agreement between the two distributions. The ratio $R$ between the two distributions is fitted with a straight line, fixing the ratio in the first bin to one. This results in the expression $R = 1 - 0.0020 \cdot E_{\text{ISR}}$, with the energy expressed in units of GeV. To estimate the systematic error due to the uncertainty in the description of ISR, each event is given a weight according to this fitted ratio. The fit to the data is also repeated using the opposite sign in the expression for $R$. The resulting errors are listed in the top part of Table 5.5. The corrections due to ISR are included in YFSWW3 up to $O(\alpha^3)$ in the leading-logarithm approximation using the YFS exponentiation. The remaining uncertainty is believed to lie well within the variation described above.

FSR is described in YFSWW3 while it is not taken into account in EXCALIBUR. Before the four-momenta of the YFSWW3 signal events are passed to EXCALIBUR to calculate the weights, the FSR photons need to be reassigned to a charged particle. This is taken to be the nearest charged particle. A crude way of testing the description of FSR is to remove all MC events that contain at least one FSR photon above a certain energy and to repeat the fit. The average of the shifts observed with energy cuts of 0.1 and 1 GeV is taken as a systematic error. The obtained systematic errors are given in the bottom part of Table 5.5.

5.5 Monte Carlo Statistics

When not enough events are used to describe the five-dimensional distribution of the phase space angles, the fit becomes sensitive to statistical fluctuations, especially in less populated regions. Ideally, a sample containing an infinite amount of MC events is preferred,
Figure 5.3: Comparison of the distribution of the total energy emitted as ISR photons between YFSWW3 (dots) and EXCALIBUR (solid line) for events generated at $\sqrt{s} = 200$ GeV and passing the qq\mu\nu selection criteria. The bottom plot shows the ratio $R$ between the two spectra and is fitted with a straight line, resulting in $R = 1 - 0.0020 \cdot E_{\text{ISR}}$.

<table>
<thead>
<tr>
<th>Systematic uncertainty due to ISR</th>
<th>channel</th>
<th>$\delta(g_1^Z)$</th>
<th>$\delta(\kappa_\gamma)$</th>
<th>$\delta(\lambda_\gamma)$</th>
<th>$\delta(g_5^Z)$</th>
<th>$\delta(\kappa_Z)$</th>
<th>$\delta(\lambda_Z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>qqqq</td>
<td></td>
<td>0.010</td>
<td>0.025</td>
<td>0.012</td>
<td>0.028</td>
<td>0.017</td>
<td>0.019</td>
</tr>
<tr>
<td>qqe\nu</td>
<td></td>
<td>0.003</td>
<td>0.016</td>
<td>0.004</td>
<td>0.010</td>
<td>0.006</td>
<td>0.007</td>
</tr>
<tr>
<td>qq\mu\nu</td>
<td></td>
<td>0.003</td>
<td>0.012</td>
<td>0.002</td>
<td>0.004</td>
<td>0.006</td>
<td>0.004</td>
</tr>
<tr>
<td>qq\tau\nu</td>
<td></td>
<td>0.020</td>
<td>0.016</td>
<td>0.015</td>
<td>0.020</td>
<td>0.022</td>
<td>0.015</td>
</tr>
<tr>
<td>combined</td>
<td></td>
<td>0.006</td>
<td>0.018</td>
<td>0.006</td>
<td>0.014</td>
<td>0.012</td>
<td>0.012</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Systematic uncertainty due to FSR</th>
<th>channel</th>
<th>$\delta(g_1^Z)$</th>
<th>$\delta(\kappa_\gamma)$</th>
<th>$\delta(\lambda_\gamma)$</th>
<th>$\delta(g_5^Z)$</th>
<th>$\delta(\kappa_Z)$</th>
<th>$\delta(\lambda_Z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>qqqq</td>
<td></td>
<td>0.001</td>
<td>0.003</td>
<td>0.005</td>
<td>0.001</td>
<td>0.004</td>
<td>0.006</td>
</tr>
<tr>
<td>qqe\nu</td>
<td></td>
<td>0.010</td>
<td>0.071</td>
<td>0.001</td>
<td>0.088</td>
<td>0.025</td>
<td>0.007</td>
</tr>
<tr>
<td>qq\mu\nu</td>
<td></td>
<td>0.009</td>
<td>0.008</td>
<td>0.010</td>
<td>0.024</td>
<td>0.014</td>
<td>0.020</td>
</tr>
<tr>
<td>qq\tau\nu</td>
<td></td>
<td>0.027</td>
<td>0.036</td>
<td>0.009</td>
<td>0.087</td>
<td>0.074</td>
<td>0.015</td>
</tr>
<tr>
<td>combined</td>
<td></td>
<td>0.002</td>
<td>0.019</td>
<td>0.005</td>
<td>0.034</td>
<td>0.001</td>
<td>0.010</td>
</tr>
</tbody>
</table>

Table 5.5: The systematic error on the TGC measurement due to the uncertainty on the description of initial and final state radiation. The errors on the couplings (columns) using the separate decay channels, as well as on the combined result (rows) are given. The errors are assumed to be fully correlated between energies and channels.
The effect on the measurement of $\lambda$, using the $qqqq$ channel (left) and on $\kappa_Z$ from $qqev$ events (right) as function of the number of events used in the signal MC. The results from 1999 and 2000 data are combined and fitted with a function having the form $a/N^b + c$.

but in practice this is impossible to acquire. For the background samples, it was shown in Section 4.3.2 that the available statistics was sufficient and therefore no systematic error is quoted.

The systematic error resulting from the use of a finite amount of signal MC events has been determined by repeating the fit using fewer and fewer MC events. All the events generated with YFSWW3 at 196 and 200 GeV are added to cover a wide range of available events. This sample, corresponding to one million generated events, is then used as baseline MC in the fit to the 1999 data. The same is done for the 2000 data, where the samples generated at 205.2 and 206.6 GeV are combined. The centre-of-mass energy differences between the data and the combined MC samples, anyhow small, are ignored since only the shifts in the couplings are of interest.

The effect on the TGC measurement can be seen in Fig. 5.4 for two specific examples. The result for each channel separately is fitted with a function of the type $a/N^b + c$. Naively, $b$ is expected to be 0.5, but that does not need to be a priori true. The phase space is not uniformly distributed and less occupied regions often are very sensitive to anomalous couplings. Therefore $b$ is left free and typical values coming out of the fit lie between 0.1 and 0.2. This function also assumes that the fit method is asymptotically free of bias. The systematic error is determined by taking the difference between the value of the fitted function at infinity, i.e. $c$, and at the actual number of events used for each channel and centre-of-mass energy.

The resulting errors on the couplings for the four decay channels when a sample of 500000 events is used are given in Table 5.6. Most of the MC samples used have this size, only at 192 GeV and 202 GeV samples of 300000 events are used, while at 189 GeV a sample of 700000 events is available. The same table also shows the total systematic error on the combined result. This is calculated by assuming that this error is uncorrelated between the channels and centre-of-mass energies. In practice, the quadratic difference is taken between the combination when the systematic error on the individual results are included and when only the statistical error is included. The method used to include and combine the systematic errors is described in Section 5.10.
5.6 Charge Confusion

Since the charge assignment is one of the key ingredients in the TGC measurement, it has been studied in great detail. This study was described in Section 4.1, where also the charge confusion was derived. In the TGC fit, a correction was made for the difference between the data and MC charge confusion, depending on the decay channel and reconstructed particle. The remaining uncertainty on the measured charge confusion is used to determine the systematic error.

For the $qqqq$ events, the W charge confusion is varied by $\pm 1.3\%$. This value was derived from the data by comparing the sum of the jet charges with the muon charge using $qq\mu\nu$ events. The charge and resulting charge confusion is known very well for muons reconstructed as an AMUI in the barrel part of the muon detector. No systematic error is assigned to this type of muons in $qq\mu\nu$ and $qq\tau\nu$ decays. For the remaining events, the charge of the W is determined from the track reconstructed in the TEC. These are events where an electron, a muon identified as a MIP, a muon in the forward or backward muon chambers or a hadronic 1-prong $\tau$ decay is reconstructed. The $\tau$-pair study resulted in a fit through the ratios between the charge confusion in the data and MC as a function of $1/p_T$. The charge confusion for these events is varied according to the relative error of this fit, which is $\delta R/R = \pm 0.096$ for the barrel part and $\pm 0.13$ for the end-caps.

The resulting systematic errors on the TGC fit values from the different decay channels, as well as on the combined result, are given in Table 5.7. The uncertainty in the charge confusion is assumed to be fully correlated between different $1/p_T$ bins and decay channels.

### Table 5.6: Uncertainty due to MC statistics

<table>
<thead>
<tr>
<th>channel</th>
<th>$\delta(g_1^Z)$</th>
<th>$\delta(\kappa_\gamma)$</th>
<th>$\delta(\lambda_\gamma)$</th>
<th>$\delta(g_5^Z)$</th>
<th>$\delta(\kappa_Z)$</th>
<th>$\delta(\lambda_Z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$qqqq$</td>
<td>0.001</td>
<td>0.032</td>
<td>0.053</td>
<td>0.063</td>
<td>0.005</td>
<td>0.074</td>
</tr>
<tr>
<td>$qq\mu\nu$</td>
<td>0.032</td>
<td>0.013</td>
<td>0.033</td>
<td>0.032</td>
<td>0.057</td>
<td>0.001</td>
</tr>
<tr>
<td>$qq\tau\nu$</td>
<td>0.216</td>
<td>0.164</td>
<td>0.328</td>
<td>0.143</td>
<td>0.043</td>
<td>0.198</td>
</tr>
</tbody>
</table>

### Table 5.7: Total systematic error due to MC statistics

<table>
<thead>
<tr>
<th>channel</th>
<th>$\delta(g_1^Z)$</th>
<th>$\delta(\kappa_\gamma)$</th>
<th>$\delta(\lambda_\gamma)$</th>
<th>$\delta(g_5^Z)$</th>
<th>$\delta(\kappa_Z)$</th>
<th>$\delta(\lambda_Z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$qqqq$</td>
<td>–</td>
<td>0.011</td>
<td>0.013</td>
<td>0.023</td>
<td>0.002</td>
<td>0.015</td>
</tr>
<tr>
<td>$qq\mu\nu$</td>
<td>0.023</td>
<td>–</td>
<td>0.019</td>
<td>0.089</td>
<td>0.064</td>
<td>0.005</td>
</tr>
<tr>
<td>$qq\tau\nu$</td>
<td>0.013</td>
<td>0.005</td>
<td>0.013</td>
<td>0.014</td>
<td>0.024</td>
<td>–</td>
</tr>
<tr>
<td>combined</td>
<td>0.063</td>
<td>0.053</td>
<td>0.105</td>
<td>0.085</td>
<td>0.019</td>
<td>0.052</td>
</tr>
</tbody>
</table>

Table 5.6: The top part of the table shows the uncertainty on the fit result of the different couplings and channels when a MC sample containing 500000 generated events are used. The bottom part gives the resulting total systematic error under the assumption that this error is uncorrelated between the channels and centre-of-mass energies.
Table 5.7: The systematic error on the various couplings (columns) in the TGC measurement due to the uncertainty in the determination of the W charge for the channels separately and combined (rows).

<table>
<thead>
<tr>
<th>channel</th>
<th>$\delta(g_1^Z)$</th>
<th>$\delta(\kappa_1)$</th>
<th>$\delta(\lambda_1)$</th>
<th>$\delta(g_5^Z)$</th>
<th>$\delta(\kappa_Z)$</th>
<th>$\delta(\lambda_Z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>qqqq</td>
<td>0.018</td>
<td>0.014</td>
<td>0.024</td>
<td>0.031</td>
<td>0.025</td>
<td>0.046</td>
</tr>
<tr>
<td>qee\nu</td>
<td>0.007</td>
<td>0.058</td>
<td>0.012</td>
<td>0.076</td>
<td>0.013</td>
<td>0.015</td>
</tr>
<tr>
<td>qq\mu\nu</td>
<td>0.003</td>
<td>0.039</td>
<td>0.006</td>
<td>0.008</td>
<td>0.010</td>
<td>0.007</td>
</tr>
<tr>
<td>qq\tau\nu</td>
<td>0.026</td>
<td>0.013</td>
<td>0.005</td>
<td>0.111</td>
<td>0.099</td>
<td>0.005</td>
</tr>
<tr>
<td>combined</td>
<td>0.010</td>
<td>0.017</td>
<td>0.013</td>
<td>0.025</td>
<td>0.016</td>
<td>0.022</td>
</tr>
</tbody>
</table>

5.7 Jet and Lepton Reconstruction

For the extraction of the coupling values from the phase space angles, it is important that the reconstruction of the jet and lepton four-momenta agrees between the data and MC. The level of agreement can be checked with fermion pair production at the Z resonance, where the energies and momenta of the produced particles are very well known and a sample with high statistics can be obtained. The absolute energy measurement of the jets agrees between the data and MC up to a level of 30 MeV, while the energy resolution is well simulated up to 0.5% and the resolution of the angles to about 0.1°. The energy scale of the leptons is reproduced by the MC to 10% of the energy resolution, while the agreement for both the energy and angular resolutions is 25%.

The reconstructed energies and angles are shifted and smeared in the Kandy sample generated at 189 GeV using values several times larger than the observed differences. These samples are then used as pseudo-data and fitted with the standard YFSWW3 baseline MC. A comparison between the unaltered Kandy sample is made to determine the effect on the TGC measurement. The shifts in the TGC fit results are then scaled down to correspond to the actual energy and angular uncertainties as given in the previous paragraph, and these are quoted as a systematic error. The values for the different channels are given in Table 5.8 where the different contributions for the jet and lepton reconstruction are added in quadrature. The table also contains the systematic uncertainties on the combined results. The systematic errors on the combination between the channels from both sources together are given in Table 5.12. The contribution from this source of uncertainty to the total error is small.

5.8 W Mass and Width

The mass $M$ and width $\Gamma$ of the W boson are used to calculate the matrix elements to obtain the coupling dependent weights, but $M_W$ and $\Gamma_W$ have a non-negligible experimental error. The events in the baseline MC are reweighted with EXCALIBUR to determine the dependence of the TGC measurement on the W mass and width. Each event is given an extra weight equal to the ratio between the squared matrix elements using the new and old masses or widths. The total weight of the event, thus with coupling dependence, is obtained by replacing the mass or width values in the numerator in Eq. (3.7), while
keeping these values fixed in the denominator. Fig. 5.5 shows the combined fit results for $\lambda_{\gamma}$ when the baseline MC is reweighted to four different $W$ masses and widths. This coupling as well as all the others show a linear dependence on these variables for all the decay channels.

To determine the systematic error, the observed shifts are scaled down to the uncertainties on the measured values. The best measurements for the mass and width come from the combined LEP results and are $M_W = 80.412 \pm 0.042$ GeV/c$^2$ and $\Gamma_W = 2.150 \pm 0.091$ GeV [135]. These results are obtained with the same type of events used in this analysis – a part of the data set is completely the same – and are hence not uncorrelated with the coupling measurements. Therefore the results from $p\bar{p}$ collisions at the Tevatron are used, which are $M_W = 80.456 \pm 0.059$ GeV/c$^2$ and $\Gamma_W = 2.115 \pm 0.105$ GeV [136]. These values are indicated with the shaded bands in Fig. 5.5 and are used to determine the corresponding error on the couplings. The resulting systematic errors from both sources on the TGC measurement in the separate decay channels as well as on the combined fit results are given in Table 5.9.

### 5.9 Fragmentation and Hadronisation

A potential source for large systematic errors is the description of the evolution from the produced quarks to the observed jets. The process consists of the fragmentation of the coloured partons due to the strong force and the subsequent hadronisation into colourless objects, which happens at typical scales of 1 fm. The hadronic channel suffers from extra uncertainties due to possible interferences between particles coming from different $W$ bosons which are not well simulated by the standard MC program. Due to the large space-
5.9. Fragmentation and Hadronisation Systematic Errors

Figure 5.5: The effect on the combined result of $\lambda_\gamma$ when the events in the baseline MC are reweighted to different $W$ masses (left) and widths (right). The central value is the one at which the events of the baseline MC are generated. The solid line represents a linear fit through the results. The shaded bands represent the 68% C.L. allowed region ($\pm \sigma$) from the combined Tevatron measurements: $M_W = 80.456 \pm 0.059$ GeV/$c^2$ and $\Gamma_W = 2.115 \pm 0.105$ GeV. The systematic error on the coupling ($\pm \delta$) is determined by the intersections of the straight line with the shaded band.

<table>
<thead>
<tr>
<th>Systematic uncertainty due to $M_W$</th>
<th>channel</th>
<th>$\delta(g_1^Z)$</th>
<th>$\delta(\kappa_\gamma)$</th>
<th>$\delta(\lambda_\gamma)$</th>
<th>$\delta(g_5^Z)$</th>
<th>$\delta(\kappa_Z)$</th>
<th>$\delta(\lambda_Z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>qqqq</td>
<td>0.001</td>
<td>0.002</td>
<td>0.002</td>
<td>0.010</td>
<td>0.002</td>
<td>0.002</td>
<td></td>
</tr>
<tr>
<td>qqe$\nu$</td>
<td>0.003</td>
<td>0.001</td>
<td>0.003</td>
<td>0.006</td>
<td>0.007</td>
<td>0.005</td>
<td></td>
</tr>
<tr>
<td>qq$\mu$$\nu$</td>
<td>0.001</td>
<td>–</td>
<td>–</td>
<td>0.002</td>
<td>0.001</td>
<td>0.001</td>
<td></td>
</tr>
<tr>
<td>q$q$$\tau$$\nu$</td>
<td>0.004</td>
<td>0.005</td>
<td>0.006</td>
<td>0.005</td>
<td>0.005</td>
<td>0.008</td>
<td></td>
</tr>
<tr>
<td>combined</td>
<td>0.002</td>
<td>0.001</td>
<td>0.003</td>
<td>0.007</td>
<td>0.002</td>
<td>0.004</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Systematic uncertainty due to $\Gamma_W$</th>
<th>channel</th>
<th>$\delta(g_1^Z)$</th>
<th>$\delta(\kappa_\gamma)$</th>
<th>$\delta(\lambda_\gamma)$</th>
<th>$\delta(g_5^Z)$</th>
<th>$\delta(\kappa_Z)$</th>
<th>$\delta(\lambda_Z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>qqqq</td>
<td>0.007</td>
<td>0.002</td>
<td>0.015</td>
<td>0.028</td>
<td>0.007</td>
<td>0.025</td>
<td></td>
</tr>
<tr>
<td>qqe$\nu$</td>
<td>0.003</td>
<td>0.001</td>
<td>0.003</td>
<td>0.007</td>
<td>0.006</td>
<td>0.006</td>
<td></td>
</tr>
<tr>
<td>qq$\mu$$\nu$</td>
<td>0.003</td>
<td>0.004</td>
<td>0.002</td>
<td>0.015</td>
<td>0.005</td>
<td>0.004</td>
<td></td>
</tr>
<tr>
<td>q$q$$\tau$$\nu$</td>
<td>0.007</td>
<td>0.024</td>
<td>0.013</td>
<td>0.071</td>
<td>0.009</td>
<td>0.015</td>
<td></td>
</tr>
<tr>
<td>combined</td>
<td>0.005</td>
<td>0.004</td>
<td>0.006</td>
<td>0.001</td>
<td>0.006</td>
<td>0.012</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.9: The systematic error on the TGC measurements due to the uncertainty on the mass (top) and the width (bottom) of the W boson, for which the combined Tevatron results have been used.
time overlap between the regions for both W’s where the fragmentation and hadronisation takes place, Bose-Einstein correlations (BEC) between identical bosons from different W parents and colour reconnection (CR) between quarks and gluons from different W’s might occur. Both phenomena are referred to as final state interconnection (FSI) effects.

These effects were studied using the fragmentation and hadronisation models available in the PYTHIA [90], ARIADNE [92] and HERWIG [91] generators. The parameters in these models have been tuned with large samples of hadronic Z decays [93]. A sample of about one million events was generated with KandY at a centre-of-mass energy of 189 GeV. Before these events are passed through the detector simulation, the fragmentation effects are handled by the different models.

To study the effects jointly among the four LEP collaborations, a similar procedure has been followed. Common samples of events were produced by ALEPH for the various models describing the fragmentation effects. In these samples, the generated four-vectors of W’s and their decay products were always the same. Each experiment then applied its own detector simulation and selection procedures. This procedure was agreed at a workshop on W-physics at LEP held in Cetraro, Italy, in October 2001, and the common event samples are therefore referred to as the Cetraro files.

All the obtained samples are used as pseudo-data and the YFSWW3 sample used in the standard fit is taken as the baseline MC. The shifts observed in the TGC measurements are used to estimate the systematic errors from the various sources. The error on these differences will be very small, since the same generated four-vectors are used with different fragmentation models and the statistical errors will be fully correlated.

### 5.9.1 Fragmentation

The different models used for the fragmentation are described in Section 1.7.2. Since JETSET is used as the default model in the baseline MC, the difference between the KandY sample with this model and KandY with ARIADNE and HERWIG are studied. Unfortunately, an error occurred in the processing of the L3 HERWIG sample, and using these events results in very large shifts. This is not only observed in this analysis, but also in the cross section and mass measurements. Further study showed large discrepancies between data and HERWIG in a number of distributions, and therefore L3 HERWIG was not used. The differences between the Cetraro files are listed in Table 5.10. The differences observed using ARIADNE with the L3 tuning are in agreement with the Cetraro files.

Since none of the models can be excluded on any ground, the average of the absolute values of the shifts are quoted as systematic error. The systematic error on the combined result, assuming that these errors are fully correlated between the decay channels and do not depend on the centre-of-mass energy, is given in Table 5.12.

### 5.9.2 Bose-Einstein Correlations

Identical bosons produced in the hadronisation process must obey Bose-Einstein statistics. An enhancement in the number of identical bosons created close in phase space is expected due to amplitude symmetrisation. This effect has been observed in e.g. $Z \rightarrow q\bar{q}$ at LEP1, thus between particles coming from the same boson. Now that boson pair production is
### Differences between fragmentation models

<table>
<thead>
<tr>
<th></th>
<th>qqqq</th>
<th>qqew</th>
<th>qqμν</th>
<th>qqτν</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta(g_{1}^{Z}) )</td>
<td>ARIADNE-JETSET</td>
<td>+0.018</td>
<td>+0.009</td>
<td>-0.004</td>
</tr>
<tr>
<td></td>
<td>HERWIG-JETSET</td>
<td>+0.008</td>
<td>+0.002</td>
<td>+0.004</td>
</tr>
<tr>
<td>( \Delta(\kappa_{\gamma}) )</td>
<td>ARIADNE-JETSET</td>
<td>+0.019</td>
<td>+0.026</td>
<td>-0.008</td>
</tr>
<tr>
<td></td>
<td>HERWIG-JETSET</td>
<td>-0.001</td>
<td>+0.009</td>
<td>-0.002</td>
</tr>
<tr>
<td>( \Delta(\lambda_{\gamma}) )</td>
<td>ARIADNE-JETSET</td>
<td>+0.021</td>
<td>+0.016</td>
<td>-0.005</td>
</tr>
<tr>
<td></td>
<td>HERWIG-JETSET</td>
<td>+0.011</td>
<td>+0.013</td>
<td>+0.003</td>
</tr>
<tr>
<td>( \Delta(g_{5}^{Z}) )</td>
<td>ARIADNE-JETSET</td>
<td>+0.003</td>
<td>+0.003</td>
<td>-0.036</td>
</tr>
<tr>
<td></td>
<td>HERWIG-JETSET</td>
<td>+0.058</td>
<td>+0.036</td>
<td>-0.031</td>
</tr>
<tr>
<td>( \Delta(\kappa_{\gamma}) )</td>
<td>ARIADNE-JETSET</td>
<td>+0.034</td>
<td>+0.012</td>
<td>-0.007</td>
</tr>
<tr>
<td></td>
<td>HERWIG-JETSET</td>
<td>+0.016</td>
<td>-0.001</td>
<td>+0.007</td>
</tr>
<tr>
<td>( \Delta(\lambda_{\gamma}) )</td>
<td>ARIADNE-JETSET</td>
<td>+0.020</td>
<td>+0.025</td>
<td>-0.008</td>
</tr>
<tr>
<td></td>
<td>HERWIG-JETSET</td>
<td>+0.011</td>
<td>+0.016</td>
<td>+0.000</td>
</tr>
</tbody>
</table>

Table 5.10: The observed shifts in the TGC measurements (rows) for the analysed decay channels (columns) due to the application of different fragmentation models. The differences for ARIADNE and HERWIG are quoted with respect to JETSET since this model was used in the baseline MC. The Cetraro files are used to determine the shifts.

Possible at LEP2 energies, the question arises whether BEC also exist between particles coming from different W’s.

The BE\(_{32}\) algorithm from the LUBOEI model \([137]\) available in JETSET is used to simulate BEC. This model just reshuffles the momenta of identical bosons and is tuned \([93]\) to hadronic Z decay data to describe BEC in these events. The fragmentation parameters are tuned simultaneously on a b quark depleted sampled, since b quark production is suppressed in W decays. The BEC are then simulated in the K\&Y samples either between particles coming from both W’s or only between particles coming from the same W. The latter option is the default used for the baseline MC. The differences between the two fits are given in Table 5.11, where the Cetraro files have been used as pseudo-data.

The Bose-Einstein correlations in W-pair events have been measured in L3 by comparing the density of identical pion pairs between semi-leptonic and hadronic decays as a function of their four-momentum difference \([138]\). No evidence was found for correlations between hadrons from different W’s. The same conclusion can be drawn from an analysis performed by ALEPH \([140]\). However, DELPHI does favour some correlation between identical pions coming from different W bosons \([141]\). The analysis performed by OPAL is less sensitive compared to the analyses of the other LEP experiments but prefers the scenario with no correlations between particles from different W’s \([142]\). The measured parameters from each experiment are rewritten as a fraction of the value observed when considering full BEC. A preliminary combination of these fractions yields a value of 0.23 ± 0.13 \([143]\). For this reason, one third of the difference observed in the TGC fits has been taken as systematic error due to BEC.
5.9.3 Colour Reconnection

In the string picture, the particles in the final state come from the decay of the colour string stretched between the decay quarks coming from the same W. When the colour flow pattern is modified, strings might span between two quarks from different W bosons and subsequently decaying particles cannot be uniquely assigned to either W. Different phenomenological models exist to describe this colour reconnection.

In the model of Sjöstrand and Khoze [144], as it is implemented in PYTHIA, CR may occur when strings overlap. In the type I model (SKI), the strings are associated to colour flux tubes with a significant transverse extension. The reconnection probability is given by $P_{\text{reco}} = 1 - \exp(-f(\sqrt{s}) \cdot k_I)$, with $f$ the volume of the overlap and $k_I$ a free parameter. The colour dipoles in ARIADNE are allowed to reconnect if this reduces the string length [145]. In the model for boson pair production, AR2, the reconnection only occurs between gluons from different W’s with energies $E_g < \Gamma_W \approx 2$ GeV. Rearrangements between dipoles within a W are permitted at all scales. In HERWIG, the fragmentation follows a space-time picture and the CR is a local phenomenon [146]. At the start of the cluster formation, right after the parton showering and gluon splitting, reconnection may occur if it reduces the space-time extension of the clusters.

The differences between the effects of these models with and without CR on the TGC measurements using the hadronic channel are listed in the bottom part of Table 5.11. Here the pseudo-data are events generated with KandY at 189 GeV using the L3 tuning and the baseline MC are YFSWW3 events with no CR. The shifts observed with the Cetraro files are of the same order.

The effects of colour reconnection in $e^+e^- \rightarrow W^+W^-$ has been measured in L3 with the particle flow method [147]. In this analysis the particle rates between jets from the same and from different W bosons are compared, since CR will result in a depletion or enhancement of particle production in inter-jet regions. An upper limit of 2.1 on $k_I$ in the SKI model at 95% confidence level is obtained which corresponds to an upper limit on the reconnection probability of 64% at $\sqrt{s} = 189$ GeV. A combination of preliminary LEP results using the same approach is performed [148] and prefers a value of $49^{+16}_{-26}$% reconnection probability. The CR schemes in ARIADNE and HERWIG can not be constrained with this method since they do not modify the inter-jet activity significantly. To determine the systematic error from CR in the hadronic channel, the average of the observed shifts are taken, but for the SKI model only half the effect is considered. The resulting errors are also given in Table 5.11.

5.10 Combination of Systematic Errors

The systematic errors determined in the previous sections need to be combined between the different channels and with each other. For correlated errors determined with the data set this is rather straightforward: the likelihoods including the changes are combined before the systematic error is determined with Eq. 5.1. When such uncertainties are derived from MC samples, the combination is not so trivial, especially when the systematic error is some average of observed shifts. A possible way to combine is to weight the systematic error for each channel with its statistical uncertainty. However, this assumes
### 5.10. Combination of Systematic Errors

#### Systematic Errors

<table>
<thead>
<tr>
<th>Shifts due to BEC between W’s</th>
<th>( \Delta(g_1^Z) )</th>
<th>( \Delta(\kappa_\gamma) )</th>
<th>( \Delta(\lambda_\gamma) )</th>
<th>( \Delta(g_5^Z) )</th>
<th>( \Delta(\kappa_\lambda) )</th>
<th>( \Delta(\lambda_\lambda) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>BE(_{32}) all - same</td>
<td>-0.003</td>
<td>-0.009</td>
<td>-0.005</td>
<td>-0.012</td>
<td>-0.007</td>
<td>-0.004</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Shifts and uncertainties due to CR</th>
<th>( \Delta(g_1^Z) )</th>
<th>( \Delta(\kappa_\gamma) )</th>
<th>( \Delta(\lambda_\gamma) )</th>
<th>( \Delta(g_5^Z) )</th>
<th>( \Delta(\kappa_\lambda) )</th>
<th>( \Delta(\lambda_\lambda) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PYTHIA SKI (100%)</td>
<td>-0.006</td>
<td>+0.008</td>
<td>-0.017</td>
<td>+0.003</td>
<td>-0.010</td>
<td>-0.009</td>
</tr>
<tr>
<td>HERWIG CR</td>
<td>-0.003</td>
<td>+0.003</td>
<td>-0.008</td>
<td>-0.024</td>
<td>+0.000</td>
<td>-0.010</td>
</tr>
<tr>
<td>ARIADNE AR2</td>
<td>-0.012</td>
<td>+0.011</td>
<td>-0.012</td>
<td>+0.021</td>
<td>-0.021</td>
<td>-0.026</td>
</tr>
<tr>
<td>syst. error</td>
<td>0.006</td>
<td>0.006</td>
<td>0.009</td>
<td>0.015</td>
<td>0.009</td>
<td>0.014</td>
</tr>
</tbody>
</table>

Table 5.11: The top part of the table shows the difference between the fit using the BE\(_{32}\) model to simulate BEC between all particles and between particles coming only from the same W. The bottom part shows the shifts due to different CR schemes within a certain fragmentation model. The systematic error extracted from these values is also given. All the results are obtained using events generated with Kandy and passing the hadronic selection criteria.

Gaussian shapes for the likelihoods and is only an approximation in other cases.

Suffering from similar problems for the LEP combination of the TGC results, i.e. combining correlated systematic errors having non-parabolic likelihoods, the following solution has been proposed [149]. In the simplest case of one likelihood for coupling \( \psi \) with one systematic error \( \delta_\xi \), this method consists of minimising the likelihood

\[
-\ln L(\psi) = -\ln L_{\text{stat}}(\psi - \Delta \cdot \delta_\xi) + \frac{\Delta^2}{2}.
\]

The assumption has been made that the systematic errors have a Gaussian distribution and induce only a shift in the central value. The simultaneous minimisation of this likelihood as function of the two parameters, \( \psi \) and \( \Delta \), will give a uncertainty on the coupling containing both the statistical and the systematic error. By construction, the minimum of \( \Delta \) will be close to zero with an uncertainty of \( \pm 1 \).

This method can easily be extended to combine multiple likelihoods by adding them and introducing one common \( \Delta \) for each correlated systematic error. The uncorrelated uncertainties are taken into account by including one \( \Delta \) for each contributing likelihood. The total systematic error can now be estimated by taking the quadratic difference with the total statistical error. Note that the central value of the TGC fit does not coincide with the value obtained from the combination without systematics because the coupling is now free to vary within both the statistical and systematic uncertainties.

This method was already applied in Section 5.5 to obtain the systematic uncertainty due to the limited MC statistics for the channels separately as well as for the combined result. This is the only source that is uncorrelated between all the channels as well as between all centre-of-mass energies. The uncertainties due to possible Bose-Einstein correlations and colour reconnection only affect the fully hadronic decays. In practice,
these sources are taken to be correlated between all channels, but a value of zero is
assigned to the semi-leptonic channels. All other sources are treated to be fully correlated
between the different channels and energies. The errors that are derived from a MC
comparison, i.e. those originating from uncertainties in the jet and lepton reconstruction
and the fragmentation modelling, are combined between the channels with the method
described above. The uncertainties from all other sources are determined using the full
data sample.

The contribution from each of these sources to the total systematic error for each of the
couplings is listed in Table 5.12. This combination is performed with the method described
above, assuming no correlations between the different sources. The total systematic error
ranges between 40 and 63% of the statistical error, depending on the coupling. One
of the most important contributions comes from the experimental uncertainty in the
charge assignment. For the couplings where this uncertainty is only the second largest
contribution, the most important error comes from the description of initial and final
state radiation (for $\kappa_\gamma$ and $g_5^Z$) or from the limited MC statistics (for $\kappa_Z$ and $\lambda_Z$). The
uncertainties coming from the reconstruction of the final state fermions and from
the modelling of the BEC and CR contribute only marginally to the total systematic error. All the other sources contribute with comparable size to the total systematic error.

Other sources that have been investigated but do not contribute significantly to the
total systematic error are: the uncertainty on the luminosity measurement, the LEP beam
energy determination and the selection efficiencies.

<table>
<thead>
<tr>
<th>source of uncertainty:</th>
<th>$\delta(g_1^Z)$</th>
<th>$\delta(\kappa_\gamma)$</th>
<th>$\delta(\lambda_\gamma)$</th>
<th>$\delta(g_5^Z)$</th>
<th>$\delta(\kappa_Z)$</th>
<th>$\delta(\lambda_Z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{WW} &amp; d\sigma_{WW}/d\cos\Theta_W$</td>
<td>0.005</td>
<td>0.016</td>
<td>0.005</td>
<td>0.01</td>
<td>0.010</td>
<td>0.010</td>
</tr>
<tr>
<td>background</td>
<td>0.006</td>
<td>0.015</td>
<td>0.007</td>
<td>0.01</td>
<td>0.011</td>
<td>0.014</td>
</tr>
<tr>
<td>ISR &amp; FSR</td>
<td>0.007</td>
<td>0.026</td>
<td>0.008</td>
<td>0.04</td>
<td>0.012</td>
<td>0.015</td>
</tr>
<tr>
<td>charge confusion</td>
<td>0.010</td>
<td>0.017</td>
<td>0.013</td>
<td>0.03</td>
<td>0.016</td>
<td>0.022</td>
</tr>
<tr>
<td>jet &amp; lepton reconstruction</td>
<td>0.001</td>
<td>0.003</td>
<td>0.002</td>
<td>0.01</td>
<td>0.002</td>
<td>0.004</td>
</tr>
<tr>
<td>$M_{WW} &amp; \Gamma_{WW}$</td>
<td>0.005</td>
<td>0.004</td>
<td>0.006</td>
<td>0.01</td>
<td>0.007</td>
<td>0.012</td>
</tr>
<tr>
<td>MC statistics</td>
<td>0.008</td>
<td>0.007</td>
<td>0.012</td>
<td>0.03</td>
<td>0.017</td>
<td>0.007</td>
</tr>
<tr>
<td>Fragmentation</td>
<td>0.008</td>
<td>0.010</td>
<td>0.008</td>
<td>0.03</td>
<td>0.014</td>
<td>0.004</td>
</tr>
<tr>
<td>BEC &amp; CR</td>
<td>0.001</td>
<td>0.002</td>
<td>0.001</td>
<td>0.01</td>
<td>0.002</td>
<td>0.003</td>
</tr>
<tr>
<td>total systematic error</td>
<td>0.019</td>
<td>0.040</td>
<td>0.022</td>
<td>0.06</td>
<td>0.035</td>
<td>0.036</td>
</tr>
<tr>
<td>statistical error</td>
<td>0.035</td>
<td>0.064</td>
<td>0.037</td>
<td>0.15</td>
<td>0.059</td>
<td>0.066</td>
</tr>
</tbody>
</table>

Table 5.12: The various sources (rows) resulting in significant contributions to the total
systematic error on the couplings (columns) in the TGC measurement combining the
hadronic and semi-leptonic channels recorded at centre-of-mass energies $\sqrt{s} \geq 189$ GeV.
The statistical error on the combined fit is also given.
Chapter 6

Results and Discussion

The previous chapters described the path followed to arrive at the results for the measurement of the triple gauge-boson couplings. This started with the theoretical background discussed in Chapter 1, followed by a description of the experiment and the WW selection procedures in Chapter 2. The fit method used to derive values for the couplings was explained and tested in Chapter 3. Several tests of quantities vital for the reconstruction and prediction of the phase space angles were performed in Chapter 4, while the remaining uncertainties were transformed into systematic errors in Chapter 5.

The results obtained at the end of this path are summarised in the next section. These results will be compared to and combined with complementary results from L3 in Section 6.2. The obtained values will be used in Section 6.3 to place limits on a possible substructure of the W boson. In Section 6.4, a comparison between and combination of the results from the four LEP experiments is performed.

6.1 Results

A measurement of the triple gauge-boson couplings $\gamma\text{WW}$ and $Z\text{WW}$ is performed using semi-leptonically and fully hadronically decaying W-pairs recorded during the last three years of LEP data taking, from 1998 to 2000. During these years, the electron and positron beams were colliding at centre-of-mass energies ranging from 189 up to 209 GeV. The total integrated luminosity collected by the L3 detector at these energies amounts to 629.4 pb$^{-1}$. Bounds on possible anomalous couplings values are derived by performing a binned fit to the five-dimensional phase space region spanned by the W$^-$ production angle and the decay angles of both W bosons.

A first set of couplings is measured under the constraints $\kappa_Z = g_1^Z - \tan^2 \theta_w (\kappa_\gamma - 1)$, $\lambda_Z = \lambda_\gamma$ and $g_5^Z = 0$, reducing the number of independent $CP$-conserving couplings to three, as explained in Chapter 1. The one-parameter fits, fixing all but one of the couplings to their Standard Model expectation, yield:
\[ g_1^Z = 0.925^{+0.036}_{-0.034} \pm 0.019 \quad \text{(SM = 1)}, \]
\[ \kappa_\gamma = 0.853^{+0.067}_{-0.062} \pm 0.040 \quad \text{(SM = 1)}, \]
\[ \lambda_\gamma = -0.058^{+0.039}_{-0.036} \pm 0.022 \quad \text{(SM = 0)}. \]

The first error is statistical and the second is systematic, both at 68\% C.L.. The latter error is obtained by taking the squared difference between the error including all systematic uncertainties, as described in Section 5.10, and the statistical error. The measured couplings deviate slightly from the Standard Model value: between 1.3\sigma for \( \lambda_\gamma \) and 1.9\sigma in the case of \( \kappa_\gamma \).

The systematic errors are included in the multi-parameter fits by folding the likelihoods with a multi-variate Gaussian. The widths of these distributions are proportional to the systematic errors which are assumed to be uncorrelated between the couplings. The one-, two- and three-parameter fits are compared in Fig. 6.1. The changes in log-likelihoods used to obtain the errors and contours are discussed in Section 3.6.2. A good agreement amongst the different fits is observed. Leaving the couplings free or fixing them to their Standard Model value does not have a large impact on the fit results. This observation implies that no cancellation occurs that could conceal the couplings measurement in lower dimensional fits. It also supports the interpretation of the deviations in the one-parameter fit results as a statistical fluctuation.

The limits at 95\% C.L. on the TGC’s from the three-parameter fit including systematic errors are:

\[
0.79 < g_1^Z < 1.06 \quad \rho(g_1^Z, \kappa_\gamma) = -0.22, \\
0.71 < \kappa_\gamma < 1.36 \quad \rho(\kappa_\gamma, \lambda_\gamma) = -0.10, \\
-0.11 < \lambda_\gamma < 0.16 \quad \rho(\lambda_\gamma, g_1^Z) = -0.69.
\]

The correlation coefficients between the couplings are given on the right.

Relaxing the constraints, the one-parameter fits to the remaining three \( CP \)-conserving couplings yield:

\[ g_5^Z = 0.00^{+0.15}_{-0.15} \pm 0.06 \quad \text{(SM = 0)}, \]
\[ \kappa_Z = 0.871^{+0.061}_{-0.057} \pm 0.035 \quad \text{(SM = 1)}, \]
\[ \lambda_Z = -0.100^{+0.071}_{-0.062} \pm 0.036 \quad \text{(SM = 0)}. \]

The statistical and systematic errors are shown separately. A similar level of agreement with the Standard Model is obtained as for the other couplings determined under the usual constraints.
6.1. Results

Figure 6.1: A comparison between the one-, two- and three-parameter fits for the couplings $g_1^Z$, $\kappa_\gamma$ and $\lambda_\gamma$, including systematic errors. The vertical and horizontal lines are the 68% C.L. intervals when all but one parameters are fixed to their Standard Model values (indicated by a star). The dark and light shaded contours are the 68% and 95% C.L. allowed regions when two couplings are varied simultaneously. The dashed line is the contour obtained from the projection of the three-parameter fit using the same value above the minimum as for the 68% C.L. contours of the two-parameter fit. The closed and open dots are the minima obtained from the two- and three-parameter fits, respectively.
6.2 Combination of L3 results

Besides the measurements presented in this thesis, TGC’s were also derived from W-pair events using the data taken at lower energies just above the W-pair threshold. These results are given in the next section, after which the results using the single-W events will be discussed. The subsequent section makes a comparison between all the results obtained with events recorded with the L3 detector. Since all these measurements are complementary, a combination will be performed at the end.

6.2.1 WW Results at 161, 172 and 183 GeV

During the 1996 and 1997 data taking periods, LEP delivered beams at lower energies but high enough to produce W-pairs. The total luminosities collected at the centre-of-mass energies of 161, 172 and 183 GeV are respectively 10.9, 10.3 and 55.5 pb⁻¹. The selection procedures for the different WW decay channels at these energies are largely similar to the ones described in Chapter 2. Values for the TGC’s have been extracted from these events [121] using the unbinned fit described in Section 4.6.1. For the hadronic channel, only the W⁻ production angle was considered while also the leptonic decay angles were included for qqℓν decays.

The results on the set 1 couplings, with the usual constraints, using these events are:

\[
\begin{align*}
    g_T^Z &= 1.13^{+0.18}_{-0.18} \pm 0.10, \\
    \kappa_\gamma &= 1.00^{+0.93}_{-0.39} \pm 0.39, \\
    \lambda_\gamma &= 0.10^{+0.22}_{-0.20} \pm 0.08.
\end{align*}
\]

The first error is statistical, while the second one is systematic. The errors, as expected from the lower luminosities and centre-of-mass energies, are much larger than those given in the previous section.

6.2.2 Single-W Results

One of the background processes in the selection of WW events is the class of single-W events, described in Section 1.6. Two of the diagrams contributing to this process contain a vertex of the type VWW. The case where the neutral boson V is a Z is suppressed because of the large Z mass, and only the γWW couplings can be measured. A dedicated analysis of these events is reported in [82].

Besides a W boson, a neutrino and an electron or positron are produced in this process. The latter particle is typically scattered under low polar angles and escapes the detector in most cases. The typical signature is thus the presence of the decay products of the W boson: two hadronic jets or a single energetic lepton with missing energy. A pre-selection based on cut variables selects candidates for the hadronic final states. These events are then presented as input to a neural network (NN) to further differentiate between signal and background. The selection of the leptonic final states is a cut based analysis with variables and cut values depending on the final state lepton.
The coupling values are extracted using a binned maximum likelihood fit to the NN output distribution, for the hadronic final states, or the lepton energy spectra, for the leptonic final states. The dependence of the likelihood on the couplings is obtained with the same reweighting technique as described in Section 3.6.1. Combining all data collected at centre-of-mass energies between 161 and 209 GeV and allowing only one coupling to vary at a time yields:

\[
\kappa_\gamma = 1.116^{+0.082}_{-0.086} \pm 0.068, \\
\lambda_\gamma = 0.35^{+0.10}_{-0.13} \pm 0.08.
\]

The first error is statistical, while the second one is systematic. The main contribution to the systematic error comes from the estimation of the signal efficiency and the theoretical uncertainty on the single-W cross section. The coupling dependence of the WW background has been taken into account in the fit.

### 6.2.3 \(Q^2\) Dependence

The introduction of a form factor taking into account a possible \(Q^2\) dependence of the couplings was discussed in Section 2. The results given above are all obtained at different values of \(Q^2\) and before they can be combined, they should be checked for a possible \(Q^2\) dependence. The results from W-pair production give a measurement at \(Q^2 = s\), while those obtained from single-W events are at \(Q^2 = M_W^2\).

Fig. 6.2 shows the results for the one-parameter fits as a function of the \(Q^2\) value of the measurement. The single-W results are combined between all centre-of-mass energies since they all have the same \(Q^2\) value. For the WW analysis, the results are separated between years, and the average of the centre-of-mass energies in each year was taken as \(Q^2\) value. The results from the years 1996 and 1997 are combined into one value. Although the WW results prefer values lower than the single-W channel, no positive evidence for a form factor behaviour as given in Eq. (1.20) is observed.

### 6.2.4 Combined Results

The combination of the results from this thesis with the WW results at \(\sqrt{s} \leq 183\) GeV takes into account the correlations between the systematic errors. Due to the much larger uncertainties on the results at lower \(\sqrt{s}\), both statistical and systematic, the gain is marginal.

The combination with the single-W results is less straightforward. The background for the single-W selection consists for a large part of W-pair events. A significant fraction of the hadronic single-W sample (156 out of 740 candidates) is also selected by the \(qq\ell\nu\) W-pair selections, and the TGC measurements from both analyses are therefore correlated. About 75% of this overlap are W-pair events, while only 7% are single-W events. The remainder of this sample are \(qq(\gamma)\) events. To avoid this double counting, data and Monte Carlo events selected in both analyses are considered in the W-pair sample only.

The effect of removing these events from the single-W sample can be seen in Table 6.1. It is expected that the statistical error on the TGC’s increases due to the reduction of the number of events, and that is the case for \(\lambda_\gamma\). The error on \(\kappa_\gamma\), however, decreases slightly.
6.2. Combination of L3 results

Results and Discussion

Figure 6.2: The measurements of the couplings as a function of $Q^2$ obtained from the single-W and W-pair analyses. The values for $g_1^Z$ (open dots), $\kappa_\gamma$ (squares) and $\lambda_\gamma$ (triangles) are derived using one-parameter fits under the assumption of the constraints $\kappa_Z = g_1^Z - \tan^2 \theta_w (\kappa_\gamma - 1)$, $\lambda_Z = \lambda_\gamma$. The inner error bar is the statistical error while the full error bar also includes systematic errors. The dashed line represents the Standard Model values at tree level.

<table>
<thead>
<tr>
<th>Coupling</th>
<th>Full Single-W Sample</th>
<th>WW Overlap Removed</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_\gamma$</td>
<td>$1.116^{+0.082}_{-0.086}$</td>
<td>$1.179^{+0.076}_{-0.080}$</td>
</tr>
<tr>
<td>$\lambda_\gamma$</td>
<td>$0.35^{+0.10}_{-0.13}$</td>
<td>$0.30^{+0.11}_{-0.19}$</td>
</tr>
</tbody>
</table>

Table 6.1: The effect on the measurement of the couplings $\kappa_\gamma$ and $\lambda_\gamma$ from the single-W channel when events also selected by the WW analyses are removed from the single-W sample.

as a consequence of the change in the central value away from the Standard Model value. The systematic errors are considered to remain equal in size since the most important sources do not affect the WW background events.

The combination of the one-parameter results from W-pair and single-W events using the full LEP2 data sample taken at centre-of-mass energies between 161 and 209 GeV leads to:

$$g_1^Z = 0.927^{+0.035}_{-0.034} \pm 0.021,$$

$$\kappa_\gamma = 0.972^{+0.066}_{-0.063} \pm 0.024,$$

$$\lambda_\gamma = -0.057^{+0.039}_{-0.036} \pm 0.023.$$

All the couplings are derived under the constraints $\kappa_Z = g_1^Z - \tan^2 \theta_w (\kappa_\gamma - 1)$ and $\lambda_Z = \lambda_\gamma$ and fixing all the other couplings to their Standard Model value. The most important
observation is that the result for $\kappa_\gamma$ moves towards the Standard Model value when more information is included.

A combination between the multi-parameter fit results from W-pair and single-W events is also performed. A comparison between the 68% C.L. contours from both types of events can be seen in Fig. 6.3. Since the coupling $g_1^Z$ is not measured in the single-W analysis, the corresponding contours are shown as bands. The Standard Model values are in each case at the edge of the 68% C.L. contours of the combined results.

When all three couplings are allowed to vary simultaneously, the 95% C.L. intervals for the couplings are:

$$0.75 < g_1^Z < 1.03 \quad \rho(g_1^Z, \kappa_\gamma) = -0.53,$$

$$0.90 < \kappa_\gamma < 1.34 \quad \rho(\kappa_\gamma, \lambda_\gamma) = 0.12,$$

$$-0.10 < \lambda_\gamma < 0.19 \quad \rho(\lambda_\gamma, g_1^Z) = -0.64,$$

where the correlation coefficients are given at the right.

### 6.3 W boson substructure

The triple gauge-boson couplings were related to the static properties of the W boson in Section 1.3.1. With the relations given in Eq. (1.14), and the L3 results of the combined two-parameter fit to $\kappa_\gamma$ and $\lambda_\gamma$, the magnetic dipole and electric quadrupole moment of the W boson are measured to be:

$$\mu_W = (1.960 \pm 0.076) \frac{e}{2M_W} = (1.245 \pm 0.048) \cdot 10^{-5} \mu_B,$$

$$Q_W = (1.06 \pm 0.11) \frac{e}{M_W} = (3.18 \pm 0.33) \cdot 10^{-36} \text{e} \cdot \text{m}^2,$$

with a correlation coefficient of 0.49.

The magnetic dipole and electric quadrupole moments of an extended non-relativistic object with a homogeneously distributed charge can be easily related to its shape. Assuming that this object is a spheroid with semi-axes $a$ and $b$, the average radius is given by $R_W \equiv (a + b)/2 = \delta \mu_W / (\mu_W M_W)$ [150] and the factor indicating the deformation from a sphere becomes $\Delta_W \equiv (a^2 - b^2)/2 = (5/4)\delta Q_W$ [151]. In these expressions, $\delta \mu$ and $\delta Q$ are the anomalous components of these moments defined as the deviation from the Standard Model values at tree level. Knowing that the W bosons are produced almost at rest and exploiting the relation between the moments and the couplings, the average radius and deformation factor can be determined using:

$$R_W = \frac{\kappa_\gamma + \lambda_\gamma - 1}{M_W}, \quad (6.1a)$$

$$\Delta_W = \frac{5 \kappa_\gamma - \lambda_\gamma - 1}{4 M_W^2}. \quad (6.1b)$$
Figure 6.3: A comparison between the two-parameter fit results for the couplings $g_Z^1$, $\kappa_\gamma$ and $\lambda_\gamma$ from $W$-pair and single-$W$ events. The 68% C.L. intervals are indicated by the hatched areas for both analyses. The combination between the two results is given by the shaded area. The star represents the Standard Model values at tree level.
Taking the values for the magnetic dipole and electric quadrupole moments as determined above, the measurement of the radius and deformation of the W yields:

\[
R_W = (-1.0 \pm 1.9) \times 10^{-19} \text{ m},
\]

\[
\Delta_W = (0.42 \pm 0.84) \times 10^{-36} \text{ m}^2.
\]

This measurement shows that the W boson is a point-like particle down to a scale of \(10^{-19}\) m and no evidence for a substructure is found.

### 6.4 Comparison with Other Results

Triple gauge-boson couplings are also measured by the other three LEP experiments. Both final and preliminary results have been submitted to conferences exploiting most of the data sample taken at centre-of-mass energies above the W-pair production threshold [152, 153, 154].

A combination of the LEP results is performed in [155]. Each experiment provides the likelihood curves including systematic uncertainties that are uncorrelated between the experiments and a table with the sizes of the LEP correlated errors. In the combination, these errors are taken into account with the same method as described in Section 5.10, treating them to be fully correlated. The uncertainty due to \(\mathcal{O}(\alpha)\) effects is assumed to be the largest difference observed by any of the experiments when YFSWW3 and RACOONWW are compared.

The results for the couplings \(g^Z_1\), \(\kappa_\gamma\) and \(\lambda_\gamma\), with the usual constraints on \(\kappa_Z\) and \(\lambda_Z\), from each experiment and the combination are given in Table 6.2 and presented in Fig. 6.4. Note that the central values and errors from each experiment can differ between the values quoted here and in the references due to a slightly different approach in the treatment of systematic errors. As can be seen, there is good agreement between the different LEP experiments and all couplings are compatible with their Standard Model expectation. The limits are an order of magnitude tighter than those from the direct measurements at DØ and slightly larger than the indirect constraints obtained from the electroweak precision measurements (both type of constraints are discussed in Section 1.3.4).

The errors obtained by the different experiments are of the same order of magnitude although some differences exist. DELPHI uses only W-pairs produced at and above centre-of-mass energies of 189 GeV, while ALEPH and OPAL also include the fully leptonic channel in the combination. In addition, the different analysis techniques and systematic error treatments cause small differences, see the references for details about these approaches. The OPAL results include for the single-W analysis only the data sample taken at 189 GeV. The other experiments include the single-W results analysing most of the LEP2 data sample. ALEPH and OPAL also include limits on \(\kappa_\gamma\) and \(\lambda_\gamma\) obtained from analysing the single-photon process, i.e. \(e^+ e^- \rightarrow \nu_e \bar{\nu}_e \gamma\). The sensitivity of this channel is typically five times smaller for \(\kappa_\gamma\) and an order of magnitude smaller for \(\lambda_\gamma\) compared to the W-pair results.

Limits on the coupling \(g^Z_5\) are also derived by ALEPH and OPAL using W-pair events. A comparison between the experiments is given in Table 6.3.
### Table 6.2

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$g_1^Z$</th>
<th>$\kappa_\gamma$</th>
<th>$\lambda_\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALEPH</td>
<td>$1.026^{+0.034}_{-0.033}$</td>
<td>$1.022^{+0.073}_{-0.072}$</td>
<td>$0.012^{+0.033}_{-0.032}$</td>
</tr>
<tr>
<td>DELPHI</td>
<td>$1.002^{+0.038}_{-0.040}$</td>
<td>$0.955^{+0.086}_{-0.088}$</td>
<td>$1.014^{+0.044}_{-0.042}$</td>
</tr>
<tr>
<td>L3</td>
<td>$0.927^{+0.042}_{-0.041}$</td>
<td>$0.978^{+0.071}_{-0.069}$</td>
<td>$-0.058^{+0.047}_{-0.044}$</td>
</tr>
<tr>
<td>OPAL</td>
<td>$0.984^{+0.035}_{-0.034}$</td>
<td>$0.929^{+0.085}_{-0.081}$</td>
<td>$-0.063^{+0.036}_{-0.036}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>68% C.L.</th>
<th>95% C.L.</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEP</td>
<td>$0.991^{+0.022}_{-0.021}$</td>
<td>$0.984^{+0.042}_{-0.047}$</td>
</tr>
</tbody>
</table>

The results from the four LEP experiments on the couplings $g_1^Z$, $\kappa_\gamma$, and $\lambda_\gamma$ under the constraints $\kappa_Z = g_1^Z - \tan^2 \theta_w (\kappa_\gamma - 1)$ and $\lambda_Z = \lambda_\gamma$ and allowing one parameter to vary at a time. All errors include statistical and systematic uncertainties. The combination between the LEP results takes the correlations between the systematic errors into account. Both the one standard deviation errors and the 95% C.L. intervals are given.

### Figure 6.4

A comparison of the measurement of $g_1^Z$ (left), $\kappa_\gamma$ (middle) and $\lambda_\gamma$ (right) between the LEP experiments as listed in Table 6.2. The triangle indicates the LEP combined result and the shaded band reflects the 68% C.L. allowed region. The vertical solid line is at the Standard Model value.

### Table 6.3

<table>
<thead>
<tr>
<th></th>
<th>ALEPH</th>
<th>L3</th>
<th>OPAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_5^Z$</td>
<td>$-0.13 \pm 0.14$</td>
<td>$0.00 \pm 0.16$</td>
<td>$-0.04 \pm 0.13$</td>
</tr>
</tbody>
</table>

The results from the measurement of the $C$ and $P$ violating coupling $g_5^Z$ using $W$-pair events from the ALEPH, L3 and OPAL experiments. The errors include both the statistical and systematic contributions.
Chapter 7

Conclusions and Outlook

7.1 Conclusions

The self-couplings of the W, Z and γ are an immediate consequence of the non-Abelian structure of the $SU(2)_L \times U(1)_Y$ gauge symmetry governing electroweak interactions. The properties of the triple gauge-boson vertex have been measured in a direct manner. The results are expressed in terms of the couplings appearing in a general expression for the Lagrangian of this vertex.

This thesis described the measurement of these couplings exploiting the full data sample of hadronically and semi-leptonically decaying W-pairs recorded with the L3 detector at centre-of-mass energies between 189 and 209 GeV. This corresponds to a total integrated luminosity of $L = 629.36 \text{ pb}^{-1}$. These results were combined with values obtained from W-pair events measured at lower centre-of-mass energies ($161 \leq \sqrt{s} \leq 183 \text{ GeV}$) and with the single-W results using the full LEP2 data sample. Under the constraints $\kappa_Z = g_1^Z - \tan^2 \theta_w (\kappa_\gamma - 1)$ and $\lambda_Z = \lambda_\gamma$, the one-parameter fit results are:

$$
\begin{align*}
g_1^Z &= 0.927^{+0.041}_{-0.039} \quad (\text{SM} = 1), \\
\kappa_\gamma &= 0.972^{+0.071}_{-0.068} \quad (\text{SM} = 1), \\
\lambda_\gamma &= -0.057^{+0.045}_{-0.043} \quad (\text{SM} = 0).
\end{align*}
$$

The errors are at 68% C.L. and include both statistical and systematic uncertainties. The Standard Model values at tree level are also given. When all LEP results are combined, the errors are further reduced by about a factor of two. The couplings $g_5^Z$, $\kappa_Z$ and $\lambda_Z$ were also measured without the aforementioned constraints.

Multi-parameter fits were also performed to check whether the one-parameter fits concealed the presence of anomalous couplings. The results from these fits are also in agreement with the Standard Model values. Furthermore, a limit on a possible substructure of the W boson is derived using the L3 combined $(\kappa_\gamma, \lambda_\gamma)$ fit result. The W is found to be compatible with a point-like particle down to a scale of $10^{-19} \text{ m}$.

The couplings of the type $\gamma W^+ W^- \text{ and } Z W^+ W^-$ predicted by the Standard Model indeed exist. Moreover, their strength is in very good agreement with the expected values.
The couplings not foreseen in the Standard Model have a measured strength compatible with zero. These observations support the belief that spontaneously broken, non-Abelian gauge theories provide a legitimate theoretical structure to describe the fundamental interactions of Nature.

7.2 Prospects for Future Experiments

The measurements of triple gauge-boson couplings at LEP have substantially improved the existing direct limits. In this section, a short overview is given of the expected sensitivities of future experiments that are able to compete with, or supersede, the LEP results. First, the limits attainable at the end of the Tevatron are discussed. Then the expectations for the LHC, currently being built at CERN, are described. Finally, the prospects for measurement of triple gauge-boson couplings at TESLA, one of the next linear colliders under study, are given. All the values discussed in the next paragraphs are compared with the LEP results in Fig. 7.1.

Tevatron

In spring 2001, the Tevatron started with the second phase of its physics programme after an upgrade of the accelerator and both detectors. The prospects for the measurements of triple gauge-boson couplings in Run II [156] were obtained by a luminosity scaling of the existing limits, which were estimated to scale like $L^{0.25}$. If a total integrated luminosity of 10 fb$^{-1}$ can be obtained and CDF performs a similar analysis as D, the limits given in (1.21) will improve by a factor of four. These bounds will then be competitive with the LEP results as can be seen in Fig. 7.1. The scaling does not take into account a possible improvement in the analyses and neglects the slightly higher cross section resulting from the increase in centre-of-mass energy from 1.8 to 1.96 TeV.

LHC

The experiments at Fermilab will continue to take data until 2009, two years after the commissioning of LHC, planned for spring 2007. The benefit of this pp collider over the Tevatron will be the higher attainable centre-of-mass energy of 14 TeV and a significant increase in luminosity. The initial plan is to run for one to three years at low luminosity, delivering about 10 fb$^{-1}$ per year, and then upgrade the machine to gain a factor of ten in luminosity.

Expected accuracies of triple gauge-boson couplings measurements are derived [157] for the ATLAS and CMS experiments, assuming a form factor scale of $\Lambda_F = 10$ TeV. After running for five years in the high luminosity mode, the 95% C.L. errors on $\kappa_\gamma$ and $g_1^Z$ will reduce by about one order of magnitude, while for the coupling $\lambda_\gamma$ an improvement of two orders of magnitude is expected.
Linear Colliders

One of the possible facilities under study for the post-LHC era is a linear $e^+e^-$ collider. Most of the studies are performed in the scope of the TESLA project at DESY [158]. The original plan is to collide the beams at $\sqrt{s} = 500$ GeV, and to deliver about $500 \text{ fb}^{-1}$ of integrated luminosity within the first five years of running.

Using polarised beams, the estimated sensitivities for the couplings are of the order of a few $10^{-4}$ [159]. The accuracy gets worse by a factor of two to three if unpolarised beams are used. Other options like running in $\gamma\gamma$ or $e\gamma$ mode were investigated [160] and were shown to be about a factor of two less sensitive to anomalous couplings.

When such sensitivities are reached, the measurement becomes particularly interesting since the vertex can now be tested at the loop level. The electroweak loop corrections to the tree level triple gauge-boson couplings are estimated to be of the order of $10^{-3}$ [48].

As discussed in Section 1.3.3, some models describing physics beyond the Standard Model give corrections of the same order of magnitude, but only for specific parameter values.

![Figure 7.1](image)

Figure 7.1: A comparison of the 95% CL errors for $g_1^Z$ (left), $\kappa_\gamma$ (middle) and $\lambda_\gamma$ (right) between the LEP results and possible future limits from different experiments. The values are derived under the constraints $\kappa_Z = g_1^Z - \tan^2 \theta_W (\kappa_\gamma - 1)$ and $\lambda_Z = \lambda_\gamma$. For the LEP limits, the errors given in Table 6.2 are shown. The other sensitivities are obtained assuming the Tevatron to deliver $10 \text{ fb}^{-1}$, and the LHC and TESLA (in $e^+e^-$ mode) a total of $500 \text{ fb}^{-1}$ each. For TESLA, a centre-of-mass energy of $\sqrt{s} = 500$ GeV is assumed with both beams polarised.
Appendix A

Performance of WW selections

The following pages contain tables with details of the performance of the various selections described in Chapter 2. The values are given separately for each channel at each centre-of-mass energy analysed. The actual centre-of-mass energy (range) and the corresponding luminosity are listed in Table 2.1.

The left part of the first table always shows the efficiencies $\epsilon$ in % for selecting a certain channel (columns) in the selection of the processes given in the rows. These numbers are obtained using YFSWW3 and are transformed into expected numbers of signal and WW background events, $N_{\text{exp}}^{\text{sig}}$ and $N_{\text{exp}}^{\text{WWbck}}$, using the CC03 cross section and luminosity at that centre-of-mass energy. The last column gives the CC03 to 4f correction factor $f_{4f}$ with which the total number of selected events should be multiplied in order to include also the singly and non-resonant diagrams containing the same final states as WW events. This factor is calculated as the average of the weights defined in Eq. (3.7) but with Standard Model couplings in the numerator.

The second table at each centre-of-mass energy shows the accepted cross sections for the known non-WW background sources. Adding all these contributions, together with the WW background events and the contribution from 4f events, results in the total number of expected background events $N_{\text{exp}}^{\text{bck}}$. The total number of expected events $N_{\text{exp}}^{\text{tot}}$ is the sum of signal and all background contributions. This value is compared with the number of observed events $N_{\text{data}}$ given in the last column.
### 189 GeV

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\epsilon(qqqq)$</th>
<th>$\epsilon(qq\ell)$</th>
<th>$\epsilon(qq\ell\ell)$</th>
<th>$\epsilon(qq\ell\ell\ell)$</th>
<th>$N_{exp}^{WW}$</th>
<th>$N_{exp}^{bg}$</th>
<th>$N_{data}$</th>
<th>$f_4f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>qqqq</td>
<td>88.08</td>
<td>0.12</td>
<td>0.05</td>
<td>0.69</td>
<td>3.7</td>
<td>1159.4</td>
<td>1157</td>
<td>1.005</td>
</tr>
<tr>
<td>qq\mu</td>
<td>0.01</td>
<td>78.07</td>
<td>0.15</td>
<td>1.53</td>
<td>7.3</td>
<td>329.0</td>
<td>327</td>
<td>1.022</td>
</tr>
<tr>
<td>qq\tau</td>
<td>0.00</td>
<td>0.12</td>
<td>77.75</td>
<td>4.15</td>
<td>18.1</td>
<td>326.9</td>
<td>325</td>
<td>1.001</td>
</tr>
<tr>
<td>qq\tau</td>
<td>0.17</td>
<td>7.33</td>
<td>10.44</td>
<td>54.23</td>
<td>77.2</td>
<td>228.6</td>
<td>227</td>
<td>1.005</td>
</tr>
</tbody>
</table>

### 192 GeV

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\sigma^{acc}_{qq(\gamma)}$ [pb]</th>
<th>$\sigma^{acc}_{ZZ}$ [pb]</th>
<th>$\sigma^{acc}_{Zee}$ [pb]</th>
<th>$\sigma^{acc}_{\gamma\gamma}$ [pb]</th>
<th>$\sigma^{acc}_{\tau\tau(\gamma)}$ [pb]</th>
<th>$N_{exp}^{WW}$</th>
<th>$N_{exp}^{bg}$</th>
<th>$N_{data}$</th>
<th>$f_4f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>qqqq</td>
<td>1.366</td>
<td>0.323</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>308.6</td>
<td>1468.0</td>
<td>1477</td>
<td>1.006</td>
</tr>
<tr>
<td>qq\mu</td>
<td>0.073</td>
<td>0.004</td>
<td>0.015</td>
<td>-</td>
<td>-</td>
<td>31.5</td>
<td>360.5</td>
<td>347</td>
<td>1.001</td>
</tr>
<tr>
<td>qq\tau</td>
<td>0.032</td>
<td>0.028</td>
<td>-</td>
<td>0.021</td>
<td>-</td>
<td>32.7</td>
<td>359.6</td>
<td>341</td>
<td>1.001</td>
</tr>
<tr>
<td>qq\tau</td>
<td>0.315</td>
<td>0.037</td>
<td>0.030</td>
<td>0.006</td>
<td>0.001</td>
<td>147.5</td>
<td>376.0</td>
<td>413</td>
<td>1.005</td>
</tr>
</tbody>
</table>

### 196 GeV

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\sigma^{acc}_{qq(\gamma)}$ [pb]</th>
<th>$\sigma^{acc}_{ZZ}$ [pb]</th>
<th>$\sigma^{acc}_{Zee}$ [pb]</th>
<th>$\sigma^{acc}_{\gamma\gamma}$ [pb]</th>
<th>$\sigma^{acc}_{\tau\tau(\gamma)}$ [pb]</th>
<th>$N_{exp}^{WW}$</th>
<th>$N_{exp}^{bg}$</th>
<th>$N_{data}$</th>
<th>$f_4f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>qqqq</td>
<td>1.226</td>
<td>0.365</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>49.2</td>
<td>246.4</td>
<td>236</td>
<td>1.006</td>
</tr>
<tr>
<td>qq\mu</td>
<td>0.071</td>
<td>0.003</td>
<td>0.018</td>
<td>-</td>
<td>0.004</td>
<td>5.5</td>
<td>61.3</td>
<td>73</td>
<td>1.001</td>
</tr>
<tr>
<td>qq\tau</td>
<td>0.030</td>
<td>0.027</td>
<td>-</td>
<td>0.009</td>
<td>-</td>
<td>5.0</td>
<td>59.4</td>
<td>63</td>
<td>1.001</td>
</tr>
<tr>
<td>qq\tau</td>
<td>0.277</td>
<td>0.043</td>
<td>0.030</td>
<td>0.009</td>
<td>0.001</td>
<td>23.9</td>
<td>62.4</td>
<td>57</td>
<td>1.001</td>
</tr>
</tbody>
</table>

### 144 GeV

| Channel | $\sigma^{acc}_{qq(\gamma)}$ [pb] | $\sigma^{acc}_{ZZ}$ [pb] | $\sigma^{acc}_{Zee}$ [pb] | $\sigma^{acc}_{\gamma\gamma}$ [pb] | $\sigma^{acc}_{\tau\tau(\gamma)}$ [pb] | $N_{exp}^{WW}$ | $N_{exp}^{bg}$ | $N_{data}$ |
|---------|----------------------------------|--------------------------|--------------------------|----------------------------------|--------------------------|---------------|---------------|-------------|--------|
| qqqq    | 1.184                           | 0.425                    | -                        | -                                | -                        | 38.1          | 110.3        | 1.002       |
| qq\mu   | 0.054                           | 0.006                    | 0.019                    | -                                | 0.004                    | 38.1          | 110.3        | 1.002       |
| qq\tau  | 0.034                           | 0.033                    | -                        | 0.027                            | -                        | 38.1          | 110.3        | 1.002       |
| qq\tau  | 0.272                           | 0.059                    | 0.030                    | 0.005                            | 0.001                    | 38.1          | 110.3        | 1.002       |

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\sigma^{acc}_{qq(\gamma)}$ [pb]</th>
<th>$\sigma^{acc}_{ZZ}$ [pb]</th>
<th>$\sigma^{acc}_{Zee}$ [pb]</th>
<th>$\sigma^{acc}_{\gamma\gamma}$ [pb]</th>
<th>$\sigma^{acc}_{\tau\tau(\gamma)}$ [pb]</th>
<th>$N_{exp}^{WW}$</th>
<th>$N_{exp}^{bg}$</th>
<th>$N_{data}$</th>
<th>$f_4f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>qqqq</td>
<td>1.184</td>
<td>0.425</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>38.1</td>
<td>110.3</td>
<td>1.002</td>
<td></td>
</tr>
<tr>
<td>qq\mu</td>
<td>0.054</td>
<td>0.006</td>
<td>0.019</td>
<td>-</td>
<td>0.004</td>
<td>38.1</td>
<td>110.3</td>
<td>1.002</td>
<td></td>
</tr>
<tr>
<td>qq\tau</td>
<td>0.034</td>
<td>0.033</td>
<td>-</td>
<td>0.027</td>
<td>-</td>
<td>38.1</td>
<td>110.3</td>
<td>1.002</td>
<td></td>
</tr>
<tr>
<td>qq\tau</td>
<td>0.272</td>
<td>0.059</td>
<td>0.030</td>
<td>0.005</td>
<td>0.001</td>
<td>38.1</td>
<td>110.3</td>
<td>1.002</td>
<td></td>
</tr>
</tbody>
</table>
## 200 GeV

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\epsilon(qqqq)$</th>
<th>$\epsilon(qq\ell\nu)$</th>
<th>$\epsilon(qq\ell\nu)$</th>
<th>$\epsilon(qq\ell\nu)$</th>
<th>$N_{WW}^{exp}$</th>
<th>$N_{exp}^{sig}$</th>
<th>$f_{lf}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>qqqq</td>
<td>85.83</td>
<td>0.07</td>
<td>0.02</td>
<td>0.52</td>
<td>1.3</td>
<td>555.0</td>
<td>1.006</td>
</tr>
<tr>
<td>qqeν</td>
<td>0.01</td>
<td>74.59</td>
<td>0.17</td>
<td>1.54</td>
<td>3.7</td>
<td>154.4</td>
<td>1.033</td>
</tr>
<tr>
<td>qqμν</td>
<td>0.00</td>
<td>0.15</td>
<td>75.85</td>
<td>4.19</td>
<td>8.7</td>
<td>151.4</td>
<td>1.000</td>
</tr>
<tr>
<td>qqτν</td>
<td>0.11</td>
<td>6.76</td>
<td>10.63</td>
<td>51.73</td>
<td>36.7</td>
<td>107.1</td>
<td>1.006</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\sigma^{acc}_{qq(\gamma)}$ [pb]</th>
<th>$\sigma^{acc}_{ZZ}$ [pb]</th>
<th>$\sigma^{acc}_{Zee}$ [pb]</th>
<th>$\sigma^{acc}_{\gamma\gamma}$ [pb]</th>
<th>$\sigma^{acc}_{\tau\tau(\gamma)}$ [pb]</th>
<th>$N_{bck}^{exp}$</th>
<th>$N_{tot}^{exp}$</th>
<th>$N_{data}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>qqqq</td>
<td>1.066</td>
<td>0.451</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>130.9</td>
<td>685.8</td>
<td>726</td>
</tr>
<tr>
<td>qqeν</td>
<td>0.052</td>
<td>0.007</td>
<td>0.016</td>
<td>–</td>
<td>–</td>
<td>15.5</td>
<td>109.9</td>
<td>152</td>
</tr>
<tr>
<td>qqμν</td>
<td>0.029</td>
<td>0.035</td>
<td>–</td>
<td>0.018</td>
<td>–</td>
<td>15.3</td>
<td>166.7</td>
<td>152</td>
</tr>
<tr>
<td>qqτν</td>
<td>0.240</td>
<td>0.066</td>
<td>0.027</td>
<td>0.004</td>
<td>0.001</td>
<td>65.8</td>
<td>172.8</td>
<td>181</td>
</tr>
</tbody>
</table>

## 202 GeV

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\epsilon(qqqq)$</th>
<th>$\epsilon(qq\ell\nu)$</th>
<th>$\epsilon(qq\ell\nu)$</th>
<th>$\epsilon(qq\ell\nu)$</th>
<th>$N_{WW}^{exp}$</th>
<th>$N_{exp}^{sig}$</th>
<th>$f_{lf}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>qqqq</td>
<td>85.61</td>
<td>0.08</td>
<td>0.03</td>
<td>0.47</td>
<td>0.5</td>
<td>247.7</td>
<td>1.005</td>
</tr>
<tr>
<td>qqeν</td>
<td>0.02</td>
<td>75.15</td>
<td>0.18</td>
<td>1.59</td>
<td>1.7</td>
<td>69.6</td>
<td>1.035</td>
</tr>
<tr>
<td>qqμν</td>
<td>0.00</td>
<td>0.15</td>
<td>75.46</td>
<td>4.05</td>
<td>3.8</td>
<td>68.3</td>
<td>1.002</td>
</tr>
<tr>
<td>qqτν</td>
<td>0.12</td>
<td>6.56</td>
<td>10.68</td>
<td>51.31</td>
<td>16.3</td>
<td>47.5</td>
<td>1.003</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\sigma^{acc}_{qq(\gamma)}$ [pb]</th>
<th>$\sigma^{acc}_{ZZ}$ [pb]</th>
<th>$\sigma^{acc}_{Zee}$ [pb]</th>
<th>$\sigma^{acc}_{\gamma\gamma}$ [pb]</th>
<th>$\sigma^{acc}_{\tau\tau(\gamma)}$ [pb]</th>
<th>$N_{bck}^{exp}$</th>
<th>$N_{tot}^{exp}$</th>
<th>$N_{data}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>qqqq</td>
<td>1.031</td>
<td>0.471</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>57.7</td>
<td>305.4</td>
<td>301</td>
</tr>
<tr>
<td>qqeν</td>
<td>0.056</td>
<td>0.007</td>
<td>0.019</td>
<td>–</td>
<td>–</td>
<td>7.4</td>
<td>77.0</td>
<td>70</td>
</tr>
<tr>
<td>qqμν</td>
<td>0.025</td>
<td>0.036</td>
<td>–</td>
<td>0.031</td>
<td>–</td>
<td>7.3</td>
<td>75.6</td>
<td>79</td>
</tr>
<tr>
<td>qqτν</td>
<td>0.177</td>
<td>0.066</td>
<td>0.032</td>
<td>0.019</td>
<td>0.006</td>
<td>19.5</td>
<td>77.0</td>
<td>77</td>
</tr>
</tbody>
</table>

## 205 GeV

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\epsilon(qqqq)$</th>
<th>$\epsilon(qq\ell\nu)$</th>
<th>$\epsilon(qq\ell\nu)$</th>
<th>$\epsilon(qq\ell\nu)$</th>
<th>$N_{WW}^{exp}$</th>
<th>$N_{exp}^{sig}$</th>
<th>$f_{lf}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>qqqq</td>
<td>84.52</td>
<td>0.07</td>
<td>0.03</td>
<td>0.40</td>
<td>1.0</td>
<td>521.9</td>
<td>1.006</td>
</tr>
<tr>
<td>qqeν</td>
<td>0.02</td>
<td>73.46</td>
<td>0.19</td>
<td>1.56</td>
<td>3.7</td>
<td>145.2</td>
<td>1.040</td>
</tr>
<tr>
<td>qqμν</td>
<td>0.00</td>
<td>0.11</td>
<td>74.55</td>
<td>4.41</td>
<td>8.4</td>
<td>137.9</td>
<td>1.001</td>
</tr>
<tr>
<td>qqτν</td>
<td>0.10</td>
<td>6.30</td>
<td>10.26</td>
<td>50.29</td>
<td>33.4</td>
<td>99.4</td>
<td>1.006</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\sigma^{acc}_{qq(\gamma)}$ [pb]</th>
<th>$\sigma^{acc}_{ZZ}$ [pb]</th>
<th>$\sigma^{acc}_{Zee}$ [pb]</th>
<th>$\sigma^{acc}_{\gamma\gamma}$ [pb]</th>
<th>$\sigma^{acc}_{\tau\tau(\gamma)}$ [pb]</th>
<th>$N_{bck}^{exp}$</th>
<th>$N_{tot}^{exp}$</th>
<th>$N_{data}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>qqqq</td>
<td>1.019</td>
<td>0.486</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>123.0</td>
<td>644.9</td>
<td>656</td>
</tr>
<tr>
<td>qqeν</td>
<td>0.048</td>
<td>0.007</td>
<td>0.021</td>
<td>–</td>
<td>–</td>
<td>15.8</td>
<td>161.1</td>
<td>176</td>
</tr>
<tr>
<td>qqμν</td>
<td>0.024</td>
<td>0.041</td>
<td>–</td>
<td>0.016</td>
<td>–</td>
<td>14.5</td>
<td>152.4</td>
<td>142</td>
</tr>
<tr>
<td>qqτν</td>
<td>0.177</td>
<td>0.066</td>
<td>0.032</td>
<td>0.019</td>
<td>0.006</td>
<td>57.8</td>
<td>157.3</td>
<td>164</td>
</tr>
</tbody>
</table>
### 206 GeV

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\epsilon(qqqq)$</th>
<th>$\epsilon(qqev)$</th>
<th>$\epsilon(qq\mu\nu)$</th>
<th>$\epsilon(qq\tau\nu)$</th>
<th>$\epsilon(l\nu\nu)$</th>
<th>$N_{WWbck}^{exp}$</th>
<th>$N_{exp}^{sig}$</th>
<th>$f_{1f}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>qqqq</td>
<td>84.07</td>
<td>0.04</td>
<td>0.02</td>
<td>0.43</td>
<td>0.00</td>
<td>1.6</td>
<td>859.3</td>
<td>1.004</td>
</tr>
<tr>
<td>qqev</td>
<td>0.02</td>
<td>72.88</td>
<td>0.19</td>
<td>1.56</td>
<td>0.08</td>
<td>6.2</td>
<td>238.5</td>
<td>1.039</td>
</tr>
<tr>
<td>qq\mu\nu</td>
<td>0.00</td>
<td>0.14</td>
<td>74.43</td>
<td>4.36</td>
<td>0.06</td>
<td>14.3</td>
<td>233.6</td>
<td>1.001</td>
</tr>
<tr>
<td>qq\tau\nu</td>
<td>0.10</td>
<td>6.26</td>
<td>10.11</td>
<td>50.01</td>
<td>0.01</td>
<td>54.7</td>
<td>163.7</td>
<td>1.005</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\sigma_{qq(\gamma)}^{acc}$ [pb]</th>
<th>$\sigma_{ZZ}^{acc}$ [pb]</th>
<th>$\sigma_{Zee}^{acc}$ [pb]</th>
<th>$\sigma_{\gamma\gamma}^{acc}$ [pb]</th>
<th>$\sigma_{\tau\tau(\gamma)}^{acc}$ [pb]</th>
<th>$N_{WWbck}^{exp}$</th>
<th>$N_{exp}^{tot}$</th>
<th>$N_{data}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>qqqq</td>
<td>0.979</td>
<td>0.490</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>197.5</td>
<td>1056.8</td>
<td>1108</td>
</tr>
<tr>
<td>qqev</td>
<td>0.038</td>
<td>0.007</td>
<td>0.022</td>
<td>–</td>
<td>–</td>
<td>24.6</td>
<td>263.1</td>
<td>269</td>
</tr>
<tr>
<td>qq\mu\nu</td>
<td>0.029</td>
<td>0.039</td>
<td>–</td>
<td>0.016</td>
<td>–</td>
<td>24.9</td>
<td>258.5</td>
<td>240</td>
</tr>
<tr>
<td>qq\tau\nu</td>
<td>0.187</td>
<td>0.071</td>
<td>0.031</td>
<td>0.019</td>
<td>–</td>
<td>95.9</td>
<td>259.6</td>
<td>287</td>
</tr>
</tbody>
</table>

### 208 GeV

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\epsilon(qqqq)$</th>
<th>$\epsilon(qqev)$</th>
<th>$\epsilon(qq\mu\nu)$</th>
<th>$\epsilon(qq\tau\nu)$</th>
<th>$\epsilon(l\nu\nu)$</th>
<th>$N_{WWbck}^{exp}$</th>
<th>$N_{exp}^{sig}$</th>
<th>$f_{1f}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>qqqq</td>
<td>83.44</td>
<td>0.06</td>
<td>0.04</td>
<td>0.34</td>
<td>0.00</td>
<td>0.1</td>
<td>54.5</td>
<td>1.004</td>
</tr>
<tr>
<td>qqev</td>
<td>0.02</td>
<td>72.45</td>
<td>0.17</td>
<td>1.65</td>
<td>0.08</td>
<td>0.4</td>
<td>15.1</td>
<td>1.039</td>
</tr>
<tr>
<td>qq\mu\nu</td>
<td>0.00</td>
<td>0.11</td>
<td>74.02</td>
<td>4.38</td>
<td>0.02</td>
<td>0.9</td>
<td>14.9</td>
<td>1.001</td>
</tr>
<tr>
<td>qq\tau\nu</td>
<td>0.14</td>
<td>6.26</td>
<td>9.72</td>
<td>49.55</td>
<td>0.01</td>
<td>3.4</td>
<td>10.4</td>
<td>1.005</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\sigma_{qq(\gamma)}^{acc}$ [pb]</th>
<th>$\sigma_{ZZ}^{acc}$ [pb]</th>
<th>$\sigma_{Zee}^{acc}$ [pb]</th>
<th>$\sigma_{\gamma\gamma}^{acc}$ [pb]</th>
<th>$\sigma_{\tau\tau(\gamma)}^{acc}$ [pb]</th>
<th>$N_{WWbck}^{exp}$</th>
<th>$N_{exp}^{tot}$</th>
<th>$N_{data}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>qqqq</td>
<td>0.949</td>
<td>0.504</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>12.5</td>
<td>66.7</td>
<td>65</td>
</tr>
<tr>
<td>qqev</td>
<td>0.044</td>
<td>0.008</td>
<td>0.024</td>
<td>–</td>
<td>0.001</td>
<td>1.7</td>
<td>16.8</td>
<td>14</td>
</tr>
<tr>
<td>qq\mu\nu</td>
<td>0.029</td>
<td>0.037</td>
<td>–</td>
<td>0.015</td>
<td>–</td>
<td>1.6</td>
<td>16.4</td>
<td>23</td>
</tr>
<tr>
<td>qq\tau\nu</td>
<td>0.184</td>
<td>0.072</td>
<td>0.034</td>
<td>0.018</td>
<td>–</td>
<td>6.1</td>
<td>16.4</td>
<td>17</td>
</tr>
</tbody>
</table>
Appendix B

Phase Space Distributions

In this appendix, all the distributions used in the fits are shown in projection for each channel at each centre-of-mass energy analysed. The definition and reconstruction of each of the angles is described in detail in Chapter 3. The dots in each figure are the data points. The light shaded histograms represent the expected number of selected WW events, including all decay channels and the contribution from 4f diagrams. The dark shaded area comes from the contamination of all the other known non-WW background sources. All the expected distributions are calculated assuming Standard Model couplings and are normalised to the measured luminosity. The corresponding numbers can be found in Appendix A.

The distributions are ordered per centre-of-mass energies, showing in each column the five phase space angles used for the different decay channels. The first row shows the cosine of the $W^-$ production angle, $\cos \Theta_{W^-}$, determined as explained in Section 3.3. The second and third row show the decay angles for the fermion that is coming from the negative $W$ boson. In case of hadronic events, this angle is folded since no flavour tagging is performed. The last two rows show the folded angles coming from the other $W$ in fully hadronic events, or from the decay of the hadronic part for semi-leptonic events. The reconstruction of the decay angles is explained in Section 3.4.
B.1 189 GeV

Phase Space Distributions
B.2 192 GeV
B.3 196 GeV
B.5 202 GeV
B.6 205 GeV

Phase Space Distributions

- $\cos \theta_{W}$ distributions for $qqqq$, $qq\nu\nu$, $qq\mu\nu$, and $qq\tau\nu$.
- $\cos \theta_{W}$ distributions for $q_1$ and $q_2$.
- $\phi$ distributions for $q_1$ and $q_2$.

Number of Events/0.2

For each phase space distribution, the number of events is plotted against the corresponding cosine angle or phi angle, with data bins indicated by vertical bars.
B.7 206 GeV

Phase Space Distributions
B.8 208 GeV
Appendix C

Likelihood Distributions

On the following pages, the negative log-likelihood distributions, $-\Delta \ln L$, are shown for the couplings measured in this thesis using each decay channel at each centre-of-mass energy separately. The likelihood curves are obtained from a binned maximum likelihood fit, as described in Section 3.6, to the five dimensional phase space spanned by the angles given in Appendix B.

The top row at each centre-of-mass energy shows the distributions for the couplings $g_1^\mathcal{Z}$, $\kappa_\gamma$, and $\lambda_\gamma$ assuming the relations $\kappa_\mathcal{Z} = g_1^\mathcal{Z} - \tan^2 \theta_w (\kappa_\gamma - 1)$, $\lambda_\mathcal{Z} = \lambda_\gamma$, while these constrains are relaxed for measurement of the couplings $g_5^\mathcal{Z}$, $\kappa_\mathcal{Z}$ and $\lambda_\mathcal{Z}$ given in the bottom row. For each of the fits, the parameters not measured are either fixed to their Standard Model expectation or to the value given by the constraints. In each plot, the contributions coming from the $qqqq$ (solid line), $qq\nu\nu$ (dashed), $qq\mu\nu$ (dashed-dotted) and $qq\tau\nu$ (dotted) channels are shown separately.
Likelihood Distributions

196 GeV

200 GeV
Likelihood Distributions

202 GeV

205 GeV
Likelihood Distributions

206 GeV

208 GeV
Bibliography


[72] DØ Collaboration, S. Abachi et al., *Limits on Anomalous WWγ Couplings from $p\bar{p} \to W\gamma + X$ Events at $\sqrt{s} = 1.8$ TeV*, Phys. Rev. Lett. 78 (1997) 3634.


[85] *yfsww3* version 1.14 is used.


[90] *PYTHIA* versions 5.722 and 6.1 are used.


[111] KK2f version 4.13 is used.


[113] BHWIDE version 1.01 is used.


[116] PHOJET version 1.05 is used.


[120] L3 Collaboration, Measurement of the Cross Section of WPair-Production at LEP, in preparation.


Summary

Many experiments conducted during the second half of the past century provided evidence that the Standard Model of electroweak interactions is a very adequate theory to describe the physics of electromagnetic and weak processes. One of the key ingredients in this quantum field theory is the requirement of gauge invariance under the non-Abelian \( SU(2) \) symmetry to generate the dynamics. A direct consequence is the appearance of couplings between three gauge bosons of the form \( \gamma WW \) and \( ZWW \). These are called triple gauge-boson couplings (TGC’s). Diagrams containing these couplings are in fact necessary to ensure a proper high energy behaviour for various processes, like W-pair production in \( e^+e^- \) collisions.

The second phase of the Large Electron Positron (LEP) collider was devoted to running at centre-of-mass energies above the W-pair production threshold. Some of the main goals of its rich physics programme were the measurement of the mass and width of the W boson and the search for the Higgs boson. However, it also became possible to study the trilinear vertices in a direct manner to great detail. Until the start of LEP2, these couplings remained poorly measured.

This thesis describes the measurement of the TGC’s using hadronic and semi-leptonic W boson pairs produced in \( e^+e^- \) collisions at the LEP collider recorded with the L3 detector from 1998 until 2000. During these years, data was collected at centre-of-mass energies between 189 and 209 GeV, amounting to a total integrated luminosity of 629 \( \text{pb}^{-1} \).

The most general Lorentz invariant effective Lagrangian for the trilinear vertex contains seven terms. The measurement is made quantitative by introducing an arbitrary factor in front of each term. Having two possible vertices, one with a photon and one with a Z boson, gives 14 couplings to be measured. This number is reduced by assuming electromagnetic gauge invariance and considering only terms that are \( CP \)-conserving. Imposing that the Lagrangian is invariant under a global \( SU(2) \) symmetry results in the additional relations \( \kappa_Z = g_1^Z - \tan^2 \theta_W (\kappa_\gamma - 1) \) and \( \lambda_Z = \lambda_\gamma \). This leaves three couplings to be measured: \( g_1^Z \), \( \kappa_\gamma \), and \( \lambda_\gamma \).

For each of the WW decay channels, a dedicated selection procedure is applied based on the event topology and kinematic characteristics. This results in a high efficiency for selecting signal events and a low background contamination. The signatures of the semi-leptonic channels qq\( \ell \nu \) are an energetic lepton, two quark jets containing a large number of particles, and missing momentum and energy carried away by the neutrino. The lepton can be reconstructed as an electron, muon or a hadronic jet from tau decays. The hadronic decay channel qqqq is characterised by the presence of four quark jets and almost all the energy and momentum will be contained in the detector.

A W-pair event can be completely described by five kinematic variables. Anomalous
values for the couplings would affect the different helicity contributions to the W-pair production process. Therefore the following angles are chosen: the production angle of the negative W and the polar and azimuthal decay angles of the lepton (anti-lepton) in the rest-frame of the $W^-$ (W$^+$). Values for the couplings are derived for each decay channel and each centre-of-mass energy by performing a binned maximum likelihood fit to the five-dimensional differential cross section. The coupling dependence is obtained by a generator level reweighting of fully simulated MC events.

Three couplings are measured under the aforementioned constraints. The combination of all one-parameter fits, i.e. fixing all but one of the couplings to their Standard Model expectation, yield:

$$g_1^Z = 0.925^{+0.036}_{-0.034} \text{(stat.)} \pm 0.019 \text{(syst.)} \quad (\text{SM} = 1),$$
$$\kappa_\gamma = 0.853^{+0.067}_{-0.062} \text{(stat.)} \pm 0.040 \text{(syst.)} \quad (\text{SM} = 1),$$
$$\lambda_\gamma = -0.058^{+0.039}_{-0.036} \text{(stat.)} \pm 0.022 \text{(syst.)} \quad (\text{SM} = 0).$$

The first error is statistical and the second is systematic, both at 68% C.L., and the Standard Model expectations are given in the last column. Multi-parameter fits, leaving two or three couplings free to vary, are also performed. The results of all type of fits are in good agreement with each other. The couplings $g_5^Z$, $\kappa_Z$ and $\lambda_Z$ are also measured relaxing the global $SU(2)$ symmetry constraints.

The results from this thesis are combined with published L3 results using W-pairs collected at lower energies and single-W events at all centre-of-mass energies above W-pair threshold. For the latter channel, only those events are considered that were not already accepted by the $q\bar{q}\ell\nu$ selections. When all three couplings are allowed to vary, the combination results in the following 95% C.L. intervals:

$$0.75 < g_1^Z < 1.03 \quad \rho(g_1^Z, \kappa_\gamma) = -0.53,$$
$$0.90 < \kappa_\gamma < 1.34 \quad \rho(\kappa_\gamma, \lambda_\gamma) = 0.12,$$
$$-0.10 < \lambda_\gamma < 0.19 \quad \rho(\lambda_\gamma, g_1^Z) = -0.64,$$

where both statistical and systematic errors are included and the correlation coefficients are given on the right.

The L3 combined results of the simultaneous fit to the couplings $\kappa_\gamma$ and $\lambda_\gamma$ were transformed into a measurement of the magnetic dipole and electric quadrupole moment of the W boson. These values also indicate that the W boson is a point-like particle down to a scale of $10^{-19}$ m.

These measurements show that the TGC’s indeed exist and their strength and structure are in agreement with the prediction of the Standard Model. Moreover, the values for the couplings not foreseen in this theory are compatible with zero.

176
Samenvatting

In de natuur bestaan er, voor zover bekend, vier fundamentele krachten: de zwaartekracht, de elektromagnetische, de sterke en de zwakke wisselwerking. Het Standaard Model voor elektro-zwakke wisselwerking is de theorie die twee van deze krachten beschrijft. In onze huidige opvattingen bestaan er in de natuur twee soorten deeltjes: de materiedeeltjes enerzijds en de krachtdeeltjes anderzijds.

De materie bestaat uit deeltjes met spin 1/2, fermionen genaamd, en kunnen onderverdeeld worden in leptonen en quarks. Het grootste verschil is dat quarks ook onderhevig zijn aan de sterke wisselwerking. Deze deeltjes kunnen verder onderverdeeld worden in drie families die elk uit twee deeltjes bestaan. In totaal zijn er dus twaalf verschillende elementaire deeltjes.

Interacties tussen deze deeltjes worden beschreven door het uitwisselen van kracht-dragers, deeltjes met spin 1, en worden ijbosonen genoemd. Voor de elektromagnetische kracht gebeurt dit enkel via het foton, maar voor de zwakke wisselwerking kan dit op drie manieren: via het neutrale Z en de twee geladen W⁺ en W⁻ bosonen.

Veel experimenten uit de tweede helft van de vorige eeuw hebben aangetoond dat dit Standaard Model een zeer goede beschrijving geeft van de waargenomen natuurkundeprocessen. Een van de belangrijkste ingrediënten van deze kwantummechanische veldentheorie is het genereren van interacties door het eisen van invariantie onder lokale ijktransformatie van de $SU(2) \times U(1)$ symmetrie. Een direct gevolg van het niet-abelse karakter van de $SU(2)$ symmetrie is het verschijnen van koppelingen tussen drie ijbosonen van de vorm $γWW$ en $ZWW$. Deze worden trilineaire ijbosonkoppelingen genoemd en afgekort als TGC’s, afkomstig van de Engelse term *triple gauge-boson couplings*. Diagrammen die zulke vertices bevatten zijn voor een aantal processen ook nodig om een goed gedrag bij hoge energieën te verzekeren, zoals dit bijvoorbeeld het geval is voor de productie van W-paren bij $e^+e^-$ botsingen.

De tweede fase van de LEP versneller bestond erin om elektronen en positronen op elkaar te laten botsen met zwaartepuntsenergieën die groter waren dan de drempelenergie voor de productie van W-paren. Enkele van de belangrijkste onderzoeksprogramma’s waren de accurate metingen van de massa en de breedte van het W boson en de zoektocht naar het Higgs deeltje. Het werd ook mogelijk om de trilineaire vertices op een directe manier nauwkeurig te onderzoeken. De sterkte van deze koppelingen waren tot voor de aanvang van LEP2 nog niet nauwkeurig gemeten.

Dit proefschrift beschrijft de meting van de TGC’s gebruik makend van W-paren geproduceerd bij $e^+e^-$ botsingen in de LEP versneller en verzameld met de L3 detector van 1998 tot en met 2000. Gedurende deze periode liepen de zwaartepuntsenergieën op van 189 GeV tot 209 GeV, hetgeen resulteerde in een totale geïntegreerde luminositeit van
De meest algemene, lorentzinvariante, effectieve lagragiaan voor de trilineaire vertex bestaat uit zeven termen. Door het introduceren van willekeurige factoren voor iedere term wordt de meting kwantitatief gemaakt. Omdat er twee mogelijke vertices bestaan, een met een foton en de andere met een Z boson, moeten er dus 14 koppelingen gemeten worden. Dit aantal kan gereduceerd worden door elektromagnetische ijkinvariantie aan te nemen en enkel termen te beschouwen die $CP$-behoudend zijn. Eens dat de lagragiaan invariant is onder een globale $SU(2)$ symmetrie resulteert in de bijkomende relaties $\kappa_Z = g_1^Z - \tan^2 \theta_W (\kappa_\gamma - 1)$ en $\lambda_Z = \lambda_\gamma$. Na al deze aannames blijven er nog drie koppelingen over: $g_1^Z$, $\kappa_\gamma$ en $\lambda_\gamma$.

Voor elk van de WW vervalskanalen wordt een specifieke selectieprocedure toegepast, die gebaseerd is op de topologische en kinematische kenmerken van de gebeurtenissen. Dit resulteert in een hoge efficiëntie voor het selecteren van signaalgebeurtenissen en houdt de achtergrondbesmetting laag. De kenmerken van het semi-leptonische kanaal $qq\ell\nu$ zijn de aanwezigheid van een energetisch lepton, twee quark-jets die een groot aantal deeltjes bevatten en ontbrekende energie en impuls ten gevolge van het niet gedetecteerde neutrino. Het lepton kan als elektron, muon of hadronische tau-jet worden gereconstrueerd. Het hadronische kanaal $qqq\bar{q}$ wordt gekenmerkt door de aanwezigheid van vier quark-jets en zo goed als alle energie en impuls wordt gemeten in de detector.

Een paar van W bosonen kan volledig worden beschreven met vijf kinematische variablen. Anomale koppelingen veranderen de relatieve contributies van de mogelijke heliciteitsconfiguraties tot de totale werkzame doorsnede voor W-paarproductie. Daarom worden de volgende hoeken gekozen: de productiehoek van het negatieve W boson en de polaire en azimutale vervalshoeken van het lepton (anti-lepton) in het ruststelsel van het $W^-$ ($W^+$). Voor elk vervalskanaal en zwaartepuntsenergie worden waarden voor de koppelingen afgeleid door de voorspelde vijfdimensionale differentiële werkzame doorsnede te vergelijken met de waargenomen grootheden. De afhankelijkheid van de werkzame doorsnede als functie van de koppelingen wordt bekomen door volledig gereconstrueerde Monte Carlo gebeurtenissen te herschalen op generatorniveau. Als resultaat worden de waarden van de koppelingen genomen die de grootste waarschijnlijkheid opleveren voor het beschrijven van de opgemeten verdelingen.

Drie koppelingen worden gemeten onder de eerder vermelde aannames. De combinatie van de een-parameter fits, waarbij slechts een koppeling vrij is om te variëren en de andere worden vastgehouden op de voorspelling van het Standaard Model, resulteert in:

\[
\begin{align*}
g_1^Z &= 0.925^{+0.036}_{-0.034} \text{(stat.)} \pm 0.019 \text{(syst.)} \quad \text{(SM = 1),} \\
\kappa_\gamma &= 0.853^{+0.067}_{-0.062} \text{(stat.)} \pm 0.040 \text{(syst.)} \quad \text{(SM = 1),} \\
\lambda_\gamma &= -0.058^{+0.039}_{-0.036} \text{(stat.)} \pm 0.022 \text{(syst.)} \quad \text{(SM = 0).}
\end{align*}
\]

De eerste fout is statistisch en de tweede系统atisch, beide op 68% betrouwbaarheidsniveau, en de waarden in het Standaard model zijn in de rechterkolom weergegeven. Fits waarbij meerdere parameters tegelijkertijd worden gemeten zijn ook uitgevoerd. Al deze resultaten zijn in zeer goede overeenstemming met elkaar. Daarnaast worden ook de kopplingen $g_5^Z$, $\kappa_Z$ en $\lambda_Z$ gemeten zonder de veronderstelling van invariantie onder een globale $SU(2)$ symmetrie.
De resultaten die in dit proefschrift zijn beschreven werden gecombineerd met andere L3 resultaten. Deze analyses maken gebruik van W-paren bij lagere zwaartepuntsenergieën en van het “single-W” kanaal (Wεν in de eindtoestand) gemeten bij alle zwaartepuntsenergieën boven de drempelenergie voor W-paarproductie. Om dit laatste kanaal te kunnen combineren werden enkel die gebeurtenissen in rekening gebracht die niet door de qqεν selecties werden aanvaard. Wanneer alle drie de koppelingen vrij worden gelaten in de fit, resulteert dit in de volgende 95% betrouwbaarheidsintervallen:

\[
0.75 < g_1^Z < 1.03 \\
0.90 < \kappa_\gamma < 1.34 \\
-0.10 < \lambda_\gamma < 0.19
\]

waarbij zowel de statistische als systematische fouten in rekening zijn gebracht. De correlatiecoëfficiënten zijn ook gegeven.

Het L3 resultaat van de simultane fit naar \( \kappa_\gamma \) en \( \lambda_\gamma \) werd omgezet naar een meting van het magnetisch dipool- en elektrisch quadrupoolmoment. Uit deze waarden kan ook worden afgeleid dat het W boson, dat in de oorspronkelijke theorie een puntdeeltje is, met zekerheid kleiner is dan afmetingen van de orde \( 10^{-19} \) m.

De metingen beschreven in dit proefschrift tonen aan dat de trilineaire ijkbosonkoppelingen inderdaad bestaan en dat hun sterkte en structuur in overeenstemming zijn met de verwachtingen binnen het Standaard Model. Bovendien zijn de waarden van de koppelingen die niet voorzien zijn in deze theorie in overeenstemming met nul.
Dankwoord

Eindelijk! Na deze kreet van opluchting zou men bijna gaan denken dat het verloop van een promotieonderzoek één grote lijdensweg is. Niets is minder waar en dat is mede dankzij de hulp en steun van velen. De volgende dankbetuigingen zullen dan ook verre van toereikend zijn.

In the first place, I would like to thank Martin Pohl for being my promotor. In particular, I enjoyed our regular meetings at CERN, which always gave me a different perspective on the problems I was dealing with. Ook ben ik mijn begeleider Paul de Jong zeer dankbaar voor alle hulp die hij heeft geboden. Alhoewel we elkaar in de beginperiode niet regelmatig spraken omdat ik op CERN zat, kon ik altijd bij hem terecht per e-mail. Ik herinner me nog levendig het versturen van lange uitweidingen over mijn problemen en vragen, waarop ik dan nog veel langere mails met uitgebreide antwoorden en suggesties ontving. I am also very grateful to both of them for the scrupulous reading and correcting of this manuscript. Verder wil ik ook de groepsleiders Gerjan Bobbink en Wes Metzger bedanken voor alle mogelijkheden die ze me hebben geboden om allerlei conferenties, workshops, zomerscholen en dergelijke bij te wonen.

During my two years stay at CERN, I learnt a lot from a large number of people. My predecessor Stefano Villa taught me all the details I needed to know to perform a TGC analysis and many things more. He was always willing to spend a considerable amount of time answering to any of my questions. Without his help this thesis would have taken much longer to accomplish. Other members of the L3 W-physics group were also a great source of information to me. In particular I am grateful to the group leaders Martin Grünewald and in a later stage Arno Straessner and Luca Malgeri. A special thanks to Luca for answering all the questions I fired at him while I was writing this thesis. I am also thankful towards the W-pair selectors for fulfilling my numerous requests: Raja, Imre, Azizur, Natalia, Arno, Luca, Evelyne and Sonia.

Tijdens mijn verblijf op CERN heb ik het grootste deel van de tijd een kantoor gedeeld met Robert en Vinod. Het voortdurend uitwisselen van handige tips en trucjes waren zeer nuttig. Samen met hen hebben Karin, Rutger, Martin, Evelyne, Cathelijne en Joost ervoor gezorgd dat ik een geweldige tijd heb beleefd in Genève. Ik heb zeer veel genoten van de talrijke dag- en avonduitstappjes.

Gedurende de periode dat ik werkzaam was op NIKHEF heb ik zowel tijdens als na de werken vele aangename momenten beleefd met vele mede-promovendi. In het bijzonder zijn de vele diepgaande discussies met Aart, Marco, Martin en René over natuurkundige, politieke en filosofische vragenstukken zeker bij mij in de smaak gevallen ...tijdens het consumeren van “één biertje”. Een constante gedurende al deze jaren waren de bijna wekelijkse, afmattende, maar spannende squashpartijtjes tegen Rutger. Dit was de ideale
manier om niet vast te roesten achter het computerscherm en opgekropte frustraties kwijt te raken. Verder wil ik het laatst overgebleven lid van het L3m-team, Vinod, nog veel succes toewensen bij het afronden van zijn proefschrift.

Toen ik opnieuw in Amsterdam woonde was het ook mogelijk om de enigszins verwaarloosde contacten met familie en vrienden weer aan te scherpen door zeer regelmatig af (en door) te zakken naar België. Ook zij hebben er mede voor gezorgd dat ik nog enige voeling heb gehouden met de wereld buiten de natuurkunde. De volgende mensen wil ik in het bijzonder bedanken: Pim, Danny, Amke, Zita, Tom, To(e)m, Elke, Guy, Marit, Hans en natuurlijk alle anderen die ik hier tot mijn grote schande over het hoofd heb gezien.

De steun, het vertrouwen en de levenservaring van mijn vader zijn voor mij van onschatbare waarde geweest. Daarom wens ik hem dan ook het allerbeste toe voor hetgeen de toekomst brengen zal. Hetzelfde geldt natuurlijk ook voor mijn broers Maarten en Kristof, mijn schoonzus Caroline en de kleine reuzen Emiel en August. Tenslotte wil ik mijn moeder bedanken voor alle liefde die ze tijdens mijn opvoeding heeft geschenken. Zij heeft dit allemaal jammer genoeg niet meer mogen meemaken, maar ik ben er zeker van dat ze trots zou zijn geweest op haar zoon.

Mark
Curriculum Vitae


Gedurende dat jaar had ik besloten om natuurkunde te studeren aan de Universiteit Gent. In 1996 haalde ik het kandidaatsdiploma en nog eens twee jaar later had ik het diploma licentiaat in de natuurkunde (met onderscheiding) op zak. Gedurende het laatste academiejaar ben ik in het kader van het *European Mobility Scheme for Physics Students* naar Zwitserland vertrokken om les te volgen aan de *Université de Genève*. Daar heb ik ook onder leiding van Prof. Dr. M.N. Kienzle-Focacci gewerkt aan een onderzoek binnen de L3 collaboratie op het *European Organisation for Nuclear Research CERN*. Dit leidde tot de scriptie *Measurement of \( \eta'(958) \) and \( \chi_{c2}(3556) \) Formation in Two-Photon Collisions at LEP at \( \sqrt{s} = 183 \) GeV*, mede onder leiding van Prof. Dr. D. Ryckbosch.

Per 1 april 1999 ben ik begonnen als onderzoeker in opleiding aan het Nederlands Instituut voor Kern en Hoge-Energie Fysica (NIKHEF) in dienst van de stichting Fundamenteel Onderzoek der Materie. Na een zeer korte periode in Amsterdam, heb ik in het kader van mijn onderzoek 2 jaar doorgebracht op het CERN. De laatste twee jaar van mijn aanstelling was ik wederom werkzaam op het NIKHEF om mijn analyse af te ronden en dit proefschrift tot stand te brengen.
